Parton branching at strong coupling from AdS/CFT

Edmond Iancu
CERN PH-TH & IPhT Saclay

based on work with Y. Hatta and Al Mueller

June 10th, 2011

Introduction

- Some RHIC data suggest that the deconfined QCD matter produced in heavy ion collisions is strongly interacting

  strong 'jet quenching' $\Longrightarrow$ enhanced parton evolution

Introduction

- Some RHIC data suggest that the deconfined QCD matter produced in heavy ion collisions is strongly interacting

  strong 'jet quenching' $\Longrightarrow$ enhanced parton evolution

- The interpretation of the data is a bit ambiguous ...

Introduction

- Some RHIC data suggest that the deconfined QCD matter produced in heavy ion collisions is strongly interacting

  strong 'jet quenching' $\Longrightarrow$ enhanced parton evolution

- The interpretation of the data is a bit ambiguous ...

  (hence my insistance in giving 2 talks !)

Introduction

- Some RHIC data suggest that the deconfined QCD matter produced in heavy ion collisions is strongly interacting

  strong 'jet quenching' $\Longrightarrow$ enhanced parton evolution

- The interpretation of the data is a bit ambiguous ...

  (hence my insistance in giving 2 talks !)

  ... but they invite to study parton evolution at strong coupling

Introduction

- Some RHIC data suggest that the deconfined QCD matter produced in heavy ion collisions is strongly interacting

  strong 'jet quenching' $\Longrightarrow$ enhanced parton evolution

- The interpretation of the data is a bit ambiguous ...

  (hence my insistance in giving 2 talks !)

  ... but they invite to study parton evolution at strong coupling

- The AdS/CFT (or 'gauge/string duality') revolution offers a convenient theoretical framework in that respect.

Introduction

- Some RHIC data suggest that the deconfined QCD matter produced in heavy ion collisions is strongly interacting

  strong 'jet quenching' $\Longrightarrow$ enhanced parton evolution

- The interpretation of the data is a bit ambiguous ...

  (hence my insistance in giving 2 talks !)

  ... but they invite to study parton evolution at strong coupling

- The AdS/CFT (or 'gauge/string duality') revolution offers a convenient theoretical framework in that respect.

- This leads to a fascinating 'parton picture' ...

Introduction

- Some RHIC data suggest that the deconfined QCD matter produced in heavy ion collisions is strongly interacting

  strong 'jet quenching' $\Longrightarrow$ enhanced parton evolution

- The interpretation of the data is a bit ambiguous ...

  (hence my insistance in giving 2 talks !)

  ... but they invite to study parton evolution at strong coupling

- The AdS/CFT (or 'gauge/string duality') revolution offers a convenient theoretical framework in that respect.

- This leads to a fascinating 'parton picture' ...

  ... which however looks very different from our daily experience with high–energy colliders

Introduction

- Some RHIC data suggest that the deconfined QCD matter produced in heavy ion collisions is strongly interacting

  strong 'jet quenching' $\Longrightarrow$ enhanced parton evolution

- The interpretation of the data is a bit ambiguous ...

  (hence my insistance in giving 2 talks !)

  ... but they invite to study parton evolution at strong coupling

- The AdS/CFT (or 'gauge/string duality') revolution offers a convenient theoretical framework in that respect.

- This leads to a fascinating 'parton picture' ...

  ... which however looks very different from our daily experience with high–energy colliders

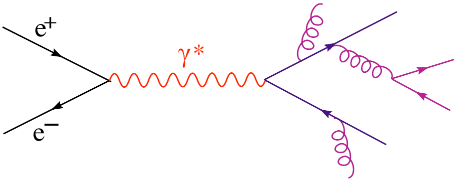
- "A perfect topics for the session on Complex Networks"

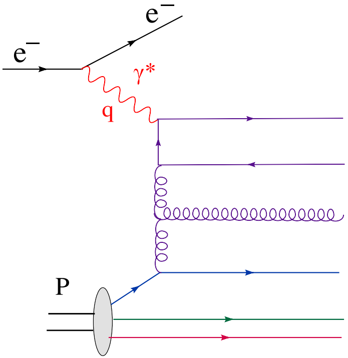
  (my friend Andrei Leonidov)

1. Weak coupling
2. Strong coupling

Parton evolution at weak coupling

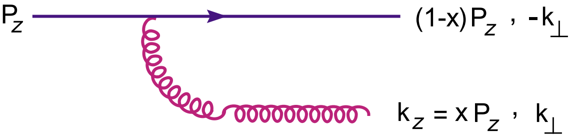
- One of the success stories of perturbative QCD





- Time–like cascades (partons branchings in the final state)
- Space–like cascades (parton structure of a hadron)
- They are all controlled by bremsstrahlung

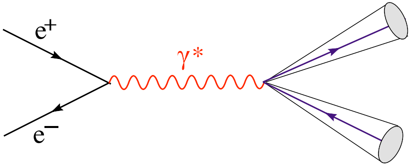
Bremsstrahlung

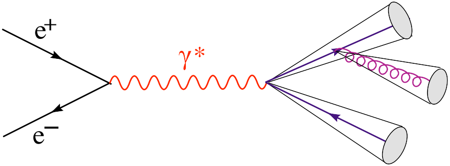


$$\mathrm{d}\mathcal{P}_{\mathrm{Brem}} \sim \alpha_s(k_\perp^2)\, N_c \,\frac{\mathrm{d}^2 k_\perp}{k_\perp^2}\, \frac{\mathrm{d}x}{x}$$

- Phase–space enhancement for the emission of
  - collinear $(k_\perp \to 0)$
  - and/or soft (low–energy) $(x \to 0)$ gluons
- Characteristic patterns for the final state & parton distributions

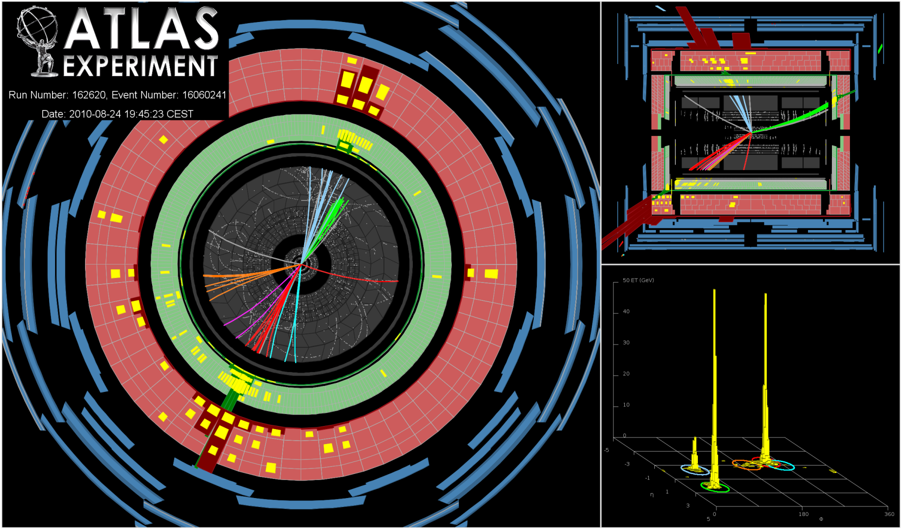
Jets in pQCD





- A few, well collimated, jets.
- The most probable configuration: a pair of back–to–back jets
- 3 jets or more: hard emissions, suppressed by $\alpha_s(k_\perp^2)$
- The respective cross–sections are well described by perturbative QCD

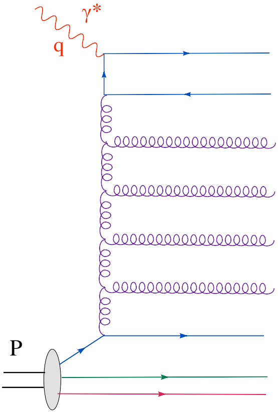
Routinely verified at the LHC

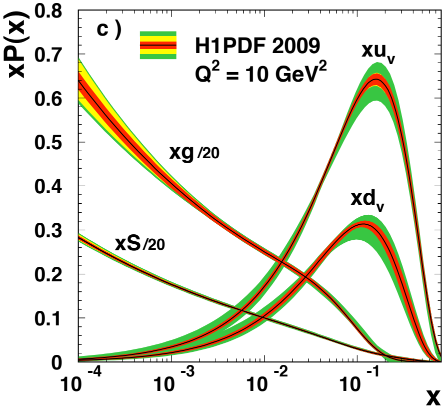


- p+p collisions at 7 TeV

Parton distributions at HERA

- Most partons are soft gluons ($x \ll 1$) ...
- They control the high energy scattering: $x \sim Q^2/s$





- Non–linear effects leading to saturation *(cf. talk by R. Enberg)*

The energy–momentum sum rule in pQCD

- HERA data confirm that gluons are numerous at small–x :

$xg(x,Q^2) \sim 1/x^{\lambda}$ with $\lambda \sim 0.2 \div 0.3$ for $Q^2 \geq 2$ GeV^2

- ... yet, they carry only a tiny fraction of the total energy !
- The energy–momentum sum rule ...

$$\int_0^1 \mathrm{d}x\, x\left[g(x,Q^2)+q(x,Q^2)+\bar{q}(x,Q^2)\right] = 1$$

- ... is dominated by the few partons remaining at $x \sim \mathcal{O}(1)$
- $Q^2 \to \infty$: the energy is carried by pointlike valence quarks

The energy–momentum sum rule in pQCD

- HERA data confirm that gluons are numerous at small–x :

$xg(x,Q^2) \sim 1/x^{\lambda}$ with $\lambda \sim 0.2 \div 0.3$ for $Q^2 \geq 2$ GeV^2

- ... yet, they carry only a tiny fraction of the total energy !
- The energy–momentum sum rule ...

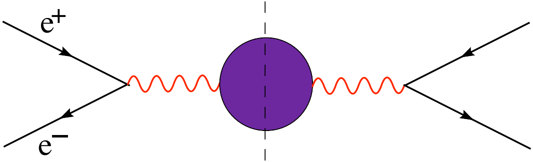
$$\int_0^1 \mathrm{d}x\, x\, [g(x,Q^2) + q(x,Q^2) + \bar{q}(x,Q^2)] = 1$$

- ... is dominated by the few partons remaining at $x \sim \mathcal{O}(1)$
- $Q^2 \to \infty$: the energy is carried by pointlike valence quarks
- What should be the corresponding picture at strong coupling ?

Current–current correlator (e^+e^-)

- How to compute parton evolution at strong coupling ?
- Optical theorem : non–pert. expression for the cross–section

$$\sigma(e^+e^-) \;=\; \frac{1}{2s}\,\ell^{\mu\nu}\;\mathrm{Im}\,\Pi_{\mu\nu}(q)$$

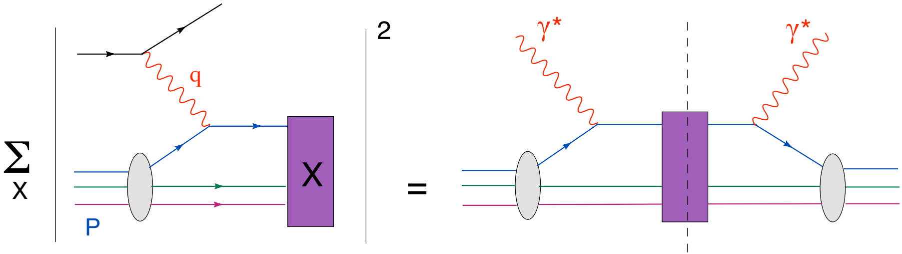


$$\Pi_{\mu\nu}(q)\;\equiv\;\int \mathrm{d}^4x\,\mathrm{e}^{-iq\cdot x}\,i\theta(x_0)\,\langle 0\,|\,[J_\mu(x),J_\nu(0)]\,|\,0\rangle$$

$$J^\mu \;=\; \sum_f e_f\,\bar q_f\,\gamma^\mu\,q_f \;\;:\;\text{quark electromagnetic current}$$

- Valid to leading order in $\alpha_{\rm em}$ but all orders in α_s

Current–current correlator (DIS)



$$F_{1,2}(x, Q^2) \sim \mathrm{Im} \int \mathrm{d}^4x\, \mathrm{e}^{-iq\cdot x}\, i\, \langle P \,|\mathrm{T}\,\{J_\mu(x) J_\nu(0)\}\,|\, P\rangle$$

- Compute the JJ correlator using the gauge/string duality
- An all–inclusive quantity:
  sum over all possible final states/parton evolutions
- How to extract some more specific information ?

Holographic RG

- Compare the AdS/CFT results with expectations from the Operator Product Expansion *(Polchinski and Strassler, 02)*
  ▷ partons = 'twist–two' operators
- Use the 'bulk' (AdS_5) picture together with the IR/UV correspondence *(Hatta, E.I., Mueller, 07)*
- Compute the final energy density : $\langle J\mathcal{E}J\rangle$ with $\mathcal{E} = T^{00}$
  ▷ finite temperature/AdS_5 Black Hole geometry
  (Gubser et al., Chesler and Yaffe, 2007)
  ▷ zero temperature: supergravity issues
  (Hofman, Maldacena, 08; HIM & Triantafyllopoulos, 10)
- Compute energy density correlators: $\langle J\mathcal{E}(x_1)\mathcal{E}(x_2)\dots\mathcal{E}(x_n)J\rangle$
  ▷ information about fluctuations in the final state
  (Hofman, Maldacena, 08)

Operator Product Expansion

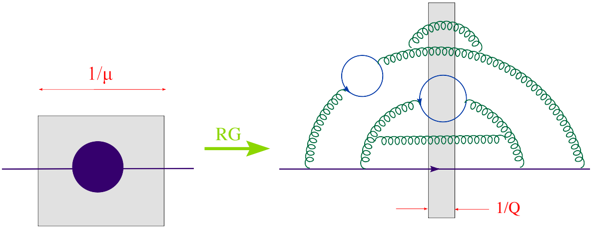
- $J(y)\, J(0) \;\sim\; \sum_n C_n(y)\, \mathcal{O}^{(n)}(0)$
- Partons $\leftrightarrow$ 'twist–2' : spin n, classical dimension $d = n + 2$

$$\mathcal{O}_f^{(n)\,\mu_1\cdots\mu_n} \;\equiv\; \bar{q}\,\gamma^{\mu_1}(iD^{\mu_2})\cdots(iD^{\mu_n})q$$

- Moments of the structure function:

$$\langle x^{n-1}\rangle_{Q^2} \;\equiv\; \int \mathrm{d}x\, x^{n-2}\, F_2(x, Q^2) \;\propto\; \langle\mathcal{O}^{(n)}\rangle_{Q^2}$$

- The operators depend upon the resolution scale Q^2



Renormalization group flow

- RG flow ⇒ negative anomalous dimensions (branching)

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}\,\mathcal{O}^{(n)} = \gamma^{(n)}\mathcal{O}^{(n)} \quad \text{with} \quad \gamma^{(n)} \le 0$$

- $\mathcal{N} = 4$ SYM at strong coupling: $\lambda \equiv g^2 N_c \gg 1$

$$\gamma^{(n)} \simeq -\sqrt{n}\,\lambda^{1/4} \quad \text{for} \quad 1 \ll n \ll \sqrt{\lambda}$$

- Only exception: energy momentum tensor for which $\gamma^{(2)} = 0$

$$\int \mathrm{d}x\, F_2(x, Q^2) \propto \langle T^{00} \rangle \quad : \quad \text{independent of } Q^2$$

- $Q^2 \to \infty$: $\langle x^{n-1} \rangle \to 0$ for any $n > 2$ while $\langle x \rangle =$ const.
- $F_2(x, Q^2)$ receives only higher twist contributions
  $\Longrightarrow$ no partons

Renormalization group flow

- RG flow $\Rightarrow$ negative anomalous dimensions (branching)

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}\, \mathcal{O}^{(n)} = \gamma^{(n)} \mathcal{O}^{(n)} \quad \text{with} \quad \gamma^{(n)} \leq 0$$

- $\mathcal{N} = 4$ SYM at strong coupling: $\lambda \equiv g^2 N_c \gg 1$

$$\gamma^{(n)} \simeq -\sqrt{n}\, \lambda^{1/4} \quad \text{for} \quad 1 \ll n \ll \sqrt{\lambda}$$

- Only exception: energy momentum tensor for which $\gamma^{(2)} = 0$

$$\int \mathrm{d}x\, F_2(x, Q^2) \propto \langle T^{00} \rangle \quad : \quad \text{independent of } Q^2$$

- Interpretation: partons keep branching, down to the smallest value of x which is consistent with energy conservation

Renormalization group flow

- RG flow ⇒ negative anomalous dimensions (branching)

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}\, \mathcal{O}^{(n)} = \gamma^{(n)} \mathcal{O}^{(n)} \quad \text{with} \quad \gamma^{(n)} \le 0$$

- $\mathcal{N} = 4$ SYM at strong coupling: $\lambda \equiv g^2 N_c \gg 1$

$$\gamma^{(n)} \simeq -\sqrt{n}\, \lambda^{1/4} \quad \text{for} \quad 1 \ll n \ll \sqrt{\lambda}$$

- Only exception: energy momentum tensor for which $\gamma^{(2)} = 0$

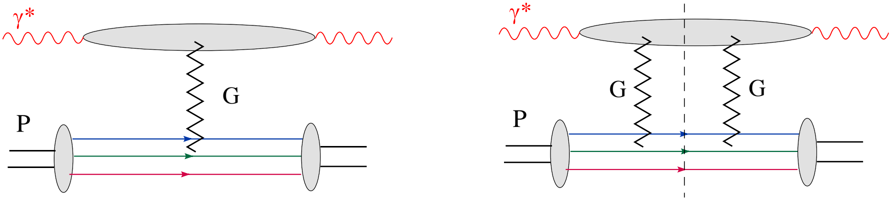
$$\int \mathrm{d}x\, F_2(x, Q^2) \propto \langle T^{00} \rangle \quad : \quad \text{independent of } Q^2$$

- OPE reduces to just one (leading–twist) operator: $T^{\mu\nu}$

$\Longrightarrow$ the effective theory for scattering must be gravity !

Where did all the partons go ?

- Is the parton branching ever stopping ? At which values of x ?



$$\mathcal{T}^{(1)} = \frac{1}{N_c^2}\frac{\Lambda^2}{x\,Q^2} \;:\; \text{real} \qquad \mathrm{Im}\,\mathcal{T}^{(2)} = \left(\frac{1}{N_c^2}\frac{\Lambda^2}{x\,Q^2}\right)^2 \;:\; \text{higher twist}$$

- The unitarity bound ('black disk limit') : $\mathrm{Im}\,\mathcal{T} = 1$

  ... is reached by $\mathcal{T}^{(2)}$ when $x = x_s(Q) \equiv \frac{1}{N_c^2}\frac{\Lambda^2}{Q^2}$

- We expect parton branching to stop at $x \sim x_s(Q)$

Parton saturation at strong coupling

- This is confirmed by various AdS/CFT calculations

  Hatta, E.I., Mueller; Brower, Strassler, Tan; Cornalba (07)

- $x \lesssim x_s(Q)$: partons saturate with occupation numbers $\mathcal{O}(1)$

$$F_2(x, Q^2) \sim N_c^2 Q^2 R^2 \sim \#(colors) \times \int^{Q^2} \mathrm{d}^2 k_\perp \int^{R^2} \mathrm{d}^2 b_\perp \times 1$$

- The energy sum rule is saturated by the small–x partons:

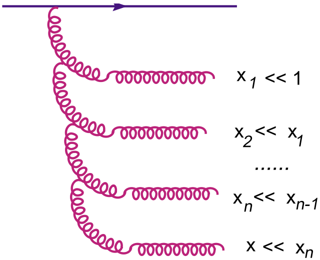
$$\int_0^{x_s(Q)} \mathrm{d}x \, F_2(x, Q^2) \sim x_s(Q) \, F_2(x, Q^2) \sim \Lambda^2 \, R^2 \sim \mathcal{O}(1)$$

- Contrast with the situation at weak coupling :
  - gluon saturate with occupation numbers $\sim 1/\alpha_s \gg 1$
  - saturation momentum grows slower: $Q_s(x) \sim 1/x^{0.3}$
  - energy sum rule is saturated by valence partons with $x \sim \mathcal{O}(1)$

Quasi–democratic parton branching

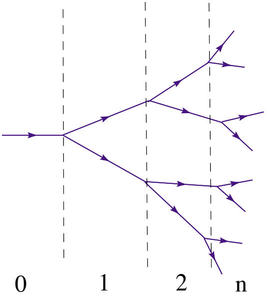
- No reason to privilege soft or collinear emissions
- Hard emissions (large $k_\perp$) are actually faster: $\Delta t \sim \omega/k_\perp^2$

- Weak coupling



- Bremsstrahlung
- Soft & collinear gluons

- Strong coupling

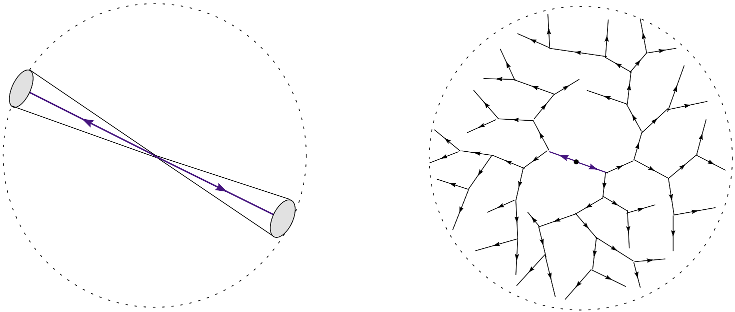


- Quasi–democratic branching :

$$\omega_n \sim \omega_{n-1}/2$$

e^+e^- annihilation (COM frame)

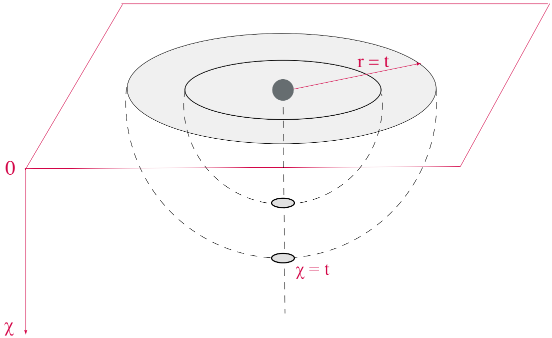
- Typical final state at weak coupling :
  a pair of back to back jets with high momenta $k \simeq \omega/2$



- Typical final state at strong coupling :
  an isotropic distribution of many soft particles $(k_i \sim \omega_i \sim \Lambda)$
- Study the evolution of a time–like wavepacket in AdS/CFT

The Backreaction

- Time–like wave–packet at rest on the boundary $\Longrightarrow$ bulk excitation falling in AdS_5 at the speed of light: $\chi = t$
- Backreaction on the AdS_5 metric: δg_{mn}



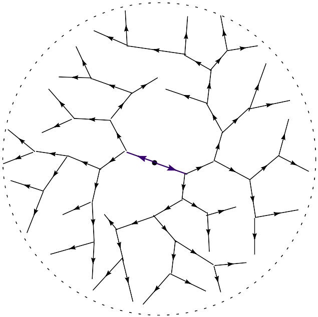
- Energy density on the boundary: $\delta g_{\mu\nu} \sim \chi^4 T_{\mu\nu}$ as $\chi \to 0$

Results

- An isotropic energy distribution in the final state ☺

$$\langle \mathcal{E}(\Theta_1)\mathcal{E}(\Theta_2)\dots\mathcal{E}(\Theta_n)\rangle = \left(\frac{\omega}{4\pi}\right)^n$$

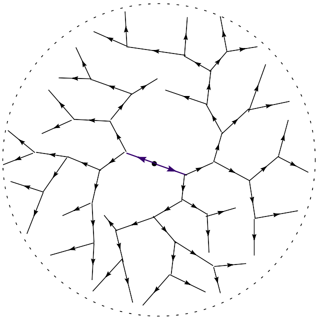
(Hofman and Maldacena, 08)

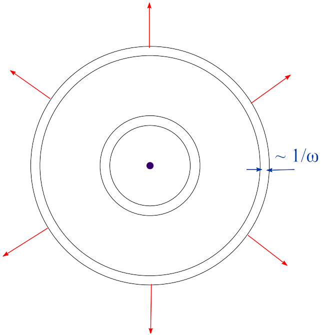


Results

- An isotropic energy distribution in the final state ☺

$$\langle \mathcal{E}(\Theta_1)\mathcal{E}(\Theta_2)\ldots\mathcal{E}(\Theta_n)\rangle = \left(\frac{\omega}{4\pi}\right)^n$$ *(Hofman and Maldacena, 08)*



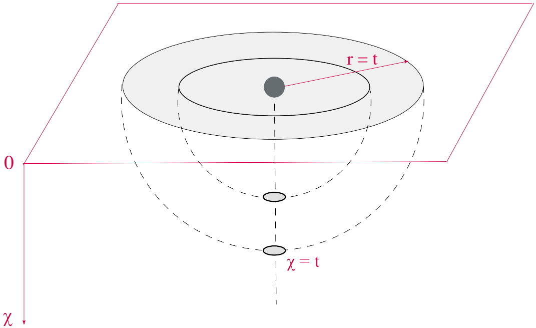


- A narrow spherical shell moving at the speed of light: $r = t$ ☹

(Hatta, E.I., Mueller, Triantafyllopoulos, 10)

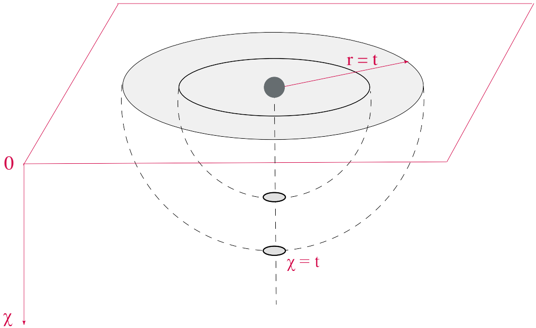
The UV/IR correspondence

- What is the rôle of the 5th dimension ? Virtuality !
- Penetration χ of the 'photon' in AdS_5 $\longleftrightarrow$ radial broadening $t-r$ of the partonic system on the boundary



The UV/IR correspondence

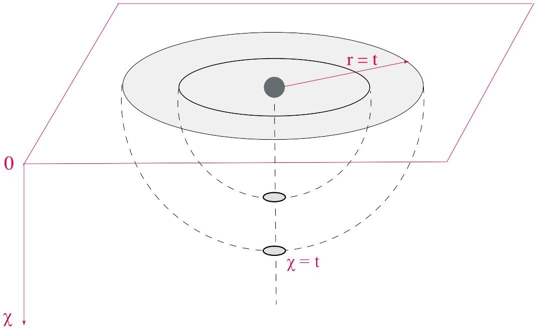
- What is the rôle of the 5th dimension ? Virtuality !
- Penetration χ of the 'photon' in AdS_5 $\longleftrightarrow$
  radial broadening $t - r$ of the partonic system on the boundary



- Supergravity approximation (semiclassical) :
  no backreaction except from $\chi = 0 \Longrightarrow$ no broadening !

The UV/IR correspondence

- What is the rôle of the 5th dimension ? Virtuality !
- Penetration χ of the 'photon' in AdS_5 $\longleftrightarrow$ radial broadening $t - r$ of the partonic system on the boundary



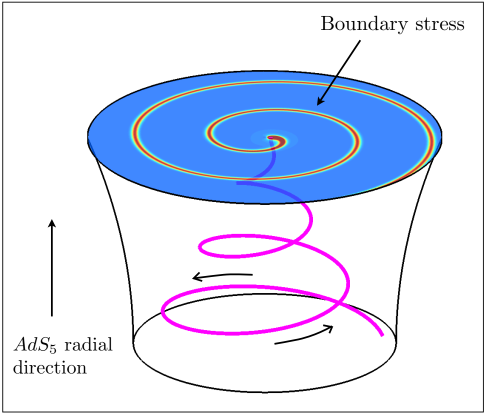
- The UV/IR correspondence is formally respected ...

  ... but the result is physically meaningless !

The rotating string (1)

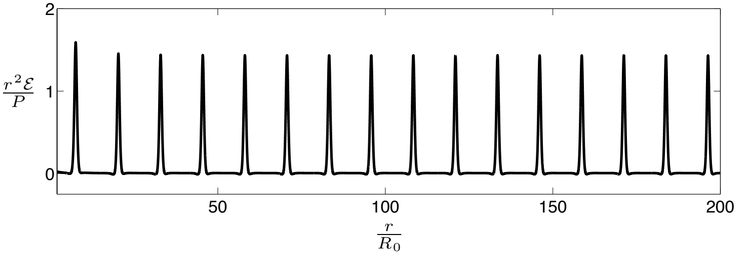
- This problem is generic: any radiation process in the vacuum
- Synchrotron radiation in 4D $\longleftrightarrow$ rotating string in AdS_5

(Athanasiou, Chesler, Liu, Nickel, and Rajagopal, 2010)



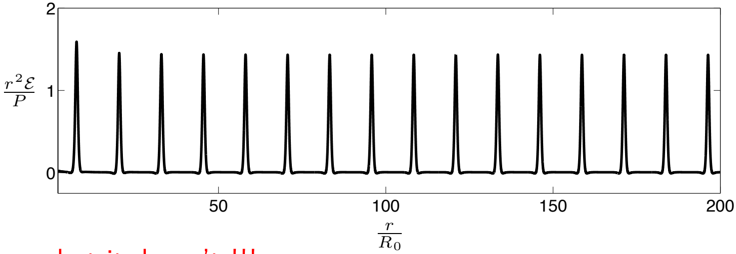
The rotating string (2)

- The width of the spiral line on the boundary should increase with r, due to quantum branching of the radiation ...



The rotating string (2)

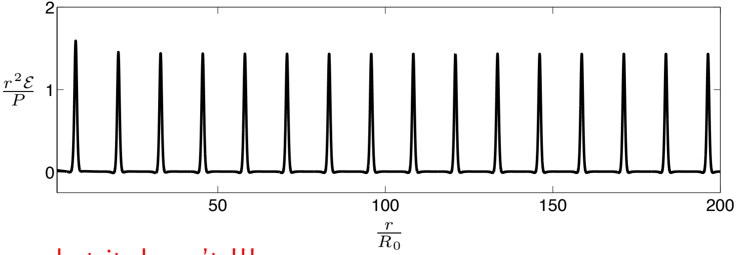
- The width of the spiral line on the boundary should increase with r, due to quantum branching of the radiation …



- … but it doesn't !!!
- The same space-time pattern as in Jackson's chapter 14: radiation propagates at the speed of light: no off–shell effects.

The rotating string (2)

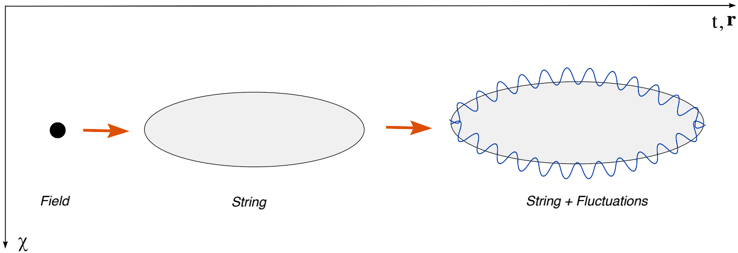
- The width of the spiral line on the boundary should increase with r, due to quantum branching of the radiation ...



- ... but it doesn't !!!
- The same space-time pattern as in Jackson's chapter 14: radiation propagates at the speed of light: no off–shell effects.
- Once again, the whole contribution to the energy density comes via backreaction from the string endpoint at $\chi = 0$.

String fluctuations

- It seems like, for some problems, the SUGRA approximation is unable to correctly capture quantum fluctuations
- The 'bulk excitation' is not just a local field (as in SUGRA), but a microscopic string, which can have fluctuations



- Standard paradigm : fluctuations are suppressed as $\lambda \to \infty$
- But this is not true for the longitudinal fluctuations

Longitudinal fluctuations in AdS_5

- String quantization in AdS_5: an outstanding open problem
- Hofman and Maldacena (08) : Heuristic treatment
- H&M : transverse flucts in the angular distribution
  $\Longrightarrow$ the effect was small and negligible as $\lambda \to \infty$
- We used it for longitudinal flucts in the radial (r) distribution
  $\Longrightarrow$ broadening $r - t \sim z$ ... as expected on physical grounds
- Problem solved (?) ... but many other problems open !

Longitudinal fluctuations in AdS_5

- String quantization in AdS_5: an outstanding open problem
- Hofman and Maldacena (08) : Heuristic treatment
- H&M : transverse flucts in the angular distribution
  $\Longrightarrow$ the effect was small and negligible as $\lambda \to \infty$
- We used it for longitudinal flucts in the radial (r) distribution
  $\Longrightarrow$ broadening $r - t \sim z$ ... as expected on physical grounds
- Problem solved (?) ... but many other problems open !
- What are the limits of SUGRA and how to go beyond ?