# The Orear regime in elastic pp-scattering at 7 TeV 

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## Unitarity equation - ZhETF Pis'ma 6, 810 (1967) etc

The diffraction cone

$$
\begin{equation*}
\frac{d \sigma}{d t} /\left(\frac{d \sigma}{d t}\right)_{t=0}=e^{B t} \approx e^{-B p^{2} \theta^{2}} \quad\left(t=-2 p^{2}(1-\cos \theta) \approx-p^{2} \theta^{2}\right) \tag{1}
\end{equation*}
$$

The amplitude

$$
\begin{equation*}
A(p, \theta) \approx 4 i p^{2} \sigma_{t} e^{-B p^{2} \theta^{2} / 2} \tag{2}
\end{equation*}
$$

The unitarity condition (at $t=0$ reduces to $\sigma_{t}=\sigma_{e l}+\sigma_{\text {inel }}$ ):

$$
\begin{gathered}
\operatorname{Im} A(p, \theta)=I_{2}(p, \theta)+F(p, \theta)= \\
\frac{1}{32 \pi^{2}} \iint d \theta_{1} d \theta_{2} \frac{\sin \theta_{1} \sin \theta_{2} A\left(p, \theta_{1}\right) A^{*}\left(p, \theta_{2}\right)}{\sqrt{\left[\cos \theta-\cos \left(\theta_{1}+\theta_{2}\right)\right]\left[\cos \left(\theta_{1}-\theta_{2}\right)-\cos \theta\right]}}+F(p, \theta
\end{gathered}
$$

The region of integration in (4)

$$
\begin{equation*}
\left|\theta_{1}-\theta_{2}\right| \leq \theta, \quad \theta \leq \theta_{1}+\theta_{2} \leq 2 \pi-\theta \tag{4}
\end{equation*}
$$

$I_{2}$ - two-particle intermediate states $\left(\sigma_{e l}\right), F$ - inelastic ones $\left(\sigma_{\text {inel }}\right)$.

For angles outside the diffraction cone one amplitude in (4) should be taken at small angles and another one at large angles. The linear integral equation outside the diffraction cone
$\operatorname{Im} A(p, \theta)=\frac{p \sigma_{t}}{4 \pi \sqrt{2 \pi B}} \int_{-\infty}^{+\infty} d \theta_{1} e^{-B p^{2}\left(\theta-\theta_{1}\right)^{2} / 2} \operatorname{Im} A\left(p, \theta_{1}\right)+F(p, \theta)$.

Analytical solution if $F(p, \theta) \ll \operatorname{Im} A(p, \theta)$ outside the diffraction cone!

It contains the exponentially decreasing with $\theta$ (or $\sqrt{|t|}$ ) term (Orear regime!) with imposed on it damped oscillations.

To account for the real parts of the amplitude one should replace $\sigma_{t}$ in Eq. (5) by $\sigma_{t} f_{\rho}$ where $f_{\rho}=\left(1+\rho_{d} \rho_{l}\right) / \sqrt{\left(1+\rho_{d}^{2}\right)\left(1+\rho_{l}^{2}\right)}$ with ratios of real to imaginary parts of the amplitude in the diffraction cone and outside it denoted as $\rho_{d}$ and $\rho_{l}$ correspondingly.

The elastic scattering differential cross section outside the diffraction cone (in the Orear regime region):

$$
\begin{align*}
& \ln \left(\frac{d \sigma}{C d t}\right) \approx-2 \sqrt{2 B \ln \left(4 \pi B / \sigma_{t} f_{\rho}\right)|t|}+ \\
& \quad 2 D \exp [-\sqrt{2 \pi B|t|}] \cos (\sqrt{2 \pi B|t|}-\phi) \tag{6}
\end{align*}
$$

The first term is exponentially decreasing with $\theta$ (or $\sqrt{|t|}$ ) and the second term demonstrates the damped oscillations.

The experimentally measured values of the diffraction cone slope $B$ and the total cross section $\sigma_{t}$ determine mostly the shape of the elastic differential cross section in the Orear region of transition from the diffraction peak to large angle parton scattering. The value of $4 \pi B / \sigma_{t}$ is so close to 1 that it becomes possible for the first time to estimate the ratio $\rho_{l}$ from fits of experimental data. It is of the order of 1 .

PRELIMINARY


The equation (4) can be used as an expression for the overlap function $F(p, \theta)$ :

$$
\begin{array}{r}
F(p, \theta)=16 p^{2}\left(\pi \frac{d \sigma}{d t} /\left(1+\rho^{2}\right)\right)^{1 / 2}- \\
\frac{8 p^{4} f_{\rho}}{\pi} \int_{-1}^{1} d z_{2} \int_{z_{1}^{-}}^{z_{1}^{+}} d z_{1}\left[\frac{d \sigma}{d t_{1}} \cdot \frac{d \sigma}{d t_{2}}\right]^{1 / 2} K^{-1 / 2}\left(z, z_{1}, z_{2}\right), \tag{7}
\end{array}
$$

where $z_{i}=\cos \theta_{i} ; \quad K\left(z, z_{1}, z_{2}\right)=1-z^{2}-z_{1}^{2}-z_{2}^{2}+2 z z_{1} z_{2}$ and the integration limits are $z_{1}^{ \pm}=z z_{2} \pm\left[\left(1-z^{2}\right)\left(1-z_{2}^{2}\right)\right]^{1 / 2}$.

At present it is not yet calculated at 7 TeV . At this high energy the angles are so small that the kernel becomes too singular. ( $K$ is very close to 0 .)

Let me refer to the shape of the overlap function $F$ at low energies.

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The ratio $F / I_{2}$ at $19.2 \mathrm{GeV} / \mathrm{c}$.

- At intermediate angles between the diffraction cone and hard parton scattering region the unitarity condition predicts the Orear regime with exponential decrease in angles and imposed on it damped oscillations. Earlier, this solution was helpful in explaining this regime at lab. energies $8-20 \mathrm{GeV}$.
- The experimental data on elastic pp differential cross section at $\sqrt{s}=7 \mathrm{TeV}$ in this region are fitted by it with well described position of the dip at $|t| \approx 0.53 \mathrm{GeV}^{2}$, its depth and subsequent damped oscillations with the predicted period about $0.3 \mathrm{GeV}^{2}$.
- The fit allows for the first time to estimate the ratio of real to imaginary parts of the elastic scattering amplitude in this region far from forward direction $t=0$. It is of the order of 1 .
- The overlap function at 7 TeV is not yet calculated. At low energies it is small and negative in the Orear region. That confirms the assumption used when solving the unitarity equation and shows that the phases of inelastic amplitudes become crucial in any model of inelastic processes.

