

# Anomalies and Transport

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in collaboration with

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arXiv:1103.5006 (Phys. Rev. Lett.)

to appear

# Outline

- Movie
- The CME (flash review)
- Kubo formulas I
- Hydrodynamics
- Kubo formulas II
  - weak coupling
  - strong coupling

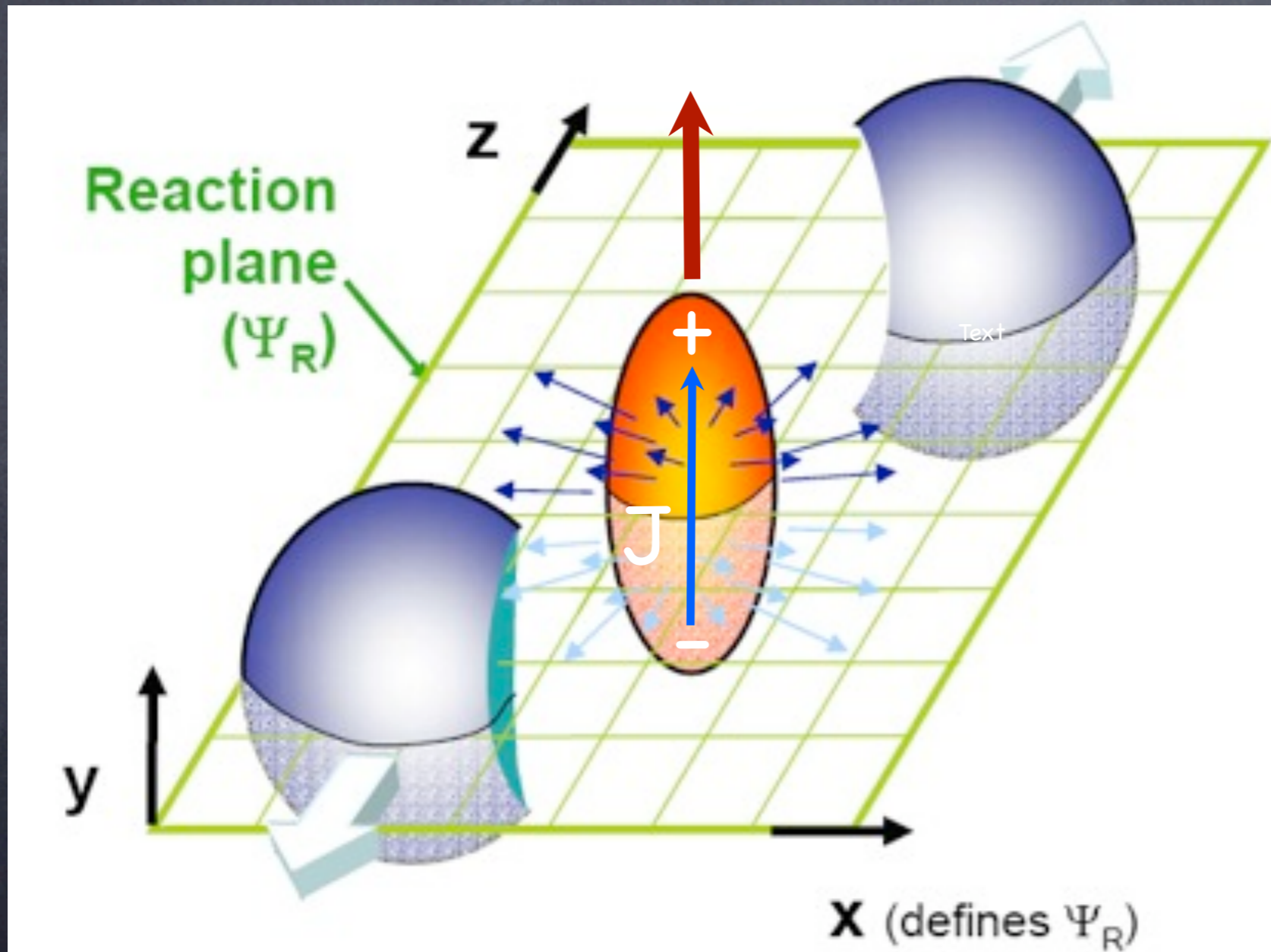
# RHIC's hot quark soup

Please click below for an introductory movie

<http://youtu.be/kXy5EvYu3fw>

# The CME

[Kharzeev, McLarren, Warringa]

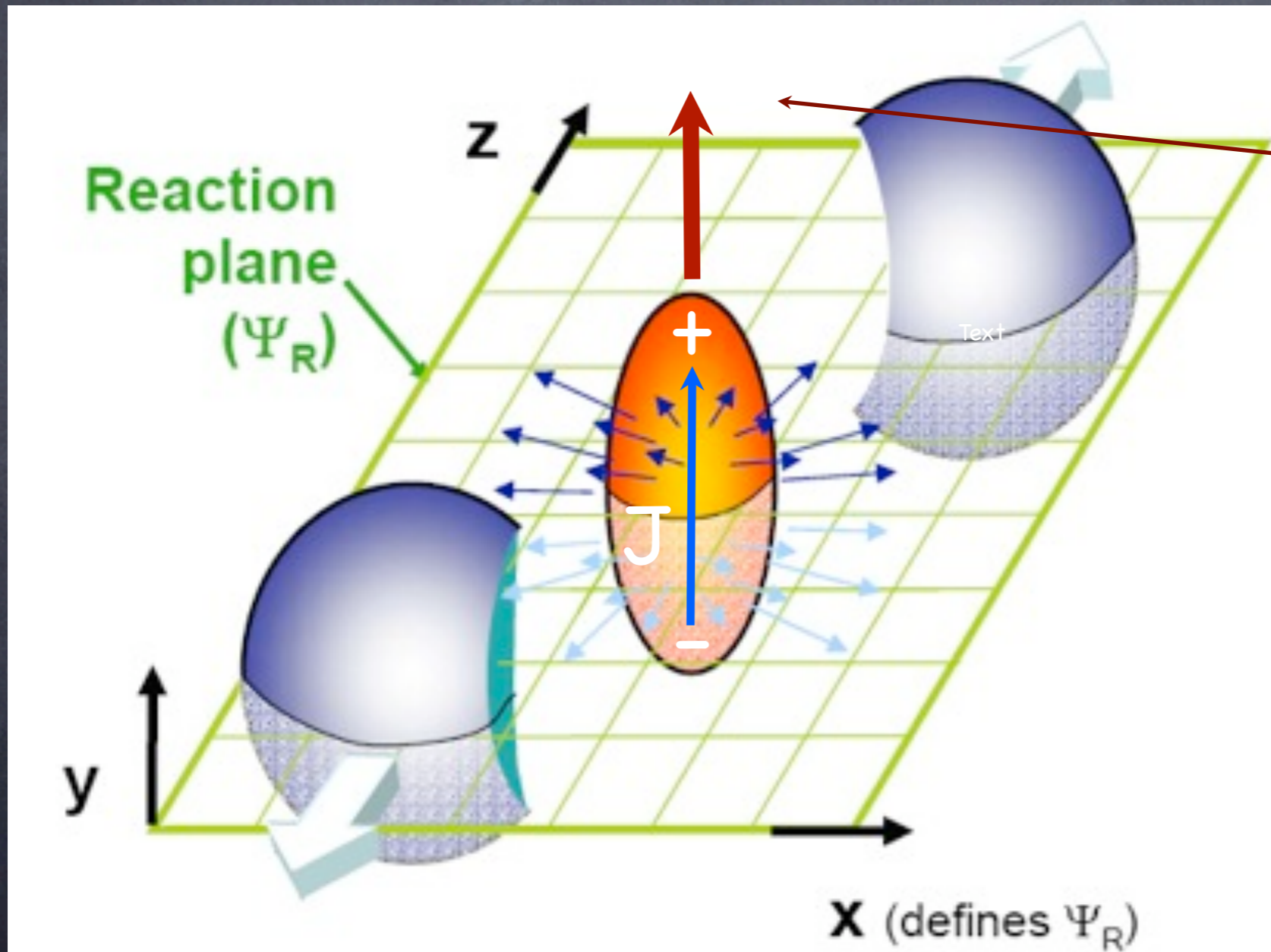


[Ruggieri's talk]

[parity violating currents: Vilenkin '80, Giovannini, Shaposhnikov '98, Alekseev, Chaianov, Frohlich '98]

# The CME

[Kharzeev, McLarren, Warringa]



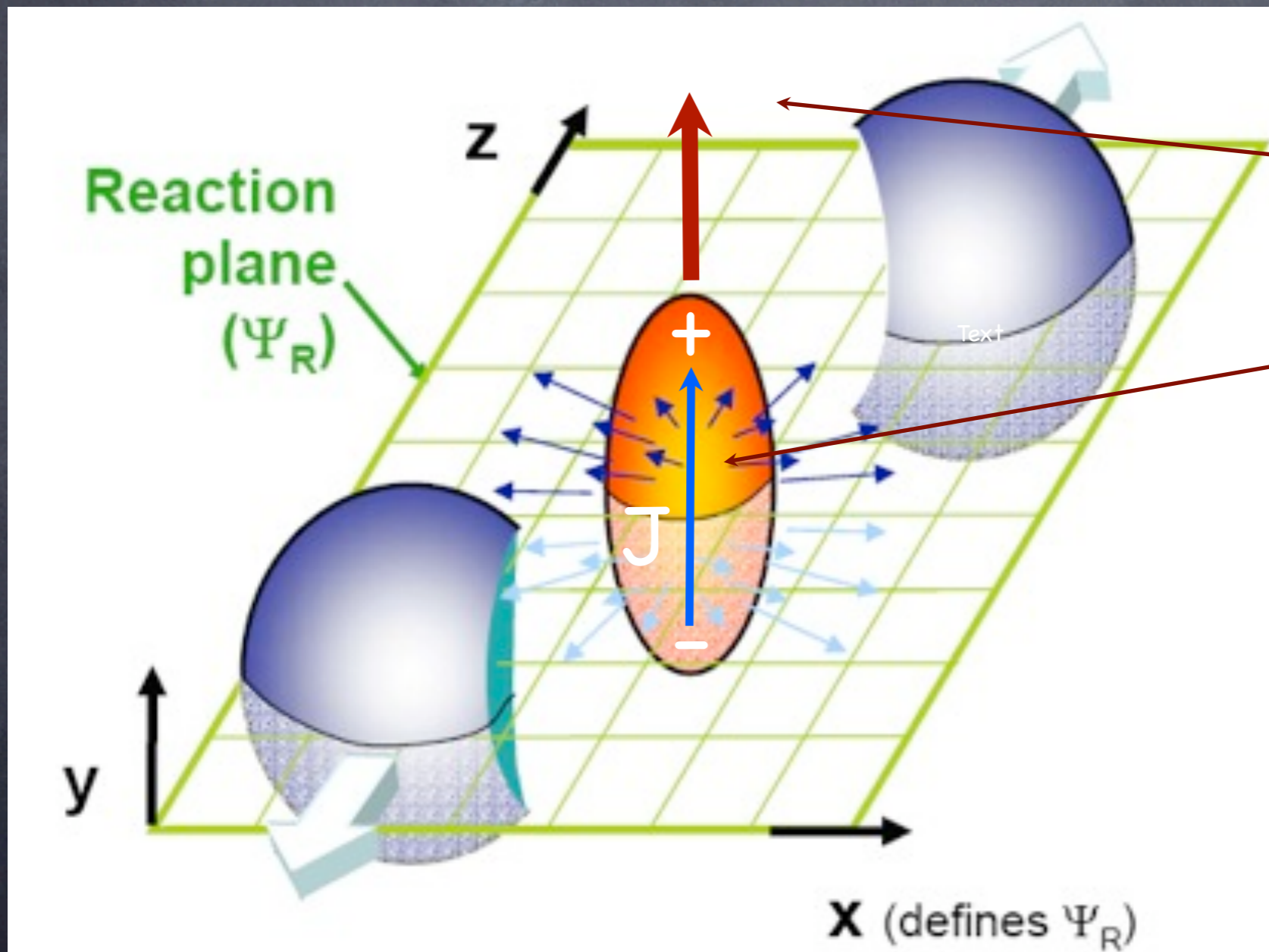
Magnetic Field

[Ruggieri's talk]

[parity violating currents: Vilenkin '80, Giovannini, Shaposhnikov '98, Alekseev, Chaianov, Frohlich '98]

# The CME

[Kharzeev, McLarren, Warringa]



Magnetic Field

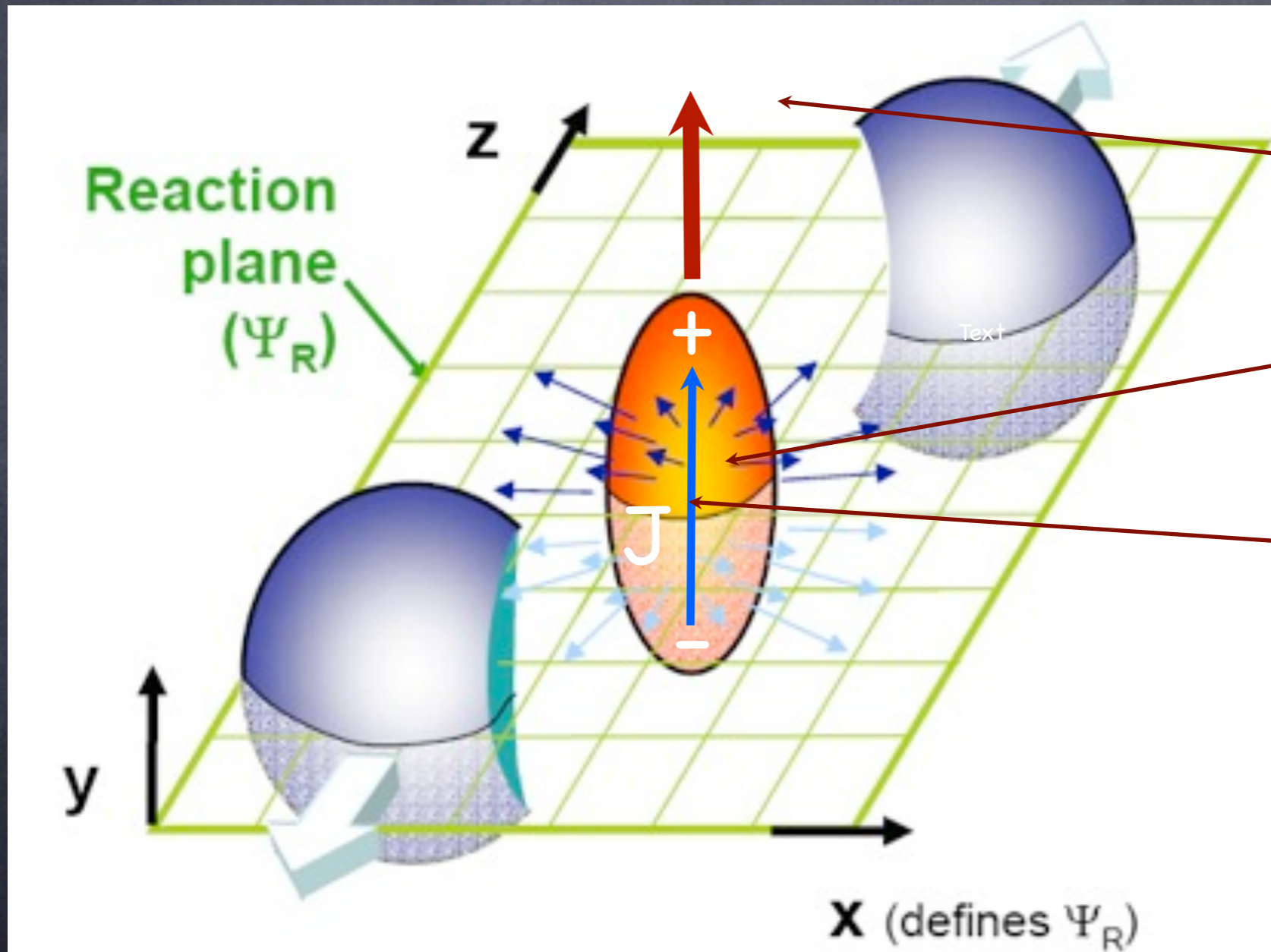
Net chirality

[Ruggieri's talk]

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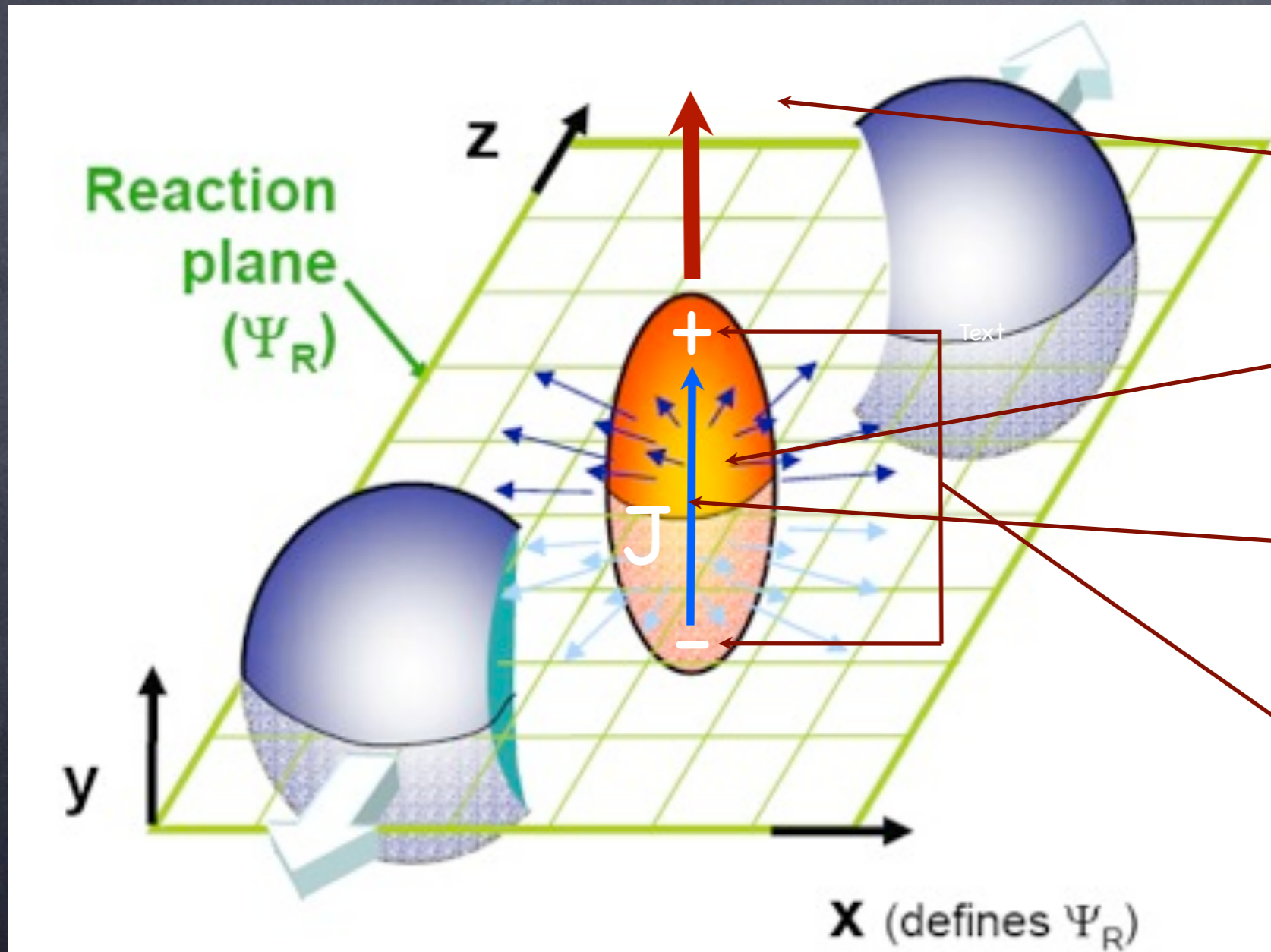
[Ruggieri's talk]

Electric current

[parity violating currents: Vilenkin '80, Giovannini, Shaposhnikov '98, Alekseev, Chaianov, Frohlich '98]

# The CME

[Kharzeev, McLarren, Warringa]



Magnetic Field

Net chirality

[Ruggieri's talk]

Electric current

P-odd charge separation

[parity violating currents: Vilenkin '80, Giovannini, Shaposhnikov '98, Alekseev, Chaianov, Frohlich '98]



# Kubo formulas I

- Chiral magnetic conductivity

$$\vec{J} = \sigma \vec{B}$$

[Kharzeev, Warringa]

$$J_i = \sigma \epsilon_{ijk} (ip_j) A_k$$

- Kubo formula, general symmetry group

$$[T^A, T^B] = if_C^{AB} T^C$$

$$\sigma^{AB} = \lim_{p_j \rightarrow 0} \sum_{i,k} \frac{i}{2p_j} \epsilon_{ijk} \langle J_i^A J_k^B \rangle \Big|_{\omega=0}$$

# Kubo formulas I

- chiral fermions

$$J_i^A = \sum_{f,g=1}^N (T^A)^g{}_f \bar{\Psi}_g \gamma_i P_+ \Psi^f$$

- chemical potentials and Cartan generators

$$H_A = q_A^f \delta^f{}_g \quad \mu^f = \sum_A q_A^f \mu_A$$

- 1-loop calculation

$$\sigma_{AB} = \frac{1}{8\pi^2} \sum_C \text{tr} (T^A \{T^B, H^C\}) \mu_C = \frac{1}{4\pi^2} d^{ABC} \mu_C$$

# Kubo formulas I

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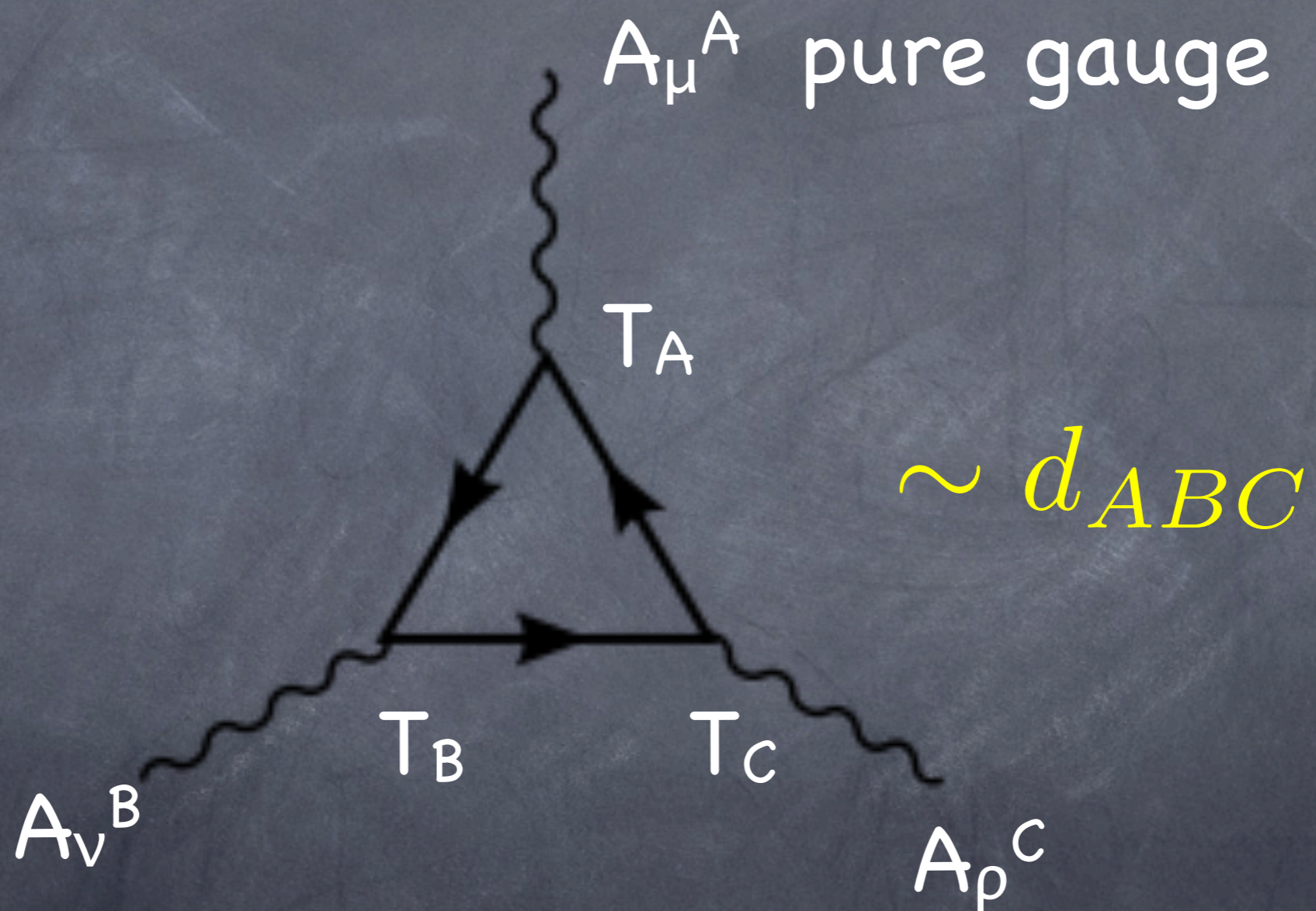
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Anomalycoeff

# Kubo formulas I



# Kubo

- finite density: charge transport  $\Rightarrow$  energy transport

$$\delta T_{0i} = \mu \delta J_i = \mu \delta \sigma B_i$$

- energy flux sourced by magnetic fields

$$\frac{i}{2p_j} \sum_{i,k} \epsilon_{ijk} \langle T_{0i} J_k \rangle |_{\omega=0} = \int \mu d\sigma + \text{const.}$$

[Neiman, Oz],

[Loganayagam], [Kharzeev, Yee]

- at  $\omega=0$  reverse order of operators

$$\sigma_V = \frac{i}{2p_j} \sum_{i,k} \epsilon_{ijk} \langle J_i T_{0k} \rangle |_{\omega=0} = \int \mu d\sigma + \text{const.}$$

# Kubo

- finite density: charge transport  $\Rightarrow$  energy transport

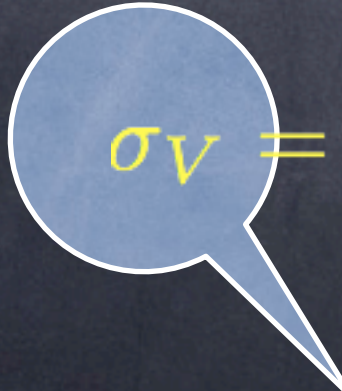
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conductivity ?

# Kubo

- $T_{\mu\nu}$  sourced by metric

$$ds^2 = -(1 - 2\Phi)dt^2 + 2\vec{A}_g dt d\vec{x} + (1 + 2\Phi)d\vec{x}^2$$

- $A_g$  "gravitomagnetic field"  $\rightarrow$  chiral gravitomagnetic effect

$$\vec{J} = \sigma_V \vec{B}_g$$

- chiral vortical effect: fluid velocities

$$u^\mu = (1, 0, 0, 0) \quad u_\mu = (-1, \vec{A}_g)$$

$$J^i = \sigma_V \epsilon^{ijk} \partial_j u_k$$

# Hydrodynamics

• also energy flux  $\lim_{p_j \rightarrow 0} \frac{i}{2p_j} \sum_{i,k} \epsilon_{ijk} \langle T_{0i} T_{0k} \rangle |_{\omega=0} \neq 0$

• hydrodynamics:  $T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$   
 $J^\mu = nu^\mu + v^\mu$ .

• Landau frame:  $T^{0i} = (\epsilon + p)u^i$

$\tau^{\mu\nu} = \text{usual}$

$$v^\mu = -\sigma T P^{\mu\nu} \nabla_\nu \left( \frac{\mu}{T} \right) + \sigma E^\mu + \xi_B B^\mu + \xi_V \omega^\mu$$



# Hydrodynamics

- $\xi$  coefficients are different from  $\sigma$ 's

$$\xi_B = \lim_{k_n \rightarrow 0} \frac{-i}{2k_n} \sum_{k,l} \epsilon_{nkl} \left( \langle J^k J^l \rangle - \frac{n}{\epsilon + P} \langle T^{tk} J^l \rangle \right) \Big|_{\omega=0}$$
$$\xi_V = \lim_{k_n \rightarrow 0} \frac{-i}{2k_n} \sum_{k,l} \epsilon_{nkl} \left( \langle J^k T^{tl} \rangle - \frac{n}{\epsilon + P} \langle T^{tk} T^{tl} \rangle \right) \Big|_{\omega=0}$$

- hydro in alternative frame:  
include "heat currents"

$$\tau_Q^{\mu\nu} = Q^\mu u^\nu + Q^\nu u^\mu$$

$$Q^\mu = \sigma^Q B^\mu + \sigma_V^Q \omega^\mu$$

$$\downarrow$$
$$\langle TJ \rangle$$

$$\downarrow$$
$$\langle TT \rangle$$

# kubo formulas II

- as before: general symmetry group

$$T^{0i} = \frac{i}{2} \sum_{f=1}^N \bar{\Psi}_f (\gamma^0 \partial^i + \gamma^i \partial^0) P_+ \Psi^f$$

$$\begin{aligned} \sigma_V^A &= \frac{1}{8\pi^2} \sum_{f=1}^N (T^A)^f_f \left[ (\mu^f)^2 + \frac{\pi^2}{3} T^2 \right] \\ &= \frac{1}{8\pi^2} \sum_{B,C} d^{ABC} \mu_B \mu_C + \frac{T^2}{24} \text{tr} (T^A) \end{aligned}$$

# kubo formulas II

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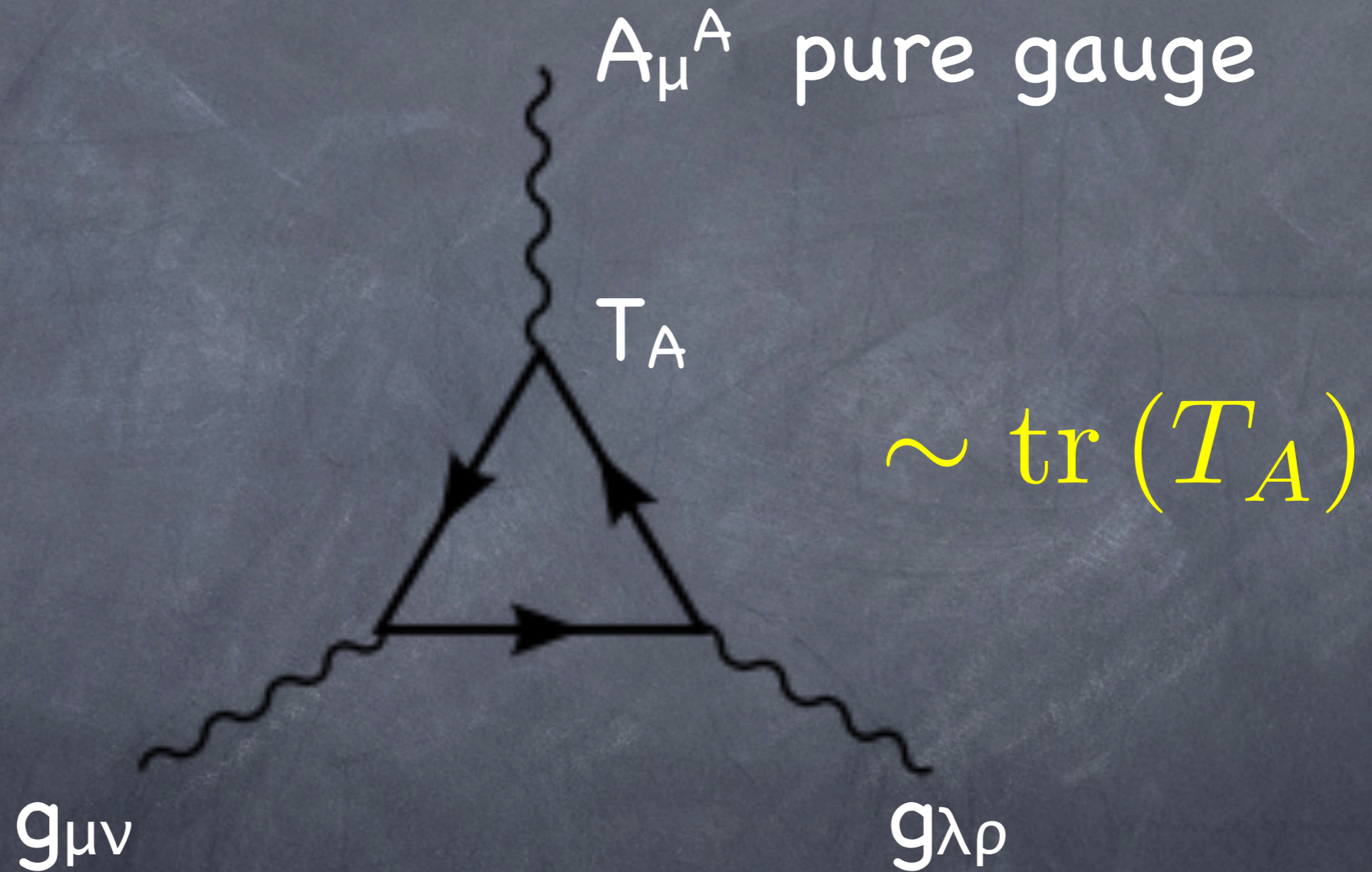
$$T^{0i} = \frac{i}{2} \sum_{f=1}^N \bar{\Psi}_f (\gamma^0 \partial^i + \gamma^i \partial^0) P_+ \Psi^f$$

$$\sigma_V^A = \frac{1}{8\pi^2} \sum_{f=1}^N (T^A)^f_f \left[ (\mu^f)^2 + \frac{\pi^2}{3} T^2 \right]$$

$$= \frac{1}{8\pi^2} \sum_{B,C} d^{ABC} \mu_B \mu_C + \frac{T^2}{24} \text{tr}(T^A)$$

Integration constant  
gravitational anomaly!

# Kubo formulas II



# Kubo formulas II

- Anomaly

$$\nabla_{\mu} J_A^{\mu} = \epsilon^{\mu\nu\rho\lambda} \left( \frac{d_{ABC}}{32\pi^2} F_{\mu\nu}^B F_{\rho\lambda}^C + \frac{b_A}{768\pi^2} R^{\alpha}{}_{\beta\mu\nu} R^{\beta}{}_{\alpha\rho\lambda} \right)$$

$$d_{ABC} = \frac{1}{2} \text{tr}(\{T_A, T_B\} T_C)_R - \frac{1}{2} \text{tr}(\{T_A, T_B\} T_C)_L$$

$$b_A = \text{tr}(T_A)_R - \text{tr}(T_A)_L$$

- axial current of one Dirac fermion

$$\sigma_V = \frac{\mu^2 + \mu_5^2}{4\pi^2} + \frac{T^2}{12}$$

# Holography

[H.U. Yee], [Gynther, KL, Pena-Benitez, Rebhan]

[Kalayhdzian, Kirsch], [Gorski, Zayakin]

- mixed gauge gravitational Chern Simons term

$$S = \frac{1}{16\pi G} \int d^5x \sqrt{-g} \left[ R + 2\Lambda - \frac{1}{4} F_{MN} F^{MN} \right. \\ \left. + \epsilon^{MNPQR} A_M \left( \frac{\kappa}{3} F_{NP} F_{QR} + \lambda R^A{}_{BNP} R^B{}_{AQR} \right) \right]$$

- current

$$16\pi G J^\mu = \frac{\sqrt{-g}}{\sqrt{-g_0}} F^{\tau\mu}$$

- on-shell we recover the anomaly

$$D_\mu J^\mu = -\frac{1}{16\pi G} \epsilon^{\mu\nu\rho\lambda} \left( \kappa F_{\mu\nu} F_{\rho\lambda} + \lambda R_{(4)}^\alpha{}_{\beta\mu\nu} R_{(4)}^\beta{}_{\alpha\rho\lambda} \right)$$

# Holography

- Kubo formulas: fluctuations

$$a_x(z), a_y(z), h_x^t(z), h_y^t(z), h_x^z(z), h_y^z(z)$$

- background: charged AdS black hole

$$ds^2 = \frac{r^2}{L^2} (-f(r)dt^2 + d\vec{x}^2) + \frac{L^2}{r^2 f(r)} dr^2 \quad A_{(0)} = \left( \beta - \frac{\mu r_H^2}{r^2} \right)$$

- correlators are

$$\langle JJ \rangle = -ip_z \left( \frac{\kappa}{2\pi G} \mu - \frac{\kappa}{6\pi G} \beta \right)$$

$$\langle JT \rangle = -ip_z \left( \frac{\kappa}{4\pi G} \mu^2 + \frac{2\lambda\pi}{G} T^2 \right)$$

$$\langle TT \rangle = -ip_z \left( \frac{\kappa}{6\pi G} \mu^3 + \frac{4\lambda\pi}{G} \mu T^2 \right)$$

coeffs consistent with weak coupling

no  $T^3$  terms !

[Neiman, Oz], [Loganayagam]

# Summary

- Anomalies  $\rightarrow$  parity violating transport
- Magnetic fields or vortices
- We have derived Kubo formulas
- (non)-renormalization
- Surprise: mixed gauge gravitational anomaly contributes
- Hydrodynamics and derivative expansion? (fluid/gravity)
- Holography with gravitational CS term
- Observable effects? [Karen-Zur, Oz], [Kharzeev, Son], [Kharzeev, Yee]



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