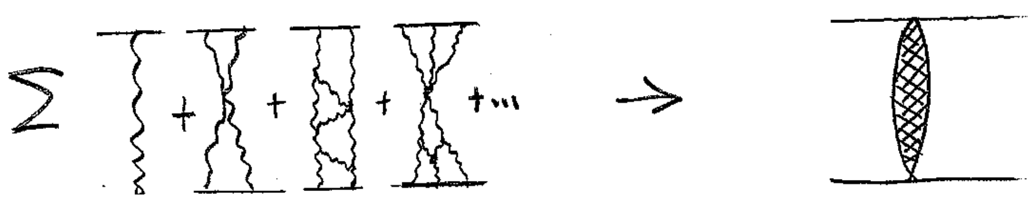
A NEW, ANALYTIC, NON-PERTURBATIVE, GAUGE-INVARIANT FORMULATION OF REALISTIC QCD

Co-Authors: T. Grandou, Y. Gabellini,
Y-M Sheu, M.Gattobigio

- A. Explaining the Title (and what is to come):
- 1. "New" => Less than 2 years old.
- 2. "Realistic" => After non-perturbative sums defining "gluon bundles" are performed, fundamental "transverse imprecision" of Qs and Qbars is necessary.
- 3. "Gluon bundles" => Correspond to the sum of Feynman graphs of an infinite number of gluons exchanged between Qs and/or Qbars (with all cubic and quartic gluon interactions included). "Bundle diagrams" replace Feynman diagrams.



- 4. "Gauge invariance" => All gluon bundle exchanges between Qs and/or Qbars are strictly gauge-invariant (via gauge independence as all intermediary, gauge-dependent gluon propagators exactly cancel). No Fadeev-Popov ghosts needed here!

  5. "Effective Locality" => A new feature of gauge-invariant, non-perturbative sums, wherein each gluon bundle appears to be emitted and absorbed at one space-time point on a Q or Qbar line (modulo transverse imprecision).

2. Quark Binding Potential: $V(r) = \xi\mu(\mu r)^{1+\xi}$,

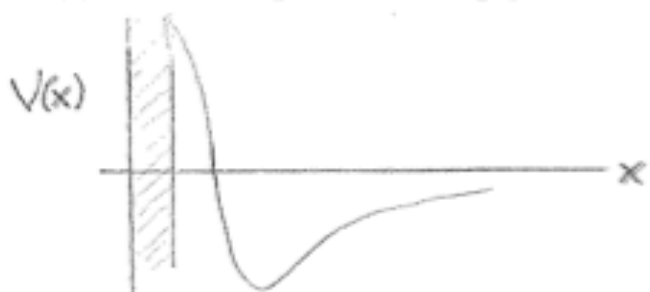
where μ is a mass scale, and ξ appears as a new, fundamental parameter, related to $[Q,Q] \neq 0$ associated with transverse imprecision. ξ is not a coupling constant, and even though small, $\xi \lesssim .1$, it plays an important role in estimating $Q\text{-}\bar{Q}$ binding to form a model "pion".

Using this potential, and a double-minimization, order-of-magnitude technique, we find

$$E_0 \simeq 2m + 3\mu\left(\frac{\xi}{2}\right)^{2/3}\left(\frac{\mu}{m}\right)^{1/3}, \quad \frac{m}{\mu} \approx \left(\frac{\pi^2}{2^3}\right)^{\frac{1}{4}} \xi^{\frac{1}{2}}, \quad m_\pi \simeq 2m + 2\mu\left(\frac{3+\xi}{1+\xi}\right),$$

giving one experimental number, m_π, to fix μ and ξ. One more condition is obtained from

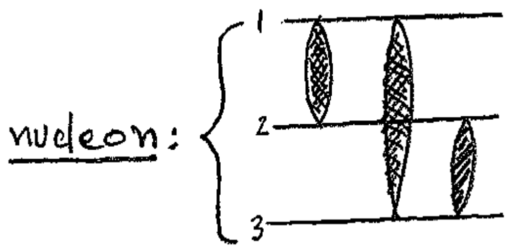
3. Nucleon-nucleon Scattering and Binding Potential: An explicit example of n-p Yukawa-type scattering and binding (to form a deuteron) potential:

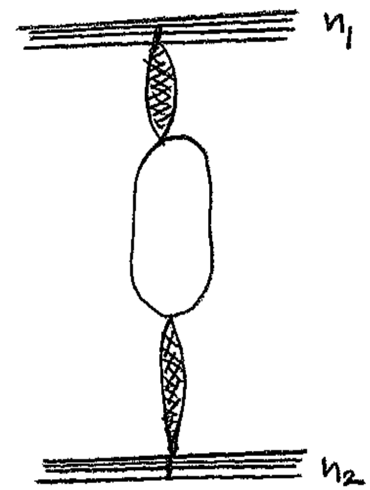


$$x = \mu r$$
$$V(x) = V_0\,(2 - x^2)\,e^{-x^2/2}$$

where the slope in the region outside the (shaded) quark binding region can be identified as $1/m_\pi$, and gives a second relation to determine μ and ξ. The potential must go negative for large r, and can correspond to n-p binding to form a deuteron.

To our knowledge, this is the first example of Nuclear Physics from basic, Realistic QCD. Essential to this process is the exchange of a gluon bundle between the (singlet) quarks comprising one nucleon and a closed-quark, "vacuum" loop, with another gluon bundle exchanged between a different point of the loop and the second nucleon. A single gluon bundle exchanged between these nucleons at $r > 1/m_\gamma$ would have little effect; but the loop can stretch in transverse directions, and the combination of loop plus two gluon bundles provides the potential shown above.





4. References:

#1: EPJC (2010) 65, 395-411 Explains rearrangements to give gauge-invariance in a quenched, eikonal model.

#2: Proof of EL in QCD, without approximations and without exception.
arXiv: hep-th:1003.2936

#3: "Ideal" vs. "Realistic" QCD: Transverse Imprecision.
arXiv: hep-th:1103.4179

#4: Quark Binding Potentials, for pion and nucleon.
arXiv: hep-th:1104.4663

#5: Nucleon-nucleon Scattering and Binding Potentials
arXiv:

6. General Procedures:

a) Use of functional methods, beginning with the Schwinger/Symanzik solution for the Generating Functional (GF) of any QFT, and applied to QCD.

(NOT the usual $\int d[A]$ which suffers from Gribov ambiguities.)

b) A special rearrangement possible for QCD, but not for QED, which guarantees gauge invariance. (Overlooked for decades.) Use of Halpern's FI representation of exp{ (i/4)$\int d^4x F^2$}, $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$.

A second, and convenient, rearrangement of this GF in terms of a functional "linkage operator".

c) How are non-perturbative sums performed?

Schwinger/Symanzik GF solutions involve the functionals G_c[A] and L[A], Both $G_c[A]$ and $L[A]$ have Fradkin functional representations, which are Gaussian in A; and functional linkage operations over Gaussians can be evaluated exactly. When combined with the Halpern FI,

$$\exp\left\{-\frac{i}{4}\int F^2\right\} = N\int d[\chi^a_{\mu\nu}]\, \exp\left\{\frac{i}{4}\int \chi^2 + \frac{i}{2}\int \chi\cdot F\right\},$$

this corresponds to Sums over all Feynman graphs of multiple gluon exchange between Qs and/or $\bar{Q}$s. These Sums are then expressed in terms of the Fradkin functional representations for G_c and L.

AND, Fradkin representations are Potential Theory constructs, and have (relatively) simple approximations in various physical situations. E.G., at high energies, $G_c[A]$ -> Bloch-Nordsieck/eikonal approximation. $L[A]$ is more difficult to approximate; but because of EL, its contributions can be readily estimated.

d) Final expressions are given in terms of appropriate expansion of L[A] effects. NOT a conventional perturbation expansion, because all gluon exchanges between Qs and/or $\bar{Q}$s have already been summed - each gluon bundle is proportional to 1/g - but there exist final summations over gluon bundles interacting with successive L[A] approximations. For the n-n potential, we expect higher corrections to be relatively unimportant.

e) How are the scattering and binding potentials calculated? It is simple, as follows. In Potential Theory, specification of V(r) -> χ(b). We calculate the QCD eikonal χ(b), and reverse the process: χ(b) -> V(r).

7. Many calculations using these techniques are waiting to be done; for example:

(i) Non-perturbative, color charge renormalization.
(ii) Quark-gluon plasma.
(iii) Insertion of flavors and electroweak interactions.
(iv) Extension to multi-nucleon, nuclear systems.
(v) QFT origin of the ξ parameter.

In Summary: We believe this analytic approach to be the QCD of the Future: It is Simple, Realistic, and it Works!