QUANTUM QUENCH ACROSS A HOLOGRAPHIC CRITICAL POINT

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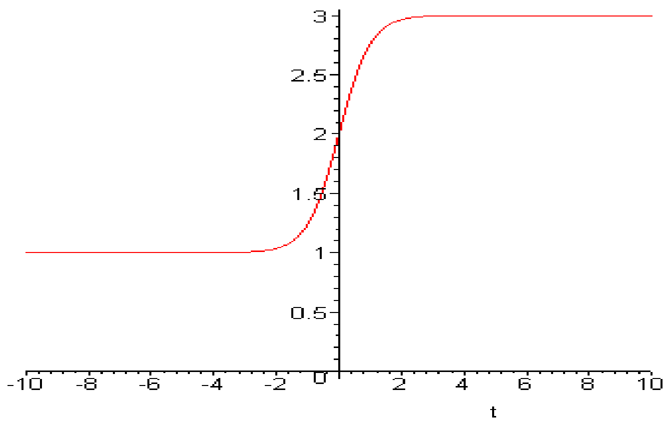
Quantum Quench

- Suppose we have a quantum field theory, whose parameters (like couplings or masses) are time dependent – e.g. going between constant values at early and late times

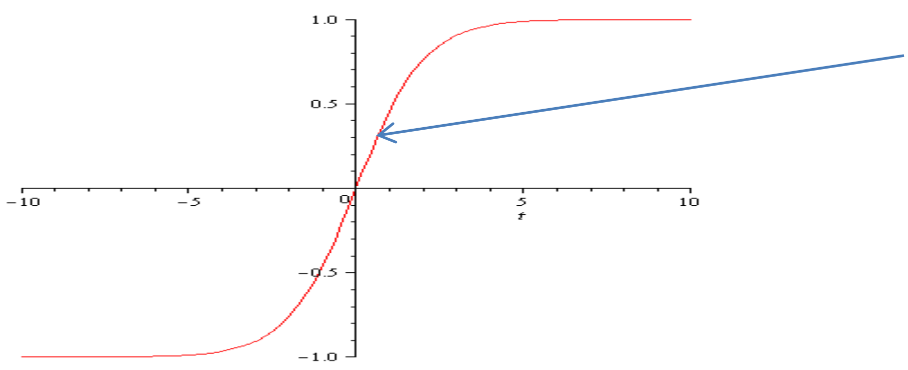


- If the system starts out in a ground state at early times what is the fate of the system at late times ?

- This is of course a standard problem in quantum mechanics and in quantum field theory - which has applications to many areas of physics - cosmological particle production, black hole radiation …….
- In recent years this problem has attracted a lot of attention due to two related reasons.

- The first relates to the question of thermalization
- Does the system evolve into a steady state at late times ? If so does the state resemble a thermal state ?
- What is the characteristic time scale for this to happen ?

- The second issue relates to dynamics near quantum critical phase transitions.
- In this case, simple scaling arguments indicate that there are some properties of the excited state which reflect universal behavior characteristic of the critical point.
- Some of these arguments are adaptations of the classic arguments of Kibble and Zurek for thermal phase transitions.
- Unlike equilibrium critical phenomena there is no conceptual framework which justified this kind of scaling hypothesis.



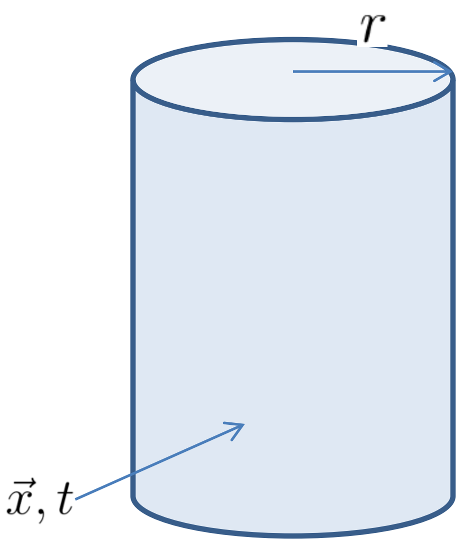
$$g(t) - g_c \sim vt$$

$$< \hat{O} >\sim (v)^{\frac{x\nu}{z\nu+1}}$$

- Much of the recent interest in quantum quench comes from the fact that the response of systems to time dependent couplings can be now experimentally measured in cold atom systems.
- On the other hand there are very few theoretical tools to study this in strongly coupled systems, particularly near quantum critical points.
- Can we use AdS/CFT to understand this kind of phenomena ?
- While it is rather unlikely that one will be able to model real systems by gravity duals, one may be able to understand any universal behavior which underlie such phenomena.

AdS/CFT

- AdS/CFT can reduce a potentially difficult quantum problem into a classical problem



Asymptotically AdS

Theory in the bulk contains gravity

Dual to a non-gravitational quantum field theory on the boundary

The boundary theory is a large-N theory

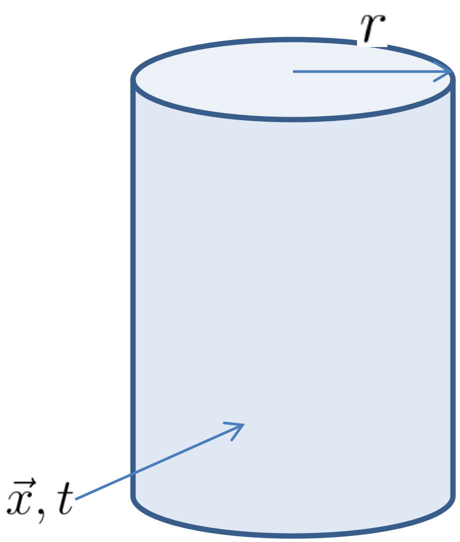
In the large-N limit the bulk theory is classical

$$ds^2 = \frac{dr^2}{r^2} + r^2[-dt^2 + d\vec{x}^2]$$

AdS/CFT

For the simplest example,

$$AdS_5 \times S^5 \to N=4 \quad SYM \quad (3+1)$$



Asymptotically AdS

$$\left(\frac{R_{AdS}}{l_s}\right)^4 = 4\pi g_{YM}^2 N$$

$$g_s = g_{YM}^2$$

‘t Hooft limit = classical limit

Strong ‘t Hooft coupling = classical supergravity

- For every field ϕ in the bulk there is a dual operator $\hat{\mathcal{O}}$

$$\phi(\vec{x},t,r) \leftrightarrow \hat{\mathcal{O}}(\vec{x},t)$$

- The dictionary is summarized by

$$Z = \int \mathcal{D}A \exp\left[iS[A] + i\int d\vec{x}dt\phi_0(\vec{x},t)\hat{\mathcal{O}}(\vec{x},t)\right] = \exp\left[iS_{cl}[\phi_0(\vec{x},t)]\right]$$

- The solution to the wave equation near the boundary $r = \infty$ is

$$\phi(r,x^\mu) \sim r^{d-\Delta}[\phi_0(x^\mu) + O(1/r^2)] + r^\Delta[\psi_0(x^\mu) + O(1/r^2)]$$

- Where Δ is the conformal dimension of the operator

$$\Delta = \frac{d}{2} + \sqrt{(d/2)^2 + m^2}$$

- Then the one point function is given by

$$< \hat{\mathcal{O}}(\vec{x},t) >= \psi_0(\vec{x},t)$$

Quantum Quench and AdS/CFT

- Since couplings of the boundary field theory are boundary conditions on the bulk field, quantum quench simply corresponds to a time dependent boundary condition.
- The response is then measured by calculating the bulk solution and extracting the sub-leading value near the boundary – the part we called ψ_0 .
- This has been studied earlier in several contexts.

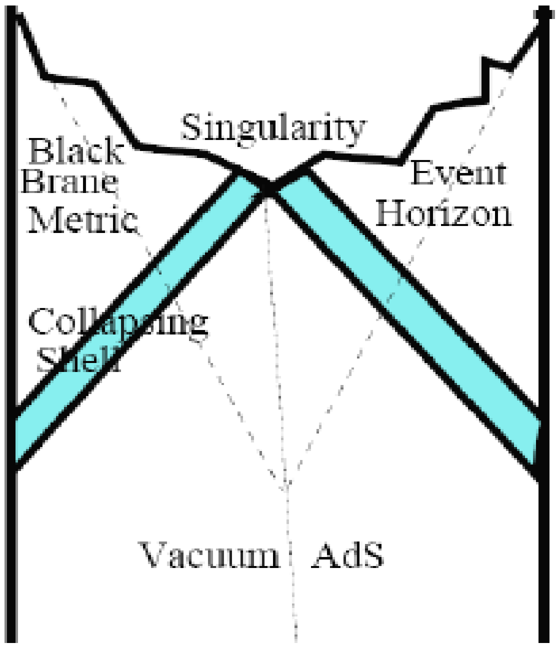
Horizon Formation

- When the change is fast enough a quench from the vacuum state leads to the formation of an apparent horizon – this is perceived as **thermalization** in the field theory.
- *(Janik & Peschanski;*

  Chesler and Yaffe;

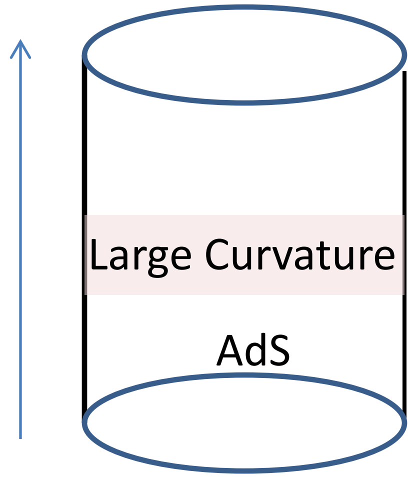
  Bhattacharyya & Minwalla;

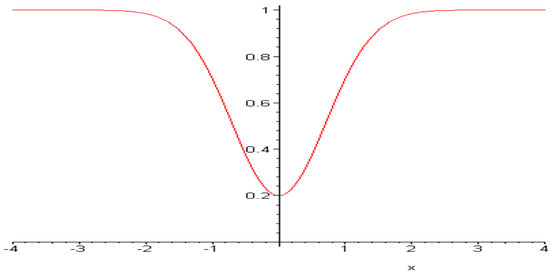
  Das, Nishioka & Takayanagi)



Cosmological Singularity

- When the change in slow enough an apparent horizon is not formed.
- However if the change takes place over a long enough times, the ‘t Hooft coupling becomes weak and the bulk develops a space-like region of large curvature – like a cosmological singularity.
- *(A.Awad, S.R.D, A. Ghosh, J-H Oh and S. Trivedi)*





- Rest of this talk :

  Such quantum quench across critical points

A Holographic Critical Point

- A model of a critical point at a finite temperature and finite chemical potential involves a single neutral scalar field

$$L = \frac{1}{2\kappa^2\lambda}\sqrt{-g}[-\frac{1}{2}(\partial\phi)^2 - \frac{1}{4}(\phi^2 + m^2)^2 - \frac{m^4}{4}]$$

in the background of a charged AdS_4 black brane

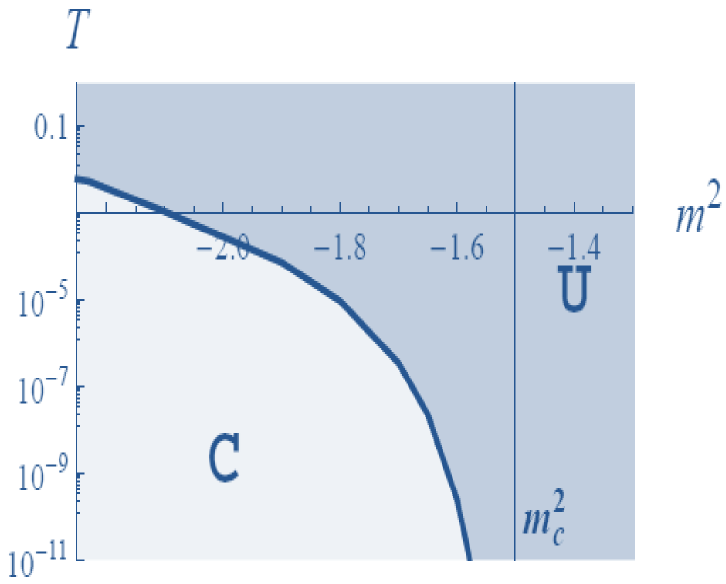
$$ds^2 = r^2[-f(r)dt^2 + d\vec{x}^2] + \frac{dr^2}{r^2 f(r)}$$

$$f(r) = 1 + \frac{3\eta}{r^4} - \frac{1+3\eta}{r^3} \qquad T = 3(1-\eta)$$

- For large λ the scalar field can be treated as a probe – the background geometry remains unchanged.

$$-9/4 \leq m^2 \leq -3/2$$

- For a given temperature T there is a critical value of the mass for below which the stable solution with vanishing ϕ_0 is non-trivial.
- This means that in the boundary theory $< \hat{\mathcal{O}} > \neq 0$ even when the source ϕ_0 vanishes – spontaneous symmetry breaking.
- The point $m^2 = m_c^2$ is a critical point, which is mean field at $T \neq 0$



$$< \hat{\mathcal{O}} > \sim (m_c^2 - m^2)^{1/2} \quad (\phi_0 = 0)$$

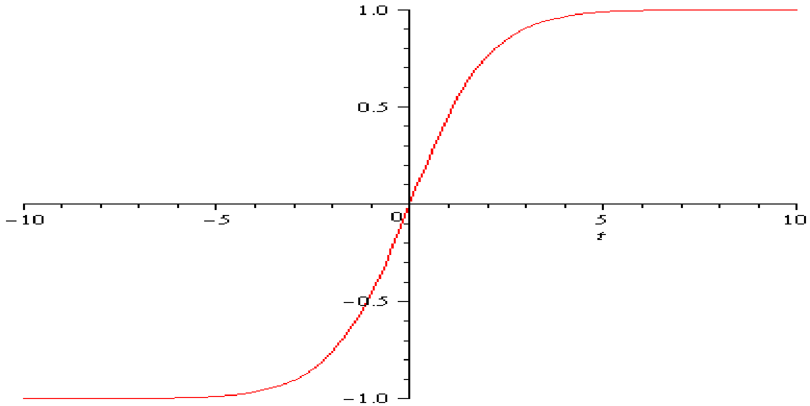
Exactly at $T = 0$ the transition is of Berezinski-Kosterlitz-Thouless type

$$< \hat{\mathcal{O}} > \sim \exp[-\pi\sqrt{6}/2\sqrt{-3/2 - m^2}]$$

Similar to BKT transitions first shown by *Karch, Jensen, Son and Thompson*

The Quench Procedure

- We will study quantum quench across this critical point by turning on a time dependent source $\phi_0(t)$ in the theory with exactly the critical mass $m^2 = m_c^2$. We only consider $T \neq 0$



$$\phi_0(t) = A \tanh(vt)$$

- In the dual theory – a time dependent boundary condition.

Loss of Adiabaticity

- At early times, the system should behave adiabatically if the rate of change v is small enough.
- As we approach the equilibrium critical point, adiabaticity should fail. To study this, it is useful to use coordinates which are well behaved at the black brane horizon. These are the Eddington-Finkelstein coordinates

$$u = \rho - t \qquad d\rho = \frac{dr}{f(r)}$$

- It is also useful to redefine the fields

$$\phi(r,t) = \frac{\chi(\rho,t)}{r}$$

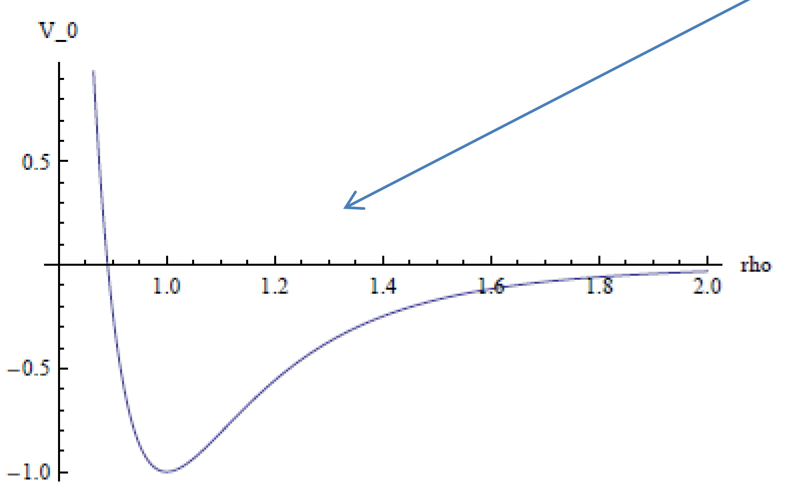
- The equation of motion then becomes

$$-2\partial_u\partial_\rho\chi = -\partial_\rho^2\chi + V(\rho,\chi)$$

- where

$$V(\rho,\chi) = V_0(\rho)\chi + f(r)\chi^3$$

$$V_0(\rho) = r^2 f(r)[(m^2+2) - \frac{6\eta}{r^4} + \frac{1+3\eta}{r^3}]$$



The Schrodinger problem in this potential has bound states for

$$m^2 < m_c^2$$

This is the instability which leads to condensation.

Exactly for $m^2 = m_c^2$ there is a zero mode

- Since the source is nonzero, the static solution is always nontrivial – call this $\chi_0(\rho, \phi_0)$
- Then an adiabatic solution may be constructed by writing

$$\chi(\rho, u) = \chi_0(\rho, \phi_0(u)) + \epsilon\chi_1(\rho, t) + \cdots$$

- The parameter $\epsilon \sim \partial_u \ll 1$ is the adiabaticity parameter.
- To any order, the correction satisfies a linear ODE in ρ with a source determined by the lower order solution. For example

$$\mathcal{D}_\rho \chi_1 = \left[(-\partial_\rho^2 + V_0(\rho)) + 3f(r)\chi_0^2\right]\chi_1 = -\partial_u\partial_\rho\chi_0$$

- Near the critical point $\chi_0 \sim \phi_0^{1/3} \approx 0$ this operator is exactly the linearized operator of the original problem.
- At $m^2 = m_c^2$ this has a zero mode – thus adiabaticity fails.
- This happens when $u \sim v^{-2/5}$

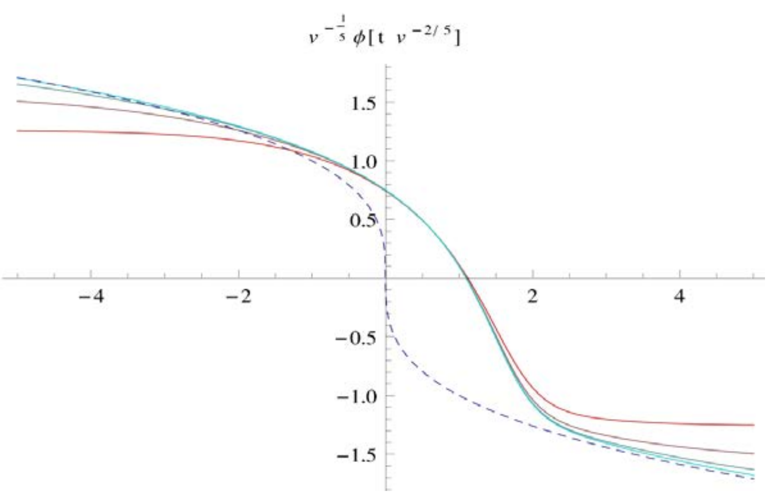
Landau-Ginsburg

- The scaling of the breakdown of adiabaticity is identical to that for a Landau-Ginsburg dynamics

$$\frac{d\varphi}{dt} + m^2\varphi + \varphi^3 + J(t) = 0$$

- For a similar source $J(t) = A\tanh(vt)$ the solution near zero has the form

$$\varphi(t, v) = v^{1/5}\varphi(tv^{2/5}, 1)$$



Dynamics in Critical Region

- First rescale the non-source part of the field

$$\chi_s \rightarrow v^{\frac{1}{5}} \tilde{\chi}_s, u \rightarrow v^{-\frac{2}{5}} \tilde{u}$$

- Expand

$$\tilde{\chi}_s(\rho, u) = \int \tilde{a}_k(u) \chi_k(\rho) dk$$

- Where

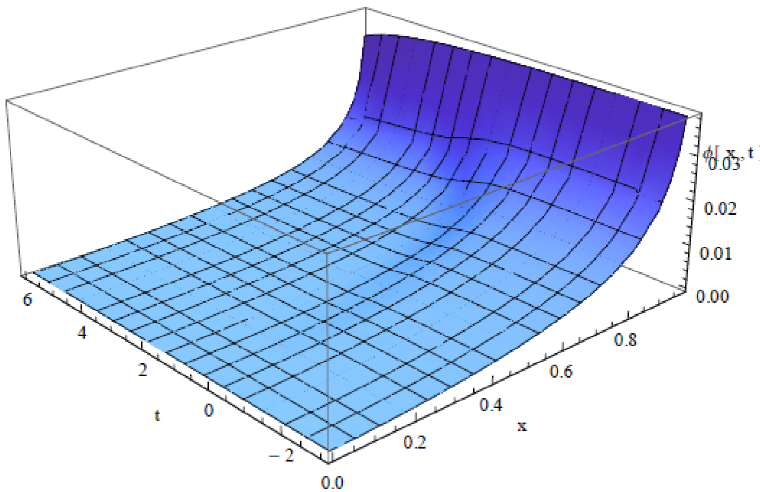
$$\mathcal{D}_\rho \chi_k = k^2 \chi_k$$

- Then one finds that there is a nice small v expansion

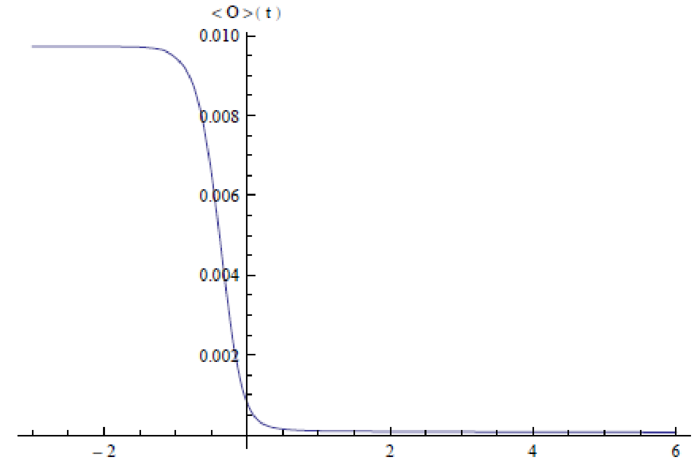
$$\tilde{a}_k(\tilde{u}) = \delta(k) \tilde{\xi}_0(\tilde{u}) + v^{\frac{2}{5}} \tilde{\eta}_k(\tilde{u}) + \cdots$$

- **ONLY when the mass is exactly at the critical value**.
- The equation satisfied by $\tilde{\xi}_0(\tilde{u})$ is the scaled LG equation.
- *For generic masses, the corrections diverge.*

- Thus in the critical region the zero mode dominates the dynamics, which then becomes Landau-Ginsburg with $z = 2$
- The fact that the dynamics involves a single time derivative rather than 2 time derivative is due to finite temperature. In the dual theory, the black hole horizon causes dissipation.
- The requirement of regularity (in EF coordinates) leads to single time derivatives – but this can be seen in usual coordinates as well as in the original work of *Kovtun, Son and Starinets*
- Beyond the critical region, all the modes become important – and the problem has to be solved numerically.
- We have performed some preliminary numerical work – but the results are consistent with the above analysis, but not accurate enough to read off the scaling exponent.
  Lot more numerical work necessary to understand late time behavior.



(a) The plot is for $\phi(x,t)$, $(x \sim \frac{1}{r})$. It is to be noted how the disturbance propagates following a light cone.



(b) Plot of boundary expectation value $< \mathcal{O} > (t)$.

Outlook

- We have shown that holographic methods can be useful in the study of quantum quench of strongly coupled field theories.
- We looked at small but finite temperature where the equilibrium transition is mean field – and indeed we found that the critical dynamics is a $z = 2$ Landau-Ginsburg dynamics.
- However, similar ideas can be used to understand zero temperature BKT transitions in this model, or in other models. This should lead to really interesting and novel results – which cannot be obtained in any other way.
- This is a rather subtle case, and is under investigation.