COMPLEX NETWORKS: A NEW SYNTHESIS

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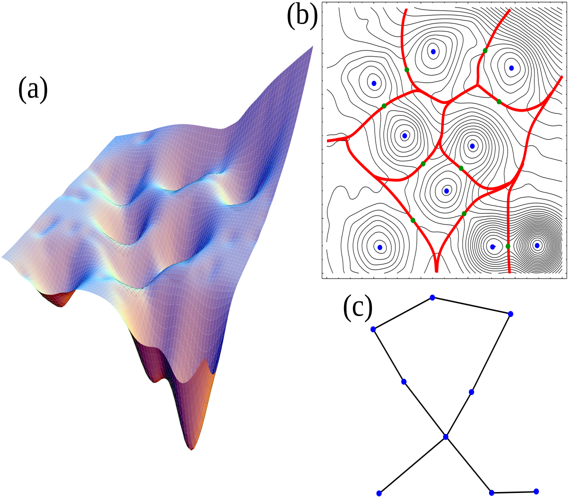
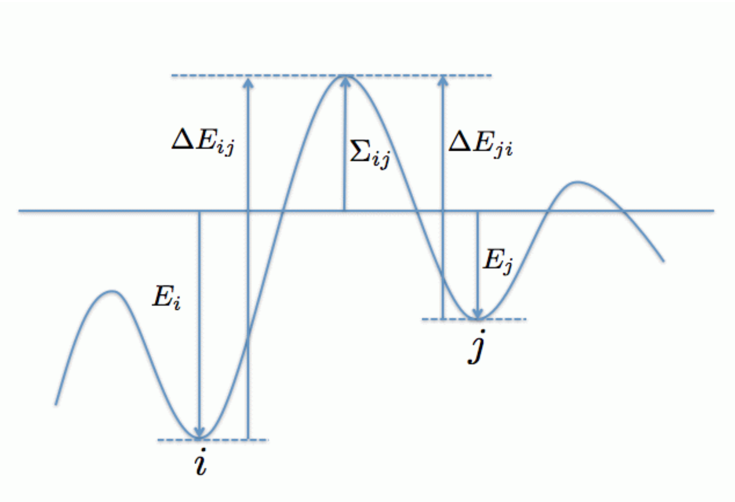
RANDOM NETWORKS IN PHYSICS

- Kinetics of spin glasses, structural glasses and supercooled liquids. Description based on considering diffusion in a complex potential landscape.
- New features of phase transitions of spin models on random networks
- ...

Mapping of a potential surface onto a network



TRANSITION NETWORK



ENERGY SURFACE OF LENNARD-JONES ENSEMBLE

- Ensemble of N atoms interacting with

$$V = 4\epsilon \sum_{i<j} \left[\left(\frac{\sigma}{r_{ij}} \right)^{12} - \left(\frac{\sigma}{r_{ij}} \right)^{6} \right]$$

- Number of minima n_{min}:

$$n_{\mathrm{min}} = e^{\alpha N}$$

- Number of transition states:

$$n_{\mathrm{ts}} = aNe^{\alpha N}$$

- Mean vertex degree

$$\langle k \rangle \approx \frac{2n_{\mathrm{ts}}}{n_{\mathrm{min}}} = \frac{2a}{\alpha} \log n_{\mathrm{min}}$$

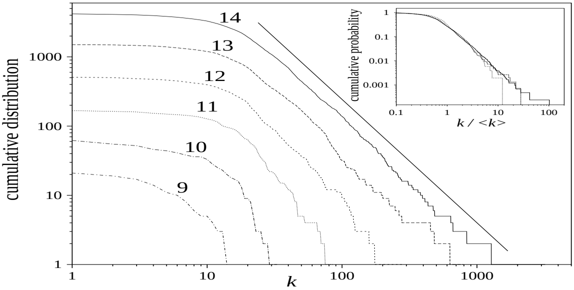
- Mean link occupation probability

$$p \approx \frac{2n_{\mathrm{ts}}}{n_{\mathrm{min}}(n_{\mathrm{min}} - 1)} \approx \frac{2a \log n_{min}}{\alpha n_{\mathrm{min}}}$$

- The distribution of vertex degrees is powerlike,

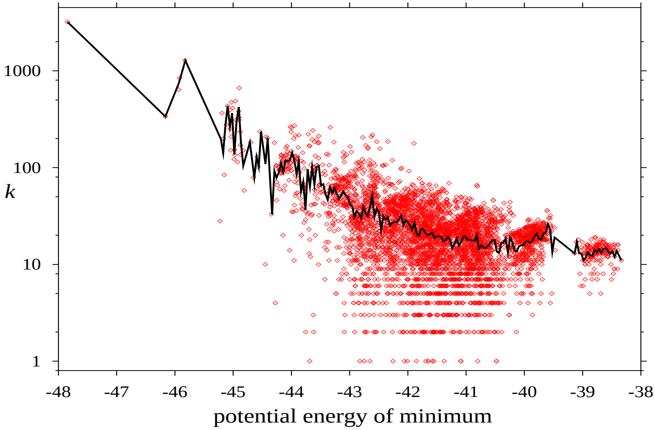
$$P(k) \approx 1/k^{\gamma}, \quad \gamma = 2.78$$

i.e. LJ networks are scale-free.



- The potential surface - to - network mapping has revealed a new dimensionless characteristics γ.

- In addition LJ ensembles are characterized by interrelation between the local topology (node degree k) and the depth of the corresponding local minimum E:



- The functional form of this correspondence is

$$E_i \approx \ln k_i$$

KINETICS AND GLASS TRANSITION

- Master equation for the probability $P(k, t_w)$ of occupying a node with the degree k at the time t_w:

$$\frac{\partial P(k,t_w)}{\partial t_w} \equiv \dot{P}(k,t_w) = -\sum_{k'} W_{kk'} P(k,t_w) + \sum_{k'} W_{k'k} P(k',t_w).$$

- Stationary distribution

$$P^{\infty}(k) = \frac{1}{\mathcal{Z}} k P(k) g(k)/f(k)$$

where $\mathcal{Z}$ is determined by $\sum_k P^{\infty}(k) = 1$

KINETICS AND GLASS TRANSITION

- For realistic transition rates (Arrhenius, Metropolis, Glauber)

$$g(k)/f(k) \equiv e^{\beta h(k)} \Rightarrow \mathcal{Z} = \sum_k k P(k) e^{\beta h(k)}.$$

- In the scale-free case $P(k) \sim 1/k^{\gamma}$ and for $h(k) = E_0 \log(k)$ the convergence condition is

$$\beta E_0 - \gamma < -2$$

- A transition between a high temperature phase in which $\mathcal{Z}$ is finite and glass phase $\mathcal{Z} = \infty$ and ergodicity is (weakly) broken takes place at

$$T_c = \frac{1}{\beta_c} = \frac{E_0}{\gamma - 2} \ .$$

- The network-based description allows to give a fairly good description of kinetic phenomena in glasses
- The probability distribution $P(k, t_w)$ was shown to be characterized by the following scaling:

$$P(k, t_w) = k_w(t_w|\gamma)^{-1}\mathcal{F}\left(\frac{k}{k_w(t_w|\gamma)}\right),$$

where $k_w(t_w|\gamma)$ characterizes the degrees that have already been equilibrated at t_w.

RANDOM NETWORKS: A NEW SYNTHESIS

- Allows to reorganize our understanding of diffusion in complex energy landscapes.
- Promising universal language for problems sensitive to event-by-event topology of random fields such as propagation through turbulent medium.
- Promising universal language for strong coupling regime. Evolution equations for parton distributions at strong coupling should include vertices (and, therefore, kernels) of arbitrary order in the number of produced partons (see the talk by E. Iancu at this Workshop).
- Interesting possibilities for studying conformal invariance/integrability for two-dimensional spin systems on scale-free random networks.
- ...

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