Nonperturbative QCD Workshop, Paris, June 2011

Forward particle production in hadronic collisions

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Thanks for collaboration to
M. Grothe (Wisconsin), S. Ostapchenko (NTU),
M. Deak (DESY), K. Kutak (Antwerp),
M. Hentschinski (UAM/CSIC-Madrid) and H. Jung (CERN/DESY)

I. General issues

▷ soft/hard particle production in the forward region

▷ QCD factorization

▷ multiple parton interactions

II. Forward jets

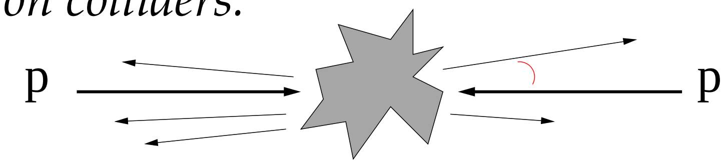
▷ soft gluons and coherence

▷ forward-central correlations

▷ energy flow

▷ b-jets, quarkonia

Particle production in the forward region at hadron colliders:



small polar angles, i.e. large rapidities

♢ Historically:

- fairly specialized subject: e.g., measurements of σ(total) and σ(elastic)
- dominated by soft, small-p$_T$ processes

♢ At the LHC:

- both soft and hard production
- phase space opening up for large $\sqrt{s}$ $\Rightarrow$ multiple-scale processes
- unprecedented coverage of large rapidities (calorimeters + proton taggers)

$\Rightarrow$ ♢ forward high-p$_T$ production

♢ central production of high p$_T$ + forward protons

♠ Forward high-p$_T$ production at the LHC involves both

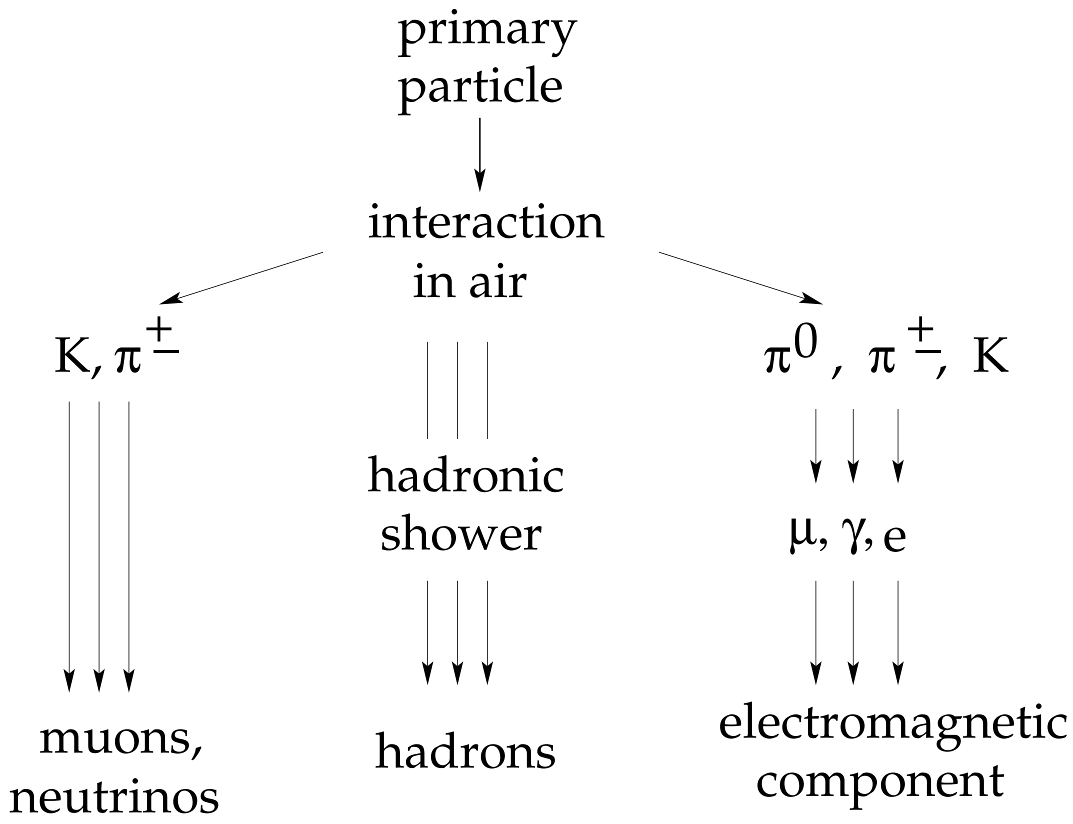
new particle discovery processes, e.g.

- Higgs searches in vector boson fusion channels
- jet studies in decays of highly boosted heavy states

and new aspects of Standard Model physics, e.g.

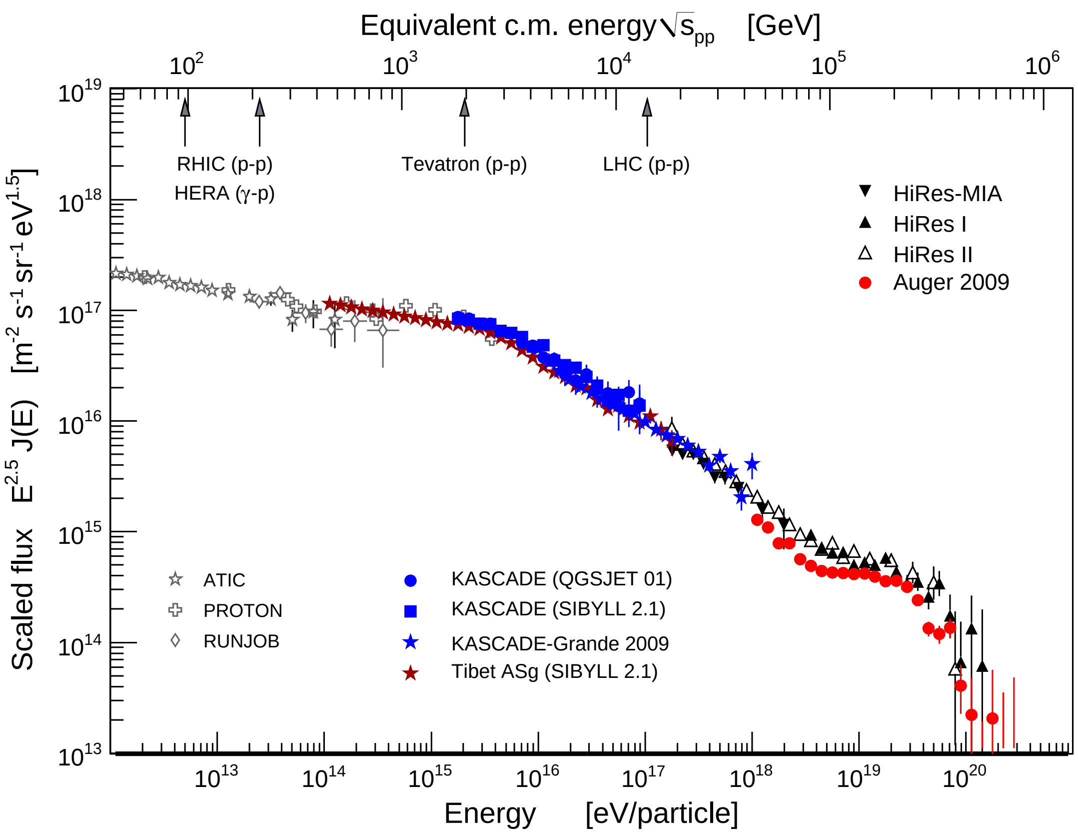
- QCD at small x and its interplay with cosmic ray physics
- new states of strongly interacting matter at high density

• Measurements of forward particle production (soft and hard) at the LHC serve as input to Monte Carlo models of high-energy showers in cosmic ray physics



• Fixed target collision in air with 10^{17} eV corresponds to pp interaction at LHC

Not only LHC physics...: The Cosmic Ray / Collider connection



R. Engel, 2010

NOTE:

♠ Nearly all topics in forward hard production processes at the LHC imply new experimental areas

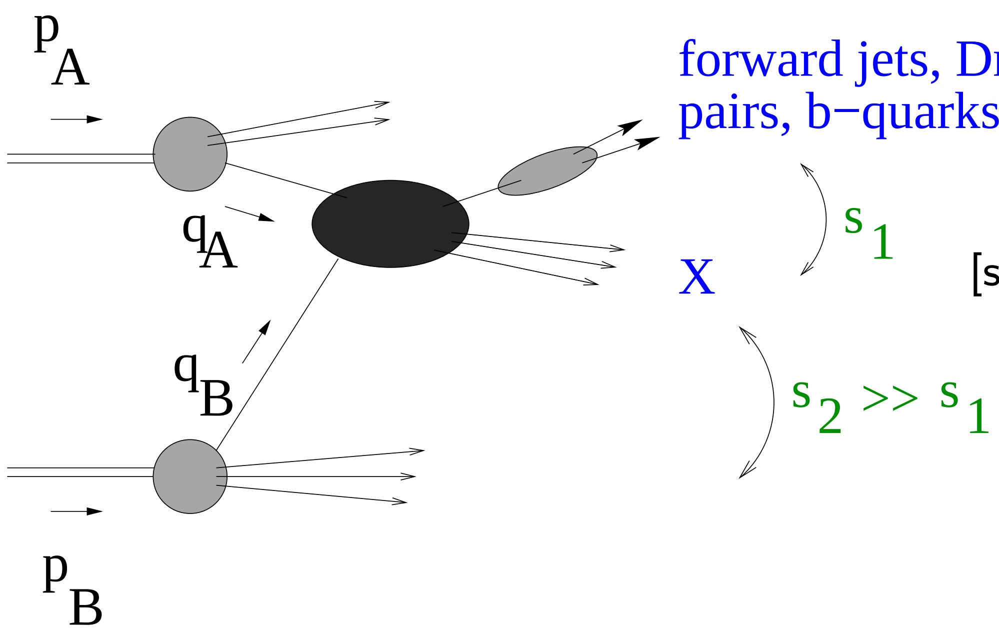
♠ Theoretical issues: LHC forward physics dominated by QCD at small x

$\Downarrow$

- Factorization/resummation for large rapidity separations
- Parton evolution / showering beyond collinear ordering
- High parton density effects

♠ Phenomenology: How well do current Monte Carlo generators simulate LHC final states in the forward region?

II. High-p_T production in the forward region

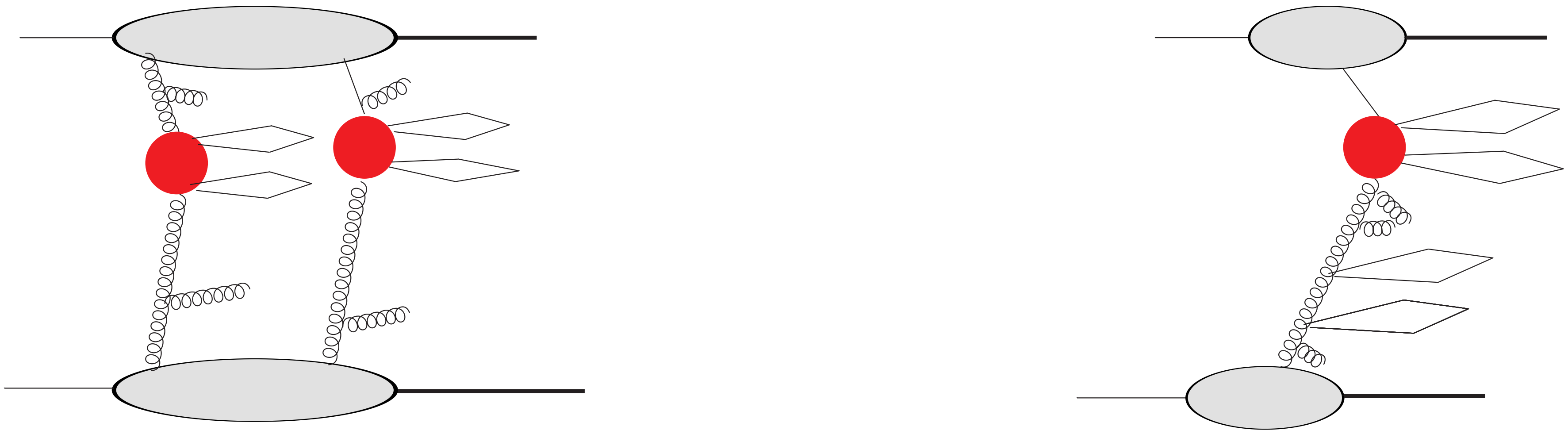


[see first results by CMS and LHCb]

- multiple hard scales
- asymmetric parton kinematics $x_A \to 1, \; x_B \to 0$

◊ Are fixed-order QCD calculations reliable in the forward region?

Multiple parton interactions



Multi-jet production by (left) multiple parton chains; (right) single parton chain.

[P. Bartalini et al., arXiv:1003.4220, Proc. 1st MPI Workshop]

- modeled by shower Monte Carlo generators

Sjöstrand & Skands, 2006; Gieseke et al., 2008

♢ Do multiple parton interactions become non-negligible in hard processes at forward rapidities?

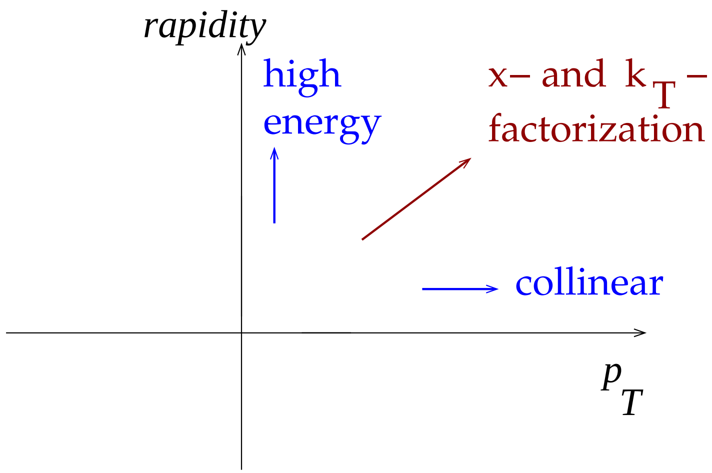
Forward jet production as a multi-scale problem

• summation of high-energy logarithmic corrections long recognized to be necessary for reliable QCD predictions

$\Rightarrow$ BFKL calculations

Mueller & Navelet, 1987; Del Duca et al., 1993; Stirling, 1994; Colferai et al., arXiv:1002.1365

• Large logarithmic corrections are present both in the hard p_T and in the rapidity interval



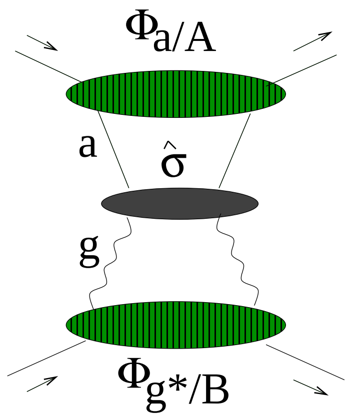
$\longrightarrow$ Both kinds of log contributions can be summed consistently to all orders of perturbation theory via QCD factorization at fixed k_T

Forward jets:

- High-energy factorization at fixed transverse momentum

$$\frac{d\sigma}{dQ_t^2 d\varphi} = \sum_a \int \phi_{a/A} \otimes \frac{d\widehat{\sigma}}{dQ_t^2 d\varphi} \otimes \phi_{g^*/B}$$

▷ needed to resum consistently both logs of rapidity and logs of hard scale *Deak, Jung, Kutak & H, JHEP 09 (2009) 121*



(a)

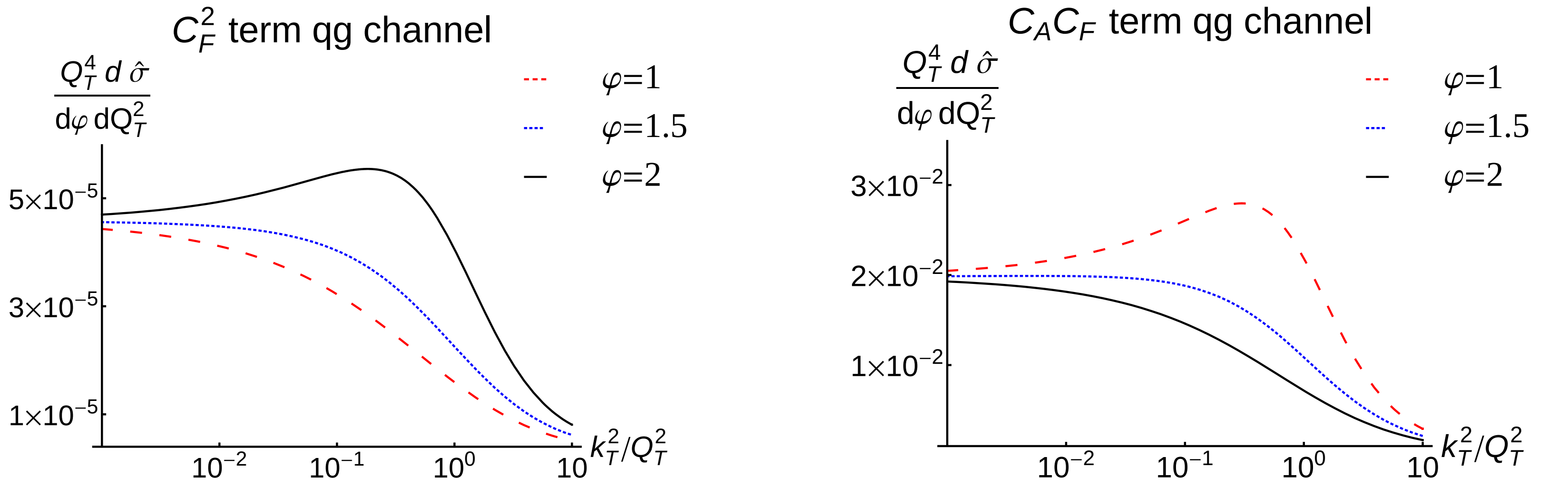
Figure 1: Factorized structure of the cross section.

♢ ϕ_a near-collinear, large-x; ϕ_{g^*} $k_\perp$-dependent, small-x

♢ $\widehat{\sigma}$ off-shell (but gauge-invariant) continuation of hard-scattering matrix elements *[Catani et al., 1991; Ciafaloni, 1998]*

FULLY EXCLUSIVE MATRIX ELEMENTS: BEHAVIOR AT LARGE $K_\perp$

Q_t = final-state transverse energy (in terms of two leading jets p_t's)
k_t = transverse momentum carried away by extra jets



- dynamical cut-off at $k_t \sim Q_t$, set by higher-order radiative effects
  - non-negligible terms from finite k_t tail
- $C_F C_A$ contribution to qg dominates at high energies $s/Q_t^2 \gg 1$

Remarks

◇ Note difference from classic Mueller-Navelet approach

$$\sigma^{(MN)} = \sum_a \int \phi_{a/A} \otimes V_{jet1} \otimes \mathcal{G}_{gg} \otimes V_{jet2} \otimes \phi_{b/B}$$

[Colferai et al., arXiv:1002.1365]

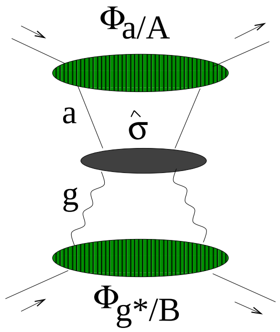
[D'Enterria, arXiv:0911.1273]

- non-collinear corrections to ϕ distributions
- no "vertex jet function" V_{jet}
- jets produced by either hard ME or parton shower (taking account of $k_\perp$)

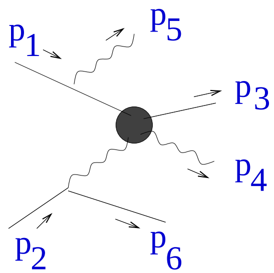
- High-energy matrix elements factorize not only in the collinear emission region but also at finite angle

◇ once coupled to distributions for parton branching at fixed $k_\perp$, can serve to take into account effects of coherence across large rapidity intervals, not associated with small angles

◇ Merging scheme defined by the factorization at high energy



(a)



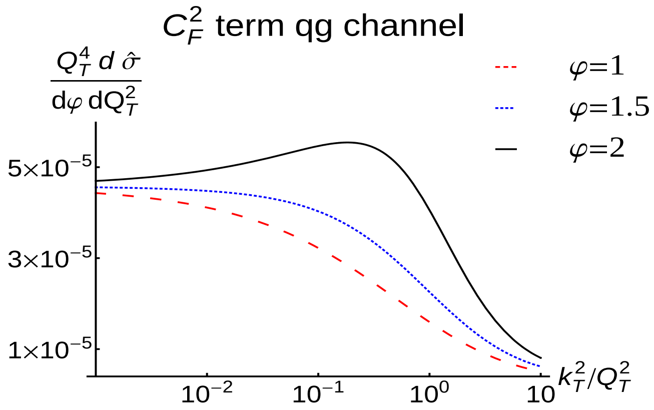
(b)

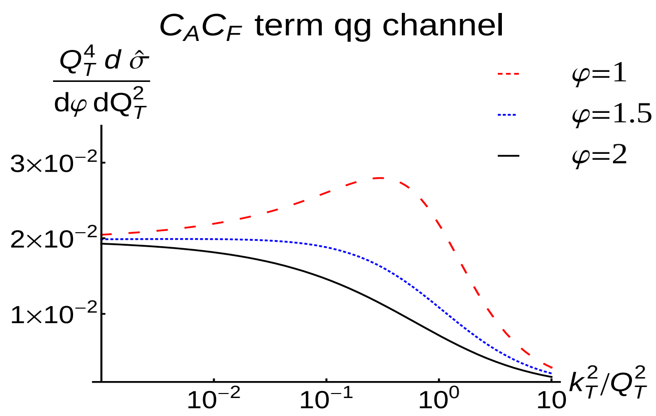
$$p_1 - p_5 = k_1 = \xi_1 p_1 + k_{\perp 1} + \overline{\xi}_1 p_2 \quad , \quad p_2 - p_6 = k_2 = \xi_2 p_2 + k_\perp + \overline{\xi}_2 p_1$$

Forward region: $(p_4 + p_6)^2 \gg (p_3 + p_4)^2$, $k_1 \simeq \xi_1 p_1$, $k_2 \simeq \xi_2 p_2 + k_\perp$

$$\Rightarrow \; p_5 \simeq (1 - \xi_1) p_1 \quad , \quad p_6 \simeq (1 - \xi_2) p_2 - k_\perp \quad , \quad \xi_1 \gg \xi_2$$

$$Q_T = (1 - \nu) p_{T4} - \nu p_{T3} \quad , \quad \text{where} \quad \nu = (p_2 \, p_4)/[(p_2 \, p_1) - (p_2 \, p_5)]$$





- small-angle limit:

$$\frac{Q_T^4\ d\widehat{\sigma}}{dQ_T^2 d\varphi} \to \alpha_s^2 f^{(0)}(p_T^2/s) \quad , \quad Q_T \to p_T = |p_{T3}| = |p_{T4}|$$

$$f^{(0)}(z) = \frac{1}{16\sqrt{1-4z}}\ \left[C_F^2 z(1+z) + 2C_F C_A (1 - 3z + z^2)\right]$$

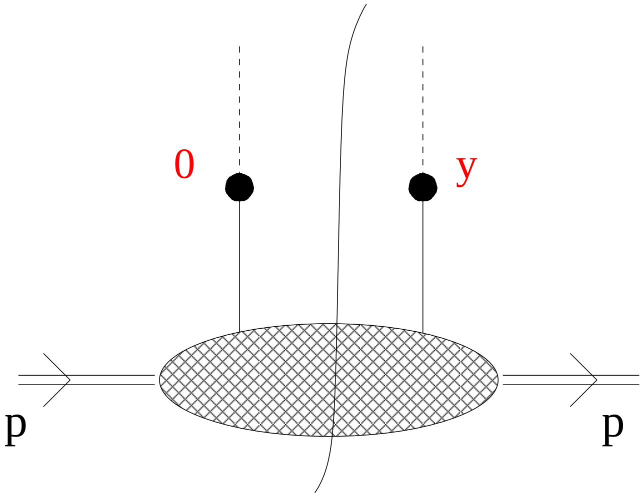
- summation of logs for large $y \sim \ln s/p_T^2$ achieved by convolution with unintegrated splitting functions

$$\int d^2 k_T \ \left(\frac{1}{k_T^2}\right)_+ \ \widehat{\sigma}(k_T) = \int d^2 k_T \ \frac{1}{k_T^2}\ \left[\widehat{\sigma}(k_T) - \Theta(\mu - k_T)\ \widehat{\sigma}(0_T)\right]$$

UNINTEGRATED (OR TRANSVERSE MOMENTUM DEPENDENT) PARTON DISTRIBUTIONS

[S. Mert Aybat and T. Rogers, arXiv:1101.5057]

[J. Collins, hep-ph/0304122]



$p = (p^+ , m^2 / 2 p^+ , 0_\perp)$

$$\widetilde{f}(y) = \langle\ P \mid \overline{\widetilde{\psi}}(y)\ \gamma^+\ \widetilde{\psi}(0) \mid P\ \rangle \quad , \qquad y = (0, y^-, y_\perp)$$

correlation of parton fields ('dressed' with gauge links) at distances y, $y_\perp \neq 0$

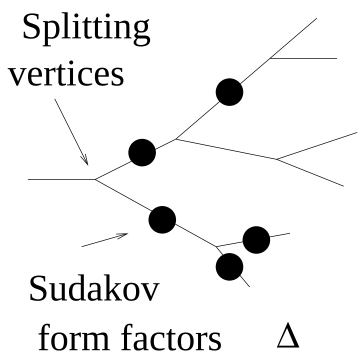
- Sudakov region ⇒ resummation $\alpha_S^n \ln^k (M/q_T)$
- high energy region ⇒ resummation $\alpha_S^n \ln^k (\sqrt{s}/E_T)$

♣ Note: exploit dominance of single gluon polarization at large $\sqrt{s}$

FROM QCD TO MONTE CARLO EVENT GENERATORS

- Factorizability of QCD x-sections $\longrightarrow$ probabilistic branching picture

$\diamondsuit$ $A)$ QCD evolution by "parton showering" methods:

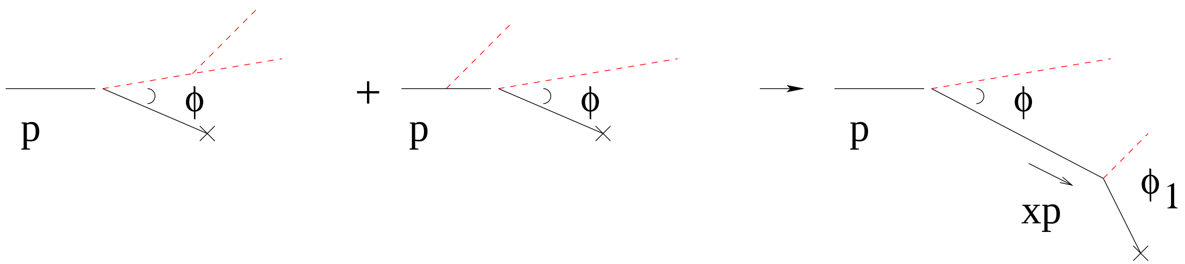


$$d\mathcal{P} = \int \frac{dq^2}{q^2} \int dz \; \alpha_S(q^2) \; P(z) \; \Delta(q^2, q_0^2)$$

$\hookrightarrow$ collinear, incoherent emission

$\diamondsuit$ $B)$ Soft emission $\longrightarrow$ interferences $\longrightarrow$ ordering in decay angles:

$\hookrightarrow$ gluon coherence for $x \sim 1$

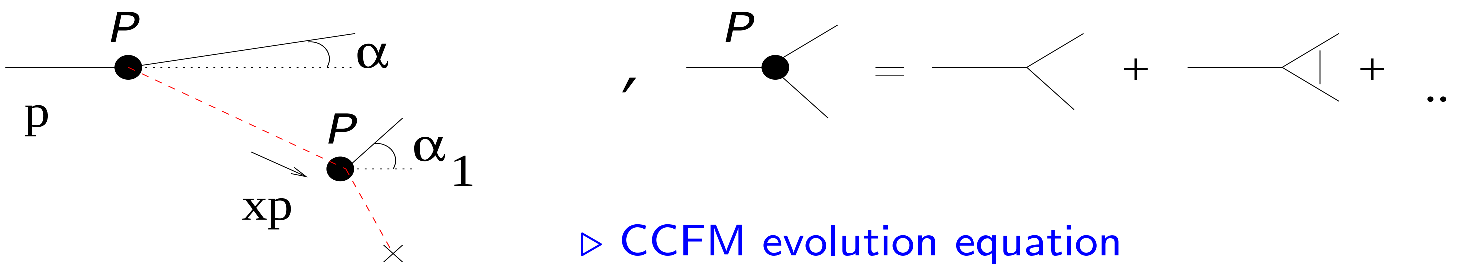


$\diamondsuit$ $C)$ Gluon coherence for $x \ll 1$ $\Rightarrow$ corrections to angular ordering:

$\hookrightarrow$ MC based on $k_\perp$-dependent unintegrated pdfs and MEs

▷ K$_\perp$-DEPENDENT PARTON BRANCHING

$$\mathcal{G}(x, k_T, \mu) = \mathcal{G}_0(x, k_T, \mu) + \int \frac{dz}{z} \int \frac{dq^2}{q^2} \, \Theta(\mu - zq) \times \underbrace{\Delta(\mu, zq)}_{\text{Sudakov}} \; \underbrace{\mathcal{P}(z, q, k_T)}_{\text{unintegr. splitting}} \; \mathcal{G}(x/z, k_T + (1-z)q, q)$$



▷ CCFM evolution equation

▷ Monte Carlo implementations: CASCADE (H. Jung et al.), LDC (G. Gustafson et al.), ...

Merging PS and ME

- Merging in high-energy limit can be done using

$$\gamma \, \frac{1}{k_\perp^2} \, \left(\frac{k_\perp^2}{\mu^2} \right)^\gamma \overset{\gamma \ll 1}{=} \delta(k_\perp^2) + \gamma \, \left(\frac{1}{k_\perp^2} \right)_{\rm R} + \gamma^2 \, \left(\frac{1}{k_\perp^2} \ln \frac{k_\perp^2}{\mu^2} \right)_{\rm R} + \dots$$

where $\int dk_\perp \, (G(k_\perp, \mu))_{\rm R} \, \varphi(k_\perp) = \int dk_\perp \, G(k_\perp, \mu) [\varphi(k_\perp) - \Theta(\mu - k_\perp) \, \varphi(0)]$

$\diamondsuit$ Unintegrated (TMD) pdf's are key ingredient for different types of QCD resummations

$\diamondsuit$ also relevant to fully take account of coherence effects in parton showers at high energy

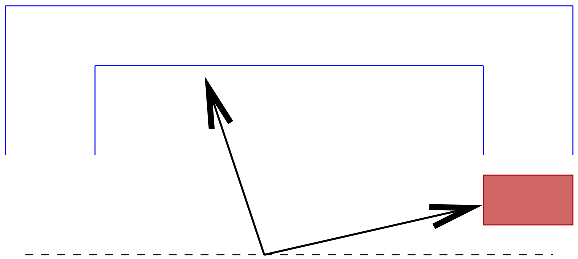
$\diamondsuit$ possibly, more natural framework

to push theory towards soft p_T physics and to treat diffraction

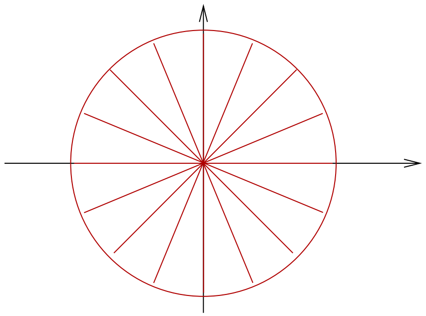
III. FORWARD JETS AT THE LHC

- polar angles small but far enough from beam axis
  - measure correlations in azimuth, rapidity, p_T

$$p_\perp \gtrsim 20 \text{ GeV} \; , \; \Delta\eta \gtrsim 4 \div 6$$



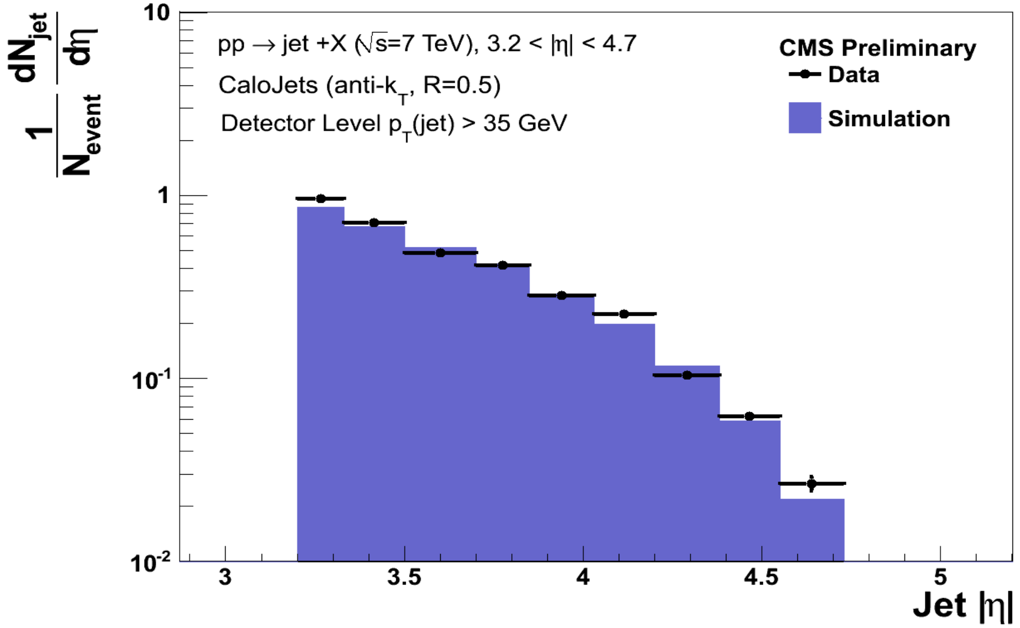
central + forward detectors



azimuthal plane

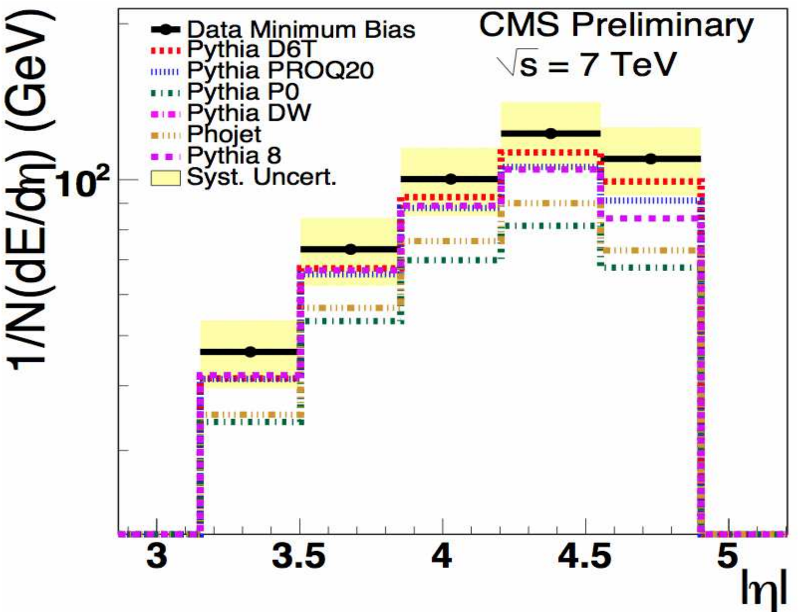
CMS jet reconstruction

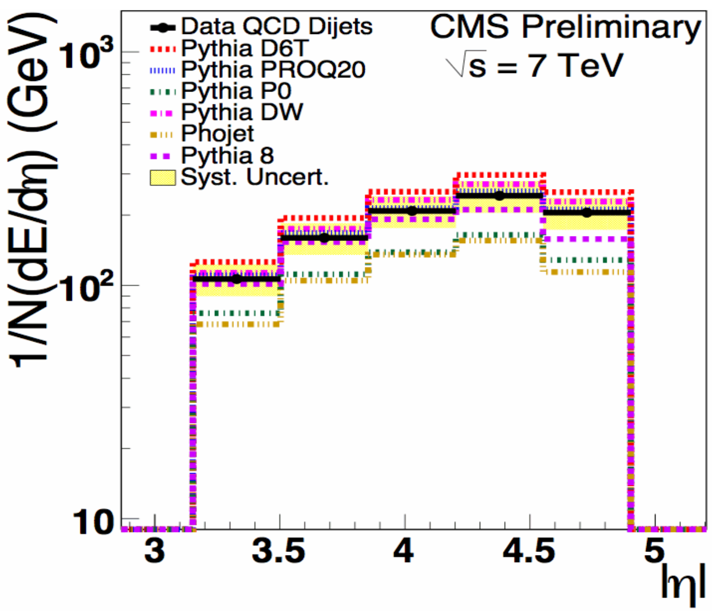
- first time jets are observed at such forward rapidities $\eta > 3$



- *talks by Flossdorf and Kousouris (CMS) at PLHC2011*

Forward energy flow in minimum bias and central dijet sample:





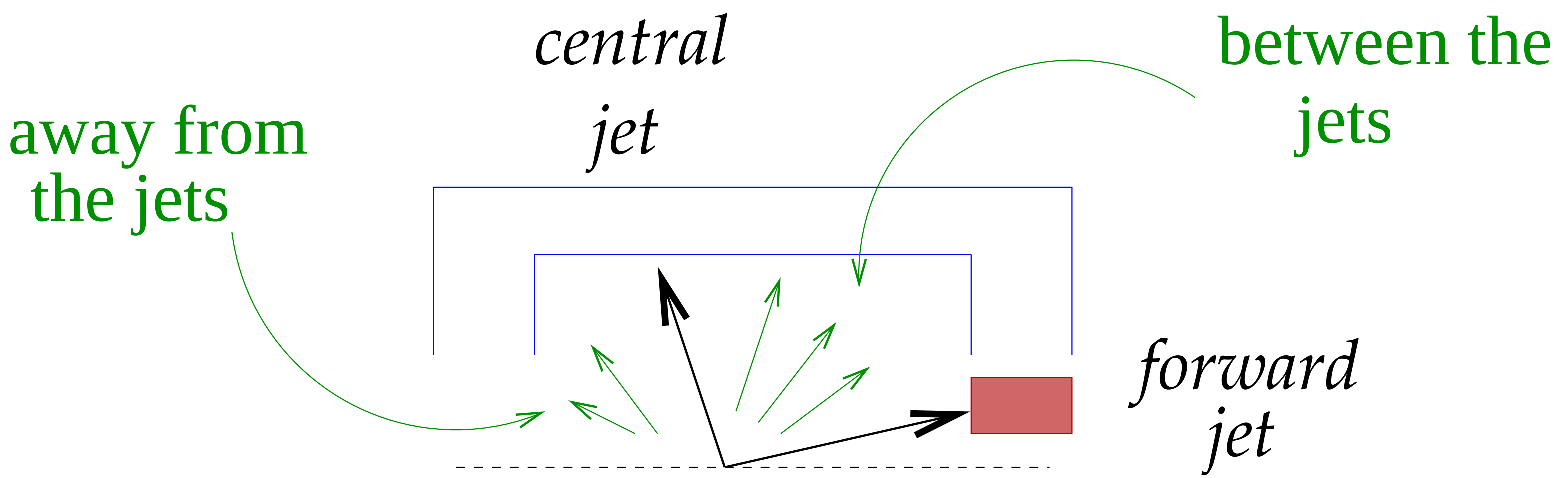
• observed increase with increasing $\sqrt{s}$

• energy flow in forward region not well described by Pythia tunes based on charged particle spectra in central region, especially for minimum bias

$\hookrightarrow$ more global UE description needed *[Bartalini & Fanò, arXiv:1103.6201]*

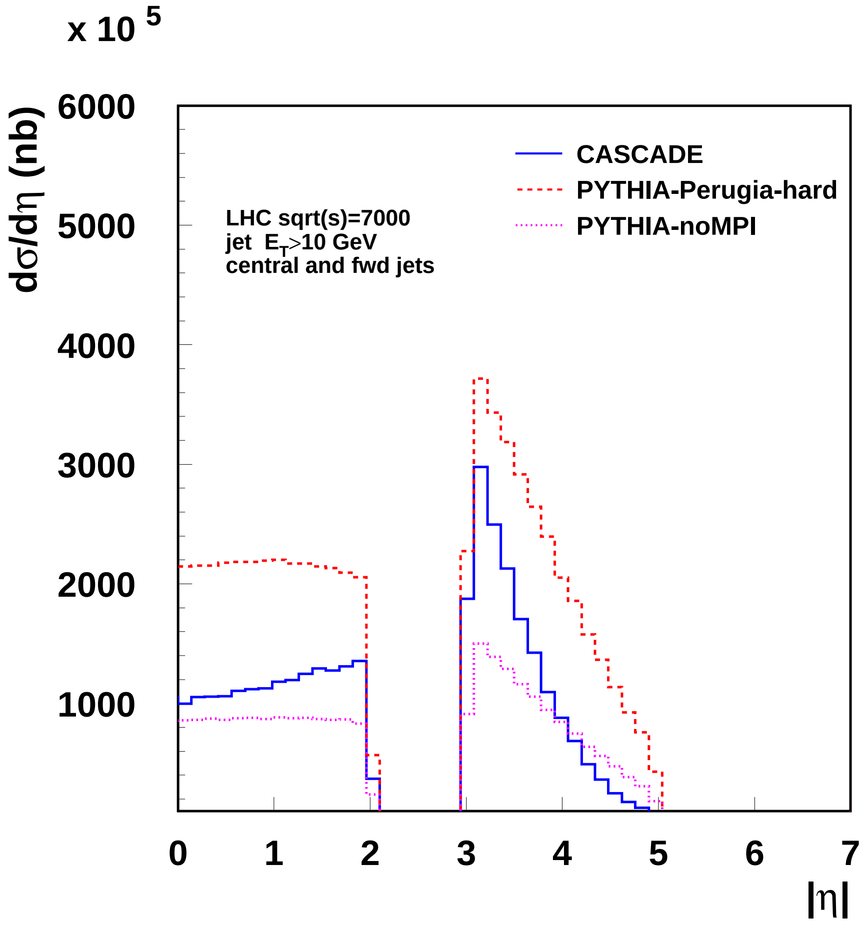
1 central + 1 forward jet:

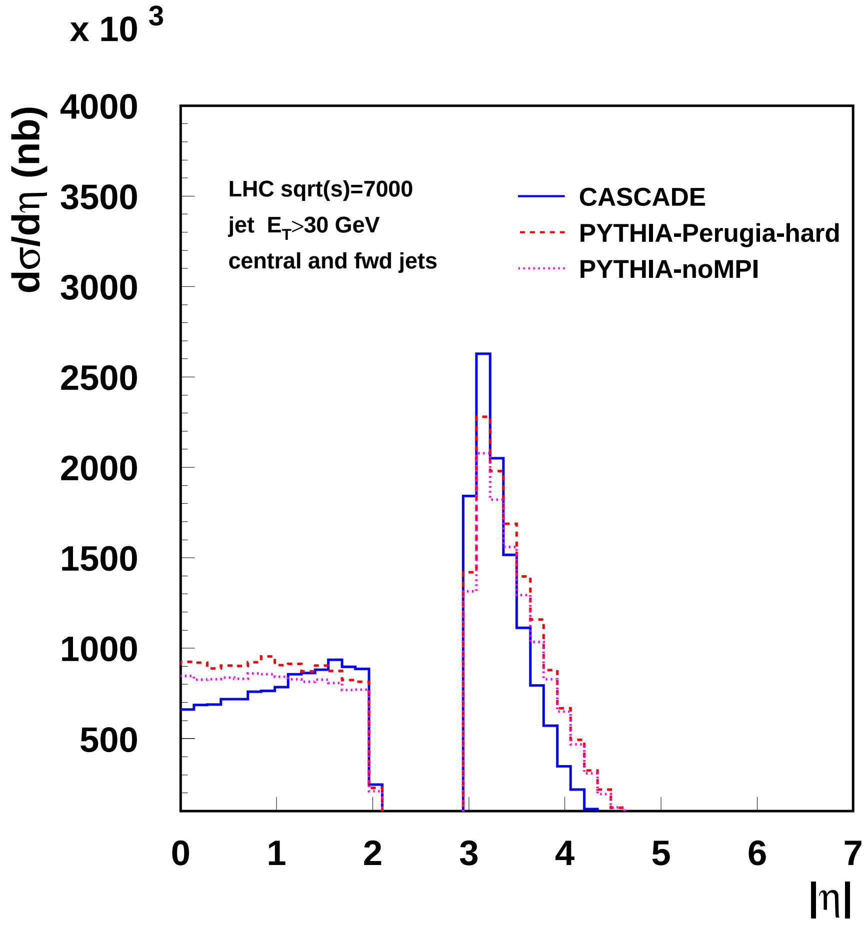
particle and energy flow in the inter-jet and outside regions



1 central $\oplus$ 1 forward jet

Rapidity spectra of produced jets

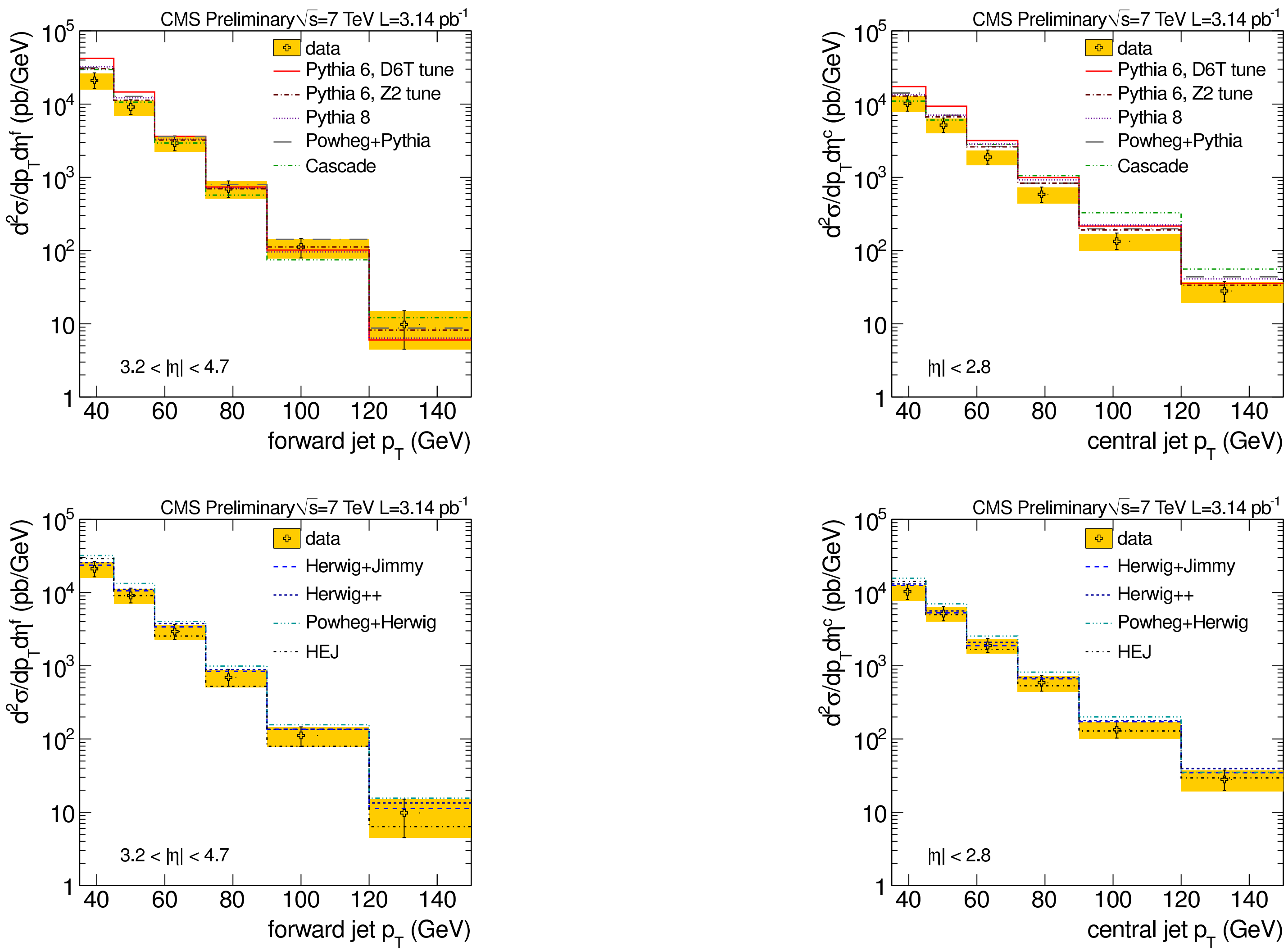




- different slopes towards forward rapidities

1 central ⊕ 1 forward jet

Transverse momentum spectra *[CMS PAS FWD-10-006 (April 2011)]*



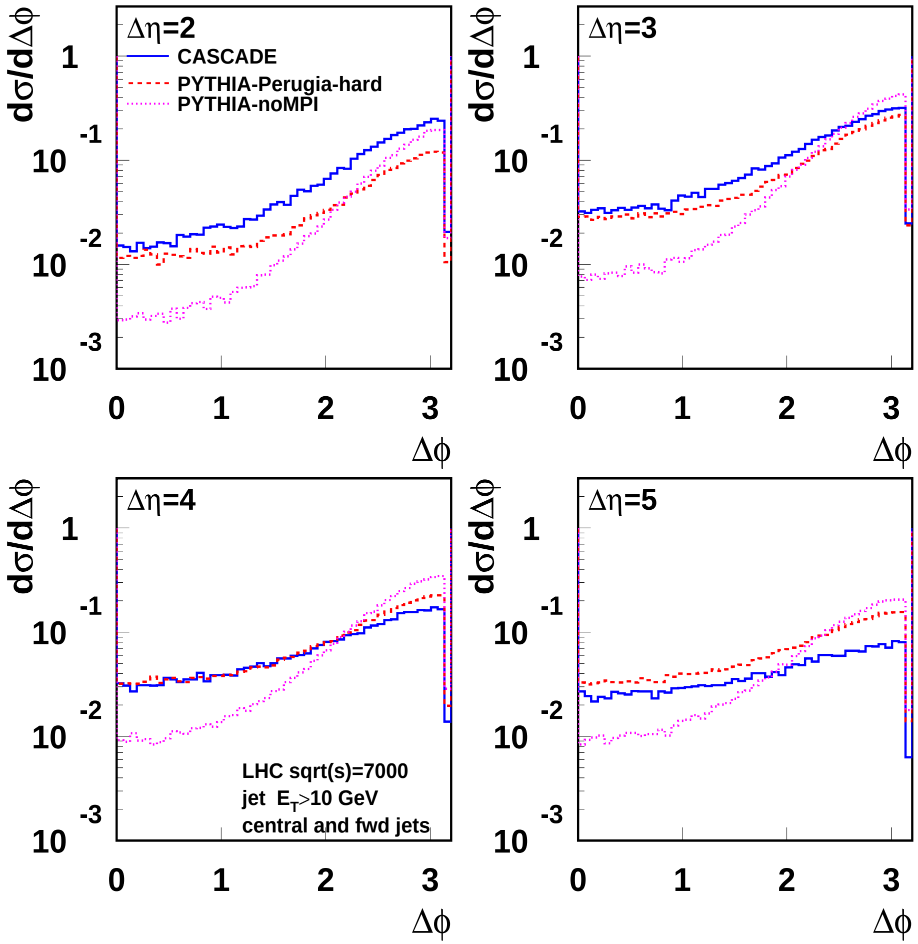
$\diamondsuit$ PYTHIA/ HERWIG MC:
no corrections to collinear ordering; model multiple parton interactions

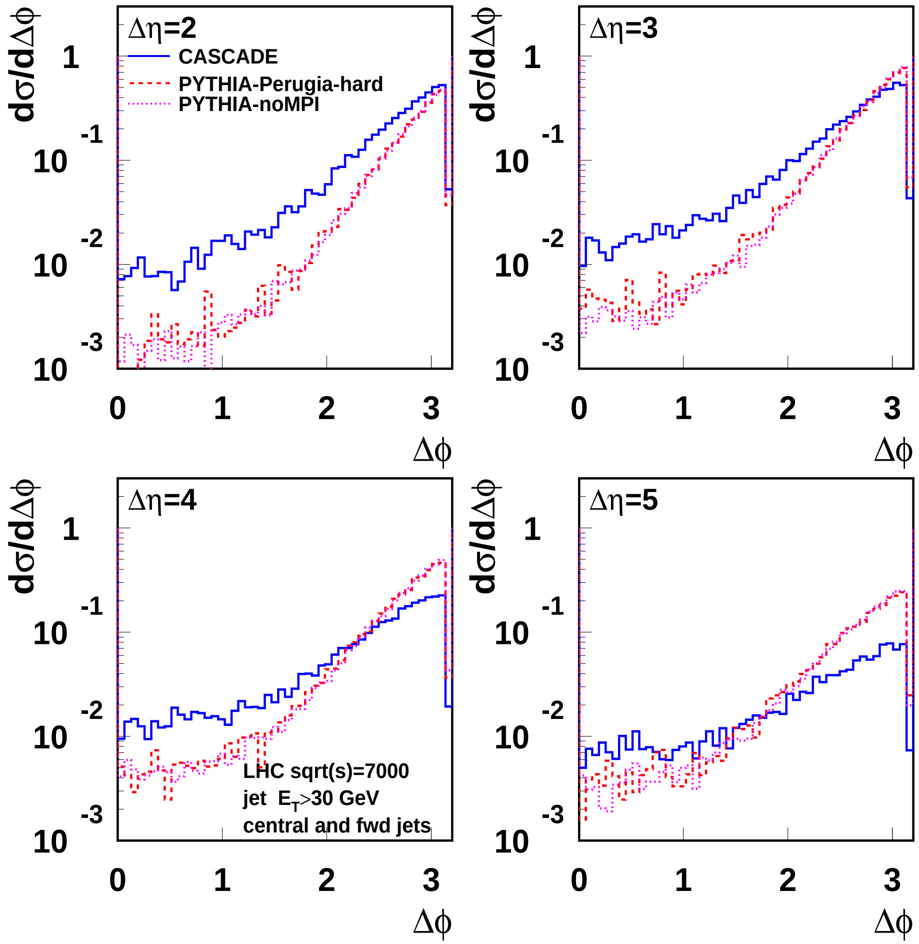
$\diamondsuit$ CASCADE/ HEJ MC:
include noncollinear corrections to single parton chain

- significant uncertainty from determination of unintegrated PDF
- to be reduced by new DIS precision data (not yet included in the plots)

Cross section as a function of the azimuthal difference $\Delta\phi$ between central and forward jet for different rapidity separations

[Deak et al., arXiv:1012.6037]

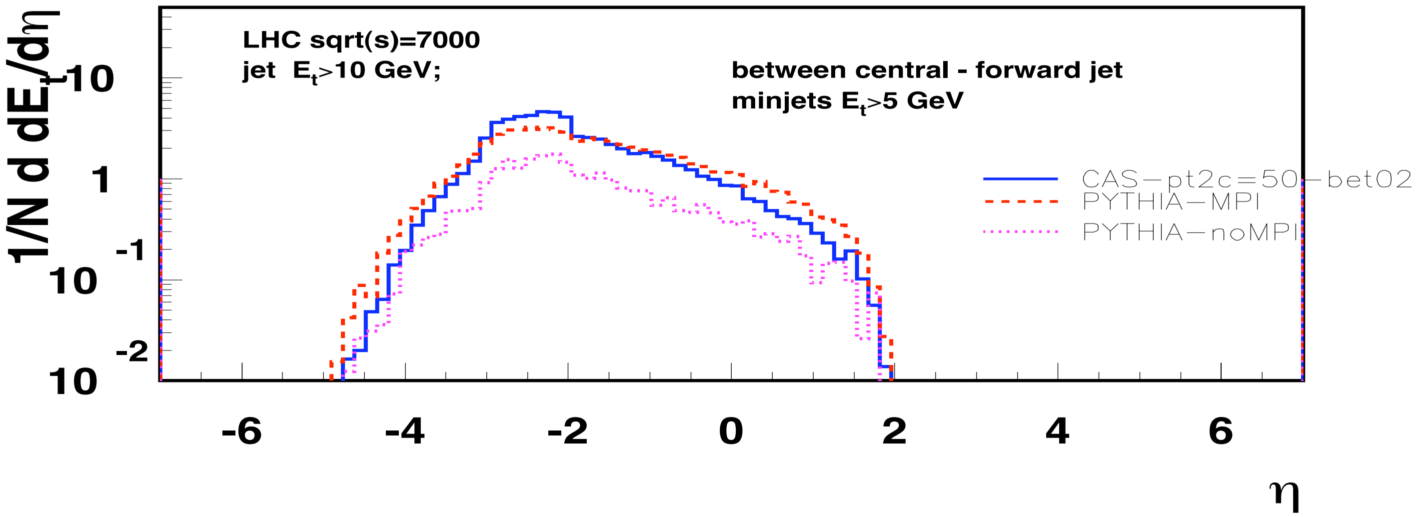


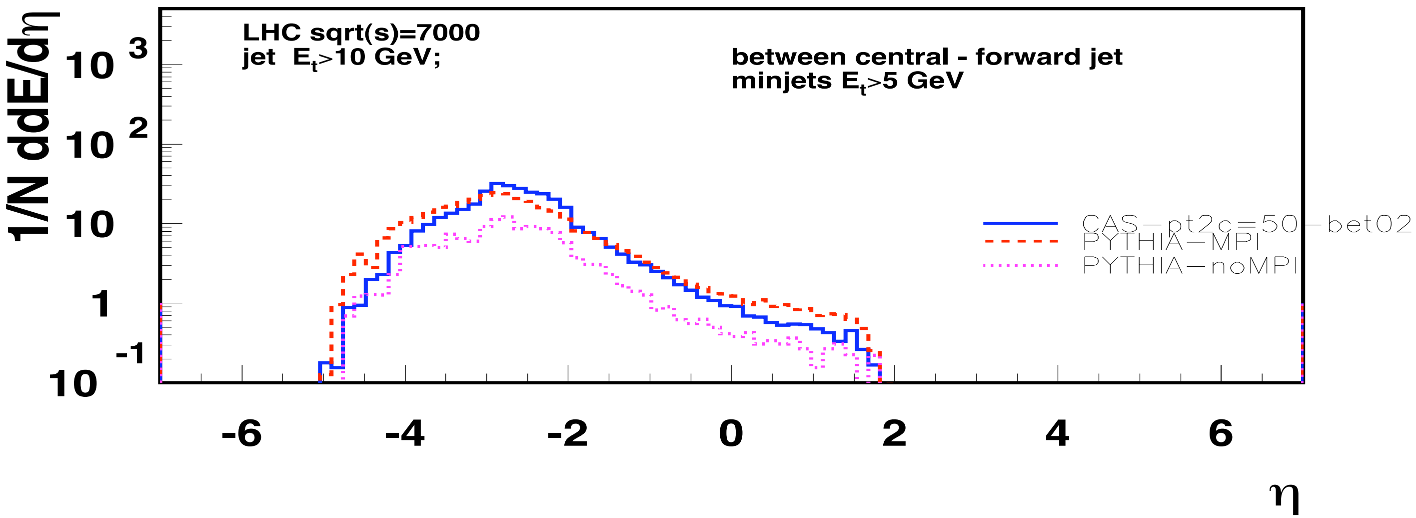


MC models: • Cascade: non-collinear radiative corrections to single parton chain

• Pythia: multiple parton interactions, no corrections to collinear approximation

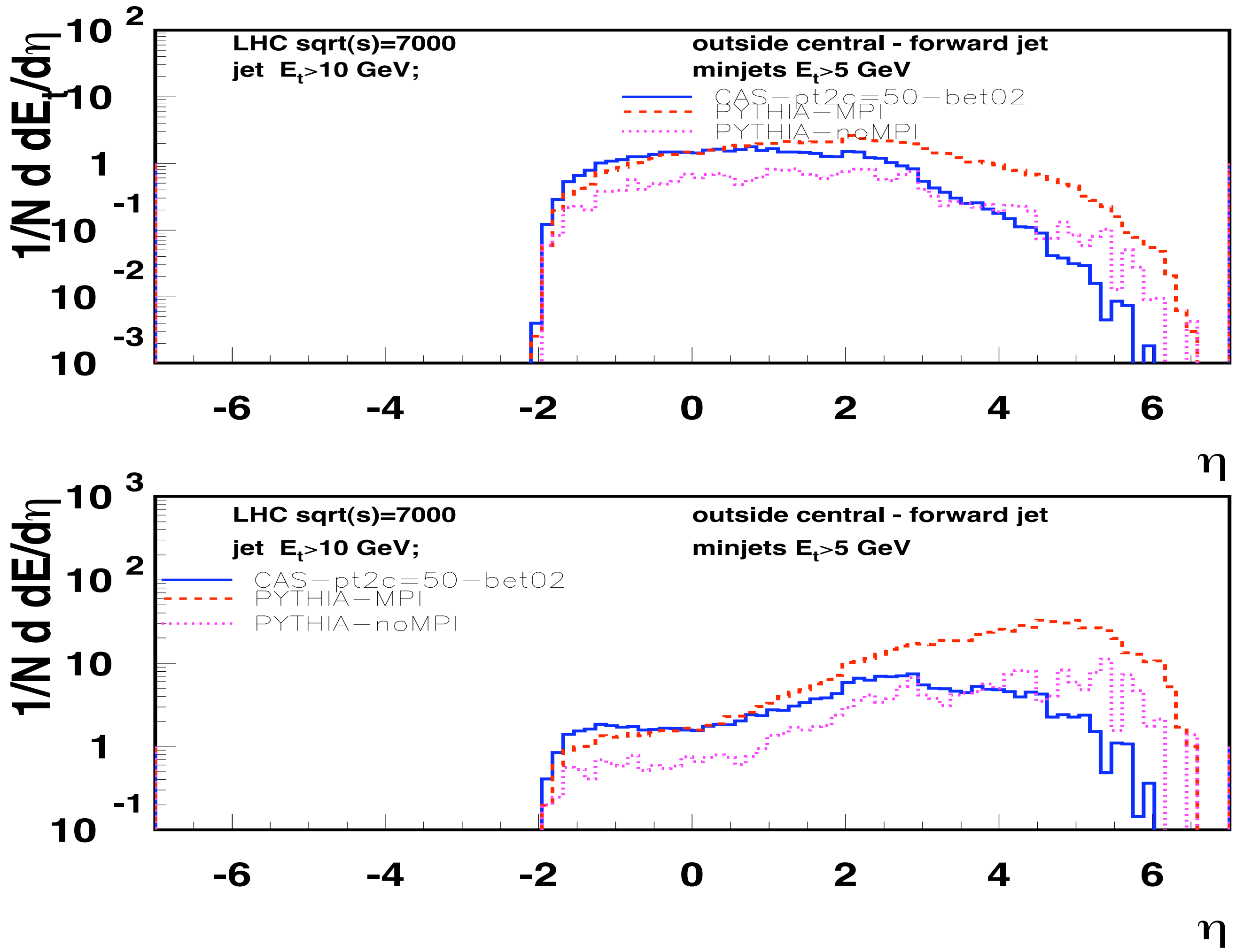
Transverse energy flow in the inter-jet region





- higher mini-jet activity in the inter-jet region from corrections to collinear ordering

Transverse energy flow in the outside region



- at large (opposite) rapidities, full branching well approximated by collinear ordering
- higher energy flow only from multiple interactions

$\diamondsuit$ Energy flow due to minijets ($E_T > 5$ GeV) $\Rightarrow$

reduced IR sensitivity

$\Rightarrow$ 'tune' (semi-)hard interaction component

$\diamondsuit$ Particle spectra will serve similar purpose

$\diamondsuit$ Distribution in $\Delta R = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2}$

also potentially useful ($\hookrightarrow$)

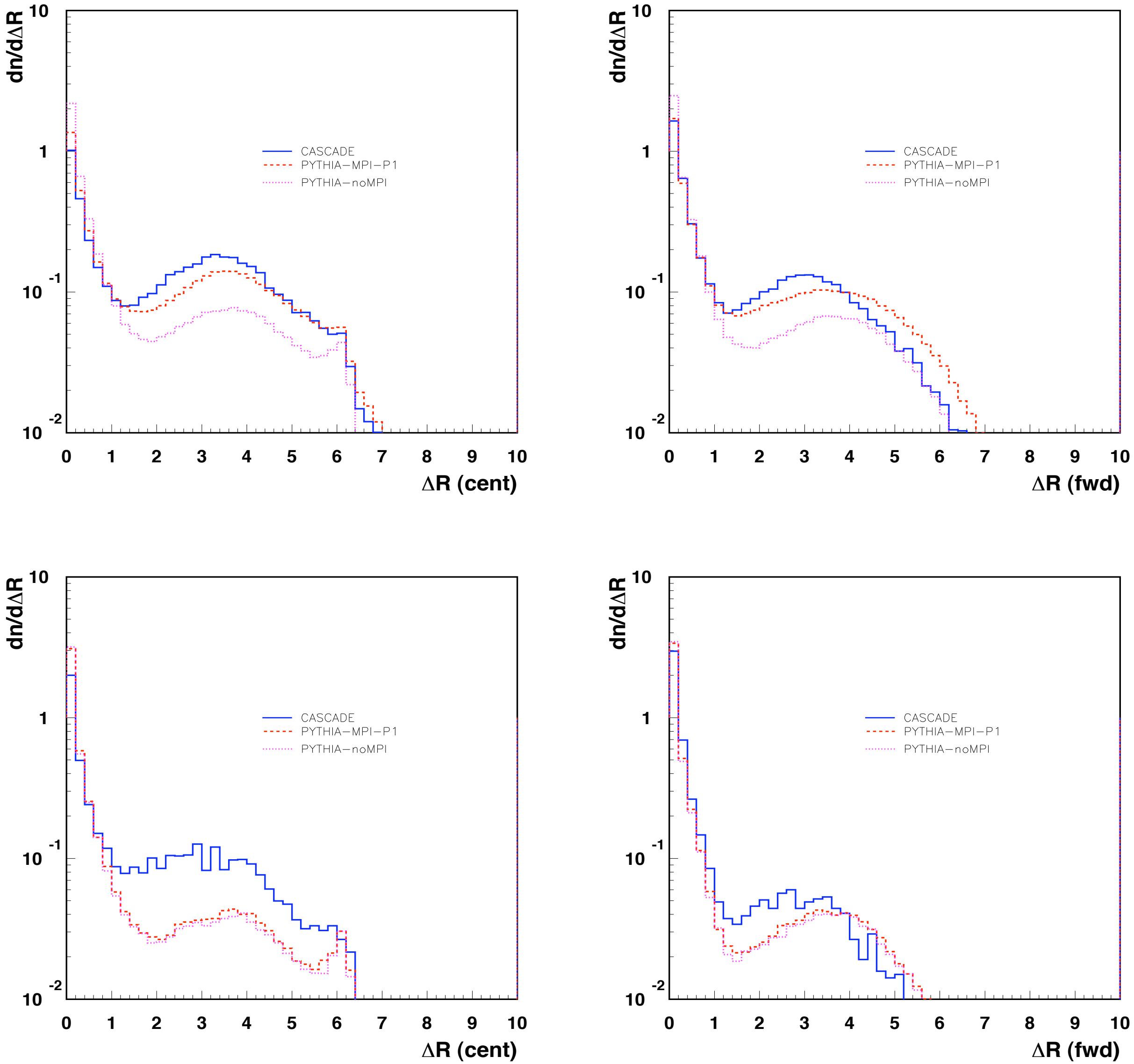
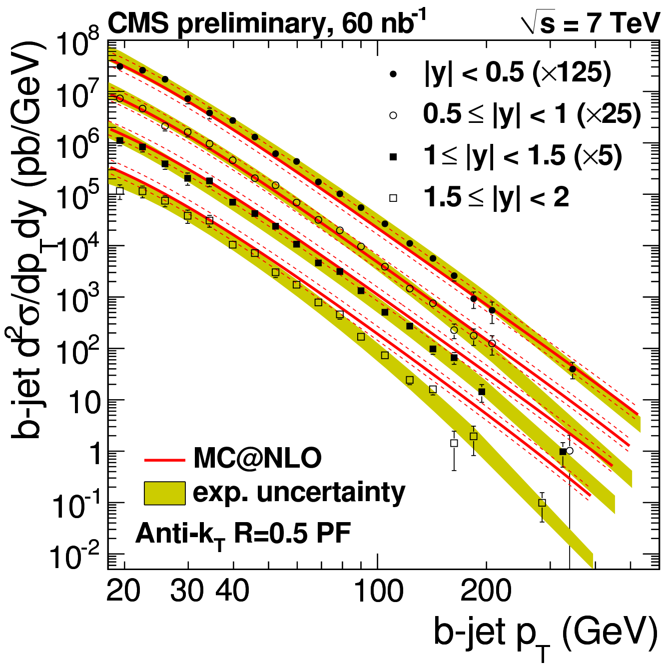
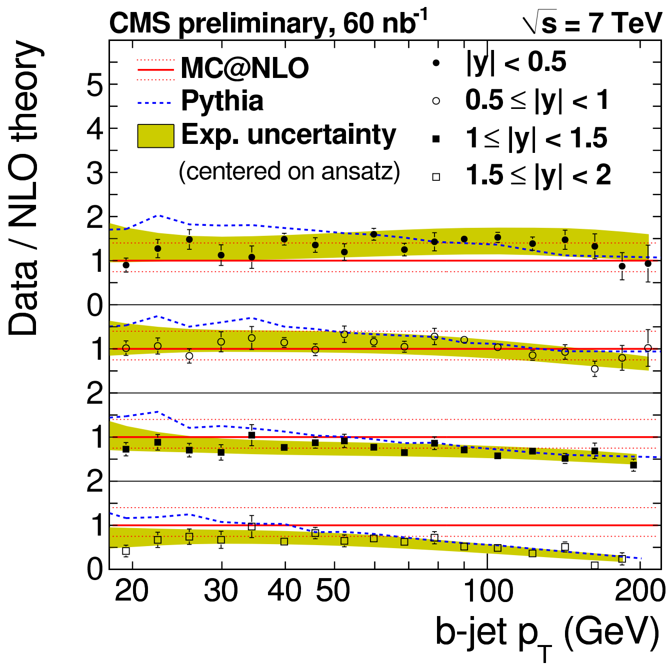


Figure 5: ΔR distribution of the central ($|\eta_c| < 2$, left) and forward jets ($3 < |\eta_f| < 5$, right) for $E_T > 10$ GeV (upper row) and $E_T > 30$ GeV (lower row). The prediction from the k$_\perp$ shower (CASCADE) is shown with the solid blue line; the prediction from the collinear shower (PYTHIA) including multiple interactions and without multiple interactions is shown with the red and purple lines. $\Delta R = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2}$, where $\Delta\phi = \phi_{jet} - \phi_{part}$, $\Delta\eta = \eta_{jet} - \eta_{part}$

b-jet cross section versus p_T *[CMS-PAS-BPH-10-009]*

- *K. Ulmer, QCD Moriond Conference, March 2011*



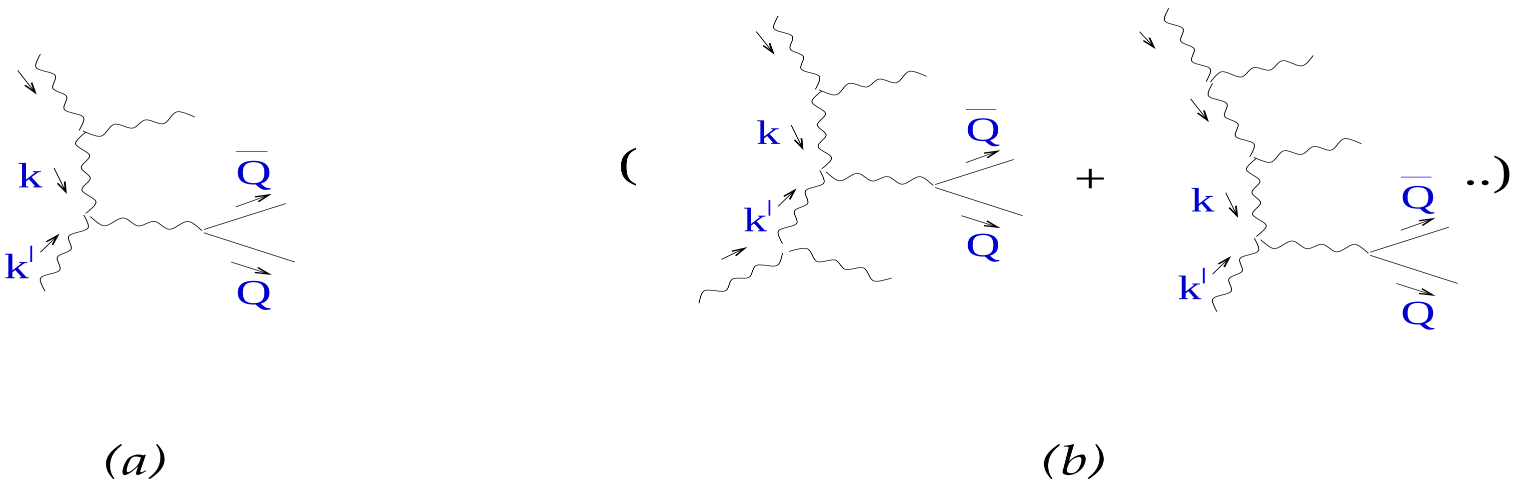


- comparison with MC@NLO $\longrightarrow$ potential effects in most forward bins

Heavy flavor production: high-energy behavior

$$\sigma_{gg,N} \simeq C \left(\frac{m_Q^2}{K_T^2} \right)^{N+1} \ln(1 + K_T^2/(4m_Q^2))$$

$\Rightarrow$ strong triple-pole singularity in moments conjugate to k_T
from $m_Q^2 \ll (k_T + k_T')^2 \ll k_T^2 \simeq k_T'^2$



(a) heavy quark hadroproduction from gluon showering;

(b) next correction from extra jet emission

• not included by collinear showers (even at NLO [MC@NLO])
• obtainable by $k_\perp$-shower (compare "multi-parton interactions"?)

$\diamondsuit$ Besides open flavor,

quarkonia can be used to analyze multiple QCD emission effects

- J/ψ production as a function of charged particle multiplicity

[Portebeuf & Granier, arXiv:1012.0719]

- J/ψ pairs as a probe of double parton interactions

[Kom, Kulesza & Stirling, arXiv:1105.4186]

[LHCb Coll., LHCb-Conf-2011-009]

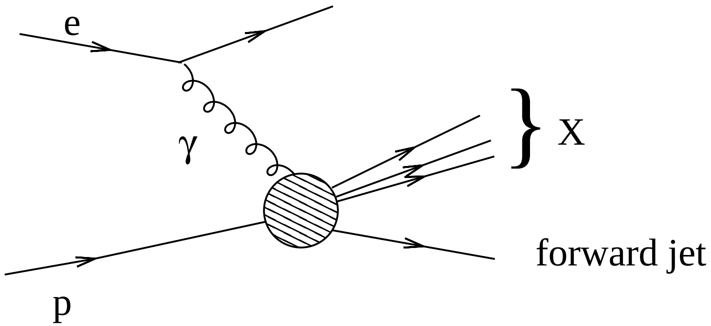
FURTHER QUESTIONS

♠ What are the implications of higher mini-jet activity in the between region for vector boson fusion search channels ?

♠ Could one include multi-parton interactions in a complete parton factorization picture?

♠ Could one achieve a unified understanding of forward hard processes including DIS? $\longrightarrow$ prospects for future LHeC *[P. Newman, Proc. DIS Workshop]*

- Note:



- neither PYTHIA Monte Carlo nor NLO calculations are able to describe forward jet HERA data

[A. Knutsson, LUNFD6-NFFL-7225-2007 (2007); L. Jönsson, AIP Conf. Proc. 828 (2006) 175]

OUTLOOK

- Summation of finite-angle multigluon emission by $k_\perp$-dependent ME + showers found to affect significantly high-p_T probes at large rapidities
  - ▷ angular jet correlations
  - ▷ energy flow in inter-jet and away regions

- Forward jet measurements can serve for QCD tuning of MC tools

- Extension to forward-backward jets:
  - ▷ look for Mueller-Navelet effects
  - ▷ backgrounds in Higgs searches from vector boson fusion
    - $\hookrightarrow$ central jet veto studies *[Cox, Forshaw and Pilkington, arXiv:1006.0986]*

- Applications to studies of event structure in minimum bias

  [Skands and Wraight, arXiv:1101.5215]

EXTRA SLIDES

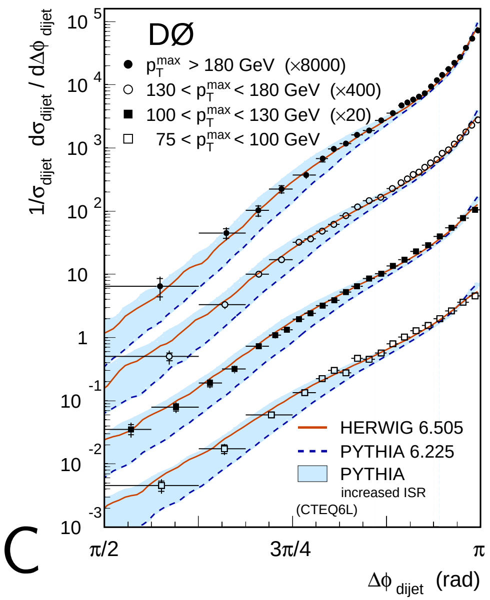
VI. FURTHER APPLICATIONS TO MULTI-JET FINAL STATES

$\Delta\phi$ correlation between two hardest jets

▷ Tevatron $\Delta\phi$ dominated by leading-order processes

- good description by HERWIG as well as by NLO
    - used for MC parameter tuning in PYTHIA

[M.G. Albrow et al., TEV4LHC Proc., hep-ph/0610012]



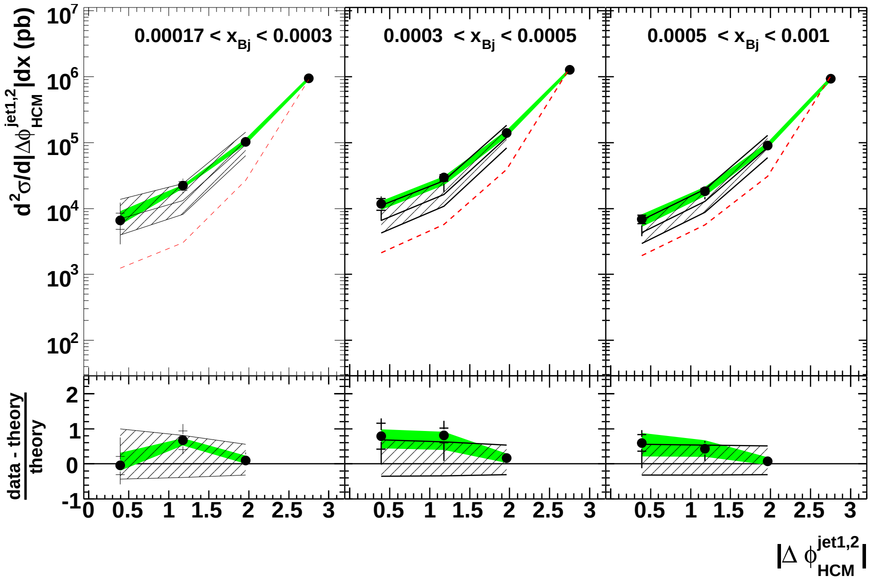
▷ HERA $\Delta\phi$ not well described by standard MC

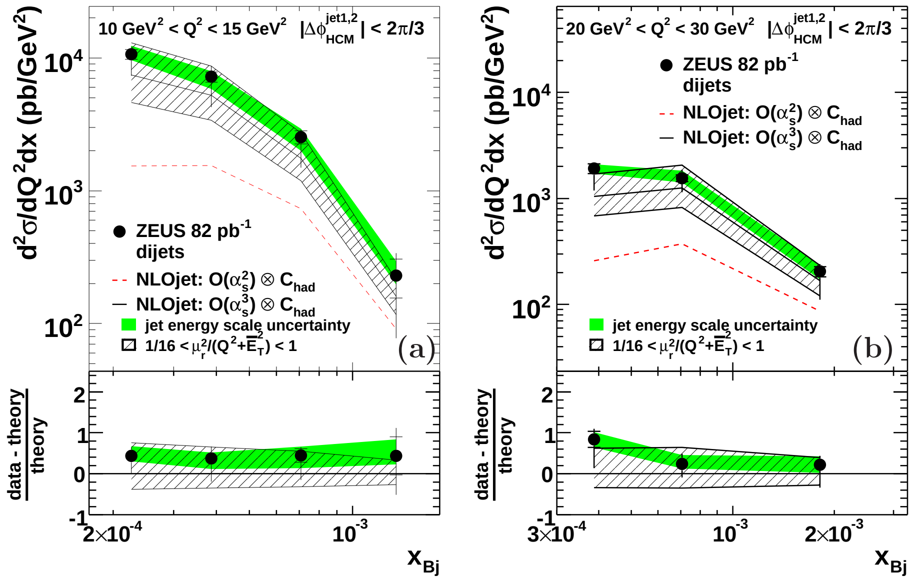
[S. Chekanov et al., arXiv:0705.1931] ↪ see next

▷ accessible at the LHC relatively early

↪ how do MC describe multiple radiation?

DI-JET EP CORRELATIONS: COMPARISON WITH NLO RESULTS





(left) Azimuth dependence and (right) Bjorken-x dependence of di-jet distributions

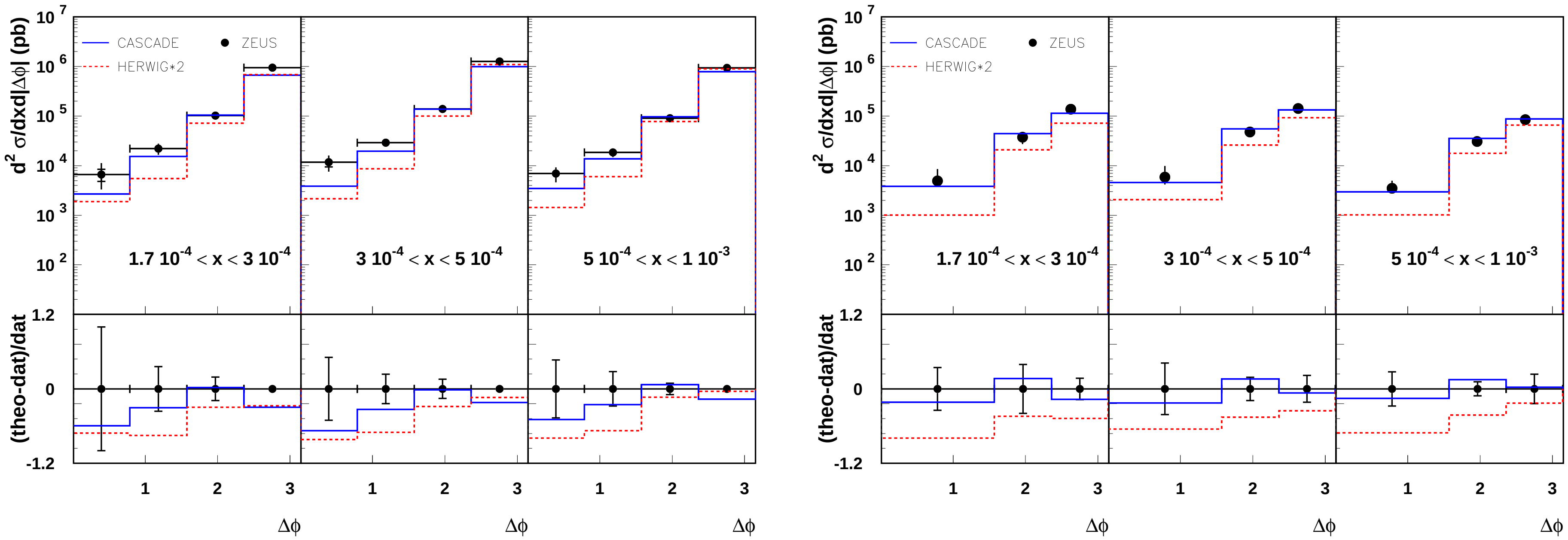
$$Q^2 > 10 \text{ GeV}^2 \quad , \qquad 10^{-4} < x < 10^{-2}$$

[S. Chekanov et al., arXiv:0705.1931]

◇ large variation from order-α_s^2 to order-α_s^3 prediction as $\Delta\phi$ and x decrease

$\Rightarrow$ sizeable theory uncertainty at NLO (underestimated by "μ error band")

ANGULAR JET CORRELATIONS FROM $k_\perp$-SHOWER (CASCADE) AND COLLINEAR-SHOWER (HERWIG)

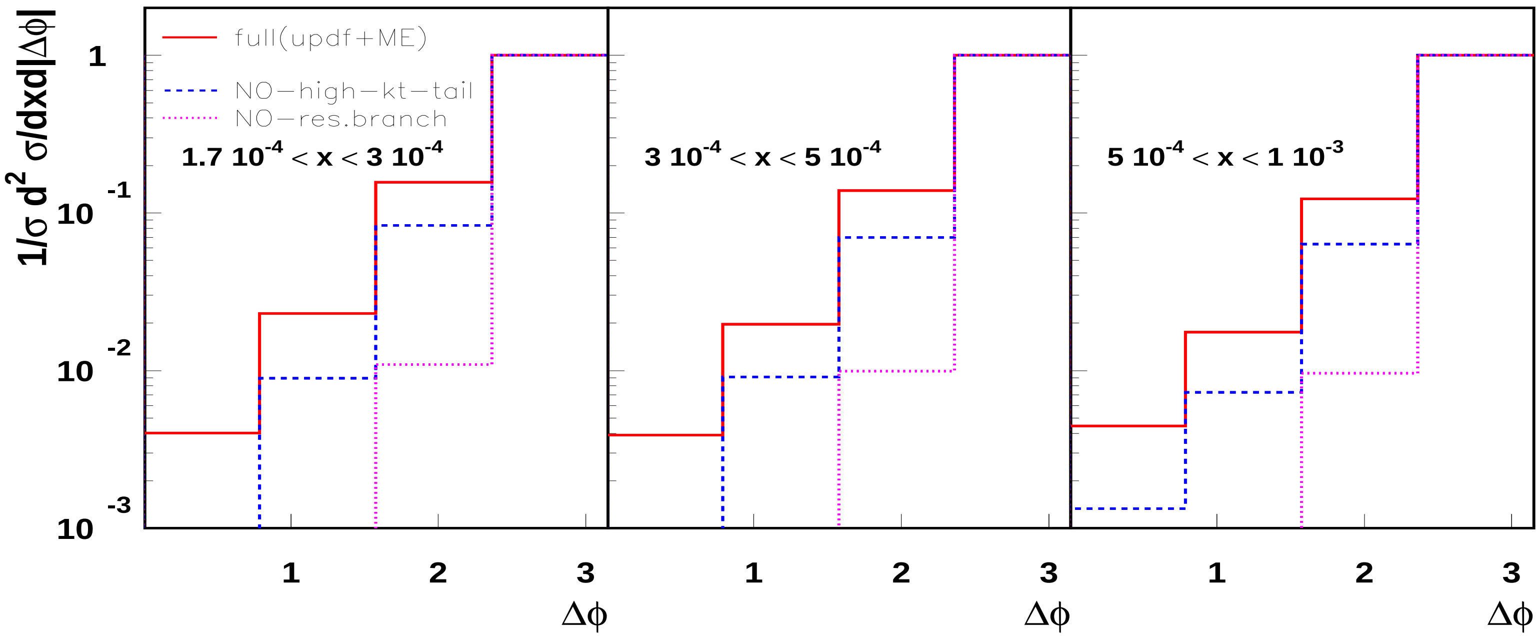


(left) di-jet cross section; (right) three-jet cross section

Jung & H, JHEP 0810 (2008) 113

- quantitative effects of small-x coherence sizeable
- largest differences at small $\Delta\phi$
- good description of shapes by $k_\perp$-shower
- HERWIG normalized to 2-jet region by K-factor

Normalize to the back-to-back cross section:



—— updf $\oplus$ ME

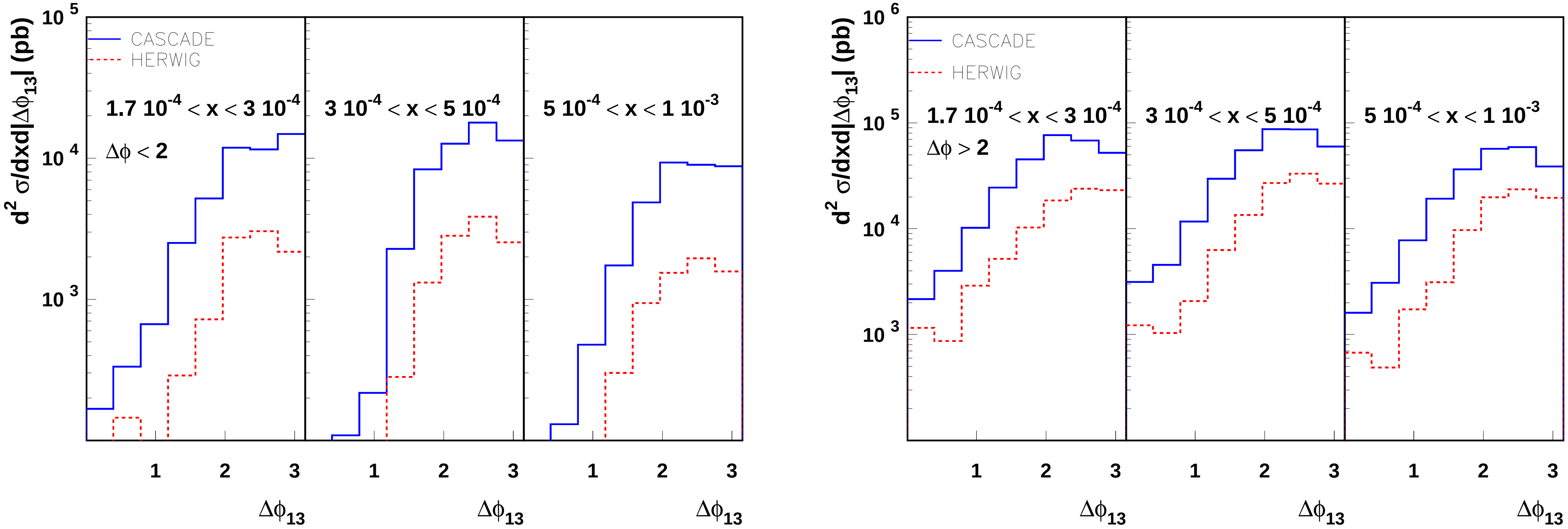
- - - updf $\oplus$ ME$_{collin.}$: $\mathcal{M} \to \mathcal{M}_{collin.}(k_T) = \mathcal{M}(0_\perp)\ \Theta(\mu - k_T)$

· · · · · · no resolved branching : $\mathcal{A} \to \mathcal{A}_{no-res.}(x, k_T, \mu) = \mathcal{A}_0(x, k_T, Q_0)\ \Delta(\mu, Q_0)$

▷ high-k$_\perp$, coherent effect essential for correlation at small $\Delta\phi$

(cfr., e.g., MC by *Höche, Krauss & Teubner, EPJC 58 (2008) 17*:
u-pdf but no ME correction)

AZIMUTHAL DISTRIBUTION OF THE THIRD JET



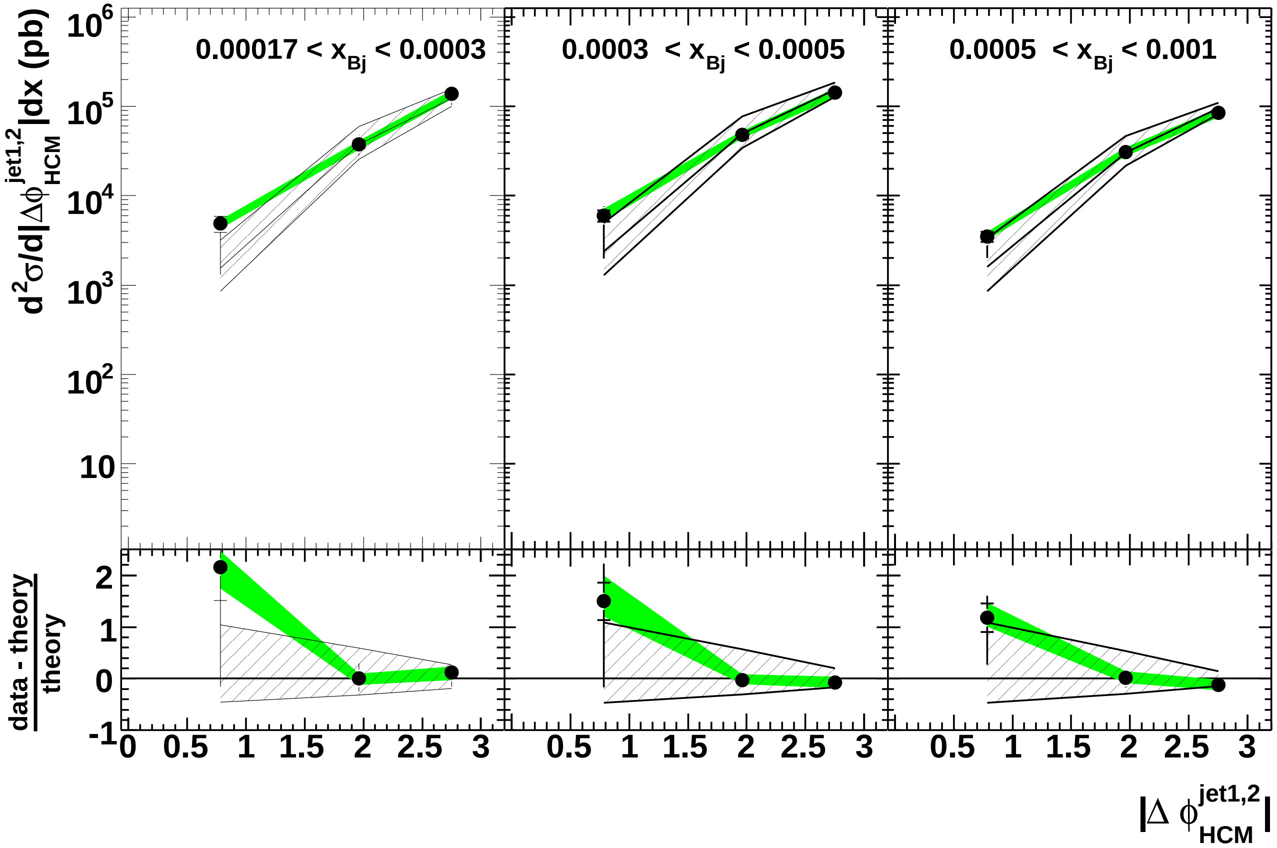
Cross section in the azimuthal angle between the hardest and the third jet for small (left) and large (right) azimuthal separations between the leading jets

Jung & H, arXiv:0805.1049 [hep-ph]

- small $\Delta\phi \Rightarrow$ non-negligible initial $k_\perp \Rightarrow$ larger corrections to collinear ordering
- curves become closer at large $\Delta\phi$

AZIMUTHAL DISTRIBUTION IN EP 3-JET CROSS SECTION

[ZEUS, 2007]



- grey dashed band: NLO result [NLOJET++]
- NLO results more stable for more inclusive distributions

VII. DEVELOPMENTS FOR HEAVY MASS PRODUCTION AND OUTLOOK

• production of b-flavor $+$ jets — what size NLO uncertainties at LHC energies?

[see MC@NLO; Nason et al.]

▷ sizeable corrections from $g \to b\bar{b}$ coupling to spacelike jet

▷ coherence effects to $b\bar{b} + 2\ jets$ for $m_b \ll p_T^{(b\bar{b})} \ll p_T^{(jet)}$

• even more complicated multi-scale effects in $b\bar{b} + W/Z$ production

[HERA-LHC Proc. arXiv:0903.3861; Mangano, 1993]

• Tevatron b-jets angular correlations

($\hookrightarrow$ CDF $\Delta\phi$ data)

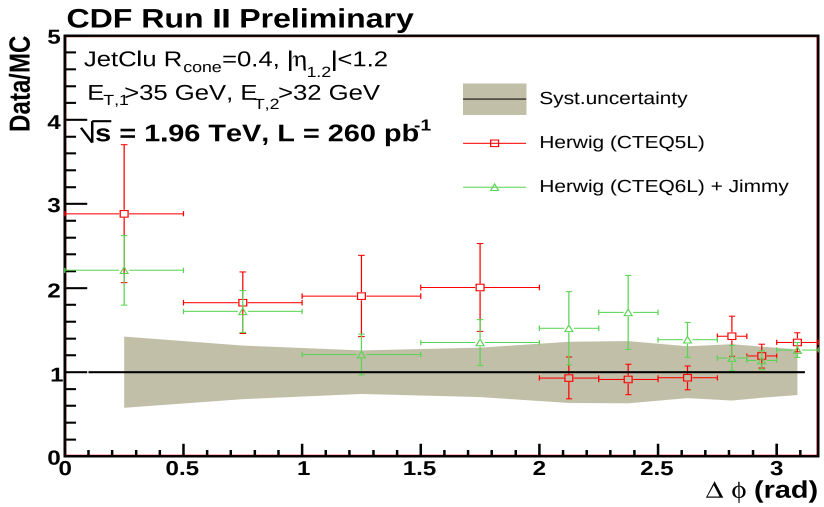
• final states with Higgs

$\to$ possibly $10 \div 20$ % effects in p_T spectrum from $x \ll 1$ terms?

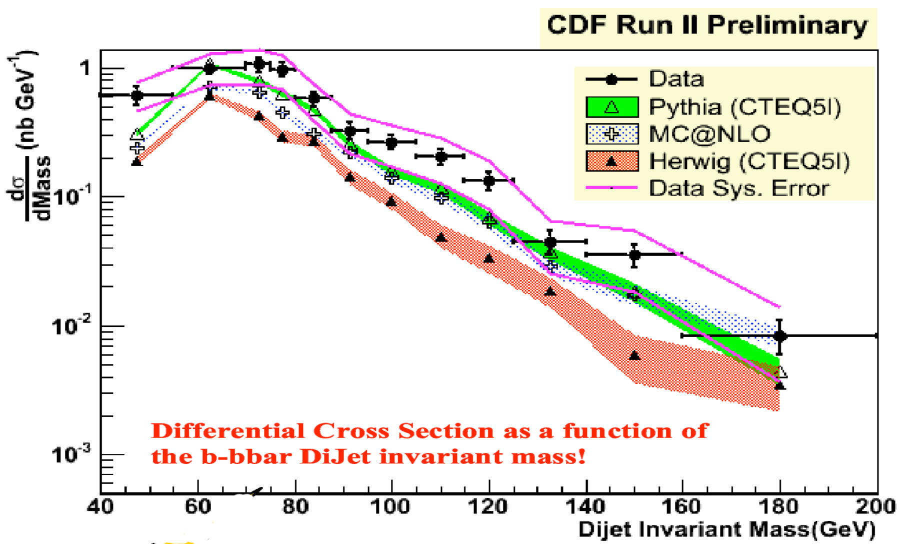
[Kulesza, Sterman & Vogelsang, 2004]

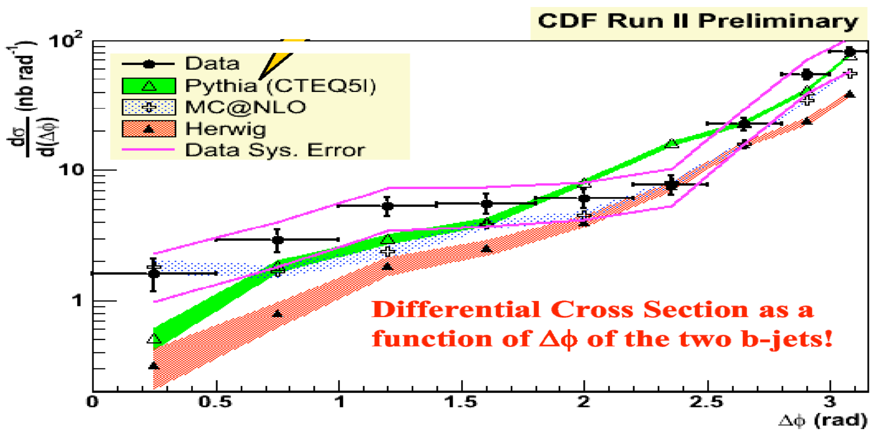
[Marzani, Ball, Del Duca et al., 2008; H, 2002]

Tevatron b-jets correlations

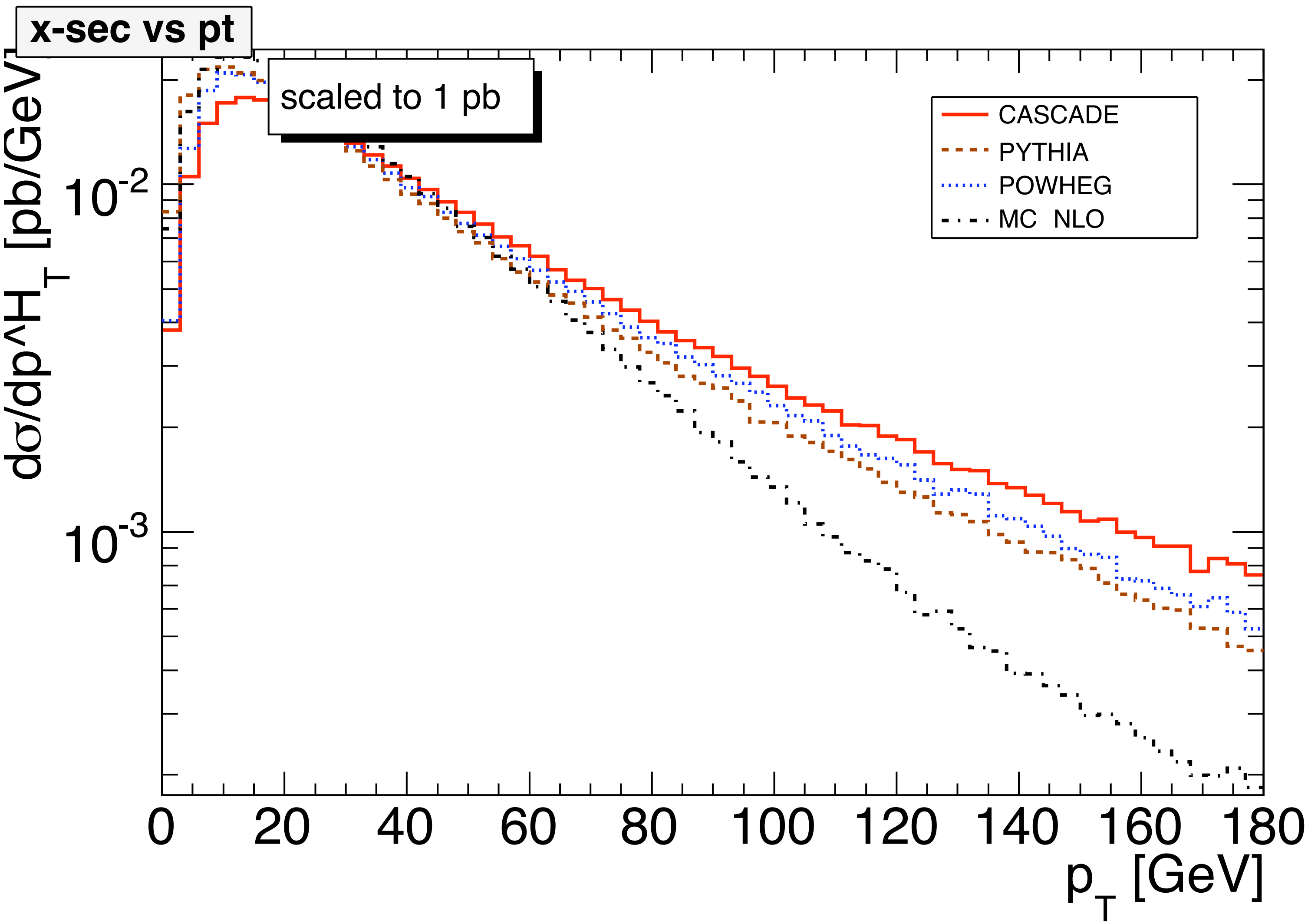


[CDF Coll., FNAL-8939 (2007)]



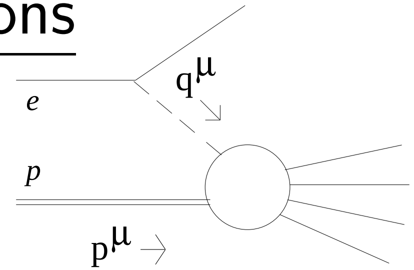


- HERWIG description not satisfactory
- $k_\perp$ distribution of underlying event?



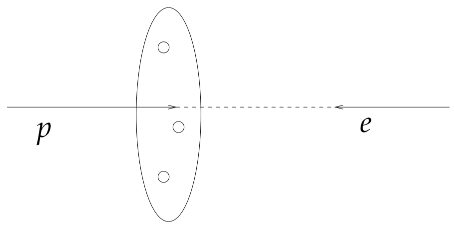
x-sec vs pt
scaled to 1 pb
CASCADE
PYTHIA
POWHEG
MC NLO
dσ/dp^H_T [pb/GeV]
10^{-2}
10^{-3}
0
20
40
60
80
100
120
140
160
180
p_T [GeV]

A) Single-scale hadron scattering. E.g., DIS structure functions

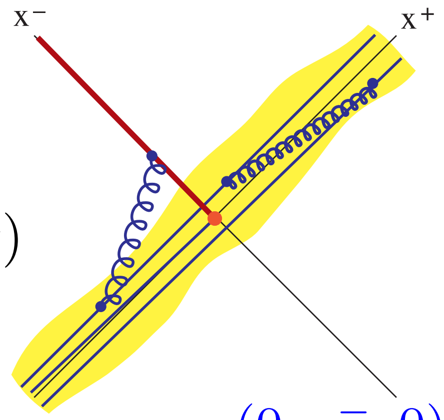


- necessarily sensitive to long timescales, BUT

σ can be written as $\sigma(Q,m) = C(Q, \text{parton momenta } > \mu) \otimes f(\text{parton momenta } < \mu, m)$



in "infinite-momentum" frame, $\delta t_{\text{scatter}} \ll \tau_{\text{parton}}$



$$\text{Pdf's}: \quad f(x,\mu) = \int \frac{dy^-}{2\pi}\, e^{-ixp^+y^-}\ \widetilde{f}(y)$$

$$\widetilde{f}(y) = \langle\ P \mid \overline{\psi}(y)\ V_y^\dagger(n)\ \gamma^+\ V_0(n)\ \psi(0) \mid P\ \rangle \quad , \qquad y = (0, y^-, 0)$$

$$V_y(n) = \mathcal{P} \exp\left(i g_s \int_0^\infty d\tau\ n \cdot A(y + \tau\ n) \right)$$

↖ correlation of parton fields at lightcone distances

◇ Renormalization group invariance ⇒

$$\frac{d}{d\ln\mu}\,\sigma = 0 \quad \Rightarrow \quad \frac{d}{d\ln\mu}\,\ln f = \gamma = -\frac{d}{d\ln\mu}\,\ln C$$

↪ DGLAP evolution equations [Altarelli-Parisi
Dokshitzer
Gribov-Lipatov]

$$f = f_0 \times \exp \int \frac{d\mu}{\mu}\,\gamma(\alpha_s(\mu))$$

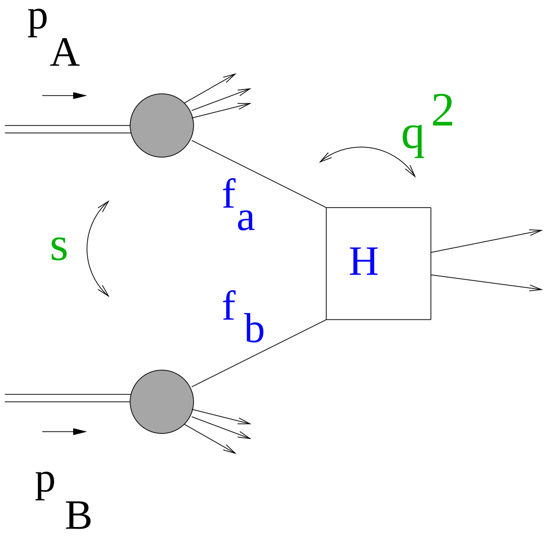
↗ resummation of $(\alpha_s \ln Q/\Lambda_{\rm QCD})^n$ to all orders in PT

Note: expansions $\gamma \simeq \gamma^{(LO)}\,(1 + b_1\alpha_s + b_2\alpha_s^2 + \ldots)$

$$C \simeq C^{(LO)}\left(1 + c_1\alpha_s + c_2\alpha_s^2 + \ldots\right)$$

give LO, NLO, NNLO, ... logarithmic corrections

B) Multiple-scale hard scattering at LHC energies



$$s \gg q_1^2 \gg \cdots q_n^2 \gg \Lambda$$

- more complex, potentially large corrections to all orders in α_s, $\sim \ln^k(q_i^2/q_j^2)$

$$e.g.\ \ \gamma \simeq \gamma^{(LO)} \left(1 + c_1 \alpha_s + \ldots + c_{n+m} \alpha_s^m (\alpha_s\ L)^n + \ldots\right)\ ,\ L = \text{“large log”}$$

$\hookrightarrow$ yet summable by QCD techniques that

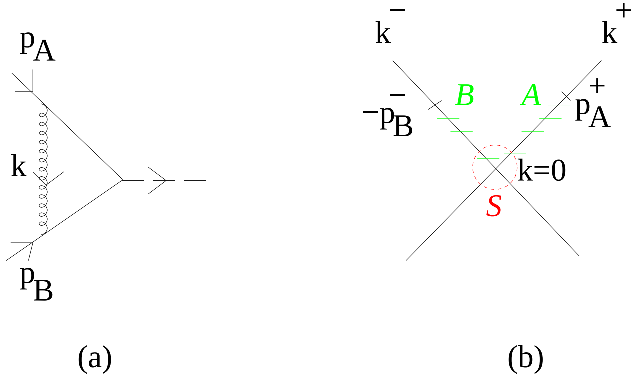
$\triangleright$ generalize renormalization-group factorization

$\triangleright$ extend parton correlation functions off the lightcone

$\Rightarrow$ unintegrated (or TMD) pdf's

Examples:

- Sudakov form factor S:

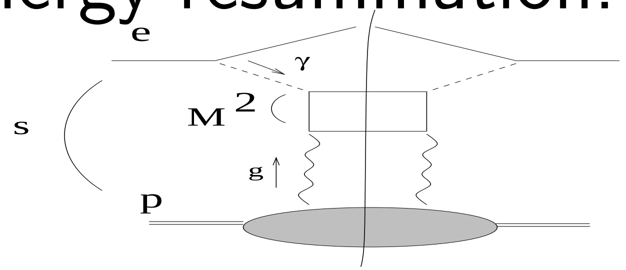


▷ entering Drell-Yan production, W-boson $p_\perp$ distribution, etc.

$\Rightarrow \partial S/\partial\eta = K \otimes S$ CSS evolution equations [Collins-Soper-Sterman]

↖ resums $\alpha_s^n \ln^m M/p_T$

- High-energy resummation: $s \gg M^2 \gg \Lambda^2_{\mathrm{QCD}}$



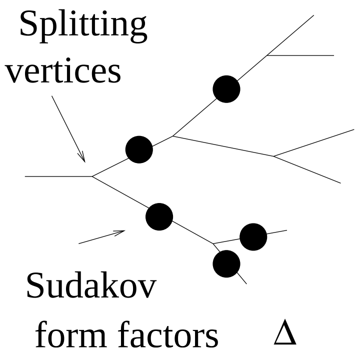
♢ energy evolution: BFKL equation [Balitsky-Fadin-Kuraev-Lipatov]

↪ corrections down by $1/\ln s$ rather than $1/M$

IV. PARTON SHOWERING

- Factorizability of QCD x-sections $\longrightarrow$ probabilistic branching picture

$\diamondsuit$ $A)$ QCD evolution by "parton showering" methods:

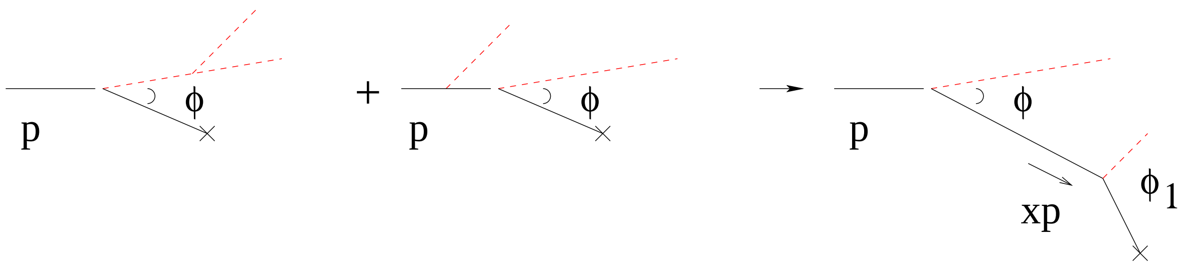


$$d\mathcal{P} = \int \frac{dq^2}{q^2} \int dz \ \alpha_S(q^2) \ P(z) \ \Delta(q^2, q_0^2)$$

$\hookrightarrow$ collinear, incoherent emission

$\diamondsuit$ $B)$ Soft emission $\longrightarrow$ interferences $\longrightarrow$ ordering in decay angles:

$\hookrightarrow$ gluon coherence for $x \sim 1$



$\diamondsuit$ $C)$ Gluon coherence for $x \ll 1 \Rightarrow$ corrections to angular ordering:

$\hookrightarrow$ MC based on $k_\perp$-dependent unintegrated pdfs and MEs

COHERENCE IN HIGH-ENERGY LIMIT

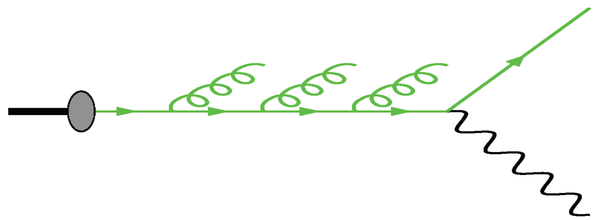
Soft vector-emission current from external legs $\rightarrow$

• leading IR singularities

[J.C. Taylor, 1980; Gribov-Low (QED)]

• fully appropriate in single-scale hard processes

Dokshitzer, Khoze, Mueller and Troian, RMP (1988); Webber, A. Rev. Nucl. Part. (1986)



multi-scale: $s = q_1^2 \gg \cdots \gg q_n^2 \gg \Lambda^2$
[e.g.: LHC final states with multi-jets]

$\Downarrow$

▷ internal emissions non-negligible

▷ current also factorizable at high-energy: *[Ciafaloni 1998; 1988]*

$$\left|M^{(n+1)}(k,p)\right|^2 = \left\{[M^{(n)}(k+q,p)]^\dagger \, [\mathbf{J}^{(R)}]^2 \, M^{(n)}(k+q,p) - [M^{(n)}(k,p)]^\dagger \, [\mathbf{J}^{(V)}]^2 \, M^{(n)}(k,p)\right\} \quad . \quad \text{BUT...} \triangleright$$

▷ … • $\mathbf{J}$ depends on total transverse momentum transmitted

⇒ matrix elements and pdf at fixed $k_\perp$ ("unintegrated")

• virtual corrections not fully represented by Δ form factor

⇒ modified branching probability $P(z, k_\perp)$ as well

▷ enhanced terms $\mathcal{O}(\alpha_S^k \ln^m s/p_T^2)$

♢ Note: superleading logs $m > k$ cancel in fully inclusive quantities

e.g: high-energy corrections to anomalous dimensions γ^{ij}

at most single-logarithmic

$$\gamma^{ij}(\alpha_s, \omega) = \frac{\alpha_s}{\omega^p} \, c_0^{ij} \, \left[1 + \sum_{n=1}^{\infty} c_n^{ij} \, \left(\frac{\alpha_s}{\omega}\right)^n + \mathcal{O}\left(\alpha_s \, (\frac{\alpha_s}{\omega})^{n-1}\right) \right]$$

ω - moment conjugate to $\ln s$ *BFKL; Jaroszewicz; Catani et al.*

♢ but cancellations do not apply in exclusive final-state correlations

Merging PS and ME

Both PS distributions and hard ME depend on $k_\perp$

- Merging in high-energy limit can be done using

$$\gamma \, \frac{1}{k_\perp^2} \, \left(\frac{k_\perp^2}{\mu^2} \right)^\gamma \stackrel{\gamma \ll 1}{=} \delta(k_\perp^2) + \gamma \, \left(\frac{1}{k_\perp^2} \right)_{\rm R} + \gamma^2 \, \left(\frac{1}{k_\perp^2} \ln \frac{k_\perp^2}{\mu^2} \right)_{\rm R} + \ldots$$

$$\text{where} \quad \int dk_\perp \, (G(k_\perp, \mu))_{\rm R} \, \varphi(k_\perp) = \int dk_\perp \, G(k_\perp, \mu) [\varphi(k_\perp) - \Theta(\mu - k_\perp) \, \varphi(0)]$$

Unintegrated quark evolution

[Jung & H, in progress]

- sea: flavor-singlet evolution coupled to gluons at small x via

$$\mathcal{P}_{g \to q}(z; q, k) = P_{qg,{\rm GLAP}}(z) \, \left(1 + \sum_{n=0}^{\infty} \, b_n(z) (k^2/q^2)^n \right)$$

all b_n known; $\mathcal{P}_{g \to q}$ computed in closed form (positive-definite)

[Catani & H, 1994; Ciafaloni et al., 2005-2006]

- valence: independent evolution (dominated by soft gluons x $\to$ 1)

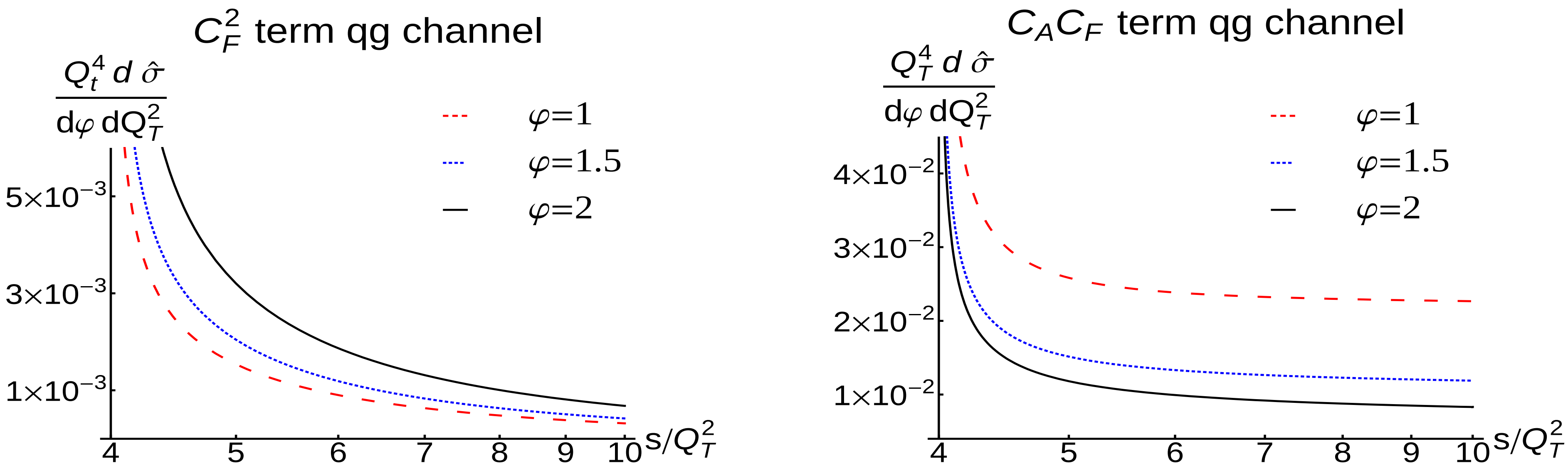
V. FORWARD JET HADRO-PRODUCTION CROSS SECTIONS

- Matrix elements for fully exclusive events with forward jets

[Deak, Jung, Kutak & H, 2009]

- Both quark and gluon channels found to be important for realistic phenomenology

$Q_t =$ final-state transverse energy (in terms of two leading jets p_t's)

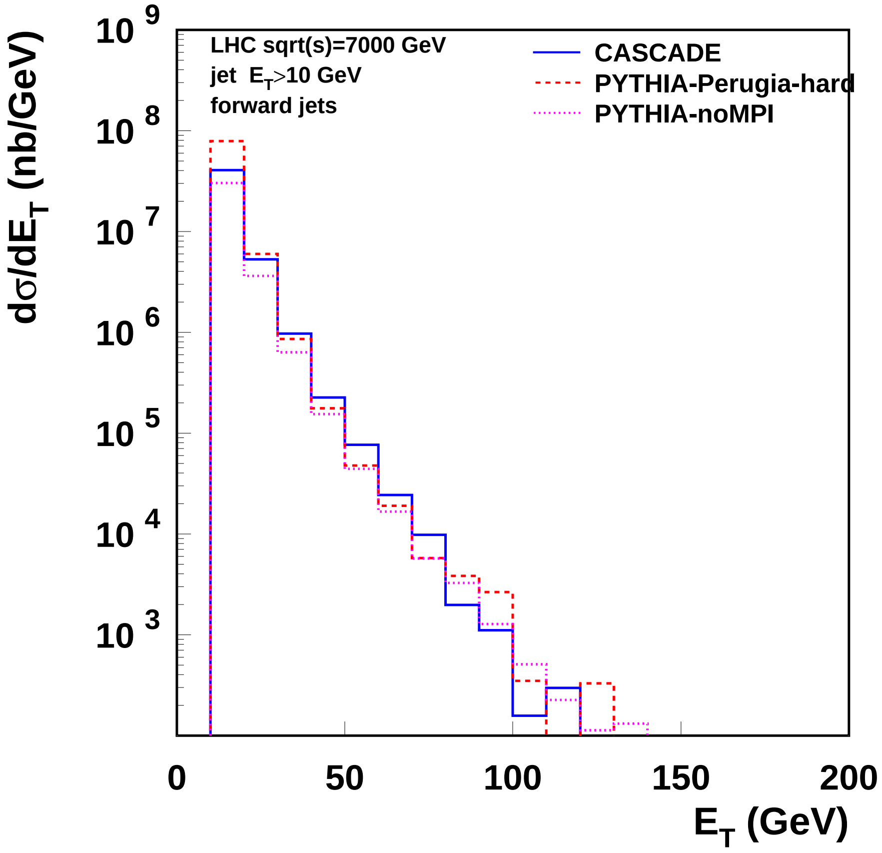


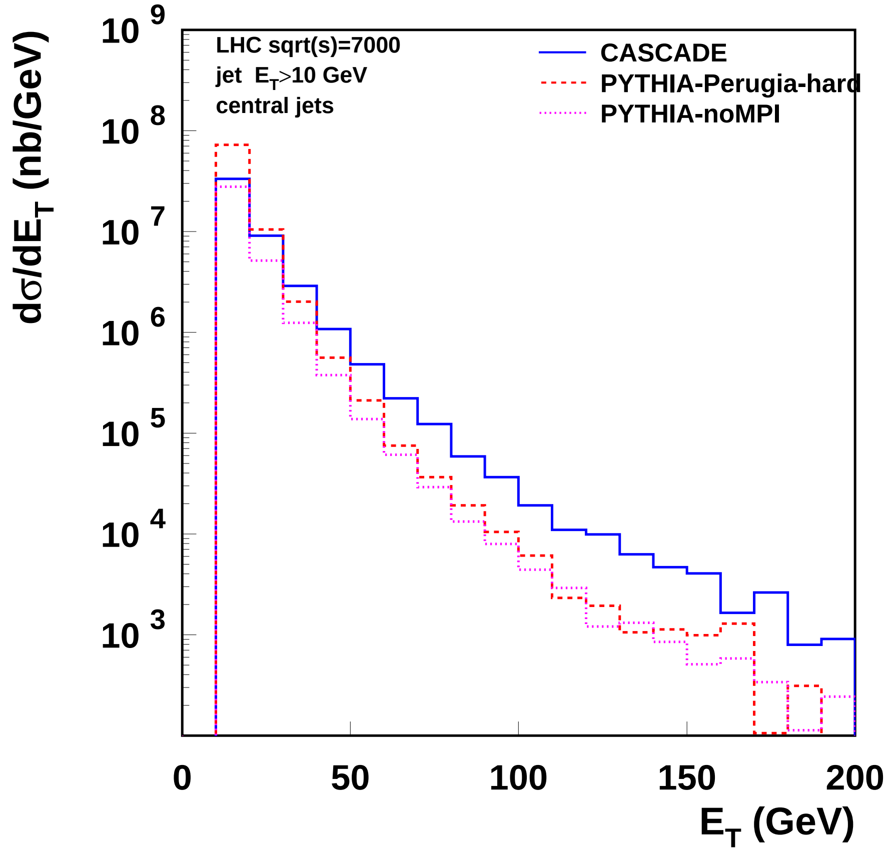
▷ $C_F C_A$ contribution to qg dominates large $\widehat{s}/Q_t^2$ (constant at large energy)

1 central ⊕ 1 forward jet

Transverse momentum spectra: $k_\perp$-shower vs. collinear shower

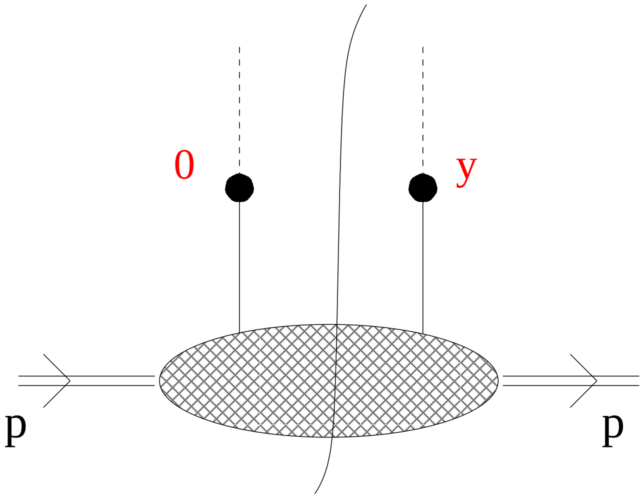
[Deak et al., arXiv:1012.6037]





- harder spectrum in central region due to small-x radiation

UNINTEGRATED PARTON DISTRIBUTIONS



$p = (p^+ \,,\, m^2 / 2 p^+ \,,\, 0_\perp)$

$$\widetilde{f}(y) = \langle \; P \mid \overline{\psi}(y) \; V_y^\dagger(n) \; \gamma^+ \; V_0(n) \; \psi(0) \mid P \; \rangle \quad , \qquad y = (0, y^-, y_\perp)$$

$V_y(n) = \mathcal{P} \exp \left(i g_s \int_0^\infty d\tau \; n \cdot A(y + \tau \; n) \right)$ eikonal Wilson line in direction n

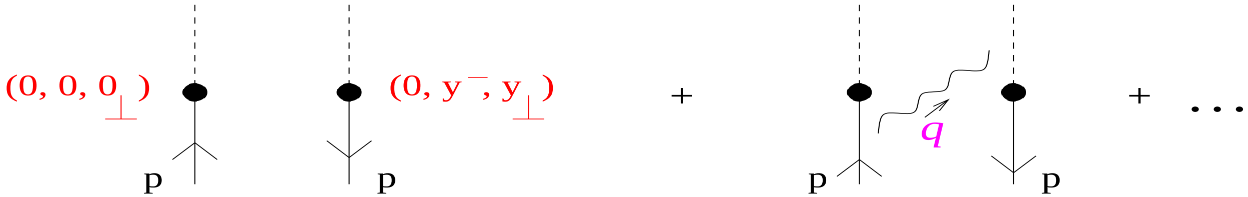
- works at tree level [Mulders, 2002; Belitsky et al., 2003]
- subtler at level of radiative corrections [Collins & Zu; H; Cherednikov et al.]

  $\hookrightarrow x \to 1 \Rightarrow$ explicit regularization method (unlike inclusive case)
- non-abelian Coulomb phase $\to$ spectator effects possibly non-decoupl.

  [Mulders, Bomhof; Collins, Qiu; Brodsky et al]

II.A LIGHTCONE DIVERGENCES

◇ Suppose a gluon is absorbed or emitted by eikonal line: $n = (0, 1, 0_\perp)$



$$f_{(1)} = P_R(x, k_\perp) - \delta(1-x)\,\delta(k_\perp) \int dx' dk'_\perp\, P_R(x', k'_\perp)$$

where $$P_R = \frac{\alpha_s\, C_F}{\pi^2} \left[\frac{1}{1-x}\, \frac{1}{k_\perp^2 + \rho^2} + \{\text{regular at } x \to 1\} \right]$$ ρ=IR regulator

↑ *endpoint singularity* $(q^+ \to 0, \ \forall\ k_\perp)$ [Brodsky et al, 2001; Collins, 2002]

◇ Physical observables:

$$\mathcal{O} = \int dx\, dk_\perp\, f_{(1)}(x, k_\perp)\, \varphi(x, k_\perp)$$
$$= \int dx\, dk_\perp\, [\varphi(x, k_\perp) - \varphi(1, 0_\perp)]\, P_R(x, k_\perp)$$

inclusive case: φ independent of $k_\perp \Rightarrow 1/(1-x)_+$ from real + virtual

general case: endpoint divergences (incomplete KLN cancellation)

CUT-OFF APPROACH

▷ cut-off in Monte-Carlo generators using u-pdf's

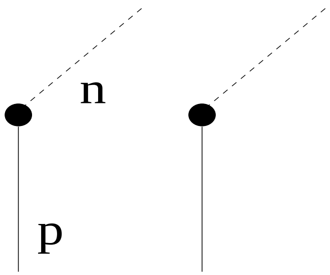
S. Jadach and M. Skrzypek, hep-ph/0905.1399 (DGLAP)

S. Höche, F. Krauss and T. Teubner, EPJC 58 (2008) 17 (KMR/BFKL)

LDCMC Lönnblad & Sjödahl, 2005; Gustafson, Lönnblad & Miu, 2002 (LDC)

CASCADE Jung, 2004, 2002; Jung and Salam, 2001 (CCFM)

▷ cut-off from gauge link in non-lightlike direction n:



$$\eta = (p \cdot n)^2 / n^2$$

Collins, Rogers & Stasto, PRD 77 (2008) 085009

Ji, Ma & Yuan, PRD 71 (2005) 034005; JHEP 0507 (2005) 020

earlier work from 80's and 90's: Collins et al; Korchemsky et al

finite $\eta \Rightarrow$ singularity is cut off at $1 - x \gtrsim \sqrt{k_\perp / 4\eta}$

∗ Note: Subtractive regularization is possible alternative to cut-off [Collins & H, 2001]

II.B UPDF's BY SUBTRACTIVE APPROACH

• Endpoint divergences $x \to 1$ from incomplete KLN cancellation

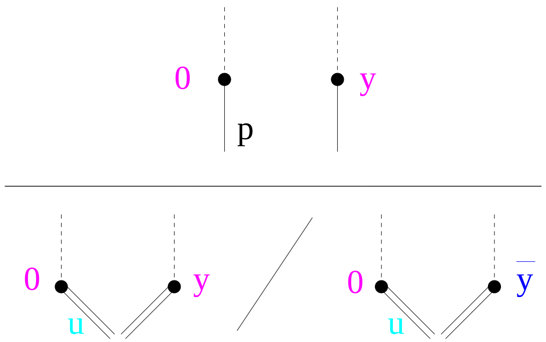
Subtractive method: more systematic than cut-off. Widely used in NLO calculations.

Formulation suitable for eikonal-operator matrix elements: Collins & H, 2001.

[See also "SCET" analog: Manohar and Stewart, 2007; J. Chiu et al, arXiv:0905.1141]

• gauge link still evaluated at n lightlike, but multiplied by "subtraction factors"

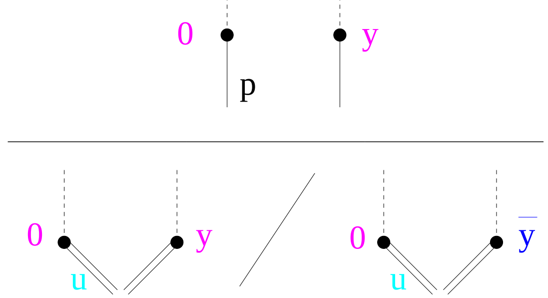
$$\widetilde{f}^{(\mathrm{subtr})}(y^-, y_\perp) = \frac{\overbrace{\langle P|\overline{\psi}(y)V_y^\dagger(n)\gamma^+ V_0(n)\psi(0)|P\rangle}^{\text{original matrix element}}}{\underbrace{\langle 0|V_y(u)V_y^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle \ / \ \langle 0|V_{\bar{y}}(u)V_{\bar{y}}{}^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle}_{\text{counterterms}}}$$



$\bar{y} = (0, y^-, 0_\perp)$; u = auxiliary non-lightlike eikonal $(u^+, u^-, 0_\perp)$

H, PLB 655 (2007) 26

◇ u serves to regularize the endpoint; drops out of distribution integrated over $k_\perp$



One loop expansion:

$$f^{(\mathrm{subtr})}_{(1)}(x,k_\perp) = P_R(x,k_\perp) - \delta(1-x)\,\delta(k_\perp)\int dx'\,dk'_\perp\,P_R(x',k'_\perp) \quad (\leftarrow\text{from numerator})$$

$$- \; W_R(x,k_\perp,\zeta) + \delta(k_\perp)\int dk'_\perp\,W_R(x,k'_\perp,\zeta) \quad (\leftarrow\text{from vev's})$$

with $P_R = \alpha_s C_F/\pi^2 \left\{1/[(1-x)\ (k_\perp^2 + m^2(1-x)^2)] + \ldots\right\}$ = real emission prob.

$W_R = \alpha_s C_F/\pi^2 \left\{1/[(1-x)\ (k_\perp^2 + 4\zeta(1-x)^2)] + \ldots\right\}$ = counterterm

- ζ-dependence cancels upon integration in $k_\perp$ $\quad [\zeta = (p^{+2}/2)u^-/u^+]$

$$\Rightarrow \mathcal{O} = \int dx\ dk_\perp\ f^{(\mathrm{subtr})}_{(1)}(x,k_\perp)\ \varphi(x,k_\perp)$$

$$= \int dx\ dk_\perp\ \{P_R\ [\varphi(x,0_\perp) - \varphi(1,0_\perp)] + (P_R - W_R)\ [\varphi(x,k_\perp) - \varphi(x,0_\perp)]\}$$

- first term: usual $1/(1-x)_+$ distribution
- second term: singularity in P_R cancelled by W_R

Note: counterterms at one loop give contributions to $f(x, k_\perp)$

$$-W_R(x, k_\perp, \zeta) + \delta(1-x)\,\delta(k_\perp) \int dx' dk'_\perp W_R$$

and

$$+\delta(k_\perp) \int dk'_\perp W_R(x, k'_\perp, \zeta) - \delta(1-x)\,\delta(k_\perp) \int dx' dk'_\perp W_R$$

ζ angle of eikonal u; W_R computed to order α_s

▷ virtual correction to gauge link does not depend on $y_\perp$

Korchemsky et al, 1992

▷ relation with cusp anomalous dimension in *Cherednikov et al arXiv:0904.2727; arXiv:0802.2821*

▷ one-loop counterterm gives extension for $k_\perp \neq 0$ of the plus-distribution regularization

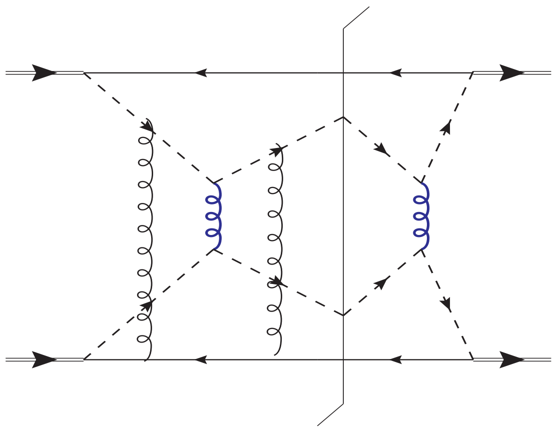
II.C COULOMB PHASE EFFECTS

- soft gluon exchange with spectator partons

Mert Aybat & Sterman, PLB671 (2009) 46

Boer, Brodsky & Hwang, PRD 67 (2003) 054003

$\Rightarrow$ factorization breaking in higher loops?



Collins, arXiv:0708.4410

Vogelsang and Yuan, arXiv:0708.4398

Bomhof and Mulders, arXiv:0709.1390

$\diamondsuit$ likely suppressed for small-x, small-$\Delta\phi$

$\diamondsuit$ could affect physical picture near large x, back-to-back region

- Note: Coulomb/radiative mixing terms also appear to break coherence in di-jet cross sections with gap in rapidity

Forshaw & Seymour, arXiv:0901.3037

Forshaw, Kyrieleis & Seymour, hep-ph/0604094

Unintegrated quark evolution

- flavor-singlet quark distribution coupled to gluons at small x via

$$\mathcal{P}_{g\to q}(z;q,k) = P_{qg,\mathrm{GLAP}}(z) \left(1 + \sum_{n=0}^{\infty} b_n(z)(k^2/q^2)^n\right)$$

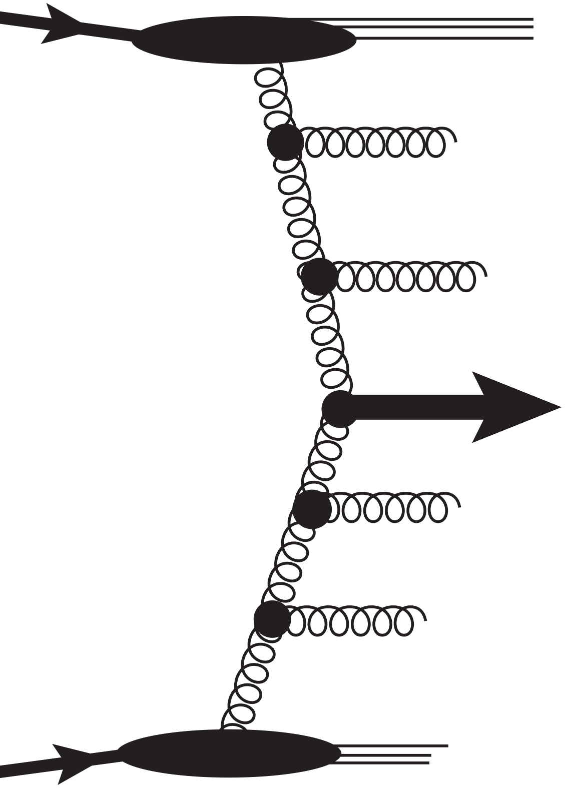
all b_n known; $\mathcal{P}_{g\to q}$ computed in closed form (positive-definite)

in *[Catani & H, 1994; Ciafaloni et al., 2005-2006]* by small-x factorization

- alternatively, $\mathcal{P}_{g\to q}(z;q,k)$ splitting function re-obtained from operator matrix element for unintegrated pdf *[A. Dafinca, in progress]*

$\Rightarrow$ verify consistency of TMD pdf with high energy factorization at small x

CCFM evolution and quark emission



CCFM evolution based on principle of color coherence
→ emissions of **gauge bosons**

unintegrated gluon and valence quark

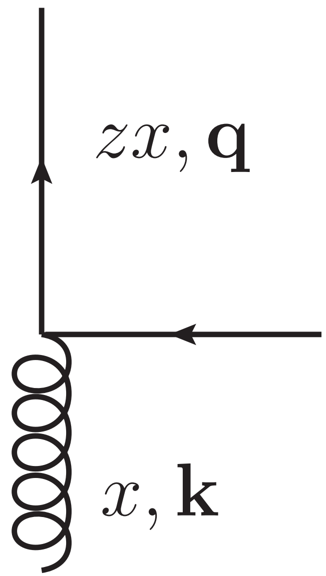
not present

Consequences: (A) Evolution (exclusive radiative corrections!):

- only gluonic emissions, no quark → jets purely gluonic
- DGLAP: naturally contained
- BFKL: through NLO corrections, not contained in (LO) CCFM evolution

Goal of this study: gluon $\longrightarrow$ quark splitting (P_{qg})

- supplement CCFM evolution by gluon $\rightarrow$ quark splitting
- restrict to splitting in the last evolution step
- keep finite transverse quark momentum q_T
  ➡ k_T factorized seaquark
- correct high energy & collinear limits,
  ➡ similar to CCFM evolution

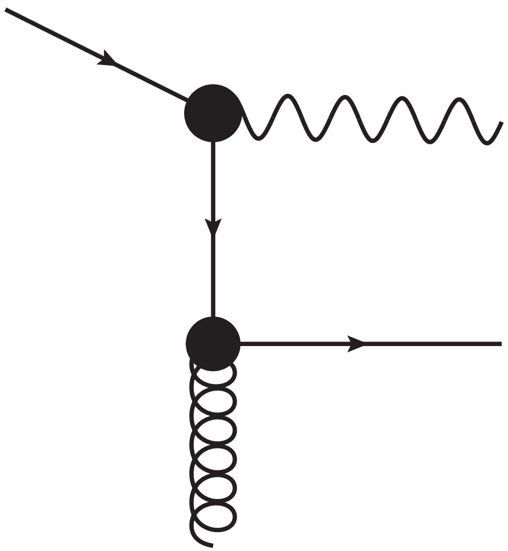


\+ test accuracy of (formal) factorization numerically

Process of interest at LHC: **forward Drell-Yan** production (γ^*, Z, W)

- probe proton at very small x, up to $3 \cdot 10^{-6}$
- investigate small x dynamics: BFKL, saturation, ...
- allows to compare exact versus factorized expression

Quark-gluon splitting and collinear factorization



- **DGLAP:** contains naturally splitting function
$$P_{qg}(z) = Tr(z^2 + (1-z)^2)$$
- no k_T dependence for seaquark distribution $q(x, \mu^2)$ and partonic cross-section $\sigma_{q\bar{q}\to Z}$
- no small x dynamics included

$$\hat{\sigma}_{q\bar{q}\to Z}(\nu = \hat{s}) = \underbrace{\sqrt{2} G_F M_Z^2 (V_q^2 + A_q^2)}_{\text{Z-coupling}} \times \frac{\pi}{N_c} \delta(\nu - M_Z^2)$$

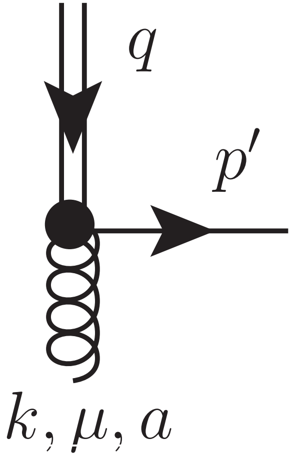
[Catani, Hautmann '94] : high energy resummation within collinear factorization: $\mathbf{k_T}$**-dependent splitting function**

$$P_{qg}^{\text{CH}}(z, \boldsymbol{k}^2, \boldsymbol{q}^2) = T_R \left(\frac{\boldsymbol{q}^2}{\boldsymbol{q}^2 + z(1-z)\boldsymbol{k}^2} \right)^2 \left[P_{qg}(z) + 4z^2(1-z)^2 \frac{\boldsymbol{k}^2}{\boldsymbol{q}^2} \right]$$

- $\otimes$ gluon Green's function: high energy resummed splitting
- universal ➡ defines small x-resummed seaquark distribution
- full k_T (gluon) dependence, but integrate out q_T (quark)

gauge invariant off-shell factorization: reggeized quarks

- **reggeized quarks** (in analogy to reggeized gluons for BFKL):
  - at high energies, effective d.o.f. in t-channel processes with quark exchange [Fadin,Sherman, 76,77], [Lipatov,Vyazovsky,'00], [Bogdan, Fadin, 06],
  - here applied to $qg^* \to Zq$ process at Born level
- **effective vertices**: re-arrangment of QCD diagrams



$$= igt^a \left(\gamma^\mu - \not{q}\frac{(n^+)^\mu}{k^+} \right) \qquad \text{etc.}$$

→ gauge invariant definition of off-shell Matrix Elements

$$\hat{\sigma}_{q\bar{q}^* \to Z}(\nu, \boldsymbol{q}^2) = \underbrace{\sqrt{2} G_F M_Z^2 (V_q^2 + A_q^2)}_{\text{Z-coupling}} \times \frac{\pi}{N_c} \delta(\nu - M_Z^2 - \boldsymbol{q}^2)$$

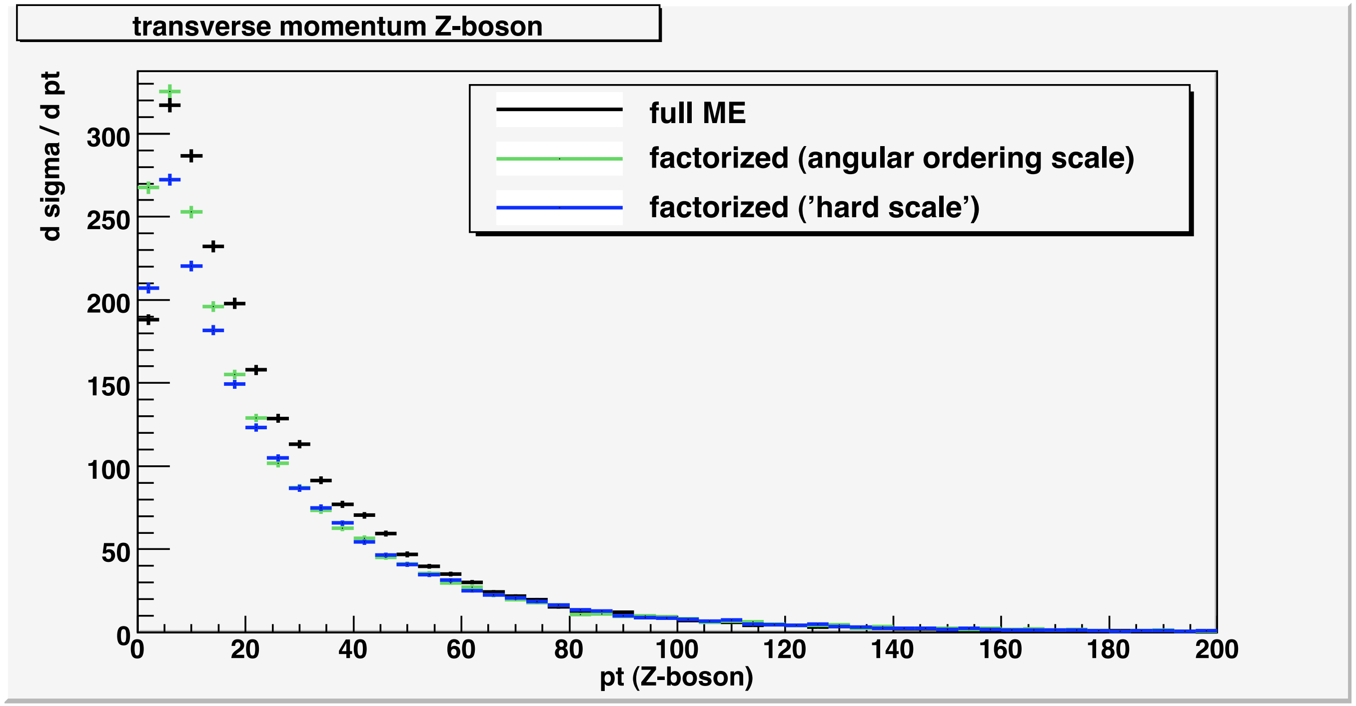
- gluon-quark splitting $= T_R$: Multi-Regge-Kinematics sets $z = 0$

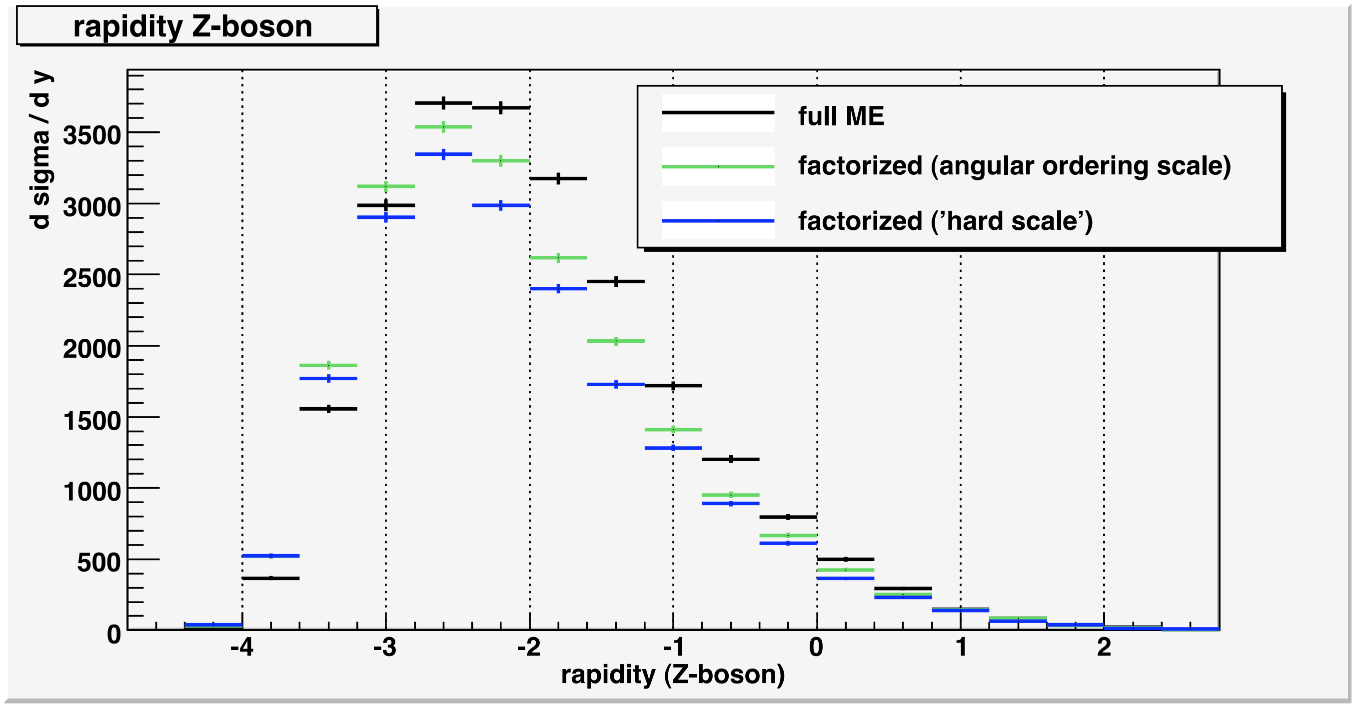
Forward DY from

CASCADE MC implementation of TMD sea quark distribution

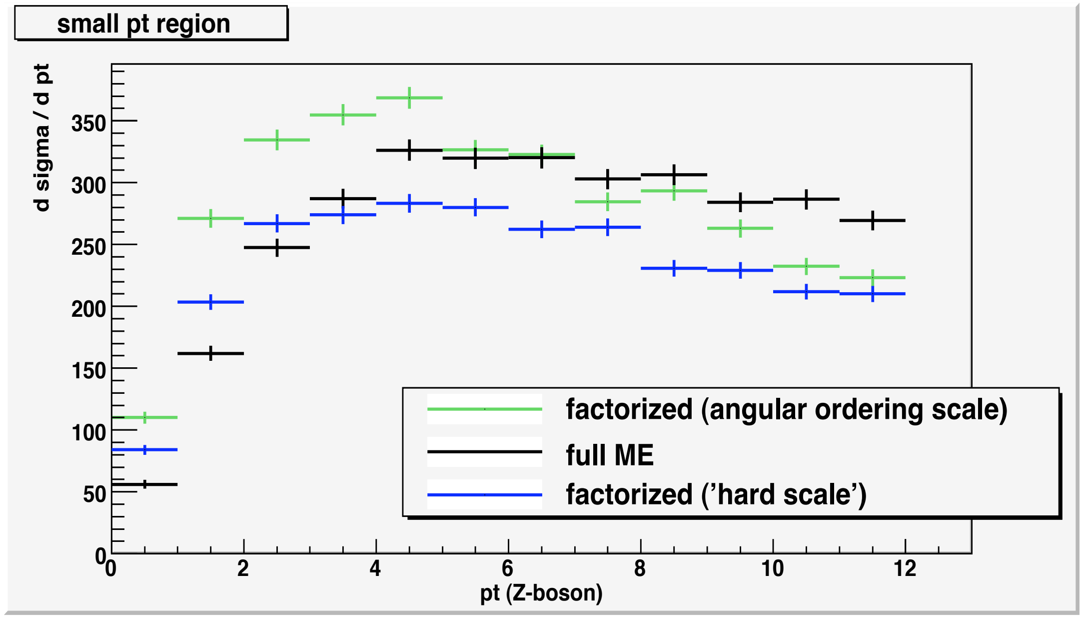
$$\text{angular ordering scale}: \quad \mu^2 = \frac{\mathbf{q}^2 + (1-z)\mathbf{k}^2}{(1-z)^2}$$

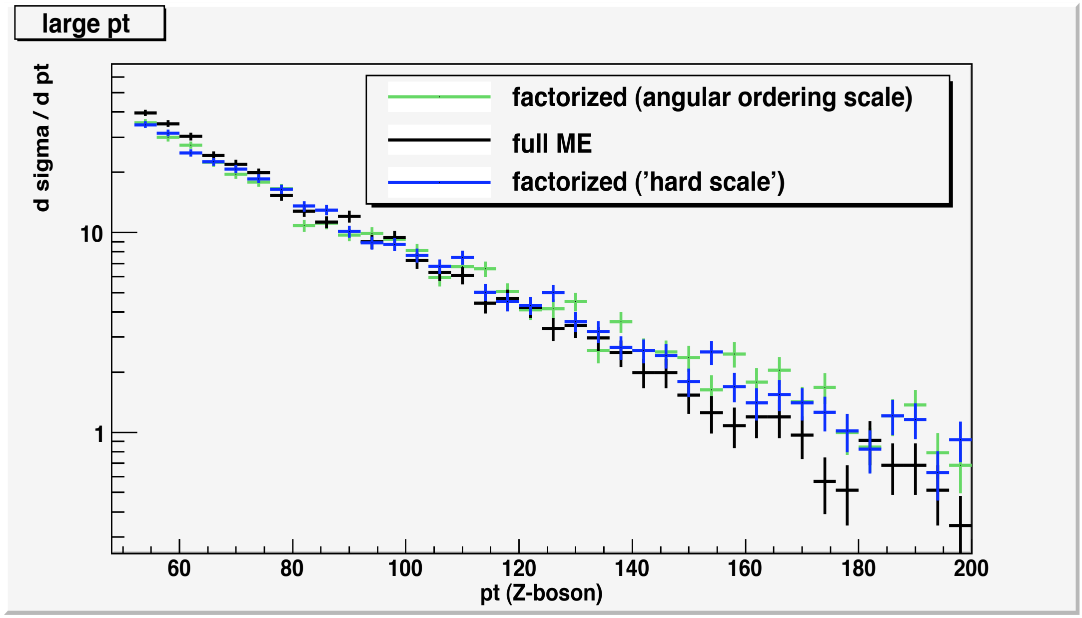
$$\text{hard scale}: \quad \mu^2 = \mathbf{p}^2 + M^2$$


transverse momentum Z-boson
d sigma / d pt
pt (Z-boson)
full ME
factorized (angular ordering scale)
factorized ('hard scale')
0
50
100
150
200
250
300
0
20
40
60
80
100
120
140
160
180
200


rapidity Z-boson
d sigma / d y
rapidity (Z-boson)
full ME
factorized (angular ordering scale)
factorized ('hard scale')
0
500
1000
1500
2000
2500
3000
3500
-4
-3
-2
-1
0
1
2

Agreement best for large p_T region





'Renormalized' $qg^* \to Zq$ cross-section

$$\bar{\sigma}(\nu, \boldsymbol{k}^2) \equiv \hat{\sigma}(\nu, \boldsymbol{k}^2) - \int_x^1 \frac{dz}{z} \int \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2} \hat{\sigma}_{q\bar{q}^* \to Z} P_{qg}^{\text{CH}}$$

yields finite $(7\% - 16\%)$ correction to factorized expression, free of large collinear logarithms