

# **Forward hadron production at collider energies and its possible application to cosmic ray physics**

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# Plan

- **Motivation**

high-energy hadron-hadron collisions

vs first impact of cosmic ray air shower

- **Necessary ingredients for the calculation**

1. Evolution eq.: rcBK - a new working paradigm

2. "Initial Cond." : AAMQS - DIS global fit (see talk by J.G.Milhano)

3. Formula: DHJ -forward hadron production

4. MC treatment of nuclei

- **MC-DHJ/rcBK:** state-of-the-art calculation

work done with Fujii, Kitadono and Nara

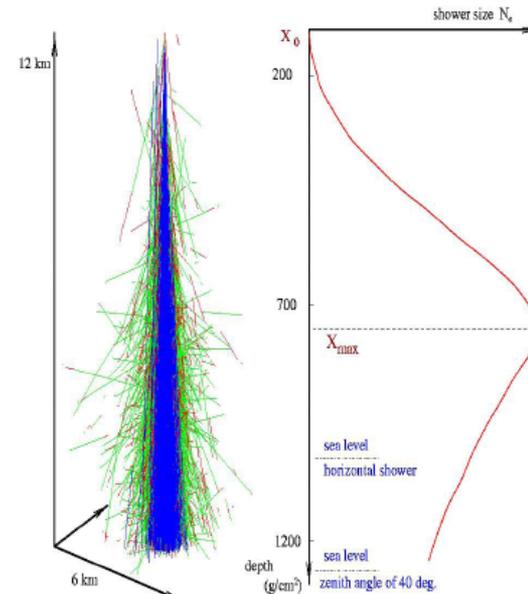
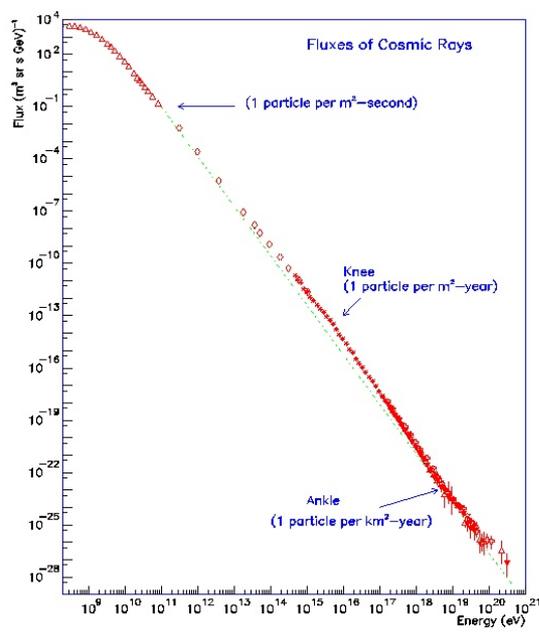
- **Future prospect towards application to CRs**

- **Summary**

# Motivation

- The first impact of cosmic ray air showers is an **extremely high energy scattering** of (probably) a proton off a nucleus in the air.

$$E_{\text{lab}} = 10^{20} \text{ eV}, \quad \sqrt{s_{pp}} = 433 \text{ TeV} \gg \sqrt{s_{pp}} = 14 \text{ TeV (LHC)}$$



- Hadronic interaction in the MC code** is crucial for determination of composition and correct energy estimation of the primary cosmic rays. (see talk by T. Pierog)
- We, theorists (working on high-energy scatt.), must provide up-to-date information on **forward hadronic cross section** based on the modern picture of theory.

# Aim of this talk

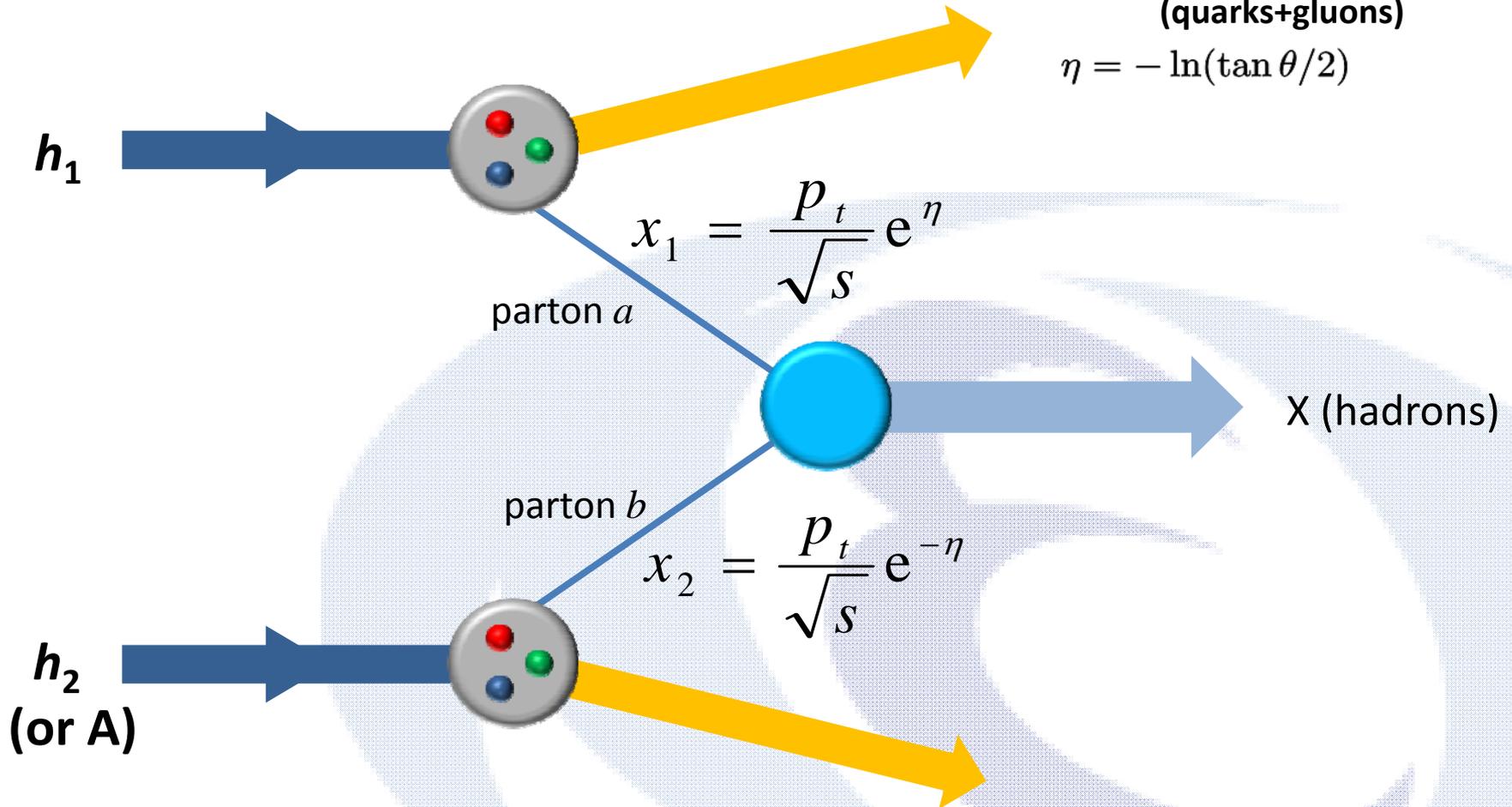
- To present (one of) the continuous efforts towards precise understanding of forward hadronic cross sections at high energies. (hadron production in hadron-hadron scattering)
- In particular, based on hard QCD picture. (could be irrelevant at very large rapidities, but not clear beyond which rapidity soft physics becomes dominant)

# What is necessary?

High-energy hadron-hadron scattering at forward rapidity  $\eta \gg 1$   
hadron-nucleus

$x_1, x_2$  : mom. fractions of partons  
(quarks+gluons)

$$\eta = -\ln(\tan \theta/2)$$



# What is necessary?

Parton distribution function  $f_a^{(h1)}(x_1)$  at large  $x$

$x_1, x_2$  : mom. fractions of partons (quarks+gluons)

$$\eta = -\ln(\tan \theta/2)$$

$h_1$

parton  $a$

$$x_1 = \frac{p_t}{\sqrt{s}} e^{\eta}$$

Cross section in coll. factorization  
 $d\sigma \sim \int dx_2 \int dx_1 f_a^{(h1)}(x_1) f_b^{(h2)}(x_2) d\hat{\sigma}_{ab}$

parton  $b$

$$x_2 = \frac{p_t}{\sqrt{s}} e^{-\eta}$$

X (hadrons)

$x_2$  becomes very small

pp @ LHC (7TeV)

$$x_1, x_2 \sim 3 \times 10^{-4} (\eta=0, p_t=2\text{GeV}),$$

$$x_2 \sim 7 \times 10^{-7} (\eta=6, p_t=2\text{GeV})$$

pp @ cosmic ray (400TeV)

$$x_1, x_2 \sim 5 \times 10^{-6} (\eta=0, p_t=2\text{GeV}),$$

$$x_2 \sim 1 \times 10^{-8} (\eta=6, p_t=2\text{GeV})$$

$h_2$   
(or A)

Parton distribution function  $f_b^{(h2)}(x_2)$  at very small  $x$

# What is necessary?

Parton distribution function  $f_a^{(h1)}(x_1)$  at large  $x$

$h_1$



$$x_1 =$$

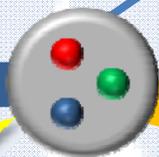
parton  $a$

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parton  $b$

$$x_2 = \frac{p_t}{\sqrt{s}} e^{-\eta}$$

$h_2$   
(or  $A$ )



Parton distribution function  $f_b^{(h2)}(x_2)$  at very small  $x$

What we really need at forward rapidity is

1. Evolution equation to go to higher energy (smaller  $x$ , instead of DGLAP)
2. Parton distribution in the target (instead of ordinary pdf's)
3. Formula useful at forward rapidity (instead of the collinear fact. formula)
4. A particular method of treating nuclei (instead of a homogeneous nucleus)

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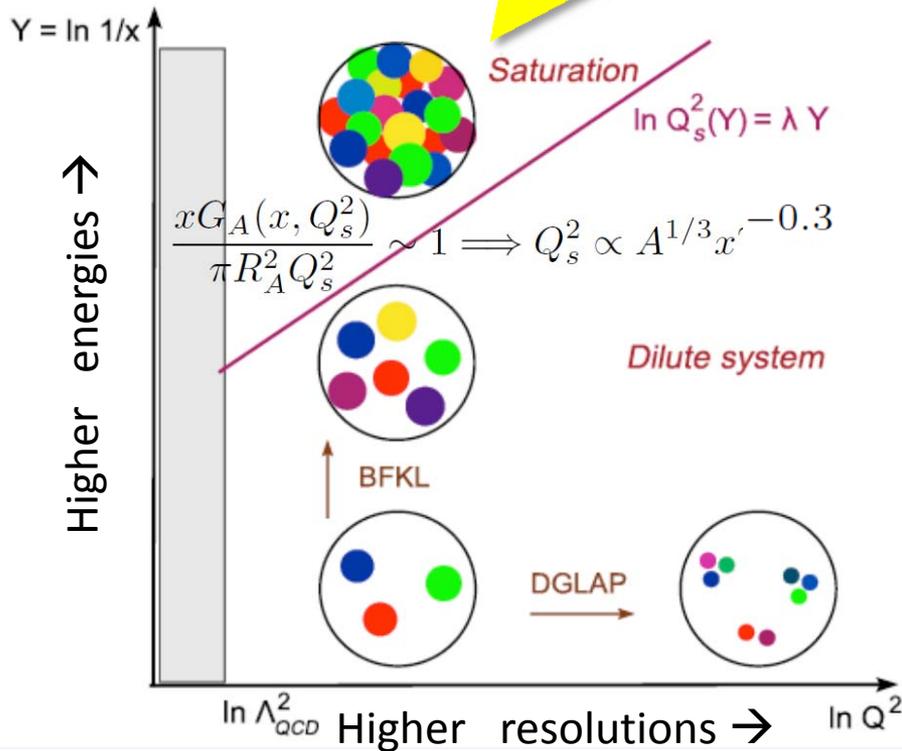
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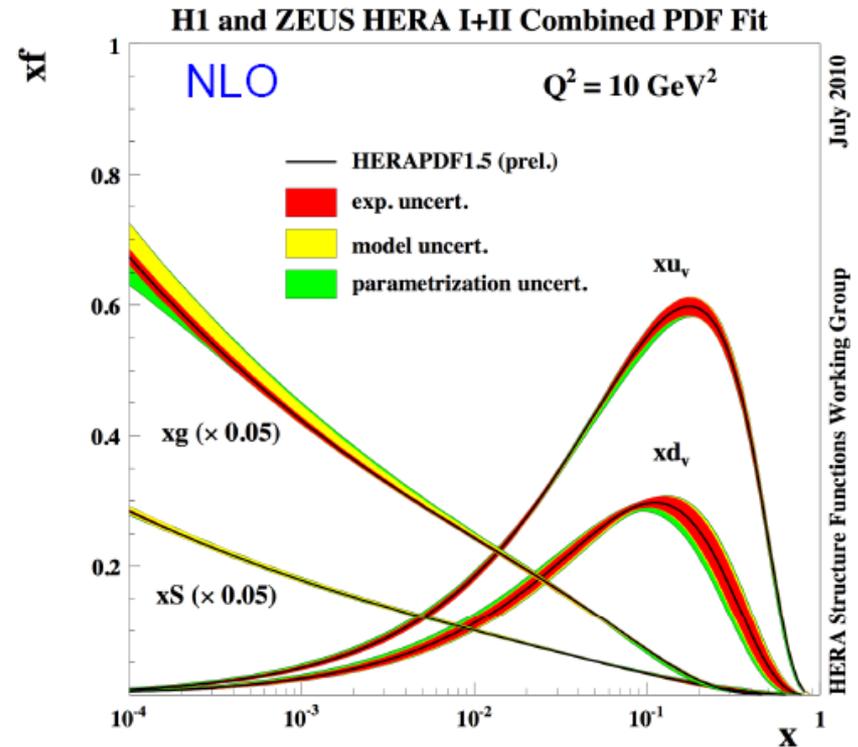
# CGC appears at small $x$

See talks by Milhano and Blaizot

**Color Glass Condensate:**  
High density gluonic states with saturation property



Internal structure of a proton



H1prelim-10-142 / ZEUS-prel-10-018

$Q_s(x, A)$ : saturation momentum  
boundary btw saturated and NON-saturated regimes

$Q_s \sim 1 \text{ GeV}$  at  $x=10^{-4}$  for a proton

# Going up higher energies: evolution eqs.

Evolution wrt  $x$  (or rapidity  $y = \ln 1/x$ ) for unintegrated gluon distribution

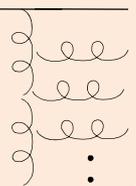
- **BFKL** (LO :  $(\alpha_s \ln 1/x)^n$ , NLO:  $\alpha_s (\alpha_s \ln 1/x)^n$ )

$$\frac{\partial \phi(\mathbf{x}, \mathbf{k}_t)}{\partial \ln(\mathbf{x}_0/\mathbf{x})} \approx \mathcal{K} \otimes \phi(\mathbf{x}, \mathbf{k}_t)$$

$K$ : gluon splitting  $g \rightarrow gg$   
 $\phi$ : unintegrated gluon distr.

**Multiple gluon emissions**

$N_g \sim e^{\omega \ln 1/x}$



**Recombination of gluons**

$N_g \leq 1$

**Unitarity**



- **BK** (includes the nonlinear effects)

$$\frac{\partial \phi(\mathbf{x}, \mathbf{k}_t)}{\partial \ln(\mathbf{x}_0/\mathbf{x})} \approx \mathcal{K} \otimes \phi(\mathbf{x}, \mathbf{k}_t) - \phi(\mathbf{x}, \mathbf{k}_t)^2$$

Known up to full NLO accuracy. [Balitsky, Chirilli 2008]

But for practical purposes, we use **BK with running coupling**  $\rightarrow$  "rcBK" [Balitsky, Gardi et al., Kovchegov-Weigert]

$$K^{\text{run}}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2) = \underbrace{\frac{N_c \alpha_s(r^2)}{2\pi^2} \left[ \frac{r^2}{r_1^2 r_2^2} \right]}_{\text{LO}} + \frac{1}{r_1^2} \left( \frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{1}{r_2^2} \left( \frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right)$$

# Fit to HERA data: AAMQS<sub>2011</sub>

- Initial Conditions : modified GBW/MV models

$$\left\{ \begin{array}{l} \mathcal{N}^{\text{GBW}}(r, x = x_0) = 1 - \exp\left[-\frac{(r^2 Q_{s0}^2)^\gamma}{4}\right], \\ \mathcal{N}^{\text{MV}}(r, x = x_0) = 1 - \exp\left[-\frac{(r^2 Q_{s0}^2)^\gamma}{4} \ln\left(\frac{1}{r\Lambda} + e\right)\right] \end{array} \right.$$

$$x_0 = 0.00893 \text{ or } 0.008$$

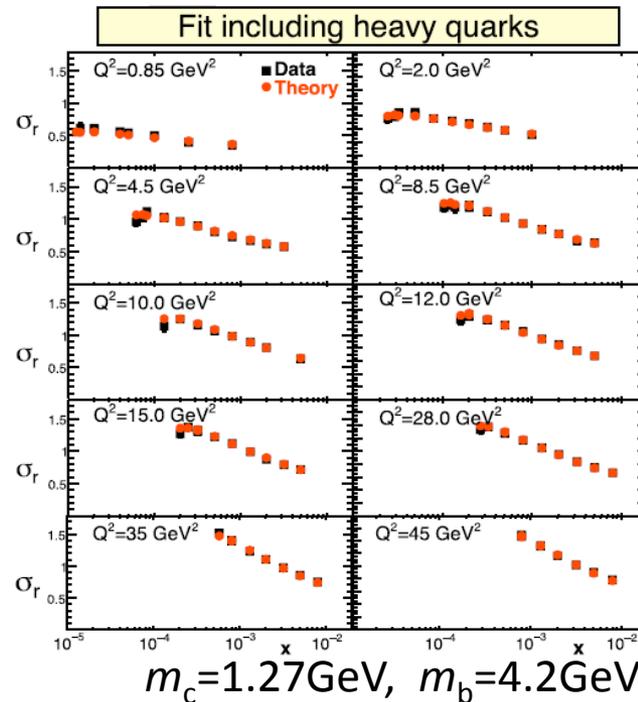
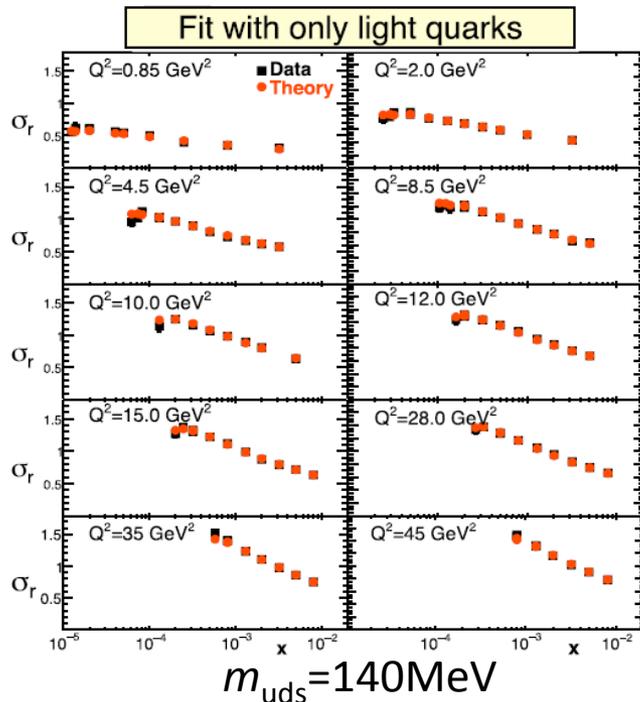
( $\gamma=1$  : ordinary GBW)

( $\gamma=1$  : ordinary MV)

- IR regularization for 1-loop running coupling

freeze the coupling at  $\alpha_s^{\text{fr}} = 0.7$

$$\alpha_{s,n_f}(r^2) = \frac{4\pi}{\beta_{0,n_f} \ln\left(\frac{4C^2}{r^2 \Lambda_{n_f}^2}\right)}$$



← Modified GBW

(Left)  $\gamma = 0.971$   
 $Q_{s0}^2 = 0.241$

(Right)  $\gamma = 0.959$   
 $Q_{s0}^2 = 0.240$

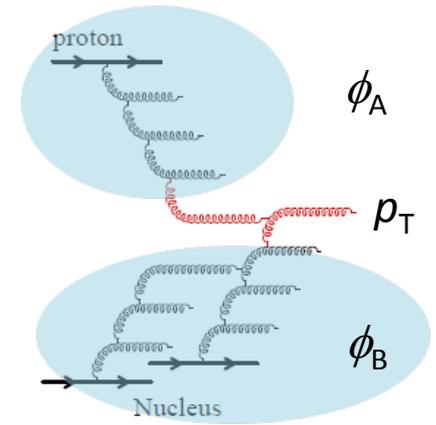
There are two more parameters ( $C, \sigma_0$ )

# Hadron collisions (pp/pA): two formulae

- $k_t$  factorization

$$\frac{d\sigma^{A+B \rightarrow g}}{dy d^2 p_T d^2 X} \sim K \frac{\alpha_s}{p_T^2} \phi_A(k_1, x_1, b) \otimes \phi_B(k_2, x_2, X - b)$$

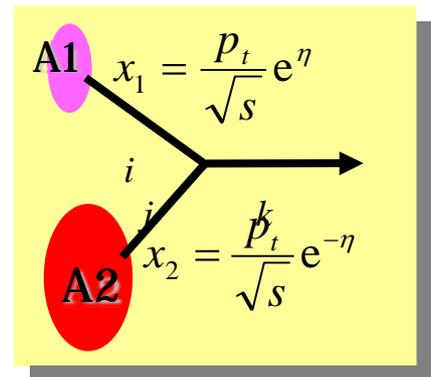
- proved for pp, pA at LO
- good when both A and B are saturated (mid rapidity at very high energy)
- used in various calculations e.g. multiplicity distribution, etc



- DHJ formalism** [Dumitru-Hayashigaki-Jalilian--Marian 2006]

$$\frac{dN}{dy_h d^2 p_T} = \frac{K}{(2\pi)^2} \sum_{ijk} \int_{x_F}^1 \frac{dz}{z^2} x_1 f_{i/p}(x_1, p_T^2) \tilde{N}_j\left(\frac{p_T}{z}, x_2\right) D_{h/k}(z, p_T^2)$$

- “Large-x / small-x” reactions: **valid at forward rapidity**  
 $x_1 \sim 1, x_2 \ll 1$
- $f_{i/p}(x)$ : pdf for valence (large x) partons in the projectile
- $D_{h/k}(z)$ : frag. func. for outgoing hadron  $h$  from a parton  $k$
- $\tilde{N}$ : un-integrated gluon distribution in the target

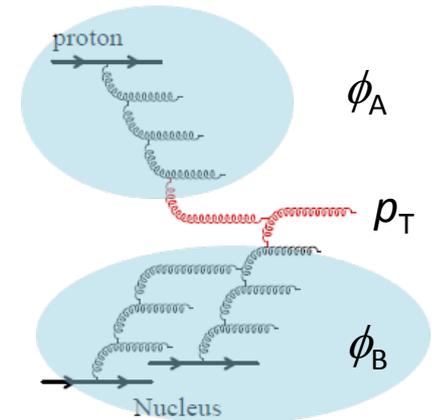


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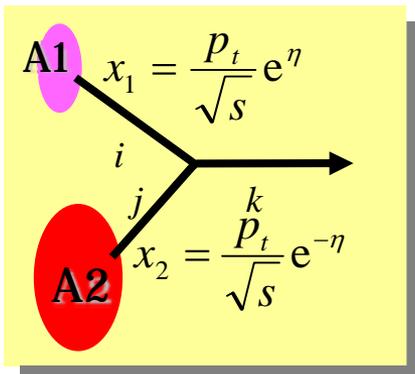
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# How to treat nuclei?

MC modeling for a nucleus:

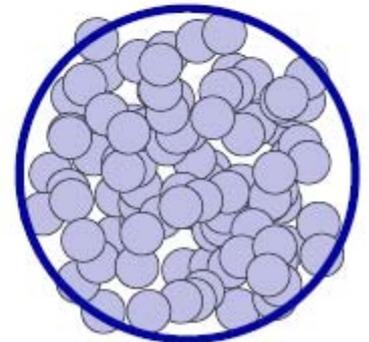
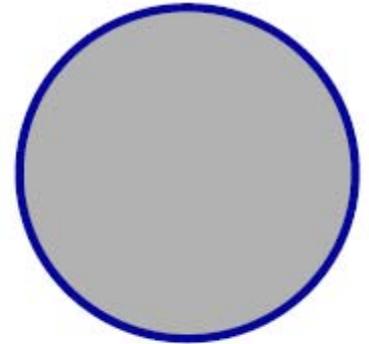
- The simplest will be a homogeneous disk  
no impact parameter dependence  
an additional parameter  $Q_{s0A}^2$  needed

may use a simple parametrization by KLN,  
or numerical solution to rcBK

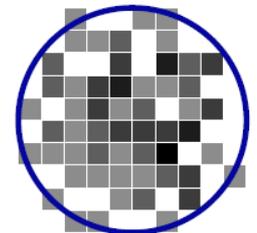
- Random nucleons w/ Woods-Saxon dist.  
fluctuating density  $\Rightarrow$   $b$ -dependence

$$Q_{s0A}^2 = Q_{s0p}^2 \times N \text{ w/o additional parameter}$$

Drescher-Nara



Apply quantum evolution locally at  
different transverse bins  $\Rightarrow$



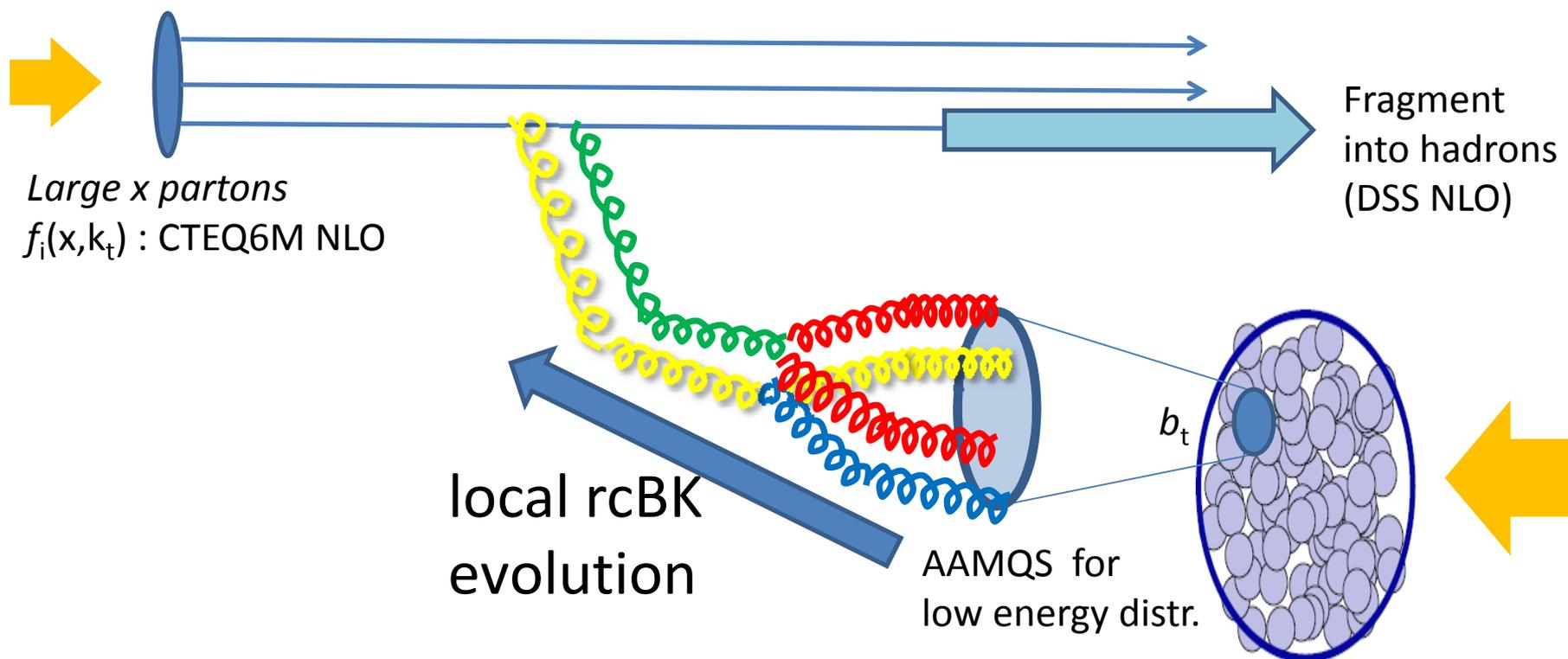
# MC-DHJ/rcBK

[Fujii, KI, Kitadono, Nara,  
arXiv:1107.1333,  
more to come soon]

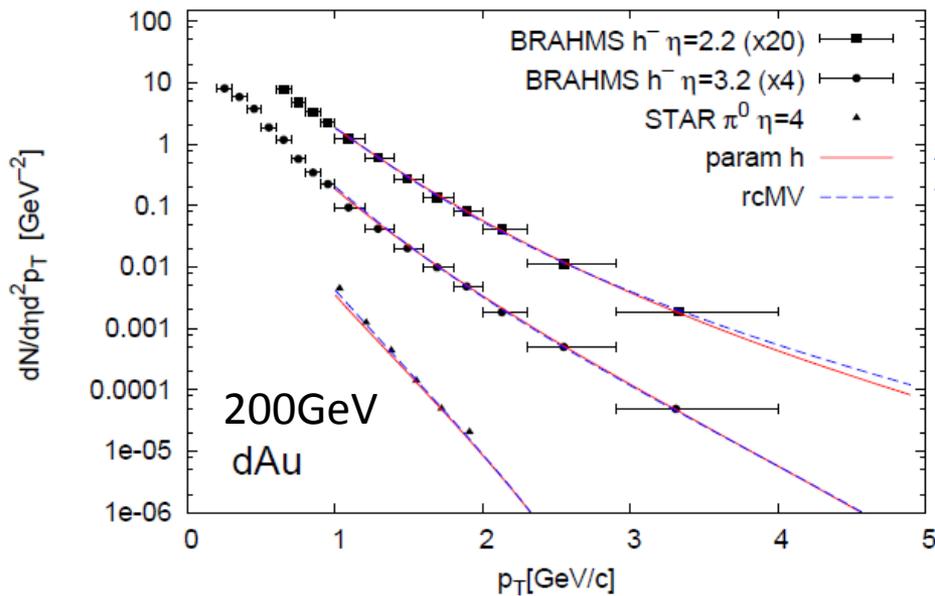
To reduce ambiguity

- construct a nucleus by randomly placing nucleons
- use AAMQS parameters for proton IC optimized for DIS at small- $x$
- quantum evolution is performed “locally” in  $b$  space with rcBK

(to avoid IR div. in  $b$ -dep BK)



# MC-DHJ/rcBK : results

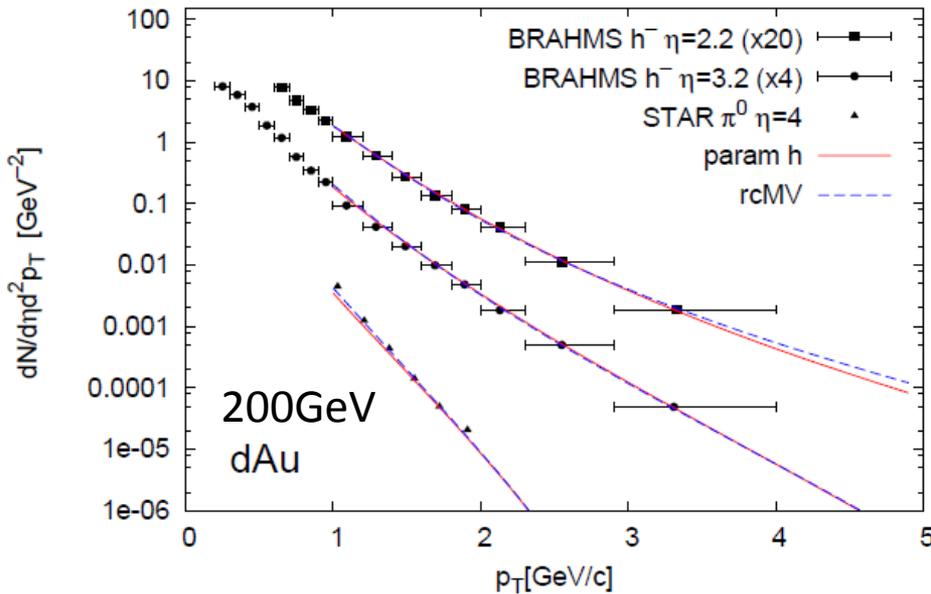


modified MV model ( $\gamma = 1.118$ )

“running coupling” version of MV model [Iancu-KI-Triantafylopoulos] : to be consistent with rcBK evolution

- reproduce the data nicely
- AAMQS set  $h$  and rcMV for  $\mathcal{N}(r, y)$
- $Q_{s0A}^2$  fixed by MC; **no additional parameter**

# MC-DHJ/rcBK : results



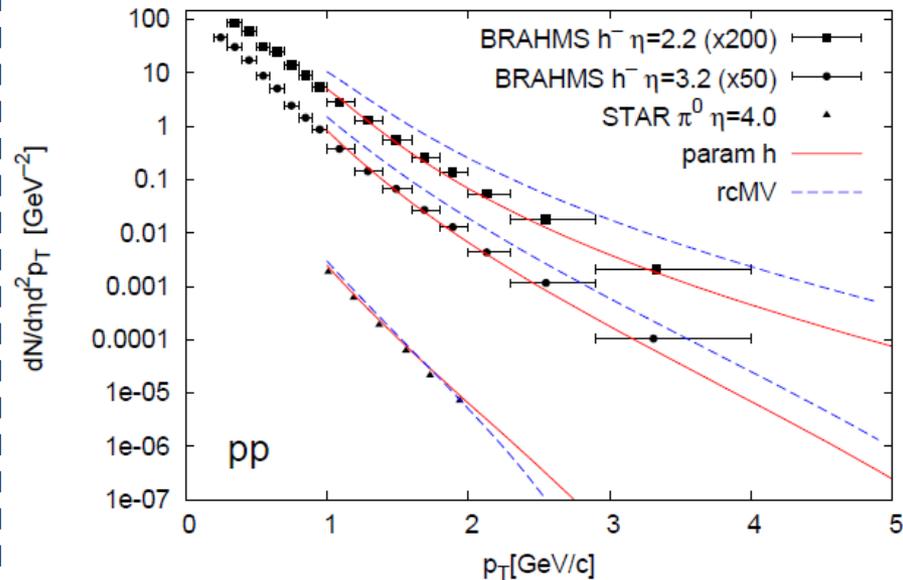
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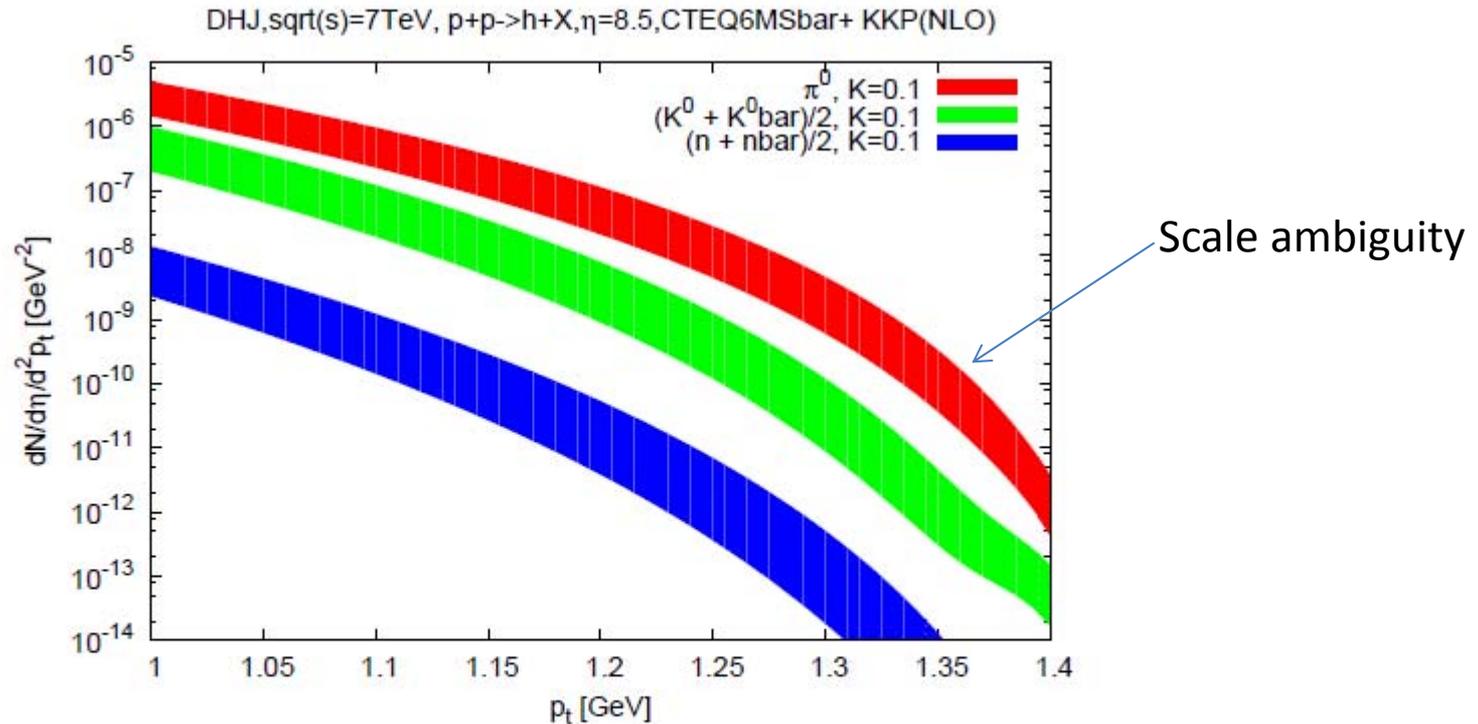
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**Best results from theoretical point of view, but still needs better (global) description including pp data (tuning of rcMV is necessary)**

- Set  $h$  works well even in **pp**
- rcMV is not “tuned” (similar param as MV)
- However, both work quite well in dAu (IC dependence reduces at high rapidity)



# MC-DHJ/rcBK extrapolated to LHC



- Hadron productions ( $\pi^0$ ,  $K^0$  and  $n$ ) at  $\eta = 8.5$  at 7 TeV (LHCf) is being studied in this framework

Very forward region could be dominated by soft interaction, but still necessary to understand how much hard contribution exists.

# Future prospect towards application to CRs

- Separation between soft and hard is not clear (model dependent)
- In several hadron interaction models (e.g. SIBYLL), IR cutoff for the hard contribution is energy dependent (very similar to  $Q_s(s)$ )

$$\text{SIBYLL2.0} \quad p_{\perp}^{cutoff} = p_{\perp}^0 + \Lambda \exp\left\{c\sqrt{\ln(s / \text{GeV}^2)}\right\}$$

with  $p_{\perp}^0 = 1 \text{ GeV}$ ,  $\Lambda = 0.065 \text{ GeV}$  and  $c = 0.9$

→ miss particle production in the semi-hard region!!

- CGC provides particle production in semi-hard region  $\Lambda_{\text{QCD}} < k_t < Q_s$  that expands with increasing energy

→ filling the gap btw soft and hard

- Calculations with CGC could help to “recover” the semi-hard contributions.

# Summary

- Theoretical description of high-energy hadron scattering based on CGC is now (almost) established up to leading log accuracy with running coupling corrections. → rcBK paradigm
- In particular, phenomenological analysis with rcBK has been making a progress enough to be compared with experimental data. → HERA DIS at small- $x$ , RHIC dAu at forward rapidity
- This approach can be, in principle, applied to higher energy collision, thus hopefully to the first impact of Cosmic Rays in the air.