Universal Rise of Total Hadronic Cross Sections and Predictions at LHC

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14th Workshop on Elastic and Diffractive Scattering
Dec.15-21, 2011
Qui Nhon, Vietnam

Ishida, Igi: PLB(2009)395, PRD79(2009)096003
Halzen, Igi, Ishida, Kim, arXiv; 1110. 1479

Contents of the talk

- We show that data on $\bar{p}(p)p, \pi^{\mp}p, K^{\mp}p$ forward scatt. support related expect. that asympt. beh. of all cross sec. is flavor ind.i.e. $, B_{pp} \simeq B_{\pi p} \simeq B_{Kp}.$
- Using most recent data from ATLAS, CMS, Auger , we predict $\sigma_{tot}^{pp}\left(\sqrt{s} = 7\ TeV\right) = 96.1 \pm 1.1 mb$ (consist. with TOTEM within exp. errors).
- We also use our results on flavor ind. to predict $\sigma_{tot}^{\pi\pi}$ a function of $\sqrt{s}.$

First Topic:Universal Rise of σ_{tot} ?

- In addition to Froissart bound, COMPETE collab.(PDG) further assumed

$$\sigma_{tot} \simeq B\left(\log s/s_0\right)^2 + Z$$

to reduce the number of adjustable parameter.

- Universality of B (flavor ind.) was theoretically anticipated.

  Jenkovszky et al. Where is "asymptopia"? (1987)

  C.-I. Tang et al. (1989)

- It was also inferred from Color Grass condensate of QCD.

  Itakura et al. (2002)

- However, no rigorous proof yet based only on QCD→Test of Universality of B is Necessary even empirically.

Particle Data Group
(by COMPETE collab)

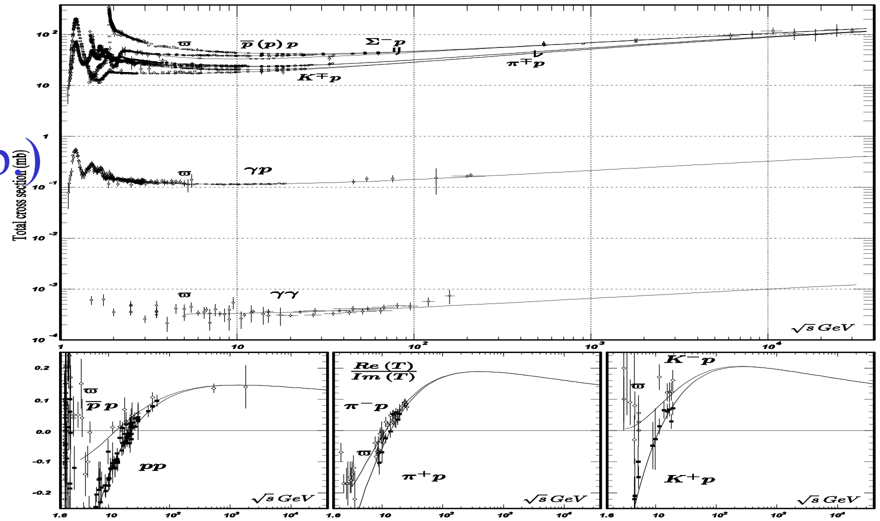


Figure 40.10: Summary of hadronic, γp, and $\gamma\gamma$ total cross sections, and ratio of the real to imaginary parts of the forward hadronic amplitudes. Corresponding computer-readable data files may be found at http://pdg.lbl.gov/xsect/contents.html (Courtesy of the COMPAS group, IHEP, Protvino, August 2005)

The upper side : σ

The lower side : ρ－ratio

B (Coeff. of $(\log s/s_0)^2$)
Assumed to be universal

Test of univ. of B:necessary
even empirically.

Kinematics

- Consider the crossing-even f.s.a.

$$F^{(+)}(\nu) \equiv \frac{f^{\bar{p}p}(\nu) + f^{pp}(\nu)}{2}$$

with $\operatorname{Im} F^{(+)}(\nu) \equiv \frac{k\sigma_{tot}^{(+)}(\nu)}{4\pi}$

- We assume $\operatorname{Im} F^{(+)}(\nu) = \operatorname{Im} R(\nu) + \operatorname{Im} F_{P'}(\nu)$

$$= \frac{\nu}{M^2}\left(c_0 + c_1 \log\frac{\nu}{M} + c_2 \log^2\frac{\nu}{M}\right) + \frac{\beta_{P'}}{M}\left(\frac{\nu}{M}\right)^{\alpha_{P'}}$$

at high energies. This correspond to the same expression as the PDG.

M : proton mass

v, k : incident proton energy, momentum in the laboratory system

$$F^{(+)}(-\nu)=\left[F^{(+)}(\nu)\right]^{*}$$

The $\rho^{(+)}$ ratio

- The $\rho^{(+)}$ ratio = the ratio of the real to imaginary part of $F^{(+)}(\nu)$

$$\rho^{(+)}(\nu)=\frac{\operatorname{Re}F^{(+)}(\nu)}{\operatorname{Im}F^{(+)}(\nu)}=\frac{\operatorname{Re}R(\nu)+\operatorname{Re}F_{P'}(\nu)}{\operatorname{Im}R(\nu)+\operatorname{Im}F_{P'}(\nu)}$$

$$=\frac{\frac{\pi\nu}{2M^2}\left(c_1+2c_2\log\frac{\nu}{M}\right)-\frac{\beta_{P'}}{M}\left(\frac{\nu}{M}\right)^{0.5}+F^{(+)}(0)}{\frac{k\sigma_{tot}^{(+)}}{4\pi}}$$

$$F^{(+)}(0)=subtraction\ const.$$

How to predict σ and ρ for pp at LHC based on duality? (as an example)

- We searched for simultaneous best fit of σ and ρ up to some energy(e.g.,ISR) in terms of high-energy parameters constrained by FESR.
- We then predicted $\sigma_{tot}^{(+)}$ and $\rho^{(+)}$ in the LHC regions.

- Both $\sigma_{tot}^{(+)}$ and $\mathrm{Re}\, F^{(+)}$ data are fitted through two formulas simultaneously with FESR as a constraint.
- FESR is used as constraint of $\beta_{P'} = \beta_{P'}\left(c_0, c_1, c_2\right)$

and the fitting is done by three parameters:

$$c_2, c_1, and\, c_0$$

giving the least χ^2 . $\left(-\infty < c_i < \infty\right)$

- Therefore, we can determine all the parameters

$$c_2, c_1, c_0, \beta_{P'}, F^{(+)}(0)$$

These predict σ, ρ at higher energies including LHC energies $\sqrt{s} = 7\, TeV$

- We attempt to obtain $B\ values$ for $pp(\bar{p}p), \pi p, Kp\ scatterings$ through search for simultaneous best fit to experimental $\sigma_{tot}\ and\ \rho\ ratios.$

- The value of B
  universal ?

New Attempt for πp

- In near future, σ_{tot}^{pp} will be measured at high energy. So, B_{pp} will be determined with good accuracy. $\bar{p}p: \sqrt{s} \leq 1.8\,TeV$
- On the other hand, $\sigma_{tot}^{\pi p}$ have been measured only up to k=370 GeV. So, one might doubt to obtain B for πp,

  with reasonable accuracy. $\pi p: \sqrt{s} < 26.4$GeV

- We attack this problem in a new light.

Practical Approach for search of B

Tot. cross sec.= Non-Reggeon comp. + Reggeon(P') comp.

$$\sigma_{\text{tot}}^{(+)} \simeq \frac{4\pi}{m^2}\left[\left(c_2\log^2\frac{\nu}{m} + c_1\log\frac{\nu}{m} + c_0\right) + \beta_{P'}\left(\frac{\nu}{m}\right)^{\alpha_{P'}-1}\right]$$

- Non-Reg. comp. shows shape of parabola as a fn. of logν with a min.
- Inf. of low-energy res. gives inf. on P' term. Subtracting this P' term from $\sigma_{tot}^{(+)}$, we can obtain the dash-dotted line(parabola).

VIP Fig.1 pp, p̄p

- We have good data for large values of log ν for det. of c_2 (pp)(or B_{pp}) with good accuracy.

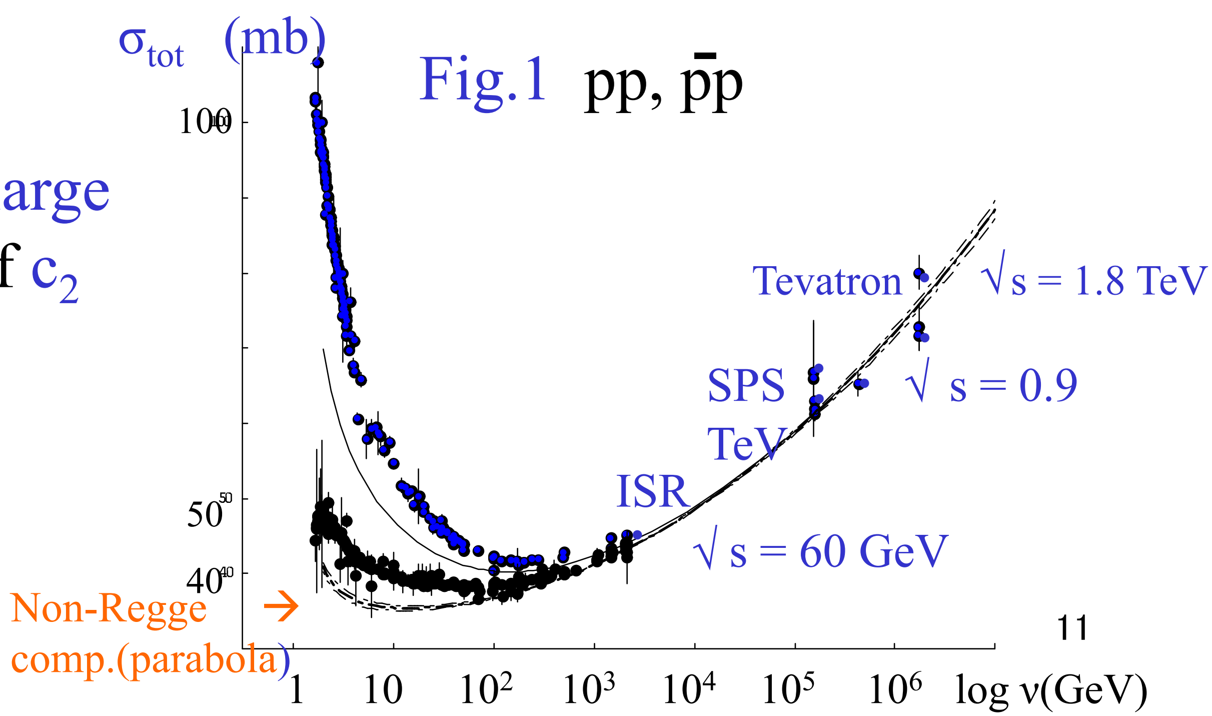
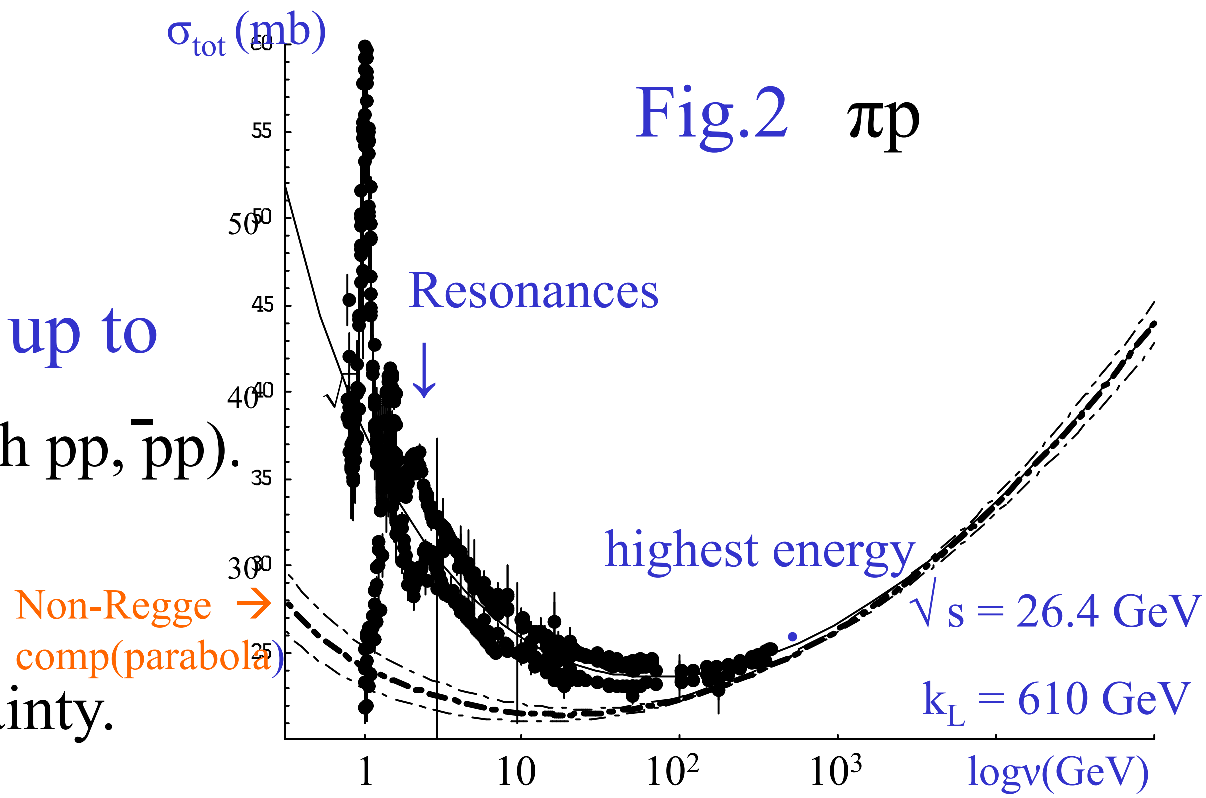


Fig.2 πp

- σ_{tot} measured only up to $\sqrt{s}$ = 26.4 GeV (cf. with pp, $\bar{p}$p).



- So, estimated $B_{\pi p}$ may have large uncertainty.

- The πp has many res. at low energies, however.

So, inf. on LHS of parabola obtained by subtracting P' term from $\sigma_{tot}^{(+)}$ is very helpful to obtain accurate value of B(πp).

(resonances with k < 10GeV turn out to be very helpful to determine shape of parabola).

•(Kp : similar to πp) .

Test of Universality of B

- Highest energy of Experimetnal data:

$\bar{p}p$: Ecm = 0.9TeV SPS; 1.8TeV Tevatron

$\pi^{-}p$: Ecm < 26.4GeV

Kp : Ecm < 24.1GeV No data in TeV→ B : large errors.

was obtained only from high-energy data.

B_{pp} = 0.273(19) mb

$B_{\pi p}$ = 0.411(73) mb

B_{Kp} = 0.535(190) mb

→ B_{pp} =? $B_{\pi p}$ =? B_{Kp} ?

No definite conclusion

- It is impossible to test of Universality of B only by using data in high-energy regions.
- We attack this problem from shape of parabola of Non-Regge component.

Kinematics

- Crossing-even amplitudes : $F^{(+)}(-\nu)=F^{(+)}(\nu)^*$

$$F^{(+)}(\nu)=\left(f^{\bar{a}p}(\nu)+f^{ap}(\nu)\right)/2$$

average of $\pi^- p$, $\pi^+ p$; $K^- p$, $K^+ p$; pp, $\bar{p}p$

$$\mathrm{Im}\, F^{(+)}_{asymp}(\nu) = \beta_{P'}/m\,(\nu/m)^{\alpha_{P'}(0)}$$
$$+(\nu/m^2)[\,c_0+c_1 \log \nu/m + c_2(\log \nu/m)^2]$$

$\alpha_{P'}(0)$

$\beta_{P'}$ term : P'trajecctory ($f_2(1275)$): $\alpha_{P'}(0) \sim 0.5$: Regge Theory

c_0, c_1, c_2 terms : corresponds to $Z + B(\log s/s_0)^2$

c_2 is directly related with B . ($s \sim 2M\nu$)

- Crossing-odd amplitudes : $F^{(-)}(-\nu)= -F^{(-)}(\nu)^*$

$$F^{(-)}(\nu)=\left(f^{\bar{a}p}(\nu)-f^{ap}(\nu)\right)/2$$

$\mathrm{Im}\, F^{(-)}_{asymp}(\nu) = \beta_V/m\,(\nu/m)^{\alpha_V(0)}$ ρ-trajecctory: $\alpha_V(0) \sim 0.5$

$\beta_{P'}$, β_V is Negligible to σ_{tot}($= 4\pi/k\ \mathrm{Im}\, F(\nu)$) in high energies.

$B = (4\pi/m^2)c_2$

FESR Duality

How to obtain the $\mathrm{Im}\, F^{(+)}_{asymp}(\nu)$ from low-energy res?

- Remind that the P' sum rule . (the first FESR, 1961,K.I.)

$$\frac{1}{2\pi^2}\int_0^N dk\sigma^{(+)}(k) - \frac{2}{\pi}\int_0^N d\nu\, \mathrm{Im}\, F^{(+)}_{asymp}(\nu)\frac{\nu}{k^2} = const.$$

- Take two N's(FESR1) $N = N_1, N = N_2 (N_2 > N_1)$
- Taking their difference, we obtain

$$\frac{1}{2\pi^2}\int_{\bar{N}_1}^{\bar{N}_2} dk\, \sigma^{(+)}_{tot}(k) = \frac{2}{\pi}\int_{N_1}^{N_2} d\nu\, \mathrm{Im}\, F_{asymp}(\nu)$$

LHS is estimated from
Low-energy exp.data.

RHS is calculable from
The low-energy ext. of Im Fasymp.

$\bar{p}p$ has open(meson) ch. below $\bar{p}p$,and div. above th.

- If we choose $\bar{N}_1$ to be fairly larger than m we have no difficulty. ($K^- p$: similar)

No such effects in πp .

Choice of N_1 for πp Scattering

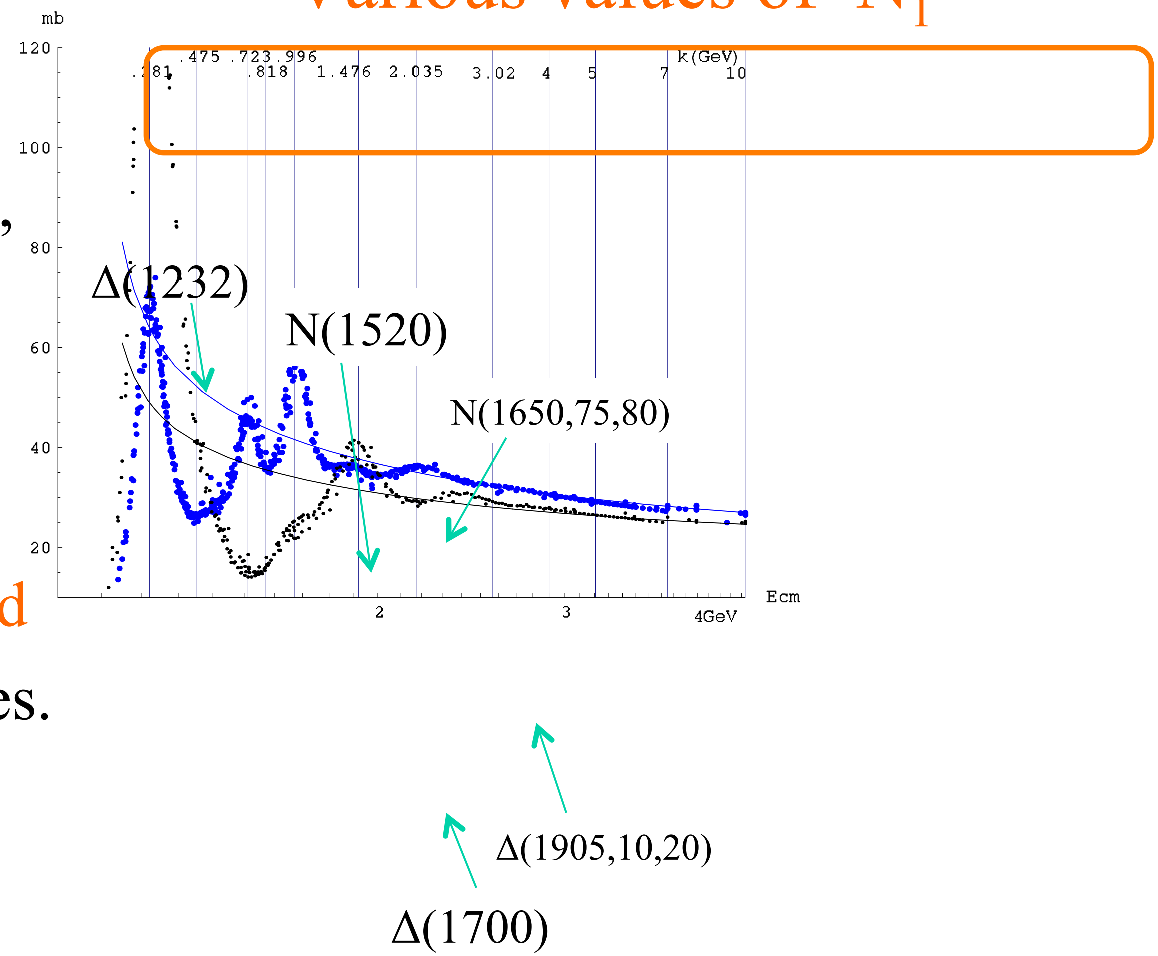
- Many resonances in $\pi^- p$ & $\pi^+ p$
- The smaller N_1 is taken, the more accurate c_2 (and $B_{\pi p}$) obtained.
- We take various N_1 corresponding to peak and dip positions of resonances.

(except for $k=\bar{N}_1=0.475$GeV)

→For each N_1,

FESR is derived. Fitting is performed. The results checked.

Various values of N_1



Test of the Universal Rise

- $\sigma_{tot} = B (\log s/s_0)^2 + Z$

	B (mb)
πp	0.304±0.034
Kp	0.328±0.045
pp	0.280±0.015

B_{Kp} improved by BargerIshida2011

$$B_{\pi p} = B_{pp} = B_{Kp}$$

$$within\ 1\sigma$$

$$\rightarrow Universality\ suggested$$

B(mb)
0.304±0.034
0.354±0.099
0.280±0.015

FESR Duality used

B(mb)
0.411±0.073
0.535±0.190
0.273±0.019

Only high-energy data

$B_{\pi p} \neq? B_{pp} =? B_{Kp}$

No definite conclusion in this case.

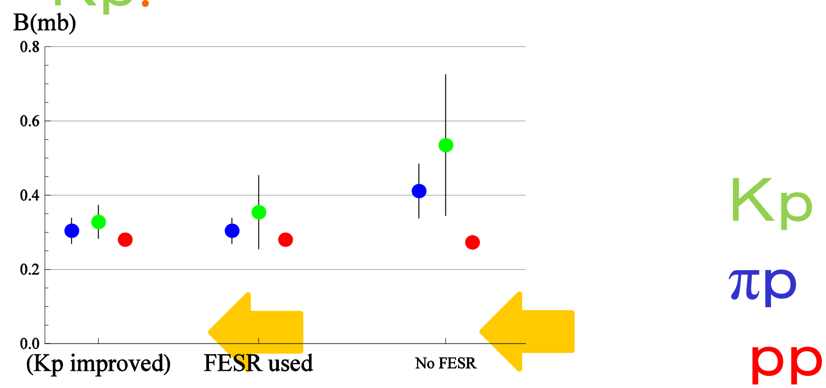
Concluding Remarks

- In order to test the universal rise of σ_{tot} , we have analyzed $\pi^{\pm}p; K^{\pm}p; \bar{p}p, pp$ independently.
- Rich information of low-energy scattering data constrain, through FESR Duality, the high-energy parameters B to fit experimental σ_{tot} and ρ ratios.
- The values of B are estimated individually for three processes.

- We obtain $B_{\pi p} = B_{pp} = B_{Kp}$.

  Universality of B suggested.

  Use of FESR is essential to lead to this conclusion.



- Universality of B suggests gluon scatt. gives dominant cont. at very high energies(flav. ind.).
- It is also interesting to note that Z for $\pi p, Kp, \overline{p}p(pp)$ approx. satisfy ratio 2:2:3 predicted by quark model.

- Our results predicts

B_{pp}=0.280(15)mb

$$\sigma_{pp}^{LHC} = 96.0(1.4)\,mb \;\; at \;\; 7\,TeV$$

$$102.0(1.7)\,mb \;\; at \;\; 10\,TeV$$

$$108.0(1.9)\,mb \;\; at \;\; 14\,TeV$$

Second Topic: Updated Analysis including LHC and Very High Energy Cosmic-Ray Data

Halzen, Igi, Ishida, Kim

- In the First Topic, we showed that universality relation $B_{pp} = B_{\pi p} = B_{Kp} \equiv B$: valid within one standard deviation.
- Now,we assume this universality from the beginning.
- Other powerful constraints:FESR Duality

- To determine the value of B more precisely, let us now include three recent measurements:
- ATLAS
- CMS
- Auger covering very high-energy region.

Total inelastic cross sections for the above: σ_{inel}.

have been employed.

We use the ratio $\sigma_{tot}/\sigma_{inel}$ of Eikonal model by Block-Halzen to obtain σ_{tot}.

- ATLAS reported σ_{inel}^{pp} at 7 TeV of

$$69.4 \pm 2.4(\exp.) \pm 6.9(extr.)$$

Using $\sigma_{tot}/\sigma_{inel}$ at 7TeV of 1.38 (from eikonal model)

$$\sigma_{tot}^{pp}(7\,TeV) = 96.0 \pm 3.3 \pm 9.5\ mb.$$

- CMS rep. $\sigma_{inel} = 68.0 \pm 2.0(syst.) \pm 2.4(lum.) \pm (extr.)\ mb.$

$$\therefore\ \sigma_{tot}^{pp}(7\,TeV) = 94.0 \pm 2.8 \pm 3.3 \pm 5.5\ mb$$

- Auger measured σ_{inel}^{pp} at 57 TeV to be

$$90 \pm 7(stat.)_{-11}^{+8}(syst.) \pm 1.5(Glauber)$$

Using $\sigma_{tot}/\sigma_{inel} = 1.45$ at 57 TeV,

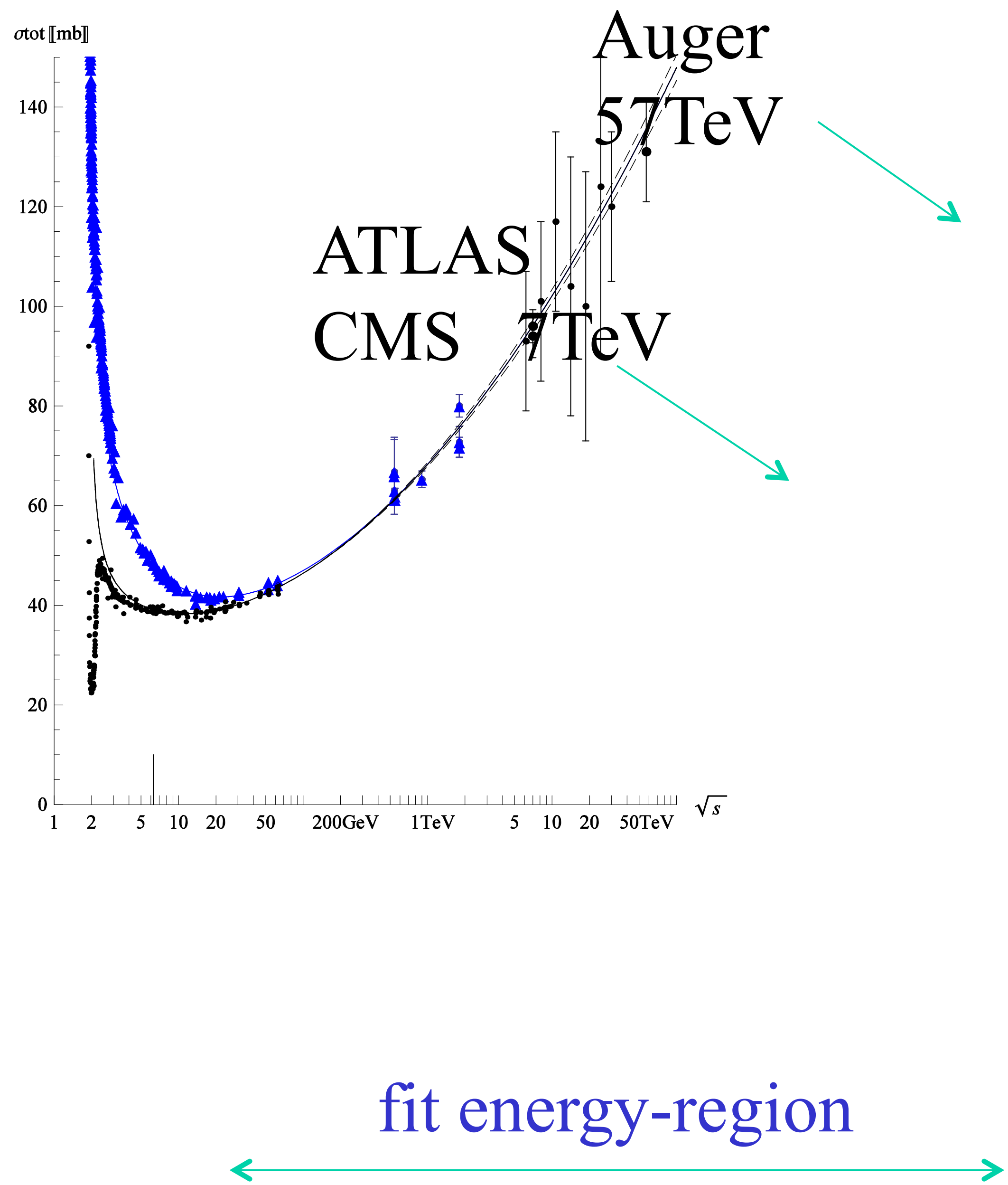
$$\sigma_{tot}^{pp}(57\ TeV) = 131 \pm 10_{-16}^{+12} \pm 2\ mb$$

- Exptl data of $\sigma_{tot}^{\bar{a}p,ap}\left(a=p,\pi,K\right)$at $k \geq 20GeV$ and $\rho^{\bar{a}p,ap} \geq 5GeV$ for $\bar{p}(p)p,\ \pi^{\mp}p,\ K^{\mp}p$ are fit simult. imposing on param. $c_{2,1,0}^{ap},\beta_{T,V}^{ap},F_{ap}^{(+)}\left(0\right)$ the constraints on B and from FESR Duality

- Highest energy data for σ_{tot} data reach

$26.4\left(25.3\right)GeV$	for $\pi^{-}p\left(\pi^{+}p\right)$
$24.1\,GeV$	for $K^{\mp}p$
$1.8\,TeV$	for $\bar{p}p$ (Tevatron)
$57\,TeV$	for pp (Cosmic-Ray)

Result of the fit to $\bar{p}p$ and pp



Best-fit parameters

ab	B(mb)	$\sqrt{s_0}$ ab (GeV)	Z_{ab}(mb)	β_T^{ab}	β_V^{ab}	$F_{ab}(0)^{(+)}$
pp	0.280(11)	4.65(42)	35.32(29)	6.71(20)	3.68(4)	10.6(6)
πp	0.280(11)	5.28(32)	21.18(14)	0.155(6)	0.040(1)	0.12(62)
Kp	0.280(11)	5.04(30)	17.85(16)	0.446(58)	0.56(1)	2.4(1.0)

$$\sigma_{tot}^{\bar{a}b,ab} = B\log^2 s/s_0^{ab} + Z_{ab} + \left(4\pi/m^2\right)\beta_T^{ab}\left(\nu/m\right)^{-0.5} \pm \left(4\pi/m^2\right)\beta_V^{ab}\left(\nu/m\right)^{-0.5}$$

$$\chi_{tot}^2/N_{DF} = 431.48/\left(517-13\right) \qquad 13 = 18\,param - 5\,constr$$

$$with \quad \chi^2\left(pp\right)/N_D = 216.17/244 \qquad \left(5 = B - universality2 + 3\,FESR\right)$$

$$\chi^2\left(\pi p\right)/N_D = 150.97/162$$

$$\chi^2\left(Kp\right)/N_D = 64.34/111$$

$$Universal \;\; value : B = 0.280(11)mb$$

$$Our \;\; prev.result \quad 0.280(15)\,mb$$

Predictions

$\sqrt{s}$(TeV)	$\sigma_{tot}^{pp}(Igi-Ishida)$	$\sigma_{tot}^{pp}(HIIK)$	$\sigma_{tot}^{pp}(\exp)(mb)$
7	96.0(1.4)	96.1(1.1)	96.0±3.3±9.5 (ATLAS)
			94.0±2.8±3.3±5.5 (CMS)
			98.3±0.2±2.8 (TOTEM)
14	108.0(1.9)	108.1(1.4)	
57	135.5(3.1)	135.7(2.2)	94.0±2.8±3.3±5.5 (Auger)

TOTEM measures the pp total cross section at 7 TeV: $98.3 \pm 0.2_{stat} \pm 2.8_{syst}$ mb.
It is somewhat large value but consistent with our prediction 96.1±1.1 mb within the errors.

Third Topic(Appendix):Theoretical Prediction of Total Pion-Pion Scatterings. HIIK

- Based on Universality of B(first & second Topic), we can predict $\sigma_{tot}^{\pi^{\mp}\pi^{+}}(s)$ at high energy as,

$$\sigma_{tot}^{\pi^{\mp}\pi^{+}}(s) = B\log^2 {s}/{s_0} + Z_{\pi\pi} + \widetilde{\beta_T^{\pi\pi}}\left({s}/{s_1}\right)^{\alpha_T(0)-1} \pm \widetilde{\beta_V^{\pi\pi}}\left({s}/{s_1}\right)^{\alpha_V(0)-1}$$

- We expect $\widetilde{\beta_{T,V}^{ab}}$ take forms in terms of Reggeon-aa(bb) couplings

$$\gamma_{Raa,Rbb} \text{ with } \widetilde{\beta_T^{ab}} = \gamma_{Taa}\gamma_{Tbb} \ , \widetilde{\beta_V^{ab}} = \gamma_{Vaa}\gamma_{Vbb}$$

- The γ -couplings are expected to satisfy SU(2) symmetry.

$\beta_T^{\pi\pi}=\beta_T^{\pi p\ 2}/(\beta_T^{pp}/2)=16.0(\pm 3.9)$mb

$\beta_V^{\pi\pi}=\beta_V^{\pi p\ 2}/(\beta_V^{pp}/2)=1.9(+1.9-1.0)$mb ← Small

- Natural to assume that Universality of B and s_0 extend to $\pi\pi$ scattering.

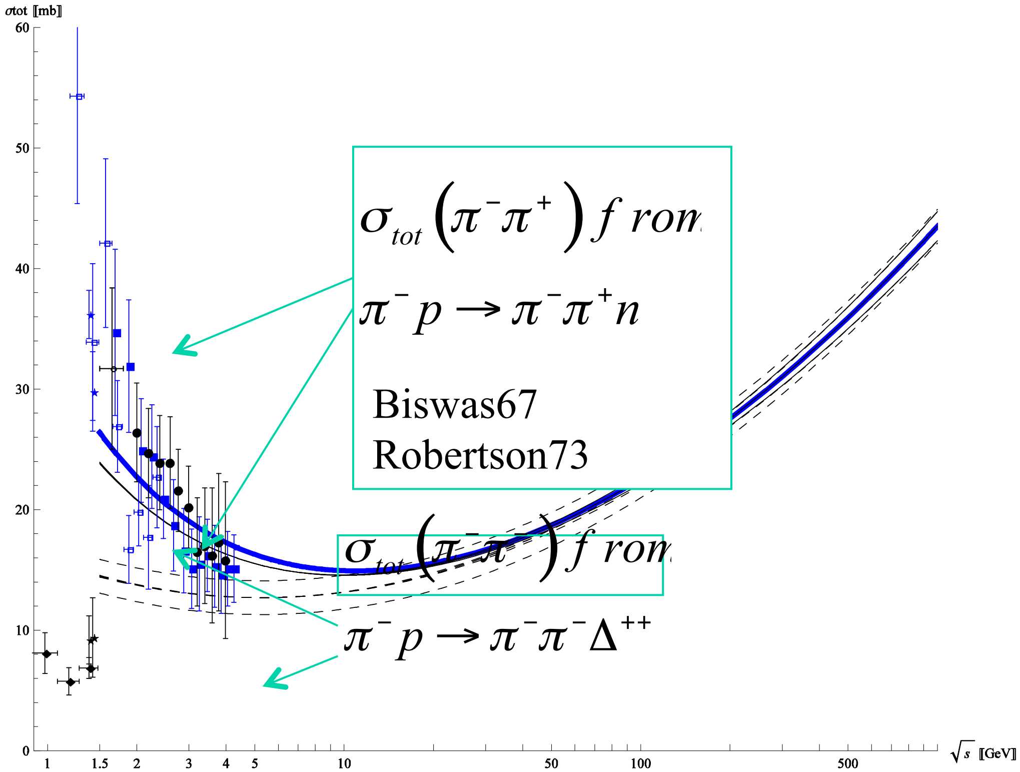
  $\sqrt{s_0^{\pi\pi}}(=\sqrt{s_0^{\pi p}})=5.28\pm0.63\mathrm{GeV}$

- $Z_{\pi p} \approx 2/3\, Z_{pp}$ →

  $Z_{\pi\pi}=(Z_{\pi p}/Z_{pp})Z_{\pi p}=12.7\pm1.4\mathrm{mb}$

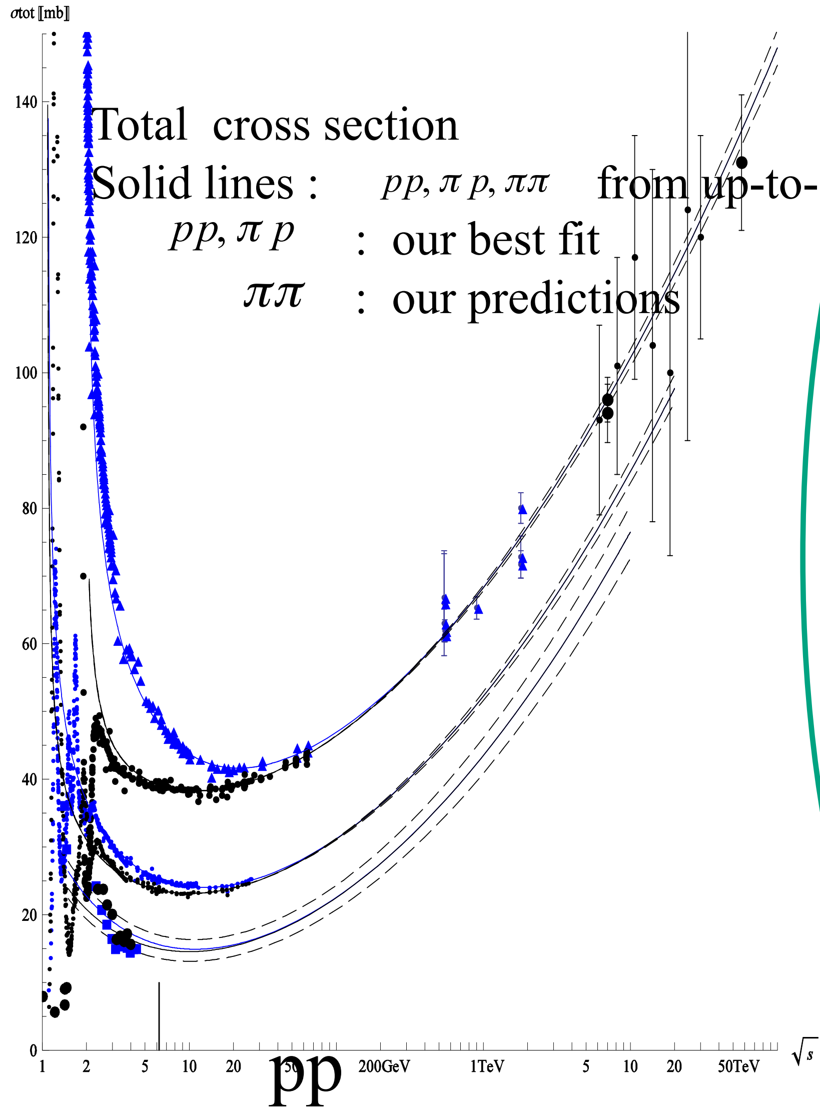
- s_1 is introduced as a typical scale for strong interactions which is taken to be $s_1 = 1\ GeV^2$
- In such a way , we can predict $\pi\pi$ total cross sections.

Comparison with Indirect experiments



Uncertainty from B

$$B \log^2 s/s_0 + Z_{\pi\pi}$$



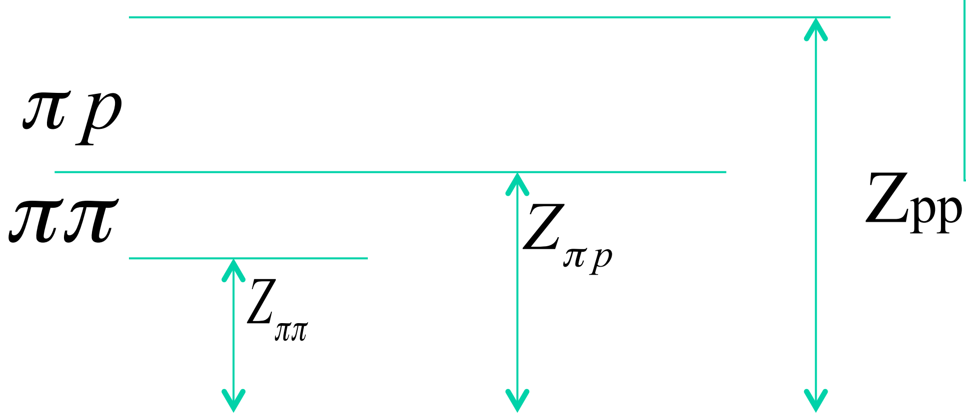
$B \log^2 s$

B=0.280(11)mb

Universal Rise

$Z_{\pi p} \approx 2/3\ Z_{pp}$

$Z_{\pi\pi} \approx 2/3\ Z_{\pi p}$



High-Energy $\pi\pi$ Experiment Possible?

- Although challenging, data on $\pi^{\mp}\pi^{+}$ collisions could be extended to higher energies exploiting high intensity proton beam accelerator beams planned worldwide, such as Project X of FNAL.
- At a later stage these may develop into muon colliders. As an example, Project X, a high intensity proton source proposed at Fermilab, would deliver proton beams at energies ranging from 2.5 to 120 GeV and second pion beams with

$$E(\pi) \approx 2 - 15\ GeV$$

- Private communication with Steven.Geer

A muon collider with Project-X-intensity pion beams would represent $\pi^+\pi^-$ collider with $\sqrt{s} = 1$ TeV and a lum. of $10^{22}\, cm^{-2}/\mathrm{sec}$,

not quite sufficient, even for measuring large cross sec. discussed here.

- Some manipulation of the secondary beams would be required.