

QCD in astrophysics: Atmospheric neutrinos

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Non-Perturbative QCD 2011, Paris

Work with Hallsie Reno & Ina Sarcevic,
Phys. Rev. D **78**, 043005 (2008)
+ work in progress

Main message

QCD is crucial for some astrophysical processes:

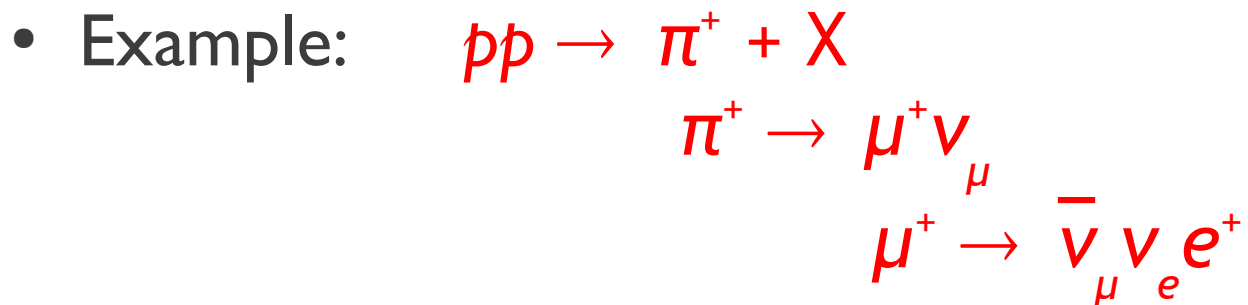
- **Atmospheric neutrinos**
- Neutrino-nucleon cross-section at large energy
- Interactions at high energy in astrophysical sources

For example:

- What happens at **small x** ? (Much smaller x than in colliders)
- Forward region (Hard to measure at colliders)
- Fragmentation of quarks \rightarrow hadrons
- Nuclear effects in pA hard interactions

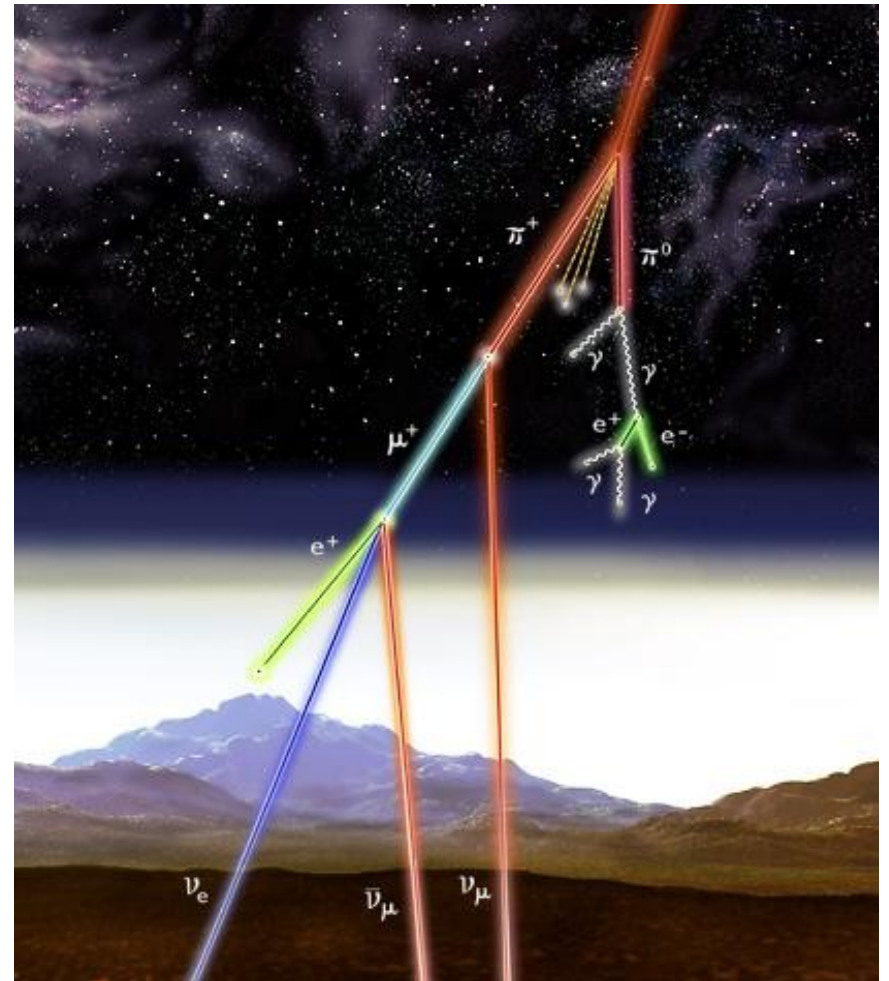
Atmospheric and extragalactic

- Processes that generate high energy neutrinos:
 - Atmospheric interactions of cosmic rays
 - Astrophysical sources (SNe, GRBs, etc.)
- Common theme:
 - Hadronic or photo-hadronic collisions produce hadrons, some of which decay to neutrinos



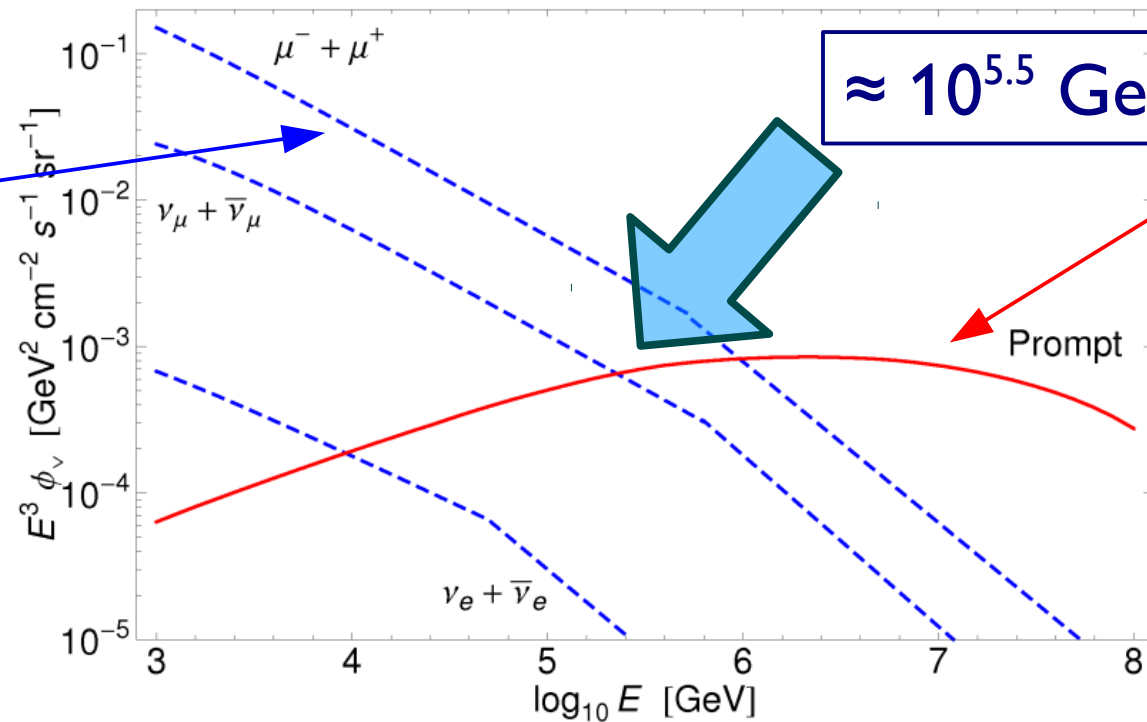
Atmospheric neutrinos

- Cosmic rays bombard upper atmosphere and collide with air nuclei
- Hadron production: pions, kaons, D-mesons ...
- Interaction & decay
⇒ cascade of particles
- Semileptonic decays
⇒ neutrino flux



Prompt vs conventional fluxes of atmospheric neutrinos

Pions & kaons:
long-lived
⇒ lose energy before decay

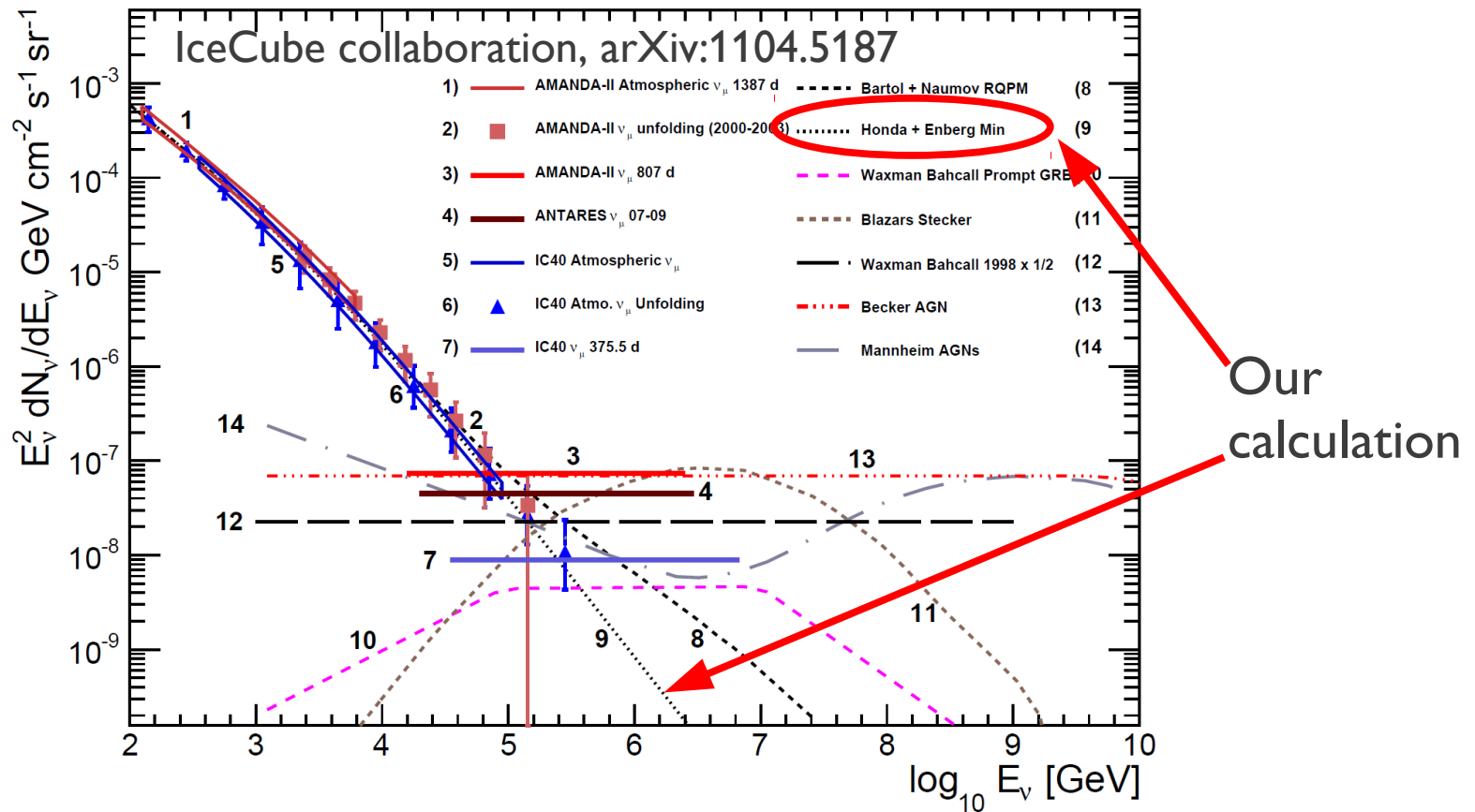


Charmed mesons:
short-lived
⇒ don't lose energy
⇒ harder spectrum

Prompt flux: R. Enberg, M.H. Reno, I. Sarcevic, arXiv:0806.0418 (in PRD)

Conventional: Gaisser & Honda, Ann. Rev. Nucl. Part. Sci. **52**, 153 (2002)

IceCube neutrino telescope data



So far the data only reach up to roughly the predicted cross-over point \rightarrow no sign of prompt contribution

(IceCube has now been completed \rightarrow much more data to come)

Prompt flux calculations

- Prompt fluxes have been calculated years ago using various phenomenological models of the charm production mechanism
- Thunman, Ingelman, Gondolo [[Astropart. Phys. 5, 309 \(1996\)](#)] did the first “real” (as in “perturbative”) QCD calculation
- Used **LO perturbative QCD** (in PYTHIA) and Monte Carlo-simulated the cascade
- Semi-analytic **NLO QCD** calc. by Pasquali, Reno, Sarcevic

Problem with QCD in this process

- Charm cross section in QCD:

$$\frac{d\sigma_{\text{LO}}}{dx_F} = \int \frac{dM_{c\bar{c}}^2}{(x_1 + x_2)s} \sigma_{gg \rightarrow c\bar{c}}(\hat{s}) G(x_1, \mu^2) G(x_2, \mu^2)$$

where

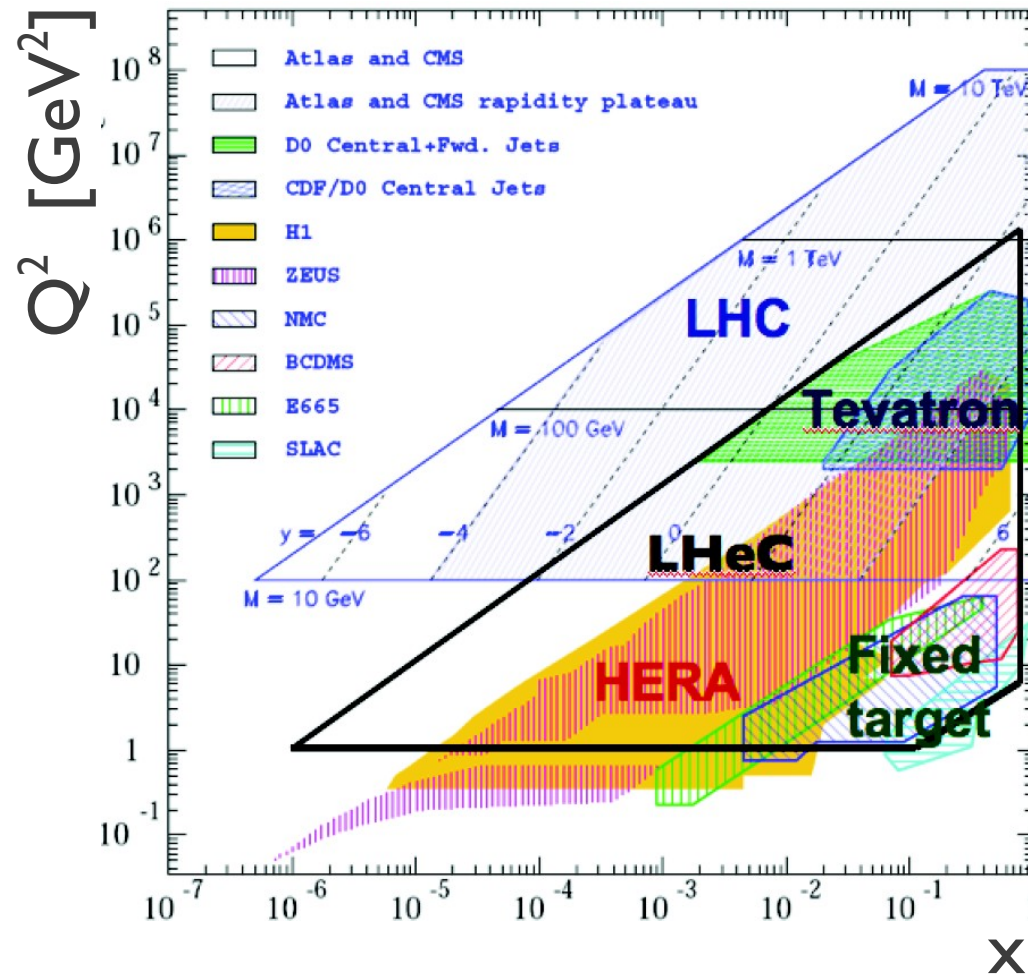
$$x_{1,2} = \frac{1}{2} \left(\sqrt{x_F^2 + \frac{4M_{c\bar{c}}^2}{s}} \pm x_F \right)$$

- CMS energy is large: $s = 2E_p m_p$ so $x_1 \sim x_F$ $x_2 \ll 1$
- For $x_F=1$,
 $E=10^5$ GeV $\rightarrow x \sim 10^{-5}$
 $E=10^6$ GeV $\rightarrow x \sim 10^{-6}$
 $E=10^7$ GeV $\rightarrow x \sim 10^{-7}$
- So very small x is needed!

How small x do we know?

- In fact we have not measured anything at such small x
- E.g. the MSTW pdf has $x_{\min} = 10^{-6}$
GJR has $x_{\min} = 10^{-9}$
- **But these are extrapolations!**
- HERA pdf fits: $Q^2 > 3.5 \text{ GeV}^2$ and $x > 10^{-4}$!

Kinematic plane



HERA: $x_{\min} \sim 10^{-4}$ used for PDF fits ($Q^2 \sim 3.5$ GeV²)

Problem with QCD at small x

- For small x fixed order QCD (LO or NLO) does not work well — there are large logarithms that must be resummed:

$$[\alpha_s \log(1/x)]^n$$

- Parton distribution functions poorly known at small x
- If logs are resummed (**BFKL equation**) one finds power growth of gluon distribution as $x \rightarrow 0$
- It grows so large that ultimately unitarity would be violated (T-matrix > 1)

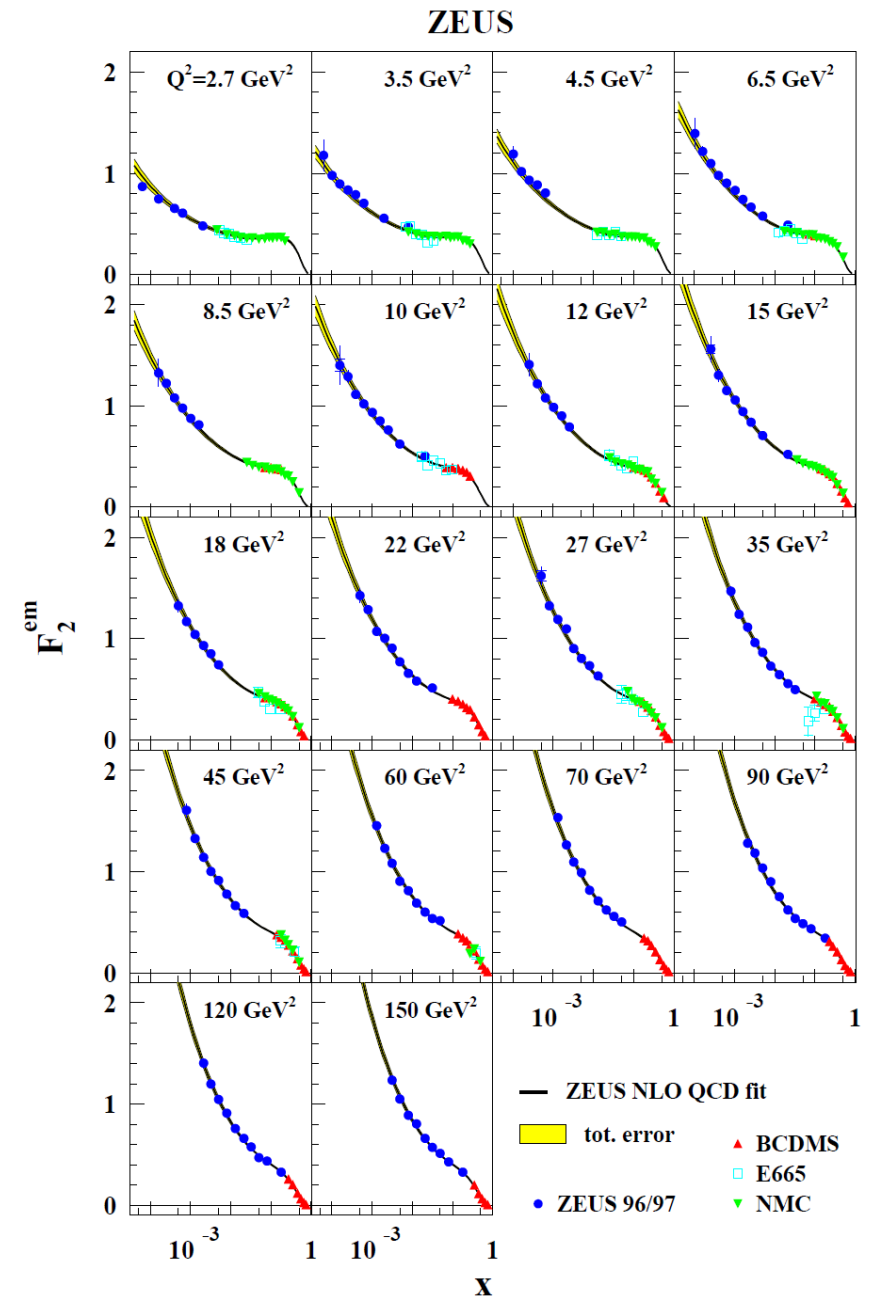
Small x

F_2 measured at HERA (ZEUS)
as a function of Bjorken- x .

Note the steep power-law rise

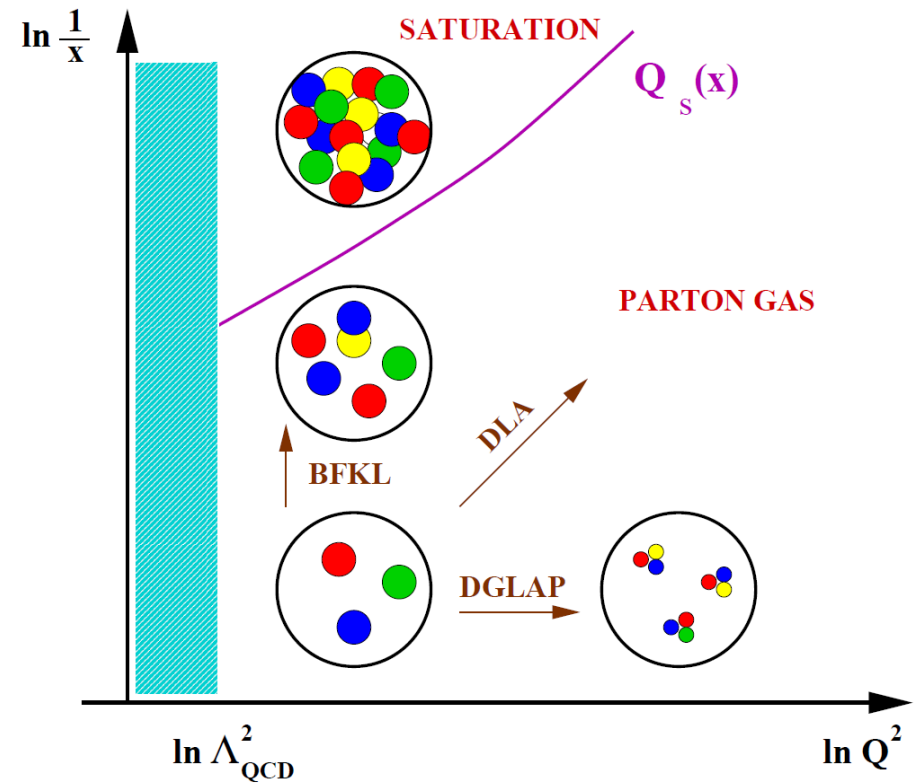
Can this rise continue?

Theoretical answer: no



Parton saturation

- Unitarity saved by **saturation**:
 - Number of gluons in the nucleon becomes so large that gluons can recombine
 - Reduction in the growth
 - Non-linear evolution eqs.



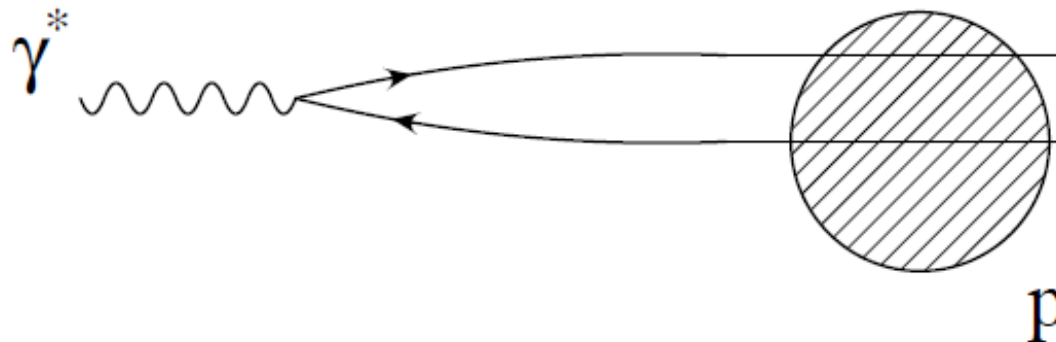
- This is sometimes called the **color glass condensate**
- The simplest evolution equation is the **Balitsky-Kovchegov equation**: BFKL with non-linear term

Dipole frame picture of DIS

It is convenient to use the **dipole frame**:

→ Go to frame where the photon has very large lightcone q^+ momentum (e.g. proton's rest frame)

Then the photon fluctuates into a **color dipole** before hitting the target and the dipole scatters on the proton:



Fluctuation is long-lived at small x :
Very useful in small- x physics

Theoretical description of saturation

- Gribov-Levin-Ryskin and Muller-Qiu
- Color Glass Condensate:
McLerran-Venugopalan and JIMWLK eqs
- Balitsky-Kovchegov equation:
The simplest description: can be more easily used
for phenomenology
- Phenomenological saturation model:
Golec-Biernat and Wüsthoff

GBW saturation model

Pheno ansatz by
Golec-Biernat and
Wüsthoff (GBW):

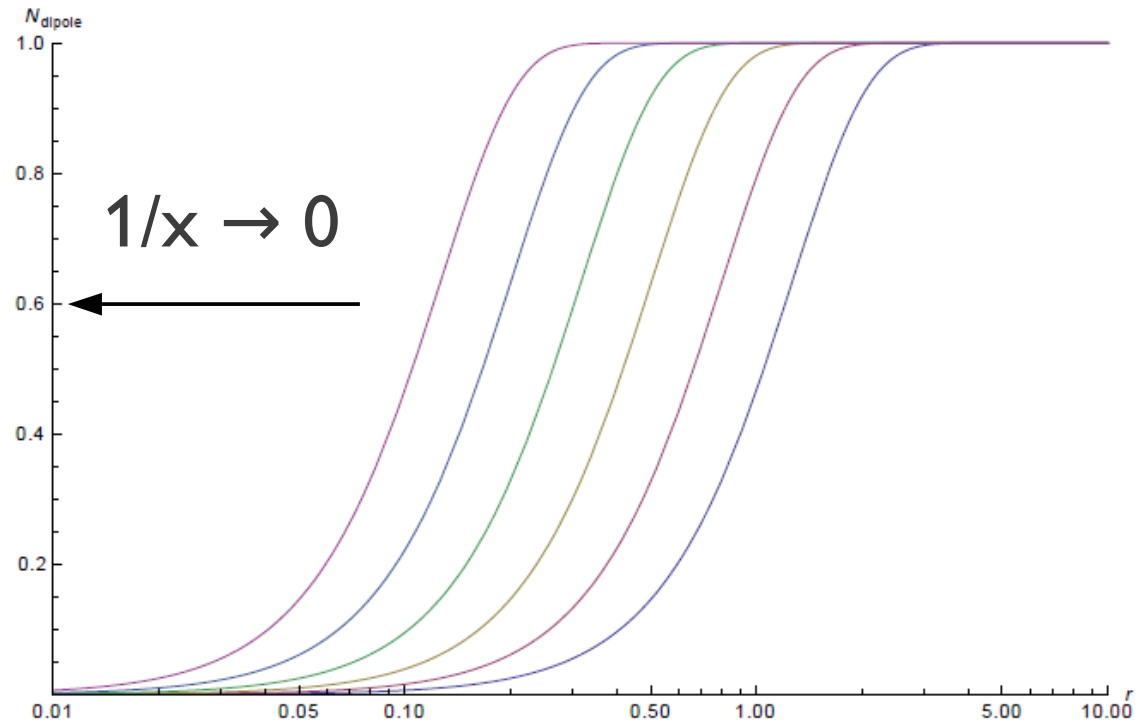
$$\sigma_d^{GBW} = \sigma_0 \left[1 - e^{-r^2 Q_s^2(x)/4} \right]$$

$$Q_s = Q_s(x) = Q_0 (x_0/x)^{\lambda/2}$$

Q_s is the **saturation scale**,
function of x (or rapidity Y)

The dipole amplitude is a
traveling wave

Was enormously
important as a pheno.
toy model



Balitsky-Kovchegov (BK) equation

$$\frac{\partial \phi(k)}{\partial Y} = \bar{\alpha}_s \int_0^\infty \frac{d\ell^2}{\ell^2} \left[\frac{\ell^2 \phi(\ell) - k^2 \phi(k)}{|k^2 - \ell^2|} + \frac{k^2 \phi(k)}{\sqrt{4\ell^4 + k^4}} \right] - \bar{\alpha}_s \phi^2(k)$$

Written here without impact parameter dependence and with fixed coupling (leading log)

Predicts the evolution of the saturation scale:

$$\ln Q_s^2(Y) = \frac{3\alpha_s}{\pi} \frac{\chi(\gamma_s)}{\gamma_s} Y - \frac{3}{2\gamma_s} \ln Y - \frac{3}{\gamma_s^2} \sqrt{\frac{2\pi}{\bar{\alpha}\chi''(\gamma_s)}} \frac{1}{\sqrt{Y}} + \mathcal{O}(1/Y)$$

Balitsky-Kovchegov (BK) equation

BFKL
equation

$$\frac{\partial \phi(k)}{\partial Y} = \bar{\alpha}_s \int_0^\infty \frac{d\ell^2}{\ell^2} \left[\frac{\ell^2 \phi(\ell) - k^2 \phi(k)}{|k^2 - \ell^2|} + \frac{k^2 \phi(k)}{\sqrt{4\ell^4 + k^4}} \right] - \bar{\alpha}_s \phi^2(k)$$

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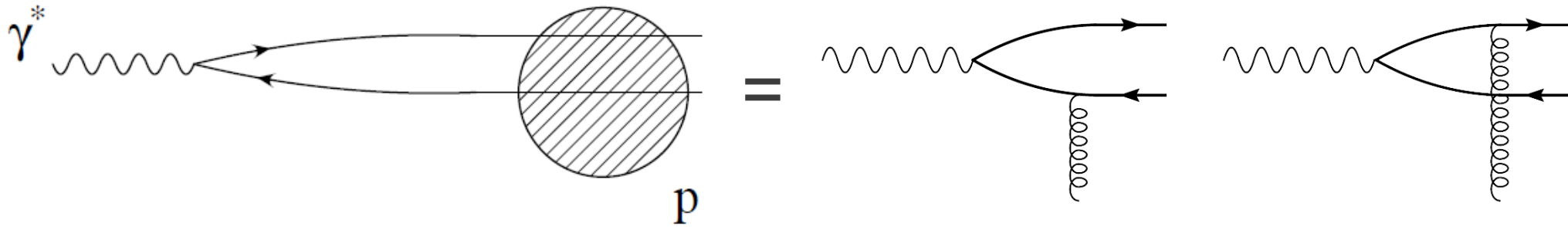
Predicts the evolution of the saturation scale:

$$\ln Q_s^2(Y) = \frac{3\alpha_s}{\pi} \frac{\chi(\gamma_s)}{\gamma_s} Y - \frac{3}{2\gamma_s} \ln Y - \frac{3}{\gamma_s^2} \sqrt{\frac{2\pi}{\bar{\alpha}\chi''(\gamma_s)}} \frac{1}{\sqrt{Y}} + \mathcal{O}(1/Y)$$

Charm production

- To calculate the atmospheric neutrino flux, we need the differential charm production cross section $d\sigma/dx_F$
- To include the effects of parton saturation, the calculation is done in the *dipole picture*, using an approximate solution of the Balitsky-Kovchegov equation
- Saturation suppresses the cross section at larger energy relative to NLO QCD

DIS at small x in dipole picture



The factorization is different from “standard” pQCD:

$$\sigma(\gamma^* N) = \int_0^1 dz \int d^2 \mathbf{r} |\Psi_T(z, \mathbf{r}, Q^2)|^2 \sigma_{q\bar{q}N}(x, \mathbf{r})$$

Dipole cross section

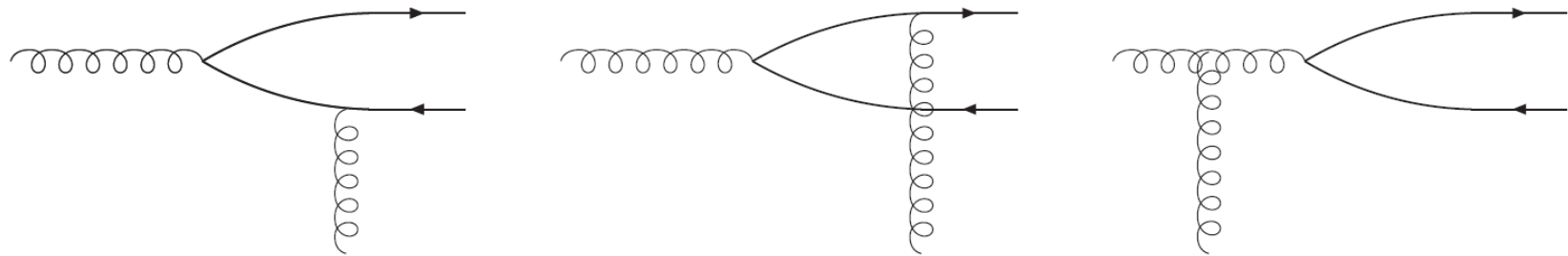
The wave function for the fluctuation is given by:

$$|\Psi_T^f(z, \mathbf{r}, Q^2)|^2 =$$

$$e_f^2 \frac{\alpha_{em} N_c}{2\pi^2} \left[(z^2 + (1-z)^2) \epsilon^2 K_1^2(\epsilon r) + m_f^2 K_0^2(\epsilon r) \right]$$

Generalize to hadron-hadron

Generalized to dipole picture for **heavy quark production** in hadron-hadron collisions by Nikolaev, Piller & Zakharov; Raufeisen & Peng; Kopeliovich & Tarasov



$$\frac{d\sigma(pp \rightarrow Q\bar{Q}X)}{dy} \simeq x_1 G(x_1, \mu^2) \sigma^{Gp \rightarrow Q\bar{Q}X}(x_2, \mu^2, Q^2)$$

→ Gluon distribution
of the projectile hadron
→ gives dipole

← Scattering of
this dipole on
the target hadron

Similar factorization as in DIS

$$\sigma^{Gp \rightarrow Q\bar{Q}X}(x, \mu^2, Q^2) = \int dz d^2\mathbf{r} |\Psi_G^Q(z, \mathbf{r})|^2 \sigma_{dG}(x, \mathbf{r})$$

$$\begin{aligned} \sigma_{GQ\bar{Q}}^N(x_2, \mathbf{r}) &= \frac{9}{8} [\sigma_d(x_2, z\mathbf{r}) + \sigma_d(x_2, (1-z)\mathbf{r})] \\ &\quad - \frac{1}{8} \sigma_d(x_2, \mathbf{r}), \end{aligned}$$

where σ_d is the same dipole cross section as in DIS !

→ if we know σ_d from DIS, we can use it here

Dipole cross section from BK

Iancu, Itakura and Munier: model for σ_d from the BK equation:

Match two analytic solutions in different regions:

- Saturated region when the amplitude approaches one
- Color transparency region when it approaches BFKL result

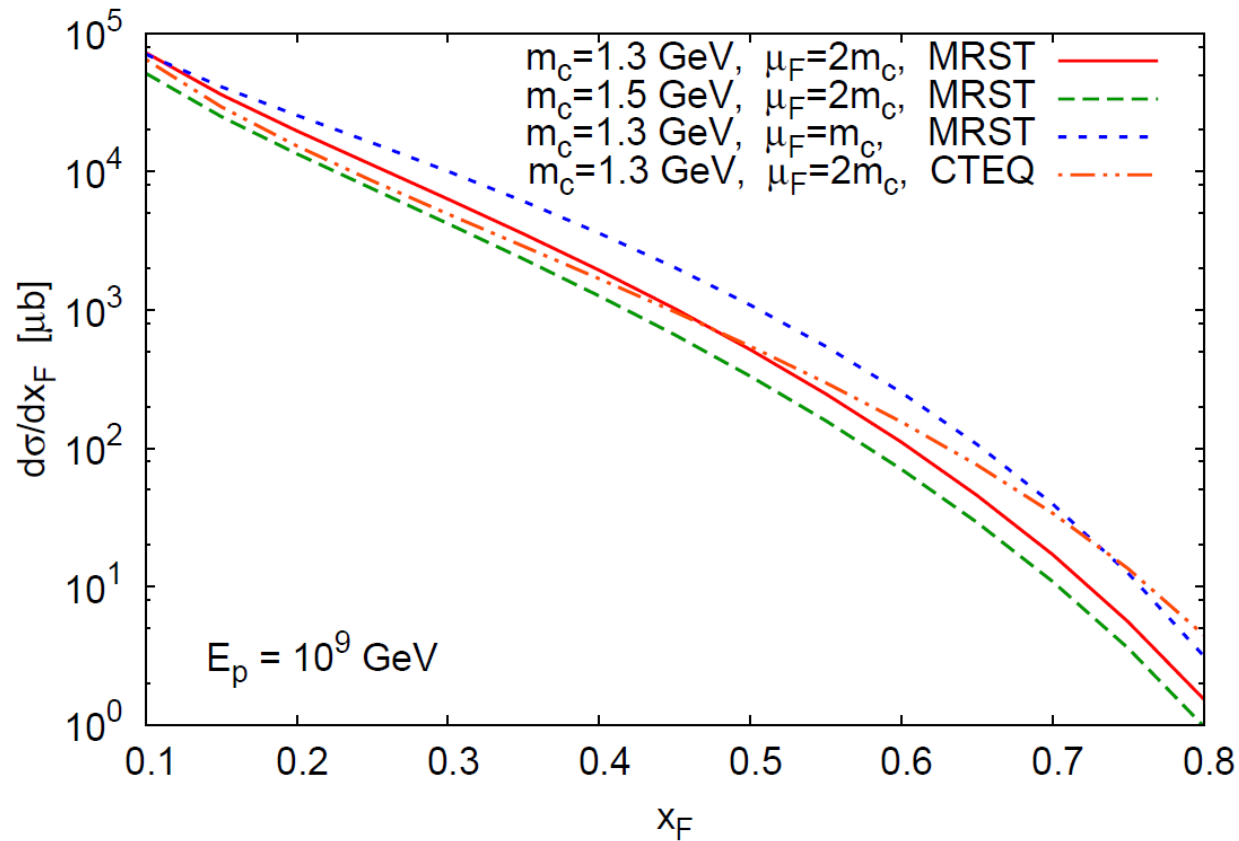
$$\mathcal{N}(rQ_s, Y) = \begin{cases} \mathcal{N}_0 \left(\frac{\tau}{2}\right)^{2\gamma_{\text{eff}}(x, r)}, & \text{for } \tau < 2 \\ 1 - \exp[-a \ln^2(b\tau)], & \text{for } \tau > 2 \end{cases}$$

where $\tau = rQ_s$, $Y = \ln(1/x)$ $\gamma_{\text{eff}}(x, r) = \gamma_s + \frac{\ln(2/\tau)}{\kappa\lambda Y}$

Then $\sigma_d(x, \mathbf{r}) = \sigma_0 \mathcal{N}(rQ_s, Y)$

Fitted to HERA data at small x : good description
(we use an update by Soyez for heavy quarks)

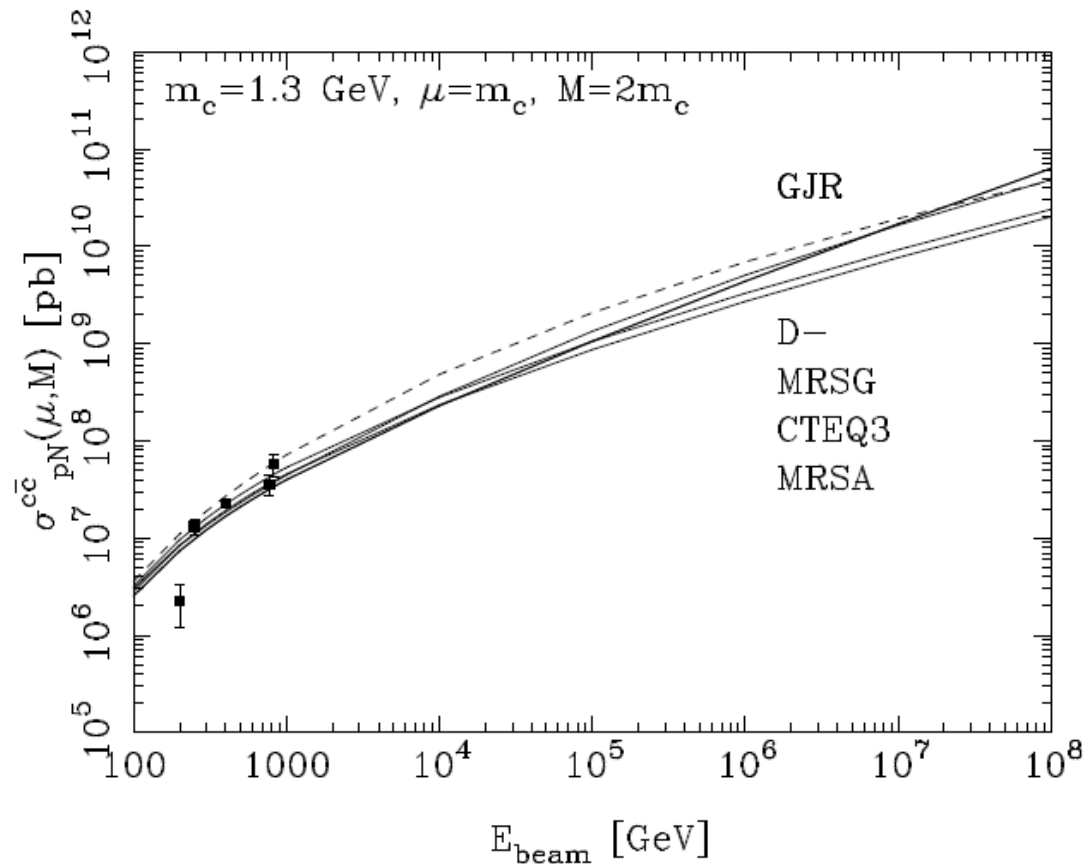
Uncertainties in charm cross section



Different charm mass, factorization scale, pdf choice

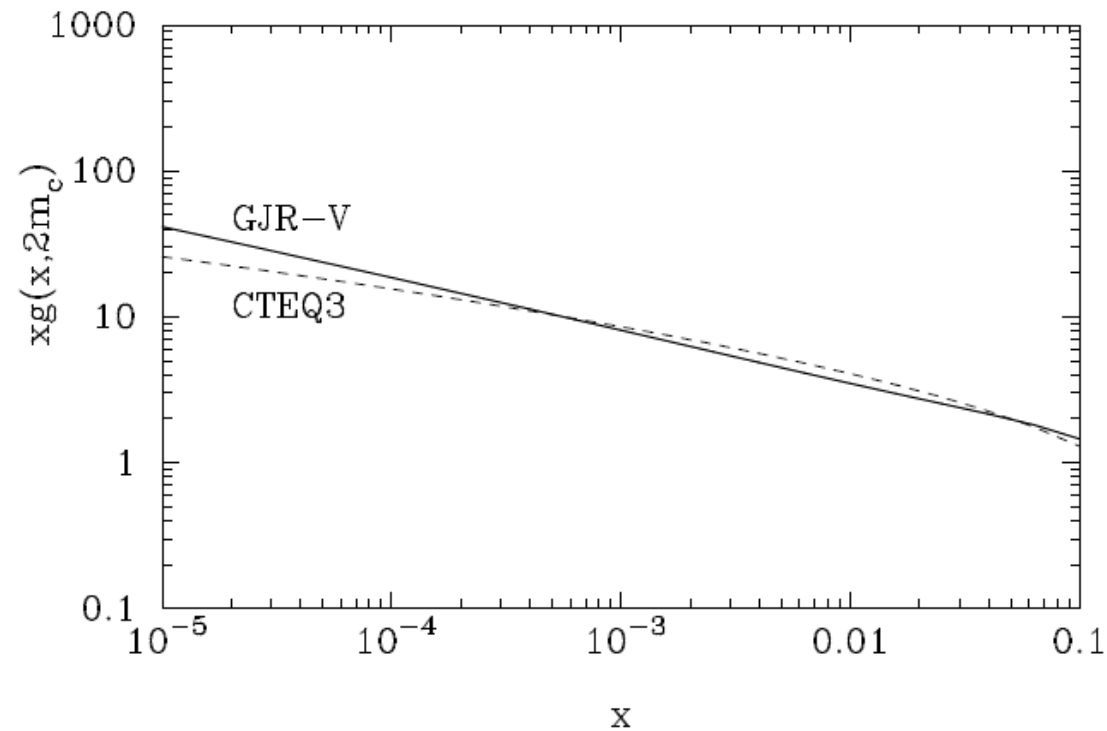
[R. Enberg, M.H. Reno, I. Sarcevic, arXiv:0806.0418 (in PRD)]

Cross section without saturation



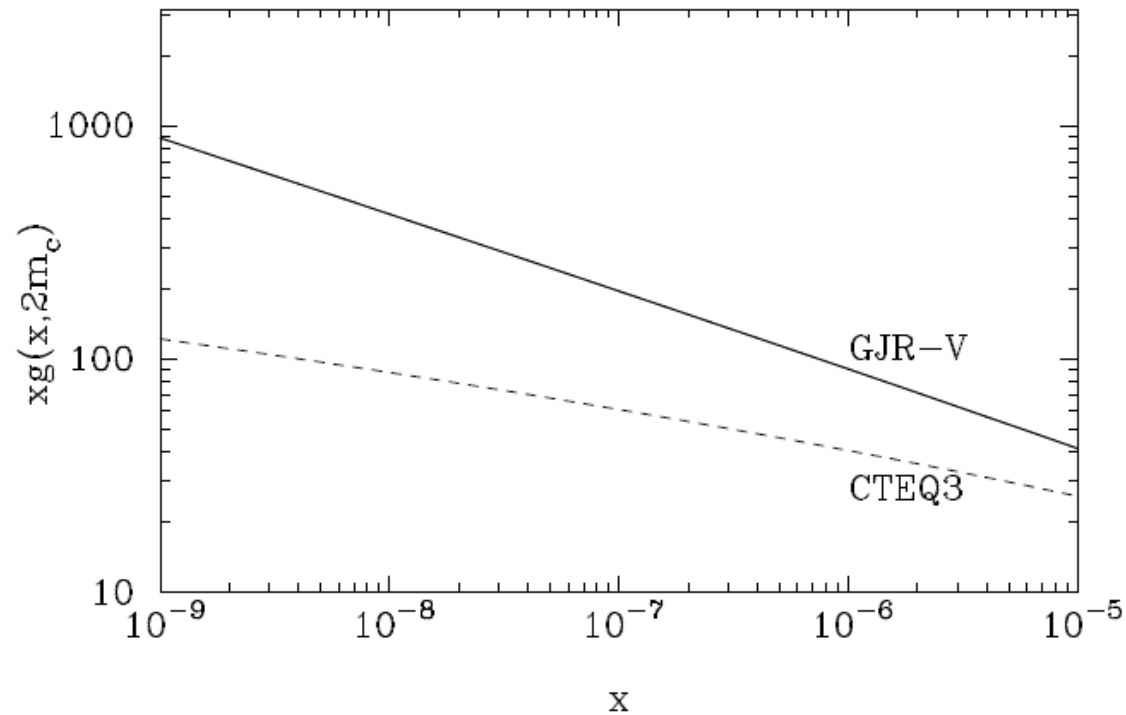
Total cross section calculated in fixed-order NLO QCD for different pdf choices – why so different?

Gluon pdfs: medium small x



GJR-V is a new pdf: **extrapolated** down to $x = 10^{-9}$
CTEQ3 was used in original calculation

Gluon pdfs: very small x



GJR-V is a new pdf: **extrapolated** down to $x = 10^{-9}$
CTEQ3 was used in original calculation

Fixed order calc at small x

- At $x \ll 10^{-4}$ the power-law extrapolation is not warranted if there is saturation
- We can take this as an upper limit on the cross section **if** there is no saturation
- We will improve this with FONLL:
NLO QCD with NLL resummation of $\log(p_T/m)$
- Saturation could in principle be included in pdf fit with data at higher energies

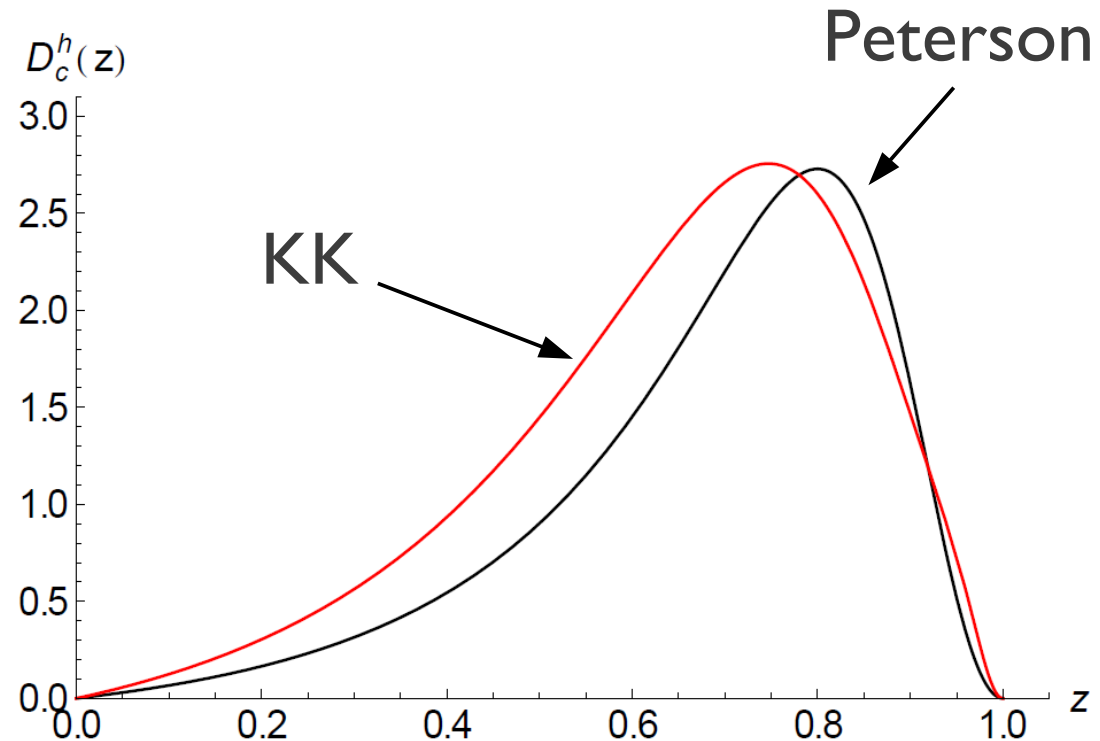
Quark fragmentation to hadrons

Hadronization degrades energy of quark compared to hadron:
Use fragmentation functions fitted to data

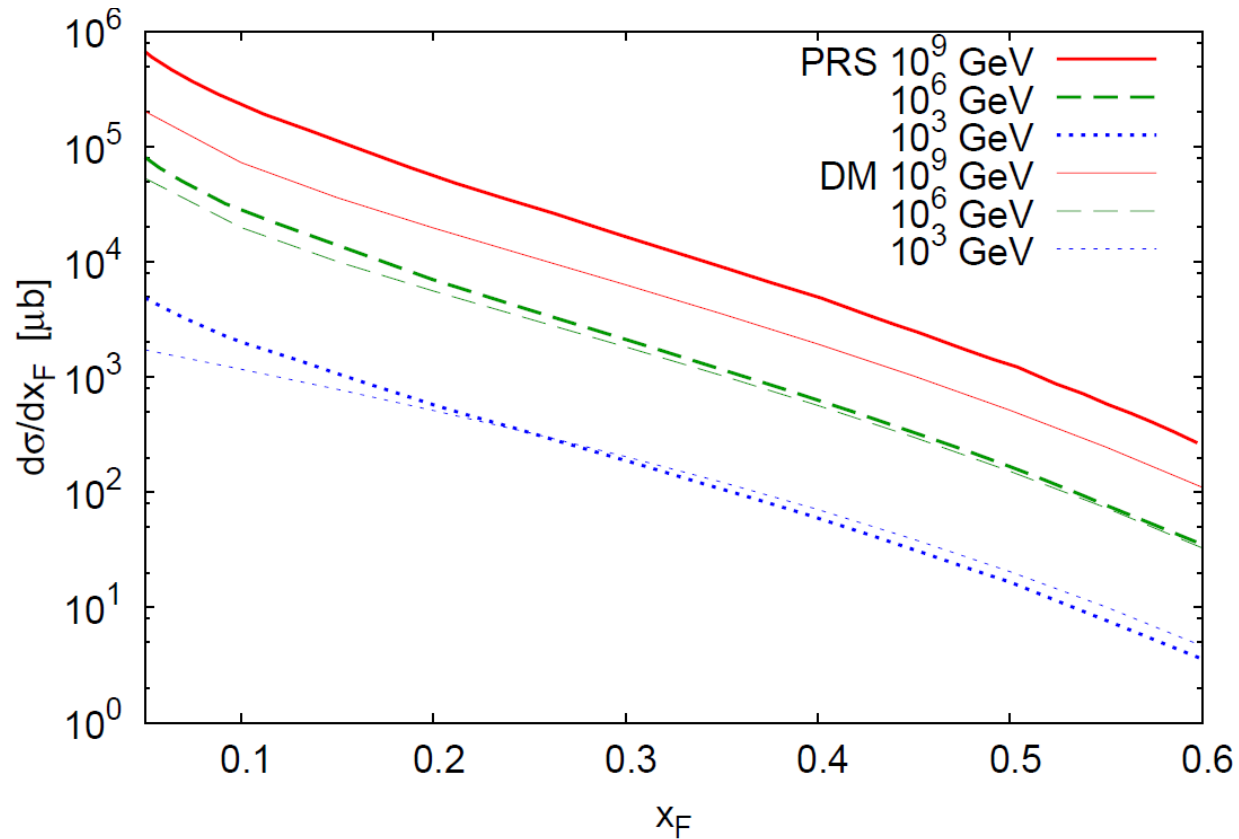
$$\frac{d\sigma(pp \rightarrow hX)}{dE_h} = \int_{E_h}^{\infty} \frac{dE_c}{E_c} \frac{d\sigma(pp \rightarrow cX)}{dE_c} D_c^h(E_h/E_c)$$

Used Kramer-Kniehl (KK)
and Peterson functions

Uncertainty in normalization
and average energy fraction



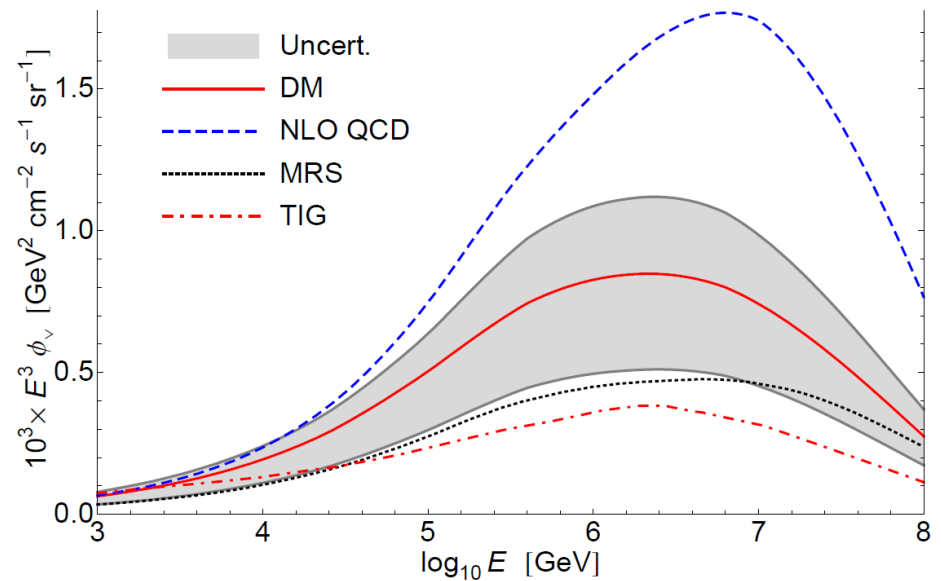
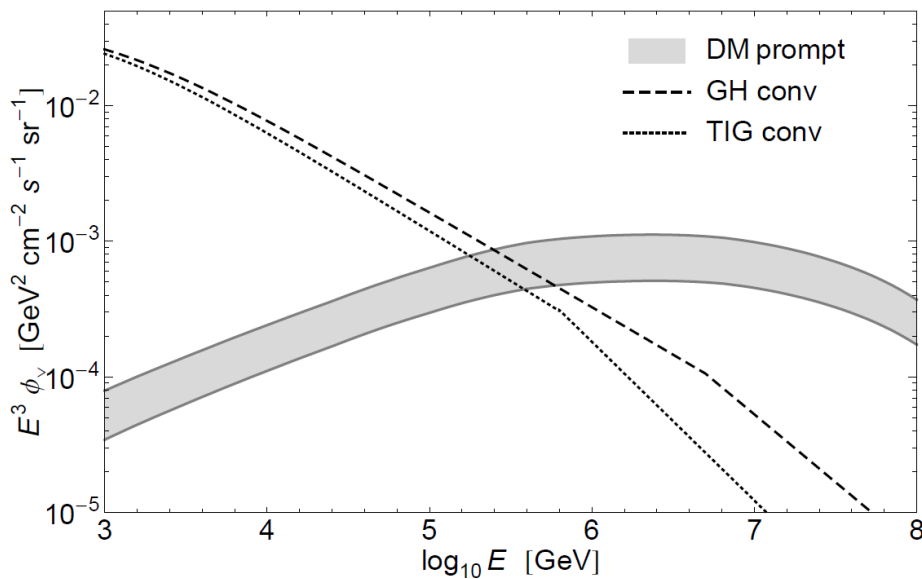
Energy dependence of cross section



Energy dependence slower in dipole model than in NLO QCD

Resulting neutrino fluxes

(Refer to Hallsie Reno's talk earlier today for details)



The band shows uncertainty of the dipole calc.
Upper line in right plot: NLO QCD

[R. Enberg, M.H. Reno, I. Sarcevic, arXiv:0806.0418 (in PRD)]

From colliders to astro

- We will need more data at smaller x to constrain neutrino fluxes:
 - Balitsky-Kovchegov calc describes HERA F2 data
 - But larger $x \rightarrow$ we don't know the energy evolution
- LHC, ALICE, LHCb
 - We saw new data on $c\bar{c}$ cross section yesterday
- LHeC !

From astro to colliders

- Maybe atmospheric neutrino data can give constraints on small- x scattering and saturation at very small x !
- Of course: numerous astrophysical and experimental uncertainties as well
 - E.g.: what is the chemical composition of cosmic rays?
- But one can hope...
- Some non-perturbative QCD models are already disfavored (RQPM...)

Main message

- QCD is crucial for some astrophysical processes:
 - Atmospheric neutrinos
 - Neutrino-nucleon cross-section at large energy
 - Interactions at high energy in astrophysical sources
- What happens at **small x** ? (Much smaller than in colliders)