On Localized Nonperturbative Solutions in 2D and 3D Gluodynamics

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Abstract: We discuss the possibilty of soliton existence in 2D and 3D SU(2) gluodynamics. Hamiltonians in terms of radial functions are presented. We are looking for localized in space YM field distributions which provide local minima to these hamiltonians. Such nontopological solitons if exist may be relevant to extended gluonic strings in mesons (in 2D) and glueball states (in 3D).

Quark-antiquark with gluonic string

The famous action density distribution between two static colour sources

[G.S. Bali, K. Schilling, C. Schlichter '95]

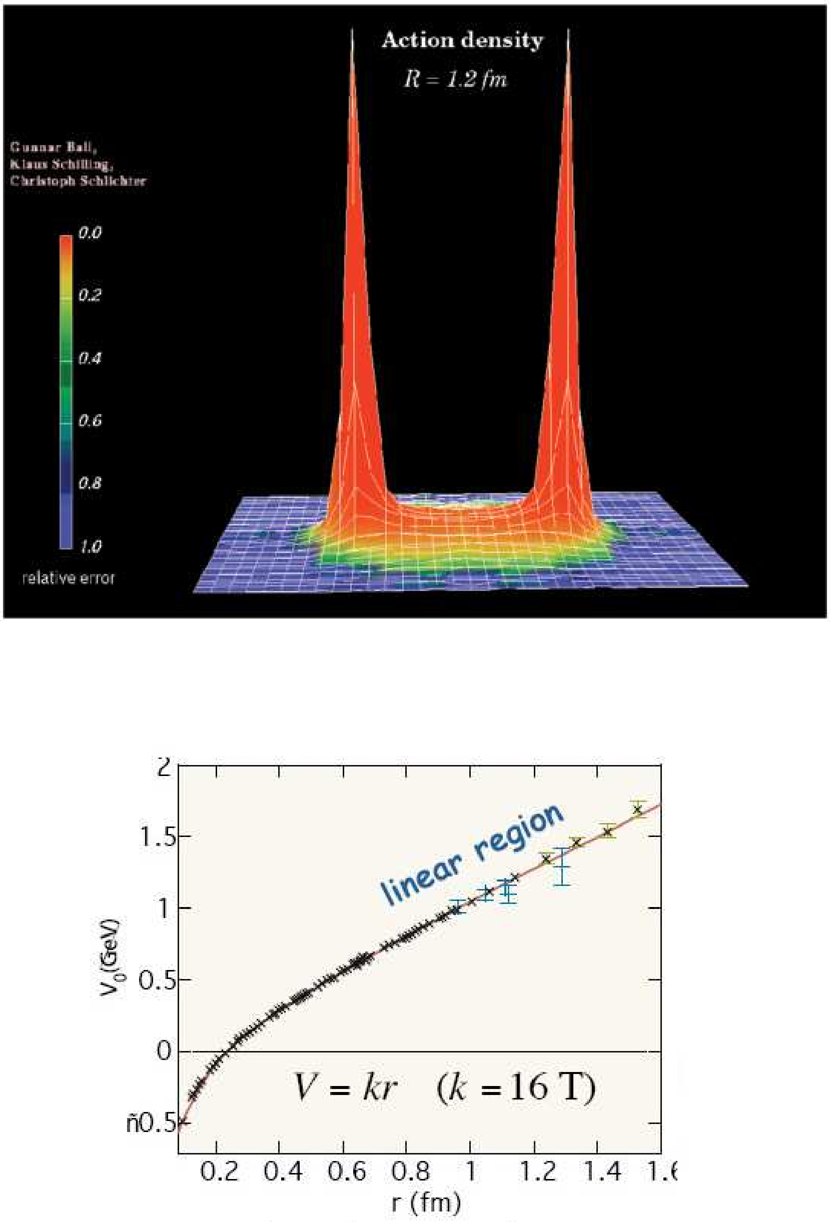

Figure 1: Structure of mesons

Introduction

- Until now there is no satisfactory theoretical description of extended string connecting quark and antiquark in mesons.

- Study of 2D solitons can clarify this issue.

- For now nobody proposed adequate ansatz for description of $2D$ Yang-Mills solitons.

- For 3D case only the simplest one-term ansatz has been studied, for it $\partial_\mu A_\mu = 0$ is valid.

- Generic 3-term ansatz requires detailed study, for it $\partial_\mu A_\mu = 0$ is not automatically satisfied.

- 3D YM solitons if exist could be viewed as classical glueballs.

- In previous studies of Yang-Mills solitons specifics of Yang-Mills fields as gauge ones has been never taken into account.

Yang-Mills in D=2

- Consider the vector $SU(2)$ Yang-Mills field $A^a_\mu(x^\nu)$

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2,$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc}A^b_\mu A^c_\nu,$$

$$D = 2, \quad \mu,\nu = 0,1,2, \quad a,b,c = 1,2,3, \quad g{-}const.$$

- We look for stationary solutions and use the following ansatz:

$$A^a_0 = 0,$$

$$gA^a_i = \delta_{a3}\varepsilon_{iak}x_k\frac{1}{R^2}s(R)+$$
$$+(\delta_{a1}+\delta_{a2})[(\delta_{ia}R^2 - x_i x_a)\frac{b(R)}{R^3} + \frac{p(R)x_i x_a}{R^4}],$$

$$i,k = 1,2 \qquad R^2 = x^2 + y^2.$$

Hamiltonian density for D=2

No gauge fixing here.

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> H_YM_2D;
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$$
\frac{1}{2}\frac{s(R)^2\,p(R)^2}{R^2 g^2} - \frac{b(R)\,p(R)}{R^3 g^2} - \frac{p(R)\left(\frac{d}{dR}b(R)\right)}{R^2 g^2} + \frac{1}{2}\frac{\left(\frac{d}{dR}b(R)\right)^2}{g^2}
$$

$$
+ \frac{p(R)^2\,s(R)}{R^3 g^2} + \frac{b(R)\left(\frac{d}{dR}b(R)\right)}{R\,g^2} + \frac{1}{2}\frac{b(R)^2}{R^2 g^2} + \frac{1}{2}\frac{\left(\frac{d}{dR}s(R)\right)^2}{g^2}
$$

$$
+ \frac{1}{2}\frac{b(R)^2\,p(R)^2}{R^2 g^2} + \frac{1}{2}\frac{s(R)^2}{R^2 g^2} + \frac{s(R)\left(\frac{d}{dR}s(R)\right)}{R\,g^2}
$$

$$
+ \frac{\left(\frac{d}{dR}s(R)\right)b(R)\,p(R)}{R\,g^2} - \frac{\left(\frac{d}{dR}b(R)\right)s(R)\,p(R)}{R\,g^2} + \frac{1}{2}\frac{p(R)^2}{R^4 g^2}
$$

Maple output 1: Hamiltonian density, D=2.

Yang-Mills in $D = 3$ (1)

- Consider the vector $SU(2)$ Yang-Mills field $A^a_\mu(x^\nu)$,

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2,$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc}A^b_\mu A^c_\nu,$$

$$D = 3, \quad \mu,\nu = 0,1,2,3 \quad a,b,c = 1,2,3, \quad g{-}const.$$

- Generic ansatz for $D = 3$ YM solitons:

$$A^a_0 = \frac{x^a}{R} q(R);$$

$$gA^a_i = \varepsilon_{iak}\frac{x_k}{R^2}s(R) +$$

$$+ \frac{b(R)}{R^3}(\delta_{ia}R^2 - x_i x_a) + \frac{p(R)x_i x_a}{R^4}.$$

$$i,k = 1,2,3 \qquad R^2 = x^2 + y^2 + z^2.$$

Yang-Mills in $D=3$ (2)

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> H_YM_3D;
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$$\frac{\left(\frac{d}{dR}\mathrm{b}(R)\right)^2}{R^2}+\frac{\left(\frac{d}{dR}\mathrm{s}(R)\right)^2}{R^2}+\frac{2\,\mathrm{s}(R)^3}{R^4}+\frac{2\,\mathrm{s}(R)\,\mathrm{b}(R)^2}{R^4}$$
$$-\frac{2\left(\frac{d}{dR}\mathrm{b}(R)\right)\mathrm{s}(R)\,\mathrm{p}(R)}{R^4}+\frac{1}{2}\frac{\mathrm{b}(R)^4}{R^4}+\frac{2\,\mathrm{b}(R)\,\mathrm{p}(R)\left(\frac{d}{dR}\mathrm{s}(R)\right)}{R^4}$$
$$+\frac{2\,\mathrm{s}(R)^2}{R^4}-\frac{2\,\mathrm{p}(R)\left(\frac{d}{dR}\mathrm{b}(R)\right)}{R^4}+\frac{\mathrm{b}(R)^2\,\mathrm{s}(R)^2}{R^4}+\frac{1}{2}\frac{\mathrm{s}(R)^4}{R^4}+\frac{\mathrm{p}(R)^2}{R^6}$$
$$+\frac{\mathrm{s}(R)^2\,\mathrm{p}(R)^2}{R^6}+\frac{\mathrm{b}(R)^2\,\mathrm{p}(R)^2}{R^6}+\frac{2\,\mathrm{p}(R)^2\,\mathrm{s}(R)}{R^6}+\frac{3}{4}\frac{\mathrm{q}(R)^2\left(\frac{d}{dR}\mathrm{p}(R)\right)^2}{R^2}$$
$$+\frac{1}{4}\mathrm{q}(R)^2+\frac{3}{2}\left(\frac{d}{dR}\mathrm{q}(R)\right)^2+\frac{3\,(\mathrm{s}(R)+1)^2\,\mathrm{q}(R)^2}{R^2}$$

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Maple output 2: Hamiltonian density, D=3, no gauge fixing.

Apply Lorentz gauge $\partial_\mu A_\mu = 0$

Now apply Lorentz gauge $\partial_\mu A_\mu = 0$.

For D=2 Hamiltonian density takes the form:

$$\mathcal{H}_{sol} = \frac{1}{2g^2}\Big[\Big(\frac{ds}{dR}+\frac{s}{R}+\frac{p}{R^3}\frac{dp}{dR}\Big)^2+\frac{1}{R^2}\Big(\frac{d^2p}{dR^2}-\frac{p}{R}(s+\frac{1}{R})\Big)^2\Big] \tag{1}$$

For D=3 Hamiltonian density reads:

$$\mathcal{H}_{sol} = \frac{1}{g^2}\Big\{\frac{1}{32\,R^4}\Big[\Big(\frac{dp}{dR}\Big)^2 + 8s + 4s^2\Big]^2 + \Big[\frac{p\,(s+1)}{R^3} - \frac{1}{2R}\frac{d^2p}{dR^2}\Big]^2 + \Big[\frac{1}{R}\frac{ds}{dR} + \frac{1}{2R^3}\frac{dp}{dR}p\Big]^2 + \Big[\frac{3}{4}\frac{\left(\frac{dp}{dR}\right)^2}{R^2} + \frac{1}{4} + \frac{3(s(R)+1)^2}{R^2}\Big]q(R)^2 + \frac{3}{2}\Big(\frac{dq}{dR}\Big)^2\Big\} \tag{2}$$

$\Rightarrow$ Numerical search for localized solutions is in progress. We plan to start with Monte-Carlo simulations.

No-Go Theorems, Coleman & Co. (1)

Coleman's study: let $A_\mu^a(x)$ - classical localized solution. Make transformatons

$$
\begin{aligned}
A_\lambda(x) &= \lambda A_k(\lambda x), \\
A_0^a(x_k;\sigma,\lambda) &= \sigma\lambda A_0{}^a(\lambda x_k), \\
A_i^a(x_k;\sigma,\lambda) &= \lambda A_i{}^a(\lambda x_k).
\end{aligned} \tag{1}
$$

Denote

$$
\begin{aligned}
H_1 &= \frac{1}{2}\int d^{\mathcal{D}}x(F_{0i}^a)^2 \\
&= \frac{1}{2}\int d^{\mathcal{D}}x(\partial_i A_0^a + e c^{abc} A_0^b A_i^c)^2,
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
H_2 &= \frac{1}{2}\int d^{\mathcal{D}}x(F_{ij}^a)^2 \\
&= \frac{1}{4}\int d^{\mathcal{D}}x(\partial_j A_i^a + e c^{abc} A_i^b A_j^c)^2.
\end{aligned} \tag{3}
$$

Then under transformation (1)

$$
H(\sigma,\lambda) = \sigma^2\lambda^{(4-\mathcal{D})}H_1 + \lambda^{(4-\mathcal{D})}H_2 \ .
$$

No-Go Theorems, Coleman & Co. (2)

Requiring stationarity:

$$\frac{\partial H}{\partial \sigma} = 0, \quad \frac{\partial H}{\partial \lambda} = 0 \quad \text{at} \quad \sigma = 1, \quad \lambda = 1,$$

Coleman has found for $D \neq 4:$ $\quad H_1 = H_2 = 0$.

For $\mathcal{D} \neq 4$ from here: $F^a_{\mu\nu} = 0$, $Q.E.D.$

Coleman's conclusion was:

"There are no classical glueballs".

⇒ Thus, Coleman has shown that there are no minima of Hamilonian in extended space of variables,corresponding to non-fixed gauge fields and including nonphysical degrees of freedom. E.g. fixing the Lorentz gauge, we get the physical space of dynamical variable, whose dimensionality is less then that of extended space of gauge field without gauge fixing.

⇒ In such physical space the existence of minima is not forbidden. Hence we can hope that $3DYM$ solitons exist.

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