Ultraviolet Complete Quantum Field Theory

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1. Introduction

• In renormalizable QFT, the divergences of the Lagrangian are eliminated to yield a finite S-matrix . Paradoxically, opposed to the finiteness of the S-matrix, the whole local relativistic QFT is based on a path integral formalism in perturbation theory with a probability amplitude, which is infinite and physically meaningless. How do we reconcile this paradox with finite and physical QFT corresponding to nature?

• Let us assume that the universe consists of two observers. The "ideal" observer sees localizable interactions of particles. This fictitious observer possesses a measuring apparatus that has no atomic structure and whose measurements are infinitely localizable. The second real observer uses a measuring apparatus (the ATLAS and CMS detectors at the LHC) that consists of quantum particles and whose measurements are limited. Due to Heisenberg's uncertainty principle, both observers cannot measure fields and particles without disturbing the local particle interactions.

• In a renormalizable QFT both the ideal and the real observers get finite values for their cross sections. However, in a non-renormalizable theory the ideal observer obtains meaningless infinities that cannot be eliminated. Observations made by the ideal observer effectively cause the wave function of the observed system to collapse into an infinitely narrow and infinitely tall peak around an eigenstate. For the real observer this peak will have finite width and height.

• Most QFTs are not renormalizable. For example, $\lambda\phi^n$, $n > 4$ and quantum gravity. Quantum mechanics is q-number physics and it is nonlocal. To be able to obtain physically finite c-number results from measurements, we must give up the notion of infinitely localizable interactions in QFT. This means that any well-defined QFT can be solved using perturbation theory with nonlocal interactions (smeared out interactions and non-idealized plane wave fields), yielding finite cross sections that can be measured by the real observer.

2. Utraviolet Complete QFT

• We construct an ultraviolet (UV) complete quantum field theory (QFT) that is valid in principle to infinite energy. The QFT is finite to all orders of perturbation theory due to coupling constant vertex operators that are nonlocal, entire functions of energy:

$$\bar{g}(x) = g\mathcal{E}(\Box(x)/\Lambda_W^2).$$

Here, Λ_W is a measurable, fundamental energy scale in the theory and $\mathcal{E}$ is an entire function of $\Box = \partial^\mu\partial_\mu$. For generally covariant quantum gravity (QG) :

$$\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu = \frac{1}{\sqrt{-g}}(\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu).$$

• The gauge invariance of massless fields (photons and gluons) is based on generalized nonlocal gauge transformations in 4-dimensional spacetime, similar to the nonlocal gauge transformations in higher-dimensional string field theory. In Minkowski spacetime, the QFT is Poincaré invariant and unitary to all orders of perturbation theory.

3. Electroweak Model

JWM, Euro. Phys. J. Plus 126, 53 (2011), arXiv:1006.1859.

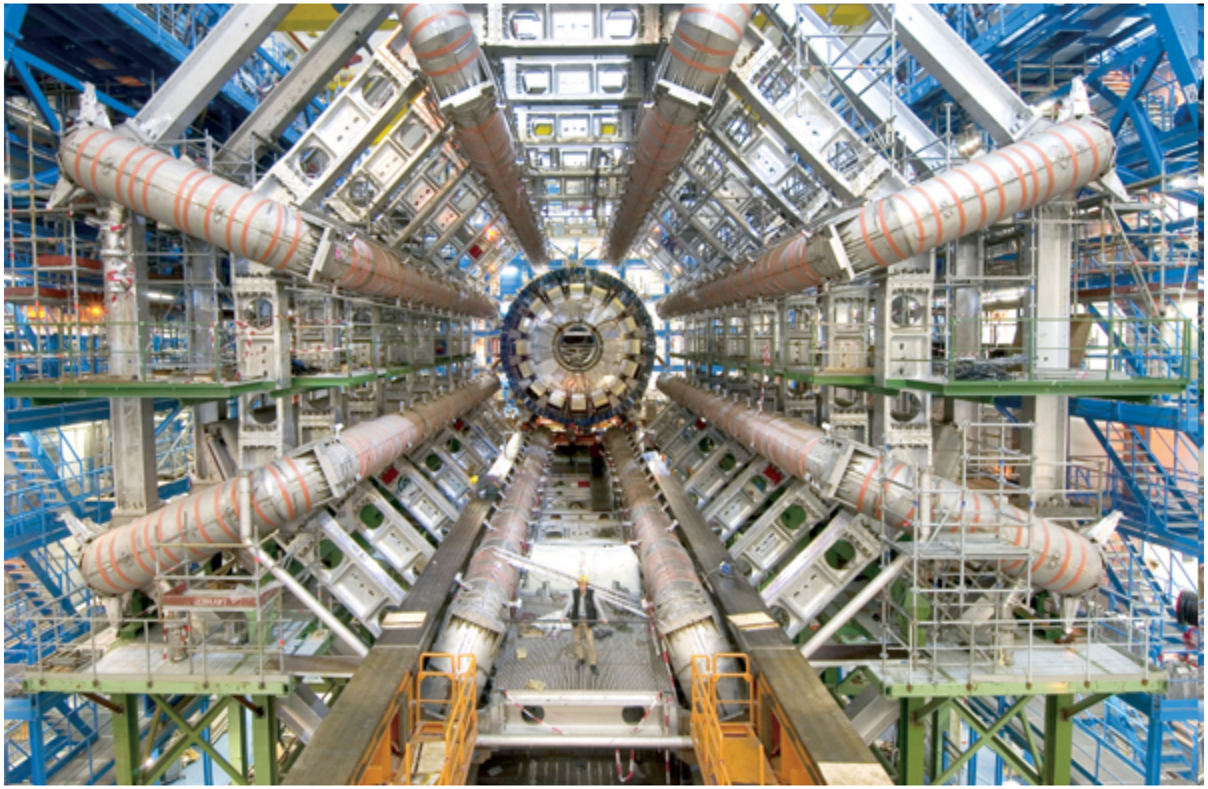
- The problems of the Higgs particle are well known. They include a possible triviality (or non-interacting scalar field) problem related to the occurrence of a Landau pole for scalar fields, the existence of the hierarchy problem that causes the Higgs mass to be unstable and the cosmological constant problem arising from the predicted vacuum energy density being many orders of magnitude greater than the expected observational value.

- Apart from these theoretical problems, a Higgs particle has not been detected experimentally. A lower bound on the Higgs mass: $m_H > 114.4$ GeV has been established by direct searches at the LEP accelerator.

- The EW precision data are sensitive to m_H through quantum corrections and yield the best fit: $m_H = 87^{+35}_{-26}$ GeV.

- Fitting all the data, yields the result $m_H = 116.4^{+15.6}_{-1.3}$ GeV at the 68% confidence level.

The ATLAS experiment at CERN. (Courtesy CERN)



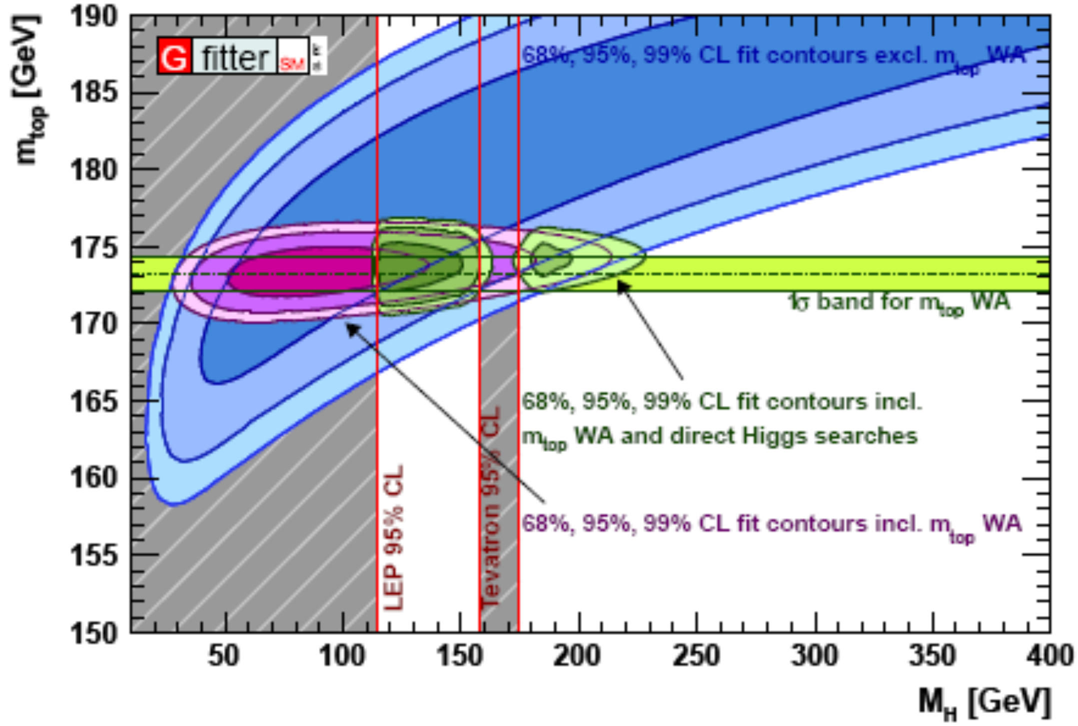


Fig. 3: Contours of 68%, 95% and 99% CL obtained from scans of fits with fixed variable pairs of m_{top} and M_H.

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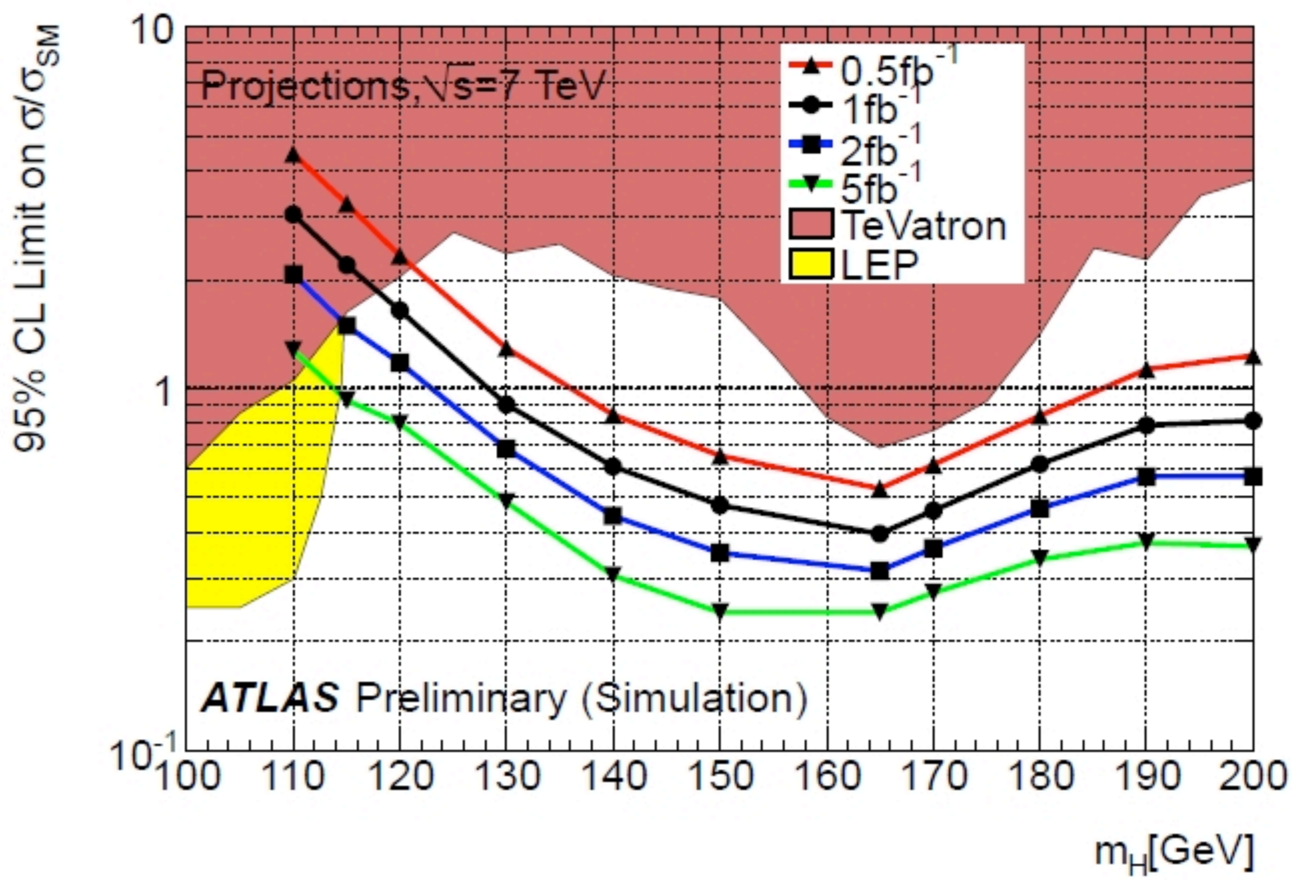


If 2/fb data at 7 TeV is collected, and expected and planned analyses are implemented, then the median exclusion region covers a range of Higgs boson masses from 114 GeV to well over 200 GeV.

ATLAS collaboration.

• The question to be considered is: can we construct a physically consistent EW model containing only the observed particles, namely, 12 quarks and leptons, the charged W boson, the neutral Z boson and the massless photon and gluon? Without the Higgs particle such a model is not renormalizable and the tree graph calculation of $W_L W_L \rightarrow W_L W_L$ longitudinally polarized scattering violates unitarity above an energy of 1-2 TeV.

• We must construct an EW model that avoids the need for it to be renormalizable and does not violate unitarity.

• In the following, we will explore an EW model which is rendered ultraviolet (UV) finite by allowing the coupling constants g and g′ associated with the $SU(2)_L$ and $U(1)_Y$ Feynman vertices, respectively, to possess a running energy dependence. This energy dependence is described by an entire function ,

$$\bar{g}(p^2) = g\mathcal{E}(p^2/\Lambda_W^2)$$

which is analytic (holomorphic) in the finite complex p^2 plane, and Λ_W > 1-2 TeV is an energy scale associated with the EW interactions. This analytic property of the entire function $\mathcal{E}$ guarantees that no unphysical poles occur in the particle spectrum, preserving the unitarity of the scattering amplitudes.

Postulate 1:

• Except for the photon, there is no phase in the universe when the fermion and boson particles have zero mass.

Postulate 2:

• The EW group $SU_L(2) \times U_Y(1)$ is not symmetric i.e. it is broken dynamically at all times.

Postulate 3:

• The EW model is based on a finite QFT with nonlocal interactions and there is no Higgs particle associated with a spontaneous symmetry breaking phase in the universe [the EBHGHK mechanism].

• The EW couplings $\bar{e}(p^2)$, $\bar{g}(p^2)$ and $\bar{g}'(p^2)$ are chosen so that off the mass-shell

$$\mathcal{E}(p^2/\Lambda_W^2) \sim 1 \text{ for } \Lambda_W < 1 \text{ TeV}$$

thereby ensuring that EW calculations at low-energies agree with experiment. On the other hand, for energies greater than 1 -2 TeV, $\mathcal{E}(p^2/\Lambda_W^2)$ decreases rapidly enough in Euclidean momentum space guaranteeing the finiteness of radiative loop corrections. A violation of unitarity of scattering amplitudes is avoided for $\mathcal{E}(p^2/\Lambda_W^2)$ decreasing fast enough as a function of the center-of-mass energy √s for √s > 1 - 2 TeV.

• In a renormalizable QFT, the coupling constants run with energy as described in the renormalization group flow scenario. The energy dependence of the coupling constants is logarithmic. In our approach, we generalize the standard renormalization group energy dependence, so that the coupling constant energy dependence realizes a QFT finite to all orders of perturbation theory for any Lagrangian based on local quantum fields, including quantum gravity. This allows for a finite quantum gravity theory [Eur. Phys. J. Plus 126:43, (2011), arXiv:1008.2482.]

• The model does not contain a fundamental scalar Higgs particle and this removes the hierarchy problem. There is no Landau pole, solving the triviality problem for scalar fields with a quartic self-coupling , and there is no Landau pole in the $U_{EM}(1)$ photon sector. Due to the absence of a spontaneous symmetry breaking Higgs mechanism, there is no cosmological constant problem and Higgs hierarchy mass problem associated with a Higgs particle.

• The theory introduced here is based on the local $SU_C(3) \times SU_L(2) \times U_Y(1)$ Lagrangian that includes leptons and quarks with the color degree of freedom of the strong interaction group SUc(3). For the EW sector

$$\mathcal{L}_{\rm EW} = \sum_{\psi_L} \bar{\psi}_L \left[\gamma^\mu \left(i\partial_\mu - \bar{g} T^a W^a_\mu - \bar{g}' \frac{Y}{2} B_\mu \right) \right] \psi_L + \sum_{\psi_R} \bar{\psi}_R \left[\gamma^\mu \left(i\partial_\mu - \bar{g}' \frac{Y}{2} B_\mu \right) \right] \psi_R - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} + \mathcal{L}_M + \mathcal{L}_{m_f}.$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \qquad W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - \bar{g} f^{abc} W^b_\mu W^c_\nu.$$

• The quark and lepton fields and the boson fields $W_{a\mu}$ and B_μ are local fields that satisfy microcausality.

The $\bar{g}$ and $\bar{g}'$ are defined by

$$\bar{g}(x) = g\mathcal{E}(\Box(x)/\Lambda_W^2), \quad \bar{g}'(x) = g'\mathcal{E}(\Box(x)/\Lambda_W^2).$$

$$\mathcal{L}_M = \frac{1}{2}M^2 W^{a\mu} W^a_\mu + \frac{1}{2}M^2 B^\mu B_\mu, \qquad \mathcal{L}_{m_f} = -\sum_{\psi^i_L, \psi^j_R} m^f_{ij}(\bar{\psi}^i_L \psi^j_R + \bar{\psi}^i_R \psi^j_L),$$

$$L_I = -\frac{\bar{g}}{\sqrt{2}}(J^+_\mu W^{+\mu} + J^-_\mu W^{-\mu}) - \bar{g}\sin\theta_w J^\mu_{\rm em} A_\mu - \frac{\bar{g}}{\cos\theta_w} J^\mu_{\rm NC} Z_\mu.$$

$$Z_\mu = \cos\theta_w W^3_\mu - \sin\theta_w B_\mu \quad \text{and} \quad A_\mu = \cos\theta_w B_\mu + \sin\theta_w W^3_\mu.$$

$$\sin^2\theta_w = \frac{g'^2}{g^2 + g'^2} \quad \text{and} \quad \cos^2\theta_w = \frac{g^2}{g^2 + g'^2}.$$

• To circumvent predicting the existence of a Higgs particle our task is two-fold:

1) We must construct a QFT that is UV complete in perturbation theory and avoids any unitarity violation of scattering amplitudes.

2) We have to invoke a symmetry breaking that is intrinsic to the initial existence of W and Z masses and yields a massless photon. We do not attempt to generate masses of the fermions and bosons as was done by the spontaneous symmetry breaking of the vacuum in the standard Higgs model.

• To solve the first problem, we invoke a generalized energy dependent coupling at Feynman diagram vertices

$$\bar{e}(p^2) = e\mathcal{E}(p^2/\Lambda_W^2), \quad \bar{g}(p^2) = g\mathcal{E}(p^2/\Lambda_W^2), \quad \bar{g}'(p^2) = g'\mathcal{E}(p^2/\Lambda_W^2).$$

Here, $\mathcal{E}(p^2/\Lambda_W^2)$ is an *entire* function for complex p^2 which satisfies on-shell $\mathcal{E}(p^2/\Lambda_W^2) = 1$.

• This allows us to obtain a Poincaré invariant, finite and unitary perturbation theory.

• To solve the second problem of adopting a correct economical breaking of *SU(2) XU(1) symmetry, we* stipulate that the massive boson Lagrangian takes the form:

$$\begin{aligned}\mathcal{L}_M =& \frac{1}{8}b^2g^2[(W^1_\mu)^2 + (W^2_\mu)^2] + \frac{1}{8}b^2[g^2(W^3_\mu)^2 - 2gg'W^3_\mu B^\mu + g'^2 B^2_\mu] \\ =& \frac{1}{4}g^2b^2W^+_\mu W^{-\mu} + \frac{1}{8}b^2(W_{3\mu}, B_\mu)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2\end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu\end{pmatrix},\end{aligned}$$

$$M_W = \frac{1}{2}bg, \quad M_Z = \frac{1}{2}b(g^2 + g'^2)^{1/2}, \quad M_A = 0. \quad \mathcal{L}_M = M_W^2 W^+_\mu W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu.$$

• We do not identify *b with the vacuum expectation value* $v = \langle\phi\rangle_0$ *in the standard Higgs model. The constant b ~ 246 GeV - the EW energy scale.*

• We do not know the origin of the symmetry breaking mechanism and scale *b. To postulate the EW* symmetry breaking is no worse than adopting the *ad hoc assumption of a scalar field Lagrangian* when motivating the Higgs mechanism.

• We circumvent the problem of the lack of renormalizability of our model by damping out divergences with the coupling vertices $\bar{g}(p^2),\ \bar{g}'(p^2)$ and $\bar{e}(p^2)$. *We emphasize that* our energy scale parameter Λ_W *> 1 -2 TeV is not a naive cutoff. The entire function property of the coupling* vertices guarantees that the model suffers no violation of unitarity or Poincaré invariance.

• The relation at the effective tree graph level is satisfied:

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_w} = 1.$$

• We do not attempt to explain the origin of the *W and Z bosons through e.g., a Higgs field spontaneous* symmetry breaking mechanism. Instead, we treat the masses of the *W and Z bosons as intrinsic to our EW* model by assuming a Proca action.

• All Feynman loop graphs are finite to all orders. The $U_{EM}(1)$ sector gauge invariance can be satisfied to all orders for the nonlocal interactions .

4. Radiative Corrections

• The one-loop radiative correction in our ultraviolet UV complete EW model without a Higgs particle is calculated. The ρ *parameter determining the ratio of the charged to neutral currents* is derived for the dominant top quark contribution with the result $\rho \sim 1.01$. *This result is compared* to the radiative correction to ρ *in the standard EW model with a Higgs particle. For the favored light* Higgs particle mass from global fits to EW data: 114.4 GeV $< m_H <$ 135 GeV, *the Higgs contribution* up to three loops is negligible and the Higgless model is consistent with EW data for the energy scale $\Lambda_W \lesssim 1-2\ \mathrm{TeV}$.

JWM, arXiv:1103.0979

JWM, arXiv:1104.5706

5. Unitarity of Scattering Amplitudes

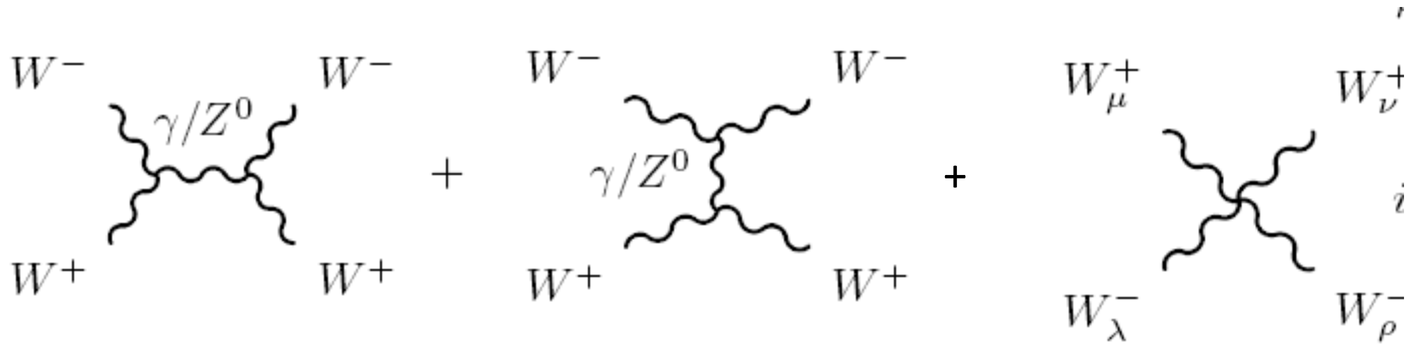
- The scattering amplitude matrix elements for the process $W_L^+ + W_L^- \to W_L^+ + W_L^-$ *is given in the SM* by

$$i\mathcal{M}_{W_L} = ig^2 \left[\frac{\cos\theta + 1}{8M_W^2} s + \mathcal{O}(1) \right] \quad \text{Higgs} \Longrightarrow \quad i\mathcal{M}_H = -ig^2 \left[\frac{\cos\theta + 1}{8M_W^2} s + \mathcal{O}(1) \right]$$

- In our EW model, the unitarity violation is canceled by the high energy behavior of $\bar{g}(s)$.

$$i\mathcal{M}_W = i\bar{g}^2(s) \left[\frac{\cos\theta + 1}{8M_W^2} s + \mathcal{O}(1) \right]$$

We require that for $\sqrt{s} > 1$ TeV, $\bar{g}(s)$ decreases as $\sim 1/\sqrt{s}$ or faster, resulting in the cancelation of the unitarity violating contribution.



We expect that a consistent choice of the entire function $\mathcal{E}(s/\Lambda_W^2)$ will lead to a different prediction for the $W_L^+ + W_L^- \to W_L^+ + W_L^-$ scattering amplitude for $\sqrt{s} > 1$ TeV compared to the Higgs EW model, providing an experimental test of our model.

JWM and V. T. Toth, arXiv: 0812.1994

6. Finite Quantum Gravity

- We have formulated a UV complete quantum gravity theory by extending the vertex coupling strength $\sqrt{G_N}$ to more general entire functions : $\sqrt{G} = \sqrt{G_N}\ \mathcal{F}$ *in spacetime and in momentum space where* $\mathcal{F}$ *is an entire function. This circumvents* the problem of the lack of finiteness and renormalizability of quantum gravity for $\sqrt{G} = \sqrt{G_N}$.

- By assuming a constant energy scale $\Lambda_{gvac} < 10^{-3}$ eV for vacuum fluctuation loops coupled to gravitons in a vacuum, and the energy scale $\Lambda_{gmattervac} > 1$ MeV for vacuum loops coupled to gravitons in material systems, the suppression of the cosmological constant λ can be achieved without cancelling the gravitational effects in the Lamb shift in hydrogen, or physical vacuum fluctuations in atomic and molecular systems such as the Casimir effect.

7. Conclusions

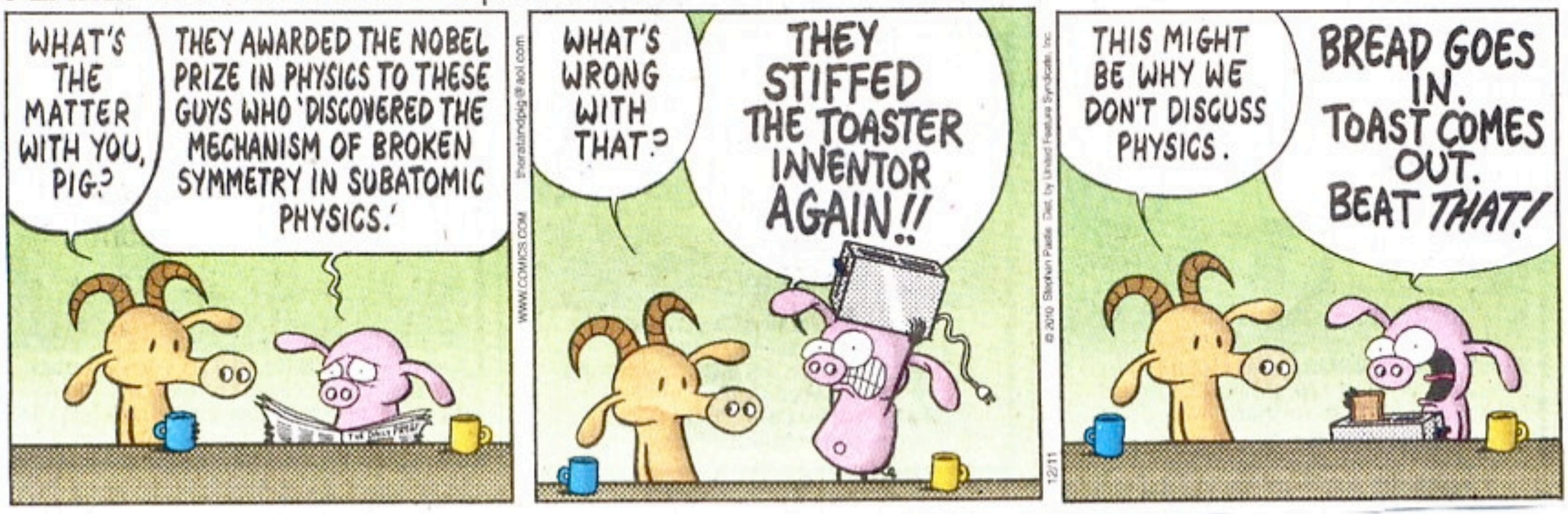
• By introducing generalized EW coupling constants $\bar{g}(p^2)$ and $\bar{g}'(p^2)$, which are energy dependent at Feynman diagram vertices with an energy scale $\Lambda_W > 1$ TeV, we can obtain a Higgless EW model that is unitary, finite and Poincaré invariant to all orders of perturbation theory, provided that the coupling functions are composed of entire functions of p^2 that avoid any unphysical particles that will spoil the unitarity of the scattering amplitudes. All the physical EW fields are local fields that satisfy microcausality, while the interactions are nonlocal. There is no Higgs particle in the particle spectrum and this removes the troublesome aspects of the standard EW model with a spontaneous symmetry breaking.

• The entire function at the Feynman diagram vertices associated with the coupling $\bar{g}(p^2)$ is nonlocal, and the vertex operators do not describe propagating particles.

• The measured masses of the particles, the measured coupling constants e, g and g′ and the energy scale Λ_W are the basic constants of the model.

• The exclusion or detection of the Higgs particle by the Tevatron and the LHC will decide whether a significant revision of the SM is required.

PEARLS BEFORE SWINE Stephan Pastis



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