PRD71(2005), JPG32(2006),
PRD77(2008), NPBPS186(2009)

THE EFFECTS DUE TO HADRONIZATION IN THE INCLUSIVE TAU LEPTON DECAY

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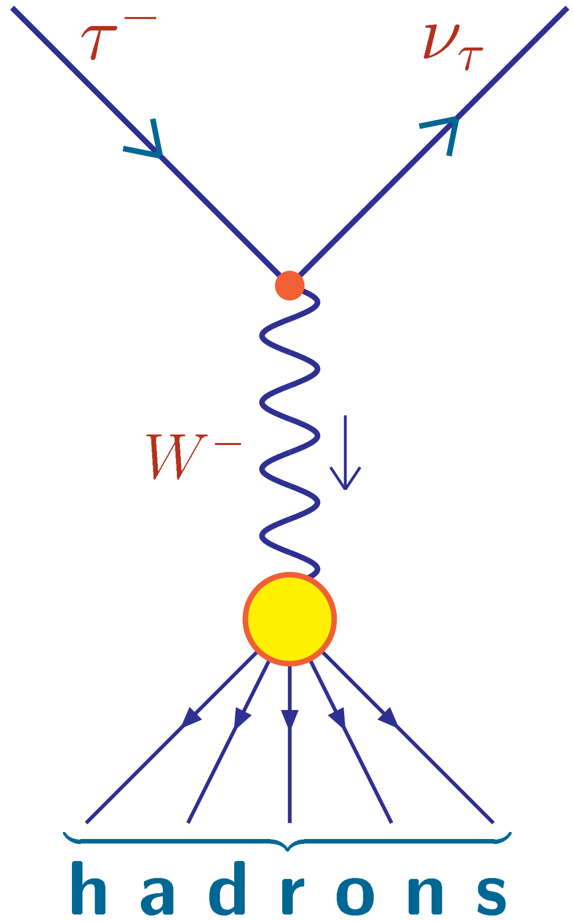
Eleventh Workshop on Nonperturbative Quantum Chromodynamics
Paris, France, 6 – 10 June 2011

INTRODUCTION

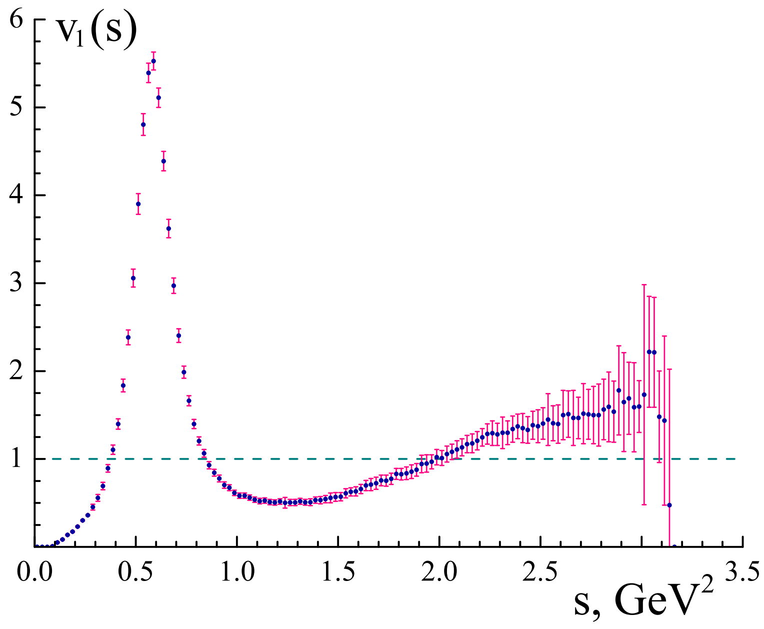
The τ lepton is the only lepton which is heavy enough ($M_\tau \simeq 1.777\,\mathrm{GeV}$) to decay into hadrons. The interest to this process is primarily due to

- Precise experimental data
- Tests of QCD and Standard Model
- No need in phenomenological models
- Probes infrared hadron dynamics



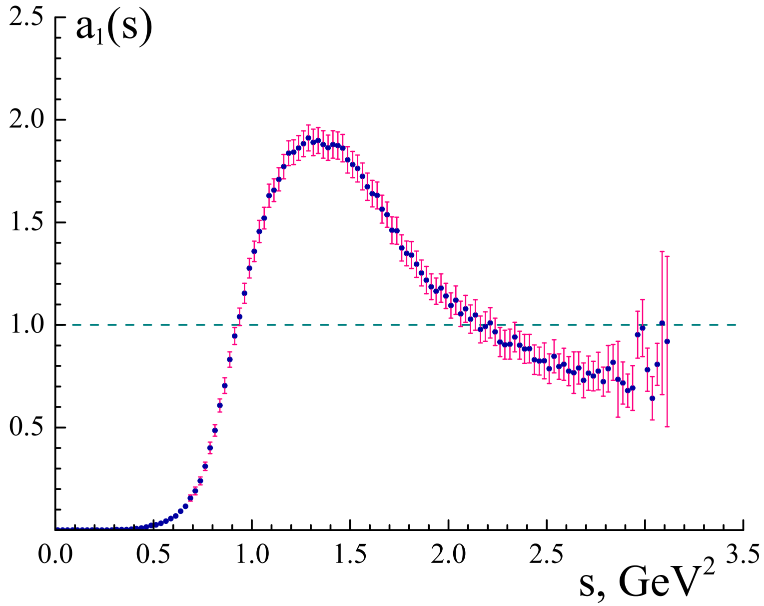
The experimentally measurable quantity here is

$$R_\tau = \frac{\Gamma(\tau^- \to \text{hadrons}^- \, \nu_\tau)}{\Gamma(\tau^- \to e^- \, \bar{\nu}_e \, \nu_\tau)}$$
$$= R_{\tau,\mathbf{V}} + R_{\tau,\mathbf{A}} + R_{\tau,\mathbf{S}}$$
$$= 3.642 \pm 0.012,$$



$$R_{\tau,\mathbf{V}} = R^{J=0}_{\tau,\mathbf{V}} + R^{J=1}_{\tau,\mathbf{V}}$$
$$= 1.787 \pm 0.011 \pm 0.007,$$

$$R_{\tau,\mathbf{A}} = R^{J=0}_{\tau,\mathbf{A}} + R^{J=1}_{\tau,\mathbf{A}}$$
$$= 1.695 \pm 0.011 \pm 0.007.$$



■ *ALEPH Collab., EPJC4(1998), PR421(2005), RMP78(2006).*

THEORETICAL DESCRIPTION

The theoretical prediction for the quantities on hand reads

$$R_{\tau,\mathbf{V/A}}^{J=1} = \frac{N_{\mathbf{c}}}{2}\,|V_{\mathbf{ud}}|^2\,S_{\mathbf{EW}}\Big(\Delta_{\mathbf{QCD}}^{V/A} + \delta'_{\mathbf{EW}}\Big).$$

In this equation $N_{\mathbf{c}} = 3$**,** $|V_{\mathbf{ud}}| = 0.9738 \pm 0.0005$**,** $\delta'_{\mathbf{EW}} = 0.0010$**,** $S_{\mathbf{EW}} = 1.0194 \pm 0.0050$**,** $M_\tau = 1.777\,\mathrm{GeV}$**, and**

$$\Delta_{\mathbf{QCD}}^{V/A} = 2\int_0^{M_\tau^2} f\left(\frac{s}{M_\tau^2}\right) R^{V/A}(s)\,\frac{ds}{M_\tau^2}\,,$$

where $f(x) = (1-x)^2\,(1+2x)$**,**

$$R^{V/A}(s) = \frac{1}{2\pi i}\lim_{\varepsilon\to 0_+}\Big[\Pi^{V/A}(s+i\varepsilon) - \Pi^{V/A}(s-i\varepsilon)\Big] = \frac{1}{\pi}\,\mathrm{Im}\lim_{\varepsilon\to 0_+}\Pi^{V/A}(s+i\varepsilon).$$

- ***Braaten, Narison, Pich, NPB373(1992).***

It is convenient to perform the theoretical analysis of τ lepton hadronic decay in terms of the Adler function

$$D(Q^2) = -\frac{d\,\Pi(-Q^2)}{d\ln Q^2}, \qquad Q^2 = -q^2 = -s.$$

■ ***Adler, PRD10(1974).***

Its ℓ–loop perturbative approximation reads

$$D^{(\ell)}_{\mathbf{pert}}(Q^2) \simeq 1 + \sum_{j=1}^{\ell} d_j \left[\alpha_{\mathbf{s}}^{(\ell)}(Q^2)\right]^j, \qquad Q^2 \to \infty,$$

where $\alpha_{\mathbf{s}}^{(\ell)}(Q^2)$ is the ℓ–loop perturbative running coupling; $\alpha_{\mathbf{s}}^{(1)}(Q^2) = 4\pi/[\,\beta_0\,\ln(Q^2/\Lambda^2)]$, $d_1 = 1/\pi$.

■ ***Gorishny, Kataev, Larin, PLB259(1991); Surguladze, Samuel, PRL66(1991); Baikov, Chetyrkin, Kuhn, PRL101(2008).***

For functions $D(Q^2)$ and $R(s)$ the following relations hold:

$$D(Q^2) = Q^2 \int_{m^2}^{\infty} \frac{R(s)}{(s+Q^2)^2}\,ds, \qquad R(s) = \frac{1}{2\pi i} \lim_{\varepsilon\to 0_+} \int_{s+i\varepsilon}^{s-i\varepsilon} D(-\zeta)\,\frac{d\zeta}{\zeta}.$$

MASSLESS LIMIT

Method I: Use of definitions of $D(Q^2)$ and $R(s)$ only

In what follows it is convenient to handle the “tree–level” terms separately from the strong corrections:

$$D(Q^2) = d^{(0)}(Q^2) + d^{(1)}(Q^2), \qquad R(s) = r^{(0)}(s) + r^{(1)}(s).$$

The quantities Δ_{QCD} can be represented as follows:

$$\Delta_{\text{QCD}} = g(1)\, R(M_\tau^2) + \int_0^{M_\tau^2} g\left(\frac{s}{M_\tau^2}\right) \rho(s)\, \frac{ds}{s},$$

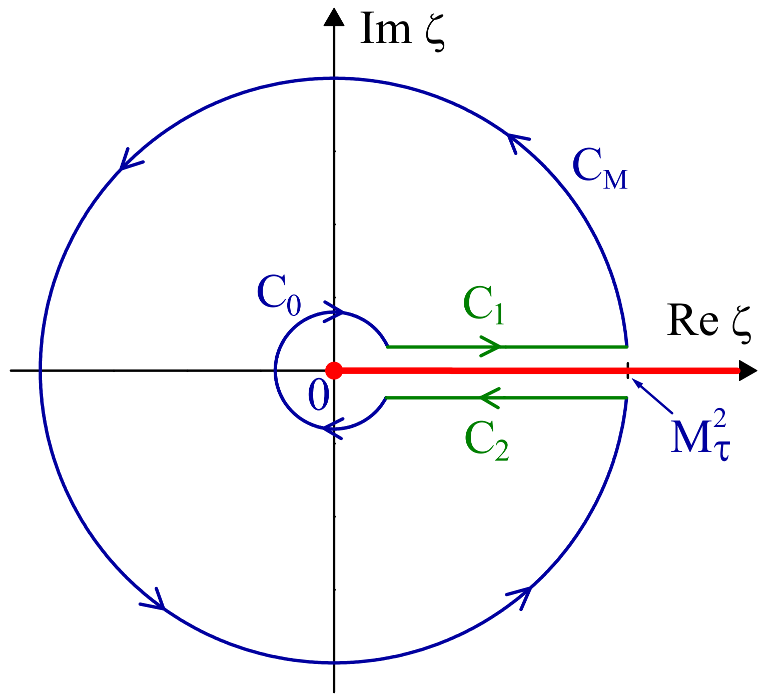
where $g(x) = x(2 - 2x^2 + x^3)$ and

$$\rho(s) = \frac{1}{2\pi i} \lim_{\varepsilon \to 0_+} \left[D(-s - i\varepsilon) - D(-s + i\varepsilon)\right]$$

is the so–called spectral density.

Method II: Additional use of Cauchy theorem

Similarly to previous case, $\Delta_{\rm QCD}$ can be rewritten as the sum of two integrals along the edges of physical cut of $\Pi(q^2)$:



$$\Delta_{\rm QCD} = \frac{1}{\pi i}\int_{0+i\varepsilon}^{M_\tau^2+i\varepsilon} f\left(\frac{\zeta}{M_\tau^2}\right)\Pi(\zeta)\,\frac{d\zeta}{M_\tau^2} + \frac{1}{\pi i}\int_{M_\tau^2-i\varepsilon}^{0-i\varepsilon} f\left(\frac{\zeta}{M_\tau^2}\right)\Pi(\zeta)\,\frac{d\zeta}{M_\tau^2}.$$

$$\Delta_{\rm QCD} = \frac{i}{\pi}\left[\int_{C_0} f\left(\frac{\zeta}{M_\tau^2}\right)\Pi(\zeta)\,\frac{d\zeta}{M_\tau^2} + \int_{C_M} f\left(\frac{\zeta}{M_\tau^2}\right)\Pi(\zeta)\,\frac{d\zeta}{M_\tau^2}\right]$$

$$= \frac{1}{2\pi}\lim_{\varepsilon\to 0_+}\int_{-\pi+\varepsilon}^{\pi-\varepsilon} D\left(M_\tau^2\, e^{i\theta}\right)\left(1+2e^{i\theta}-2e^{i3\theta}-e^{i4\theta}\right)d\theta.$$

All the mentioned above is valid for the massless limit of "true physical" functions $\Pi_{\mathrm{phys}}(q^2)$ and $D_{\mathrm{phys}}(Q^2)$. However, one has to deal with their perturbative approximations, which are inconsistent with dispersion relations for these functions.

THE PERTURBATIVE RESULTS

NEED TO BE MERGED WITH

RELEVANT DISPERSION RELATIONS

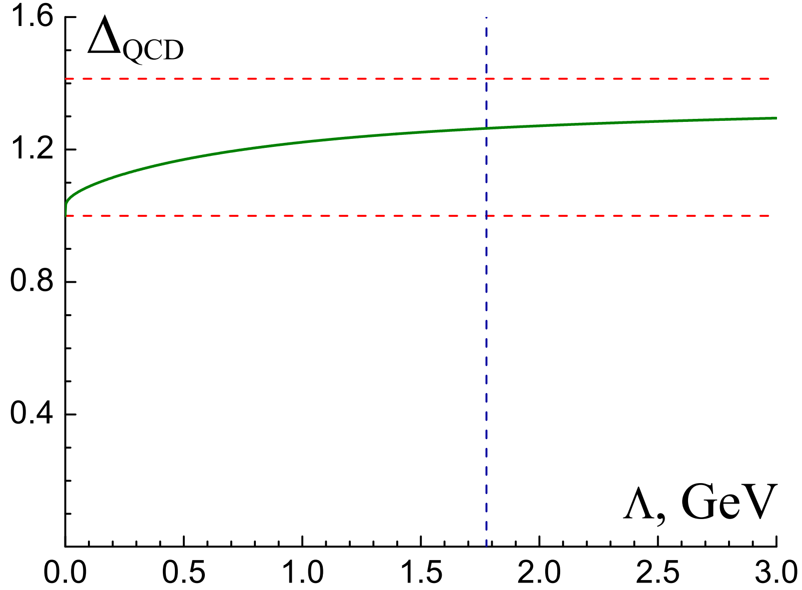
Hadronic decay of τ lepton: the only available option here is to directly use in the obtained expressions for Δ_{QCD} perturbative approximations $\Pi_{\mathrm{pert}}(q^2)$ and $D_{\mathrm{pert}}(Q^2)$.

Method I + one–loop QCD:

$$\Delta_{\mathbf{QCD}} = 1 + \frac{4}{\beta_0}\int_0^{\infty} h\left(\frac{\sigma}{M_\tau^2}\right) \rho^{(1)}_{\mathbf{pert}}(\sigma)\frac{d\sigma}{\sigma},$$

$$h(x) = g(x)\,\theta(1-x) + g(1)\,\theta(x-1),$$

$$\rho^{(1)}_{\mathbf{pert}}(\sigma) = \left[\ln^2(\sigma/\Lambda^2) + \pi^2\right]^{-1}.$$



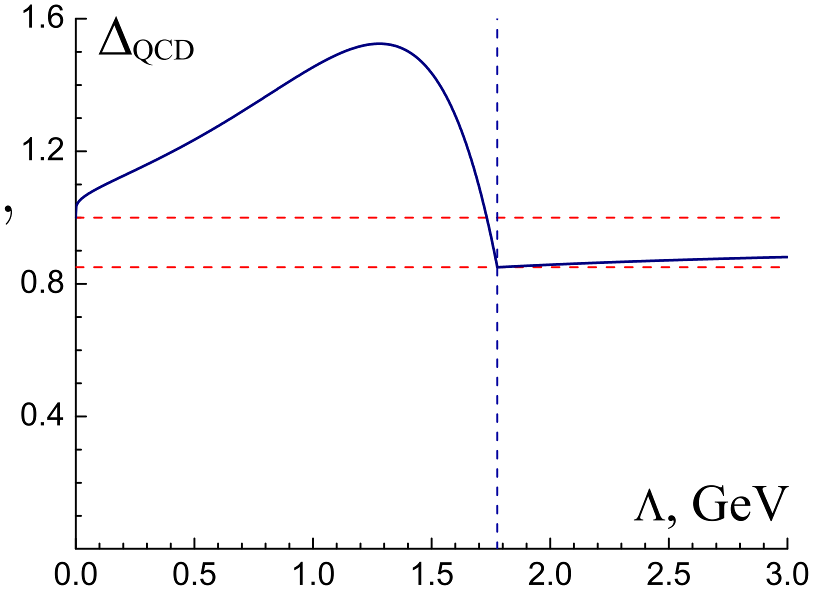
Method II + one–loop QCD:

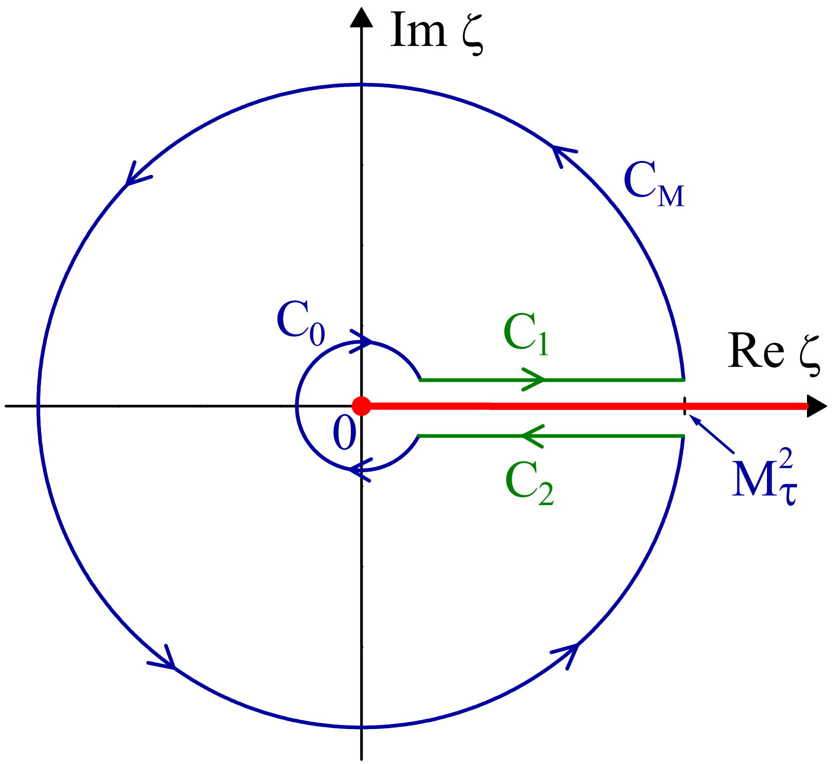
$$\Delta_{\mathbf{QCD}} = 1 + \frac{4}{\beta_0}\int_0^{\pi} \frac{a_0\,A_1(\theta) + \theta\,A_2(\theta)}{\pi(a_0^2+\theta^2)}\,d\theta,$$

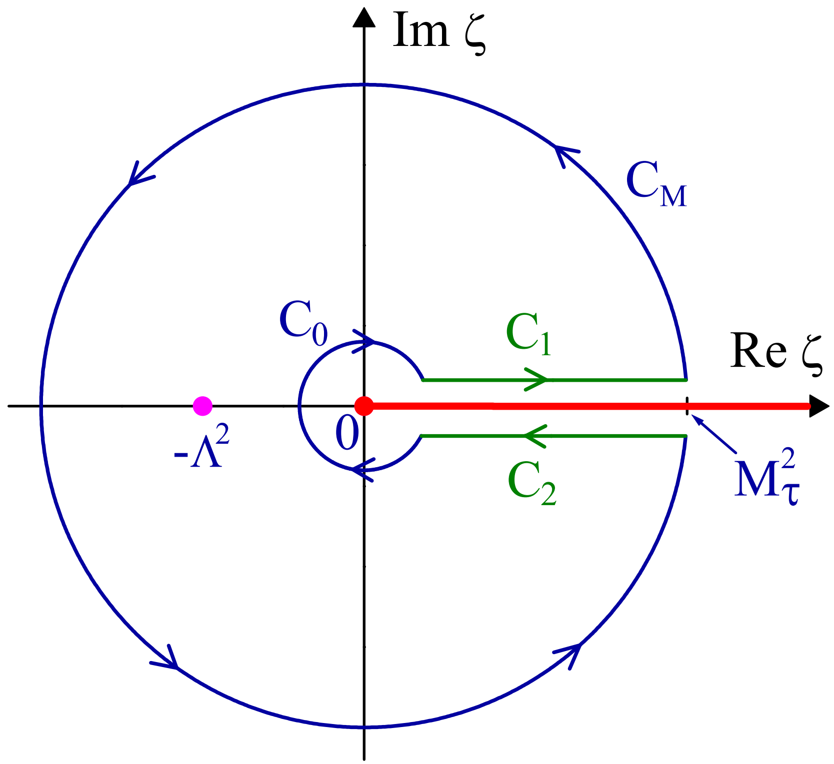
$$A_1(\theta) = 1 + 2\cos(\theta) - 2\cos(3\theta) - \cos(4\theta),$$

$$A_2(\theta) = 2\sin(\theta) - 2\sin(3\theta) - \sin(4\theta),$$

$$a_0 = 4\pi/\left[\beta_0\,\alpha^{(1)}_{\mathbf{pert}}(M_\tau^2)\right].$$







Unknown "true physical" Adler function $D_{\mathrm{phys}}(Q^2)$:

The use of any of two integration contours would have led to the same result.

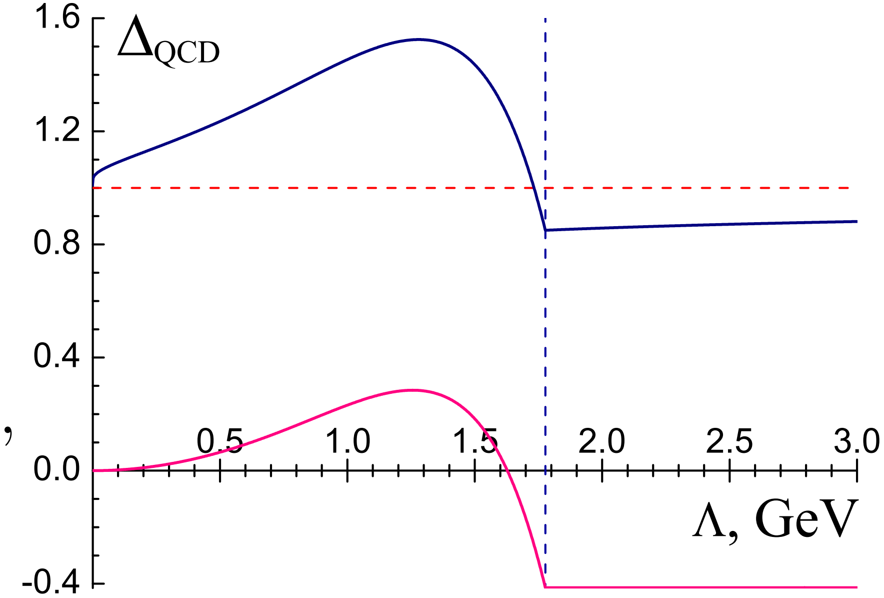
One–loop perturbative Adler function $D^{(1)}_{\mathrm{pert}}(Q^2)$:

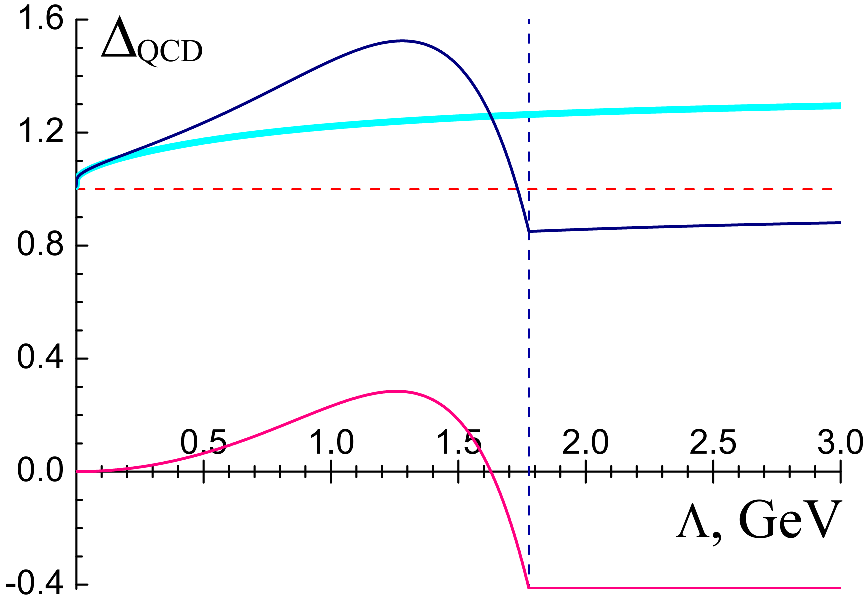
The integration contours used within methods I and II are not equivalent.

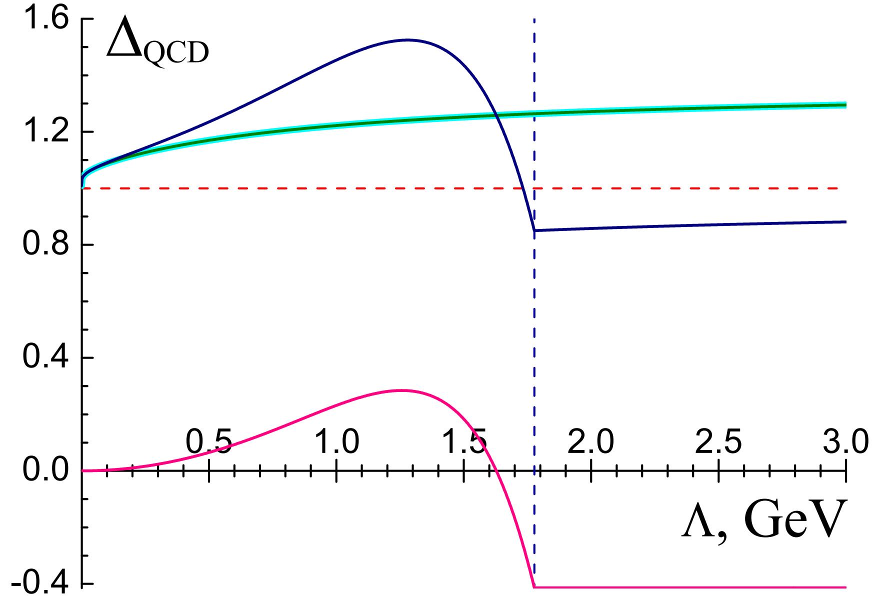
Residue term in method II:

$$\Delta_{\mathbf{res}} = \frac{4}{\beta_0} h_1\!\left(\frac{\Lambda^2}{M_\tau^2}\right), \quad \textbf{where}$$

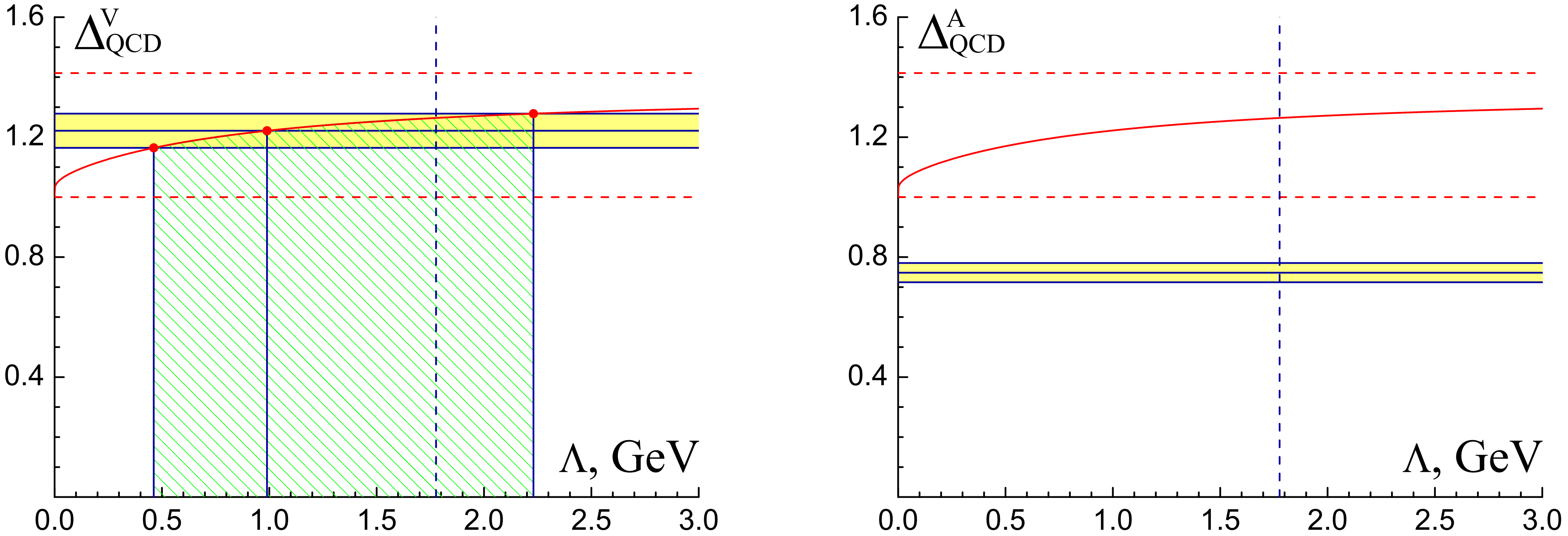
$$h_1(x) = h_2(x)\theta(1-x) + h_2(1)\theta(x-1),$$
$$h_2(x) = x(2 - 2x^2 - x^3).$$







Method I: One solution for V-channel, none for A-channel



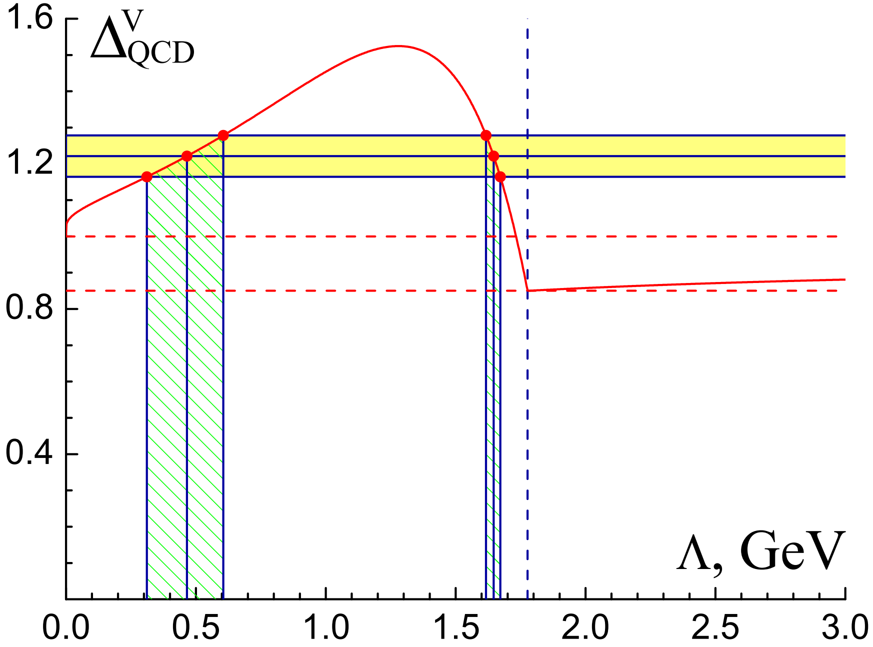
$$\Lambda = \left(989^{+1242}_{-527}\right)\,\mathrm{MeV}$$

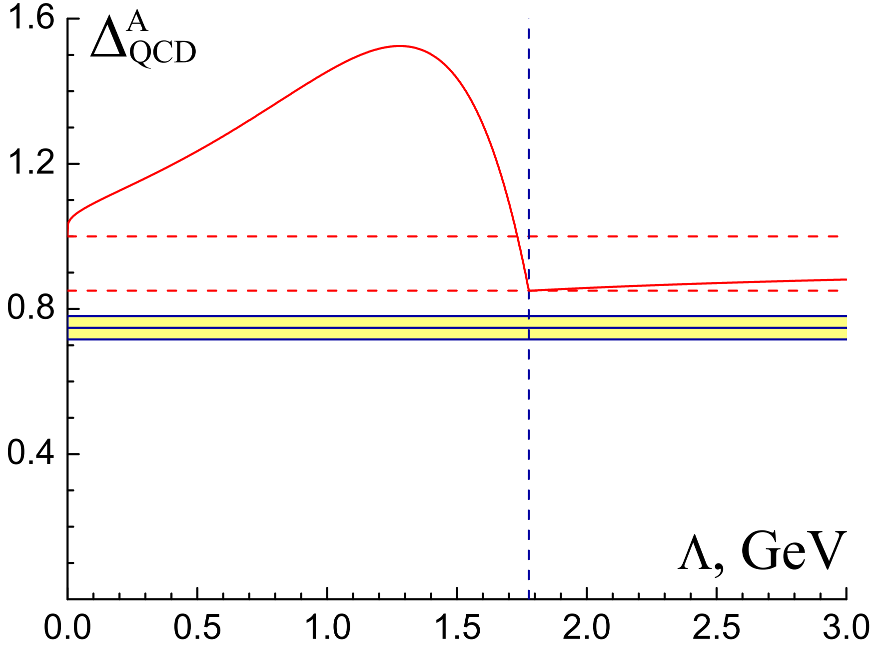
no solution

From ALEPH data: $\Delta^{V}_{\mathbf{exp}} = 1.221 \pm 0.057$, $\Delta^{A}_{\mathbf{exp}} = 0.748 \pm 0.032$.

In the framework of perturbative approach vector and axial–vector channels are indistinguishable: $\Delta^{V}_{\mathbf{pert}} \equiv \Delta^{A}_{\mathbf{pert}}$.

Method II: Two solutions for V-channel, none for A-channel





$$\Lambda = (458 \pm 147)\,\text{MeV}$$

$$\Lambda = (1644 \pm 27)\,\text{MeV}$$

no solution

DISPERSIVE APPROACH TO QCD

The dispersion relation imposes a number of stringent nonperturbative constraints on Adler function:

$$D(Q^2) = Q^2 \int_{m^2}^{\infty} \frac{R(s)}{(s+Q^2)^2}\, ds$$

- **Since** $R(s)$ **assumes finite values and** $R(s) \to \mathrm{const}$ **when** $s \to \infty$**, then** $D(Q^2) = 0$ **at** $Q^2 = 0$ **(valid for** $m \neq 0$ **only)**
- **Adler function possesses the only cut** $Q^2 \leq -m^2$ **along the negative semiaxis of real** Q^2

BASIC IDEA: **merge perturbative approximation for Adler function with these nonperturbative constraints.**

$$D(Q^2) = d^{(0)}(Q^2) + \frac{Q^2}{Q^2+m^2} \int_{m^2}^{\infty} \rho(\sigma)\, \frac{\sigma - m^2}{\sigma + Q^2}\, \frac{d\sigma}{\sigma}\,,$$

$$R(s) = r^{(0)}(s) + \theta(s-m^2) \int_{s}^{\infty} \rho(\sigma)\, \frac{d\sigma}{\sigma}\,.$$

- ***Nesterenko, Papavassiliou, PRD71(2005); JPG32(2006); NPBPS164(2007).***

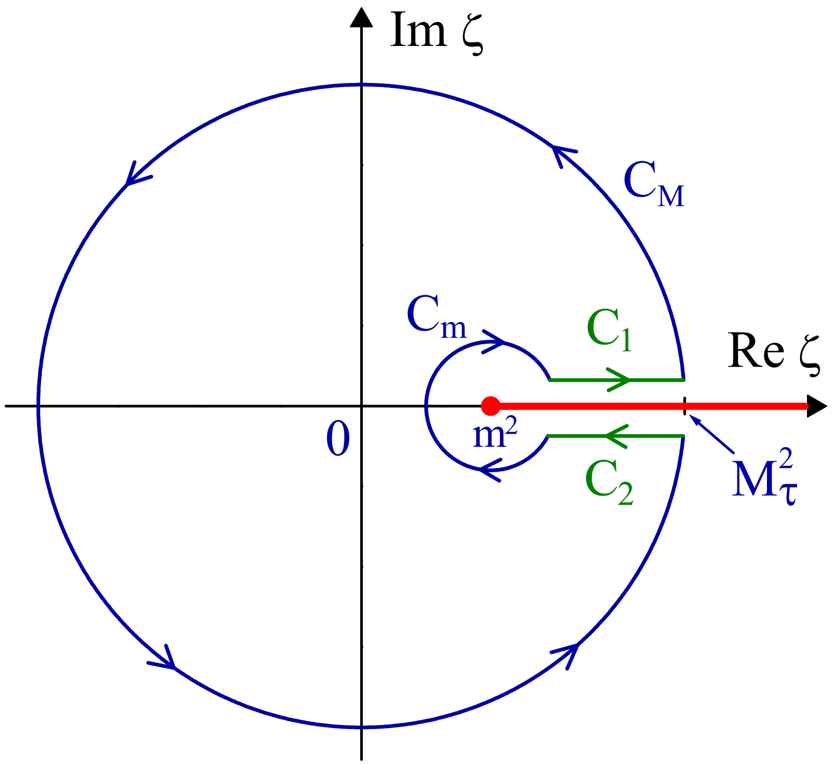
In the limit $m = 0$ these expressions become identical to those of the so–called Analytic perturbation theory:

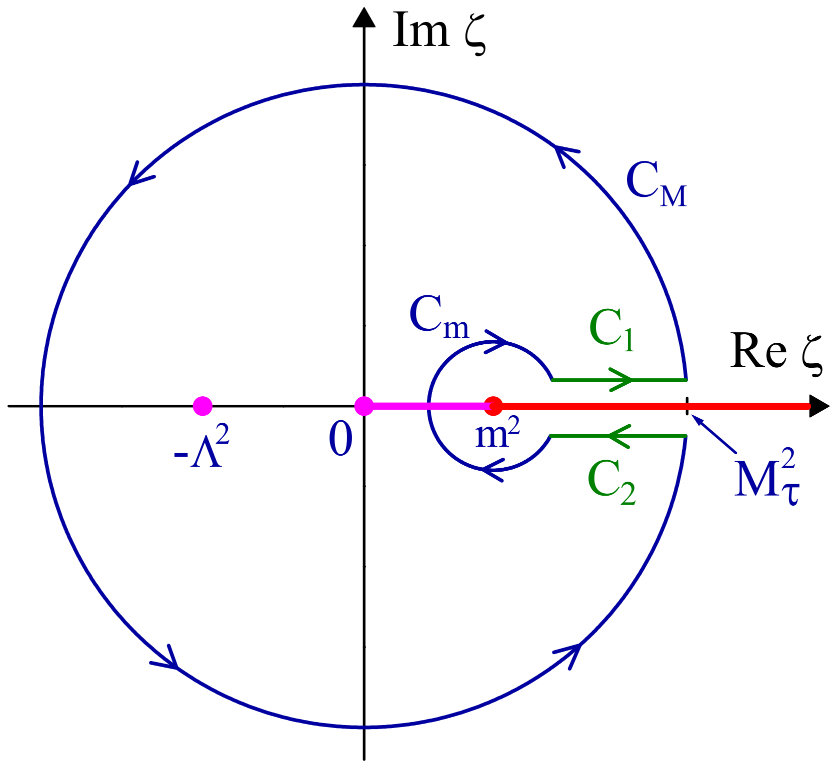
- ***Shirkov, Solovtsov, PRL79(1997); TMP150(2007).***

Nonperturbative model for the spectral density:

$$\rho(\sigma) = \frac{1}{\ln^2(\sigma/\Lambda^2) + \pi^2} + 2\,\frac{\Lambda^2}{\sigma}\,.$$

- ***Nesterenko, PRD62(2000); PRD64(2001).***





Unknown “true physical” Adler function $D_{\mathrm{phys}}(Q^2)$:

The use of any of two integration contours would have led to the same result.

One–loop perturbative Adler function $D^{(1)}_{\mathrm{pert}}(Q^2)$:

Method I is compatible with perturbative input, whereas method II is not.

ABRUPT KINEMATIC THRESHOLD

Here, the parton model prediction for $R(s)$ is approximated by the step–function:

$$r^{(0)}_{\mathbf{V/A}}(s) = \theta\left(1 - \frac{m^2_{\mathbf{V/A}}}{s}\right) \quad \longleftrightarrow \quad d^{(0)}_{\mathbf{V/A}}(Q^2) = \frac{Q^2}{Q^2 + m^2_{\mathbf{V/A}}}.$$

- ***Feynman (1972).***

Eventually this leads to the following expression for $\Delta^{V/A}_{\mathbf{QCD}}$:

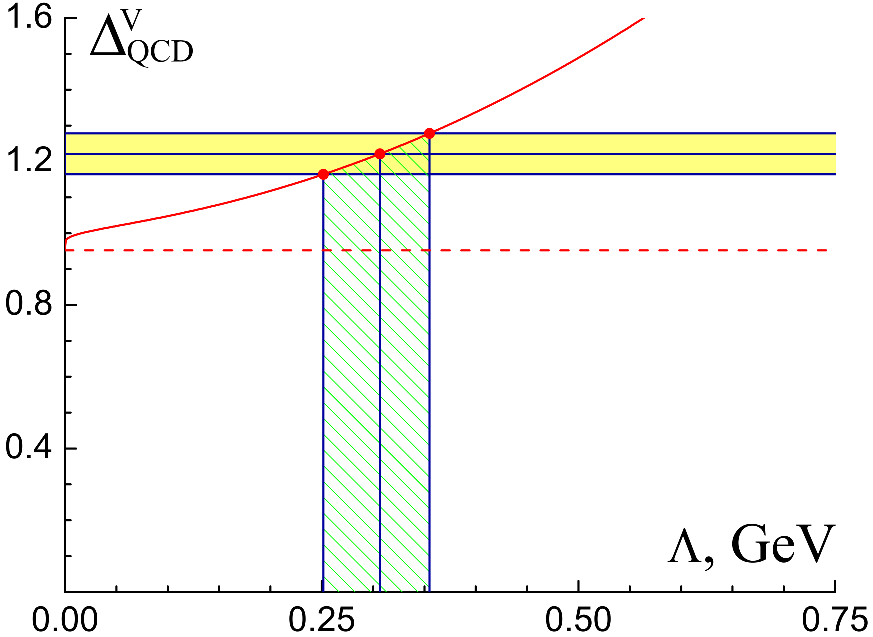
$$\Delta^{V/A}_{\mathbf{QCD}} = 1 - \zeta_{\mathbf{V/A}}\left(2 - 2\zeta^2_{\mathbf{V/A}} + \zeta^3_{\mathbf{V/A}}\right) + \int_{m^2_{\mathbf{V/A}}}^{\infty} H\left(\frac{\sigma}{M^2_\tau}\right)\rho(\sigma)\,\frac{d\sigma}{\sigma},$$

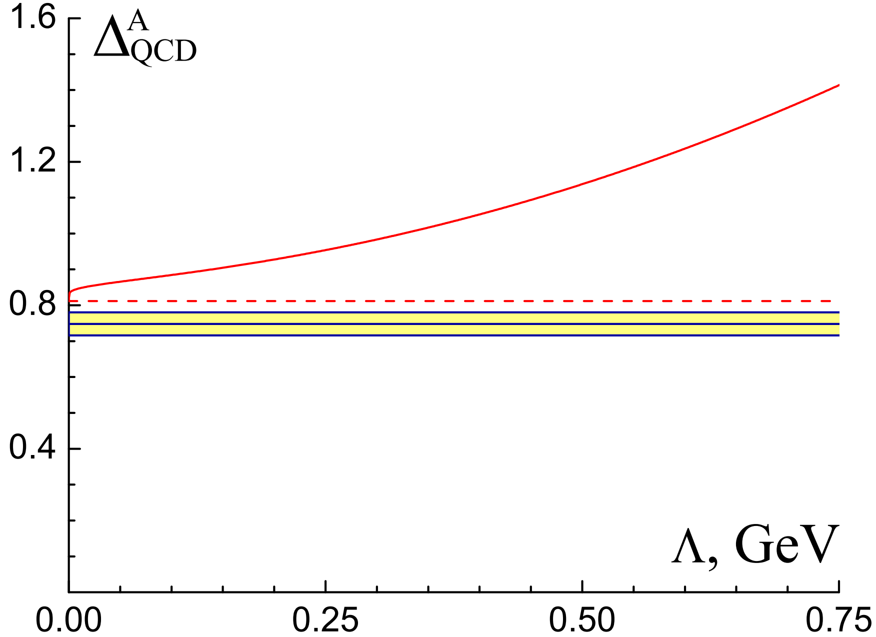
where $\zeta_{\mathbf{V/A}} = m^2_{\mathbf{V/A}}/M^2_\tau$ **and**

$$H(x) = g(x)\,\theta(1-x) + g(1)\,\theta(x-1) - g\big(\zeta_{\mathbf{V/A}}\big).$$

- ***Nesterenko, Papavassiliou, JPG32(2006); Nesterenko, NPBPS186(2009).***

Result: one solution for V-channel, none for A-channel





$$\Lambda = \left(304 \pm 51\right) \,\mathrm{MeV}$$

no solution

SMOOTH KINEMATIC THRESHOLD

In this case the parton model prediction for $R(s)$ reads

$$r^{(0)}_{\mathbf{V/A}}(s) = \left(1 - \frac{m^2_{\mathbf{V/A}}}{s}\right)^{3/2} \longleftrightarrow d^{(0)}_{\mathbf{V/A}}(Q^2) = 1 + \frac{3}{z} + \frac{3u(z)}{2z} \ln\left[1 + 2z\left(1 - u(z)\right)\right],$$

where $u(z) = \sqrt{1 + \frac{1}{z}}$**,** $z = Q^2/m^2_{\mathbf{V/A}}$**.**

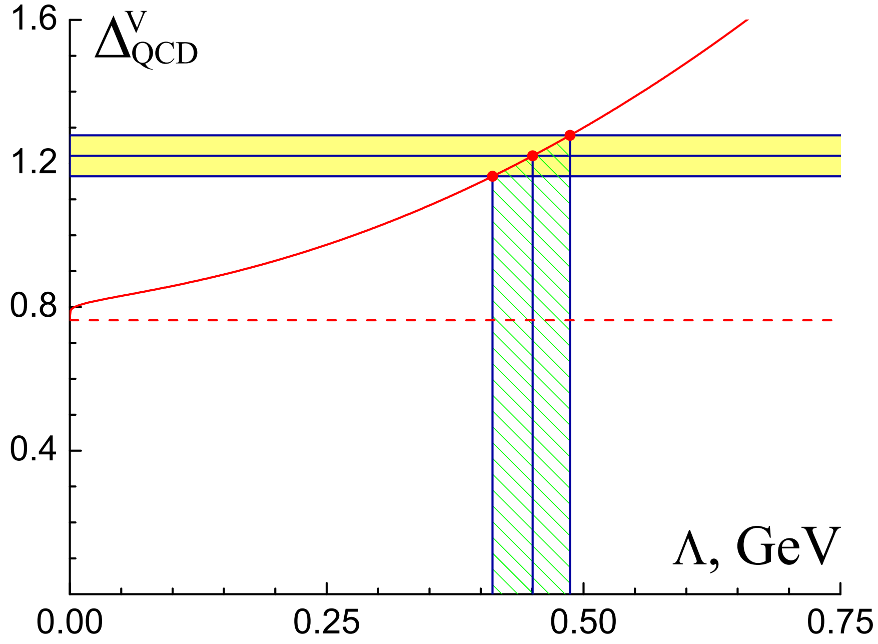
- ***Feynman PR76(1949).***

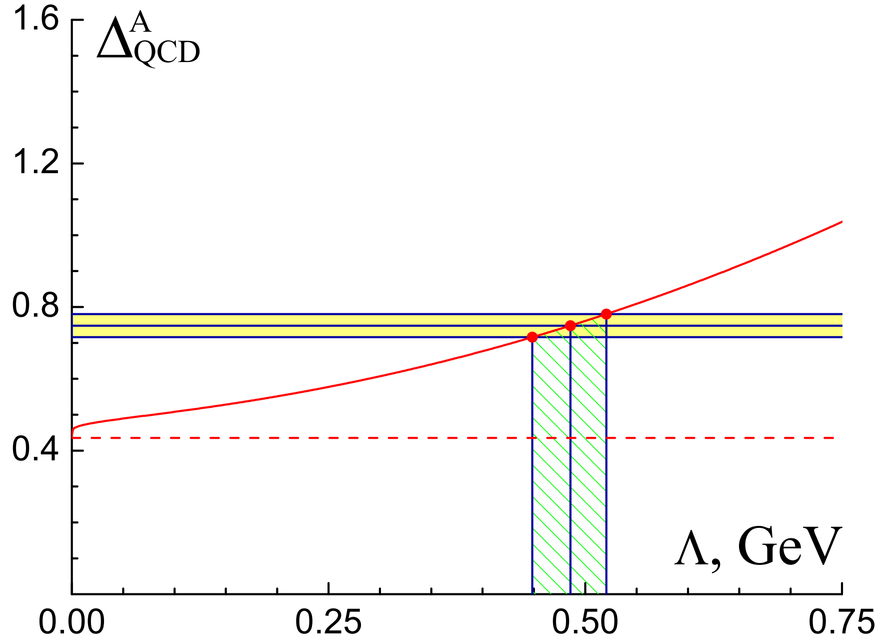
In turn, the prediction for $\Delta^{V/A}_{\mathbf{QCD}}$ takes the following form:

$$\Delta^{V/A}_{\mathbf{QCD}} = \sqrt{1 - \zeta_{\mathbf{V/A}}}\left(1 + 6\zeta_{\mathbf{V/A}} - \frac{5}{8}\zeta^2_{\mathbf{V/A}} + \frac{3}{16}\zeta^3_{\mathbf{V/A}}\right) + \int_{m^2_{\mathbf{V/A}}}^{\infty} H\left(\frac{\sigma}{M^2_\tau}\right)\rho(\sigma)\frac{d\sigma}{\sigma}$$
$$- 3\zeta_{\mathbf{V/A}}\left(1 + \frac{1}{8}\zeta^2_{\mathbf{V/A}} - \frac{1}{32}\zeta^3_{\mathbf{V/A}}\right)\ln\left[\frac{2}{\zeta_{\mathbf{V/A}}}\left(1 + \sqrt{1 - \zeta_{\mathbf{V/A}}}\right) - 1\right].$$

- ***Nesterenko (2011).***

This results in nearly identical solutions for both channels:

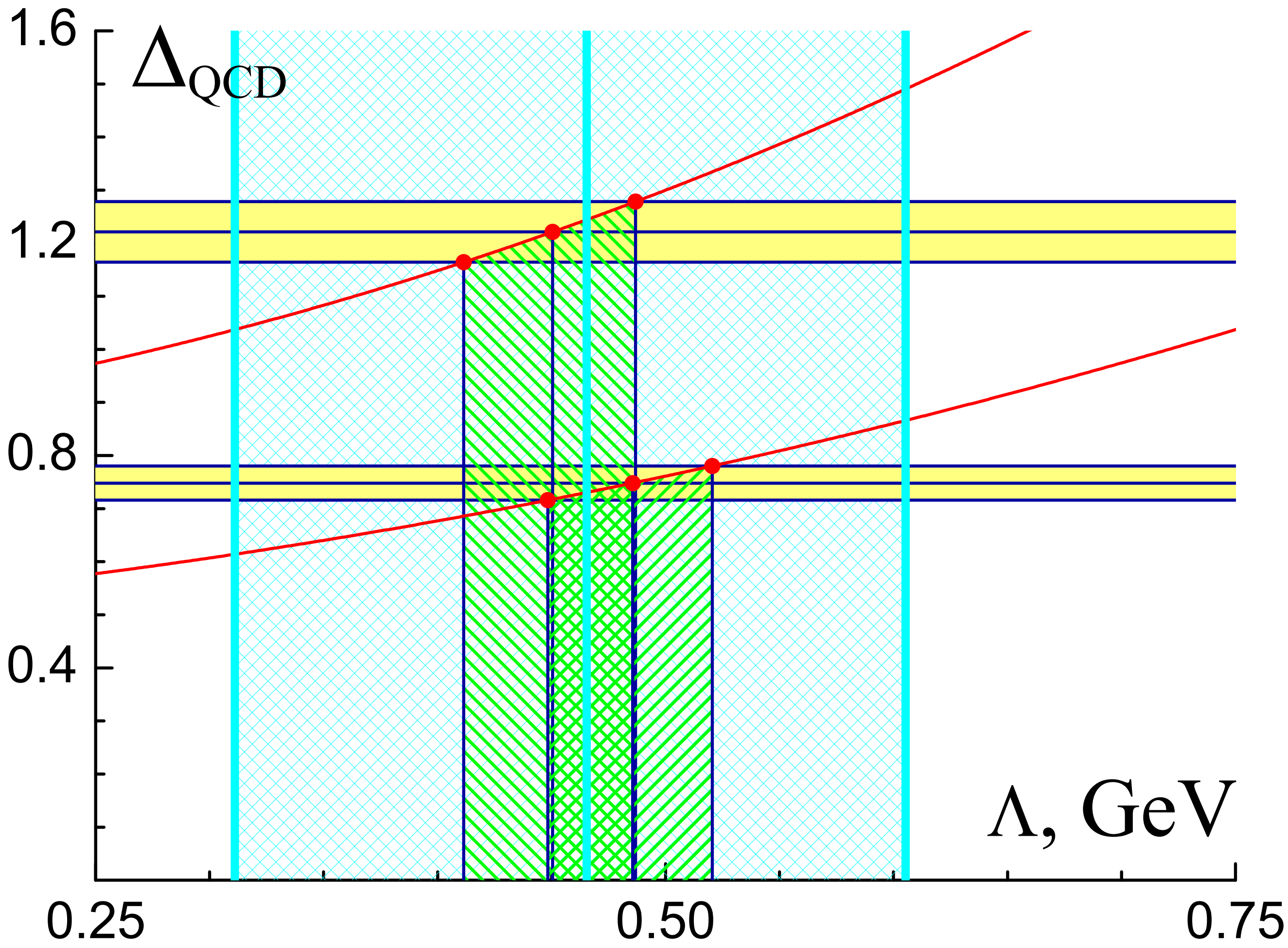




$$\Lambda = \left(449 \pm 38\right) \, \text{MeV}$$

$$\Lambda = \left(484 \pm 36\right) \, \text{MeV}$$

The obtained solutions agree with perturbative one:



Dispersive approach, V-channel: $\Lambda = \left(449 \pm 38\right)$ MeV

Dispersive approach, A-channel: $\Lambda = \left(484 \pm 36\right)$ MeV

Perturbative approach, V-channel: $\Lambda = \left(458 \pm 147\right)$ MeV

SUMMARY

- **Theoretical description of τ lepton hadronic decay is performed in the framework of Dispersive approach to QCD**

- **The significance of effects due to hadronization is demonstrated**

- **The developed approach provides nearly identical values of Λ_{QCD} in vector and axial–vector channels**