# Anti-de-Sitter AVATARS of LARGE N QCD DIPOLES

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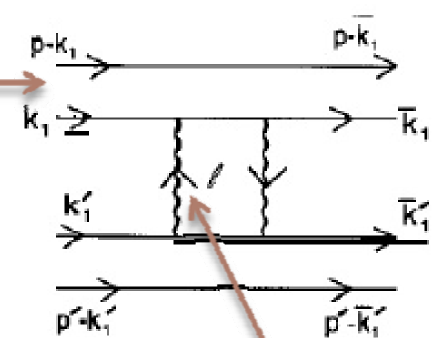
# OVERVIEW

- Large N expansion implemented through Maldacena's AdS/CFT correspondence is at present our main analytical tool for understanding non-perturbative features of YM Gauge theory
- *N*=4 SuperYM: exact results summarized by the Bethe ansatz(defining Gauge/String Duality)
- Vector models, result in $AdS_4/CFT_3$ correspondence [Klebanov & Polyakov '02] with Higher Spin gravity in pure AdS spacetime
- Dipole Model of AdS/CFT[S.Das+A.J,R.d.M.Koch, K.Jin ,J.P.Rodrigues, Q.Ye

# QCD Dipole

- Forward quarkonium-quarkonium scattering can be studied using the dipole picture at large $N_c$ limit.
- [A. H. Mueller & B. Patel '94; M. Li & C. I. Tan '95 ]
- At high energy and large $N_c$ limit, each gluon acts like a quark-antiquark pair.
- So the quarkonium can be viewed as a collection of quark-antiquark dipoles.
- The dipole-dipole scattering amplitude is found through a gluon-pair exchange diagrams



# Scheme for quarkonium scattering

- Basic dipole-dipole scattering amplitude:

$$F^{(0)} = -\frac{1}{2}\alpha^2 \int \frac{d^2\underline{l}}{(\underline{l}^2)^2}[2 - exp(-i\underline{l}\cdot\underline{x}_{01}) - exp(i\underline{l}\cdot\underline{x}_{01})][2 - exp(-i\underline{l}\cdot\underline{x}'_{01}) - exp(i\underline{l}\cdot\underline{x}'_{01})]$$

- Plus incorporation of the dipole density: integral eqs

[A. H. Mueller & B. Patel '94] $n(\underline{x}_{01}, \underline{x}, Y)$

- Alternatively the amplitude can be found through the BFKL pomeron exchange scheme.

- Relevance of AdS for the Pomeron[ R. C. Brower, J. Polchinski, M. J. Strassler & C. I. Tan '06 ]

# Talk:

- Review the collective Dipole Mechanism which will lead to

AdS4 HS gravity from Field theory

S. Das, AJ '03; R. de Mello Koch, K, Jin, AJ, J. Rodrigues ,Q.Ye'2010,'1

Origin of Anti de Sitter space-time

Emergence of Gravity (and of Higher Spin ) Degrees of Freedom

Scheme for Quarkonium Scattering through the AdS representation

# Collective dipole representation:

- Consider the example of a 3d field theory, with
- O(N) symmetry $\vec{\phi}(x) = (\phi_1, \phi_2, \ldots \phi_N)$
- And Conformal invariance

O(N) invriant

- The Dipole is represented by the Composite;

$$\Phi(x,y) = \phi(x) \cdot \phi(y) = \sum_{a=1}^{N} \phi^a(x)\phi^a(y)$$

↑ ↑

3d + 3d

# Relativistic Dipole(in Light-coneDescription):

- Hamiltonian and momentum can be written in terms of bi-local fields as

$$H^{(2)} = \int d\vec{x} d\vec{y} \psi^\dagger(\vec{x}, \vec{y}) \Big( \sqrt{-\nabla_x^2} + \sqrt{-\nabla_y^2} \Big) \psi(\vec{x}, \vec{y}),$$

$$P^{(2)} = \int d\vec{x} d\vec{y} \psi^\dagger(\vec{x}, \vec{y}) (\partial_{\vec{x}} + \partial_{\vec{y}}) \psi(\vec{x}, \vec{y}).$$

For light-cone quantization, we have:

$$P^-_{(2)} = H_{(2)} + P_{(2)} = \int dx_1^- dx_2^- dx_1 dx_2 \psi^\dagger \Big( -\frac{\nabla_1^2}{2p_1^+} - \frac{\nabla_2^2}{2p_2^+} \Big) \psi$$

# Conformal group

- To establish its $AdS_4$ manifestation, we will concentrate on the realization of the conformal group

- In light-cone quantization: $x^+ = \tau$ is the propagating time

$$\phi(x^-, x^i) = \int_0^\infty \frac{dp^+}{\sqrt{2\pi}} \frac{1}{\sqrt{2p^+}} \left( a(p^+, x^i) e^{ip^+x^-} + a^\dagger(p^+, x^i) e^{-ip^+x^-} \right),$$

(In d=3, we have only one transverse coordinate: $x^i = x$ )

$$x^\mu = (x^+, x^-, x) = (\tau, x^-, x)$$

# Conformal group transformations:

$$P^- : \qquad \delta a(p^+, x^i) = \frac{\partial_i^2}{2p^+} a(p^+, x^i),$$

$$M^{+-} : \qquad \delta a(p^+, x^i) = \Big(t\frac{\partial_i^2}{2p^+} - i\sqrt{p^+}\frac{\partial}{\partial p^+}\sqrt{p^+}\Big) a(p^+, x^i)$$

$$M^{-i} : \qquad \delta a(p^+, x^i) = \Big(-\partial_i \frac{\partial}{\partial p^+} - \frac{\partial_j x^i \partial_j}{2p^+}\Big) a(p^+, x^i)$$

$$D : \qquad \delta a(p^+, x^i) = \Big(t\frac{\partial_i^2}{2p^+} + i\Big[d_\phi + x^i\partial_i + \sqrt{p^+}\frac{\partial}{\partial p^+}\sqrt{p^+}\Big]\Big) a(p^+, x^i)$$

$$K^+ : \qquad \delta a(p^+, x^i) = \Big\{t^2\frac{\partial_i^2}{2p^+} + it(d_\phi + x^i\partial_i) - \frac{1}{2}x^i x^i p^+\Big\} a(p^+, x^i)$$

- and similarly for other conformal generators.

# For the bi-local field (dipole):

- These induce transformations of the bi-local fields.
- In creation-annihilation form

$$A(x_1^-, x_2^-, \vec{x}_1, \vec{x}_2) = a(x_1^-, \vec{x}_1) a(x_2^-, \vec{x}_2)$$

we deduce the bi-local transformations as:

$$\delta A(1,2) = \delta a(1) a(2) + a(1) \delta a(2)$$

giving the bi-local generators:

$$G = \int dx_1^- dx_2^- dx_1 dx_2 A^\dagger \hat{g} A = \int dx_1^- dx_2^- dx_1 dx_2 A^\dagger (\hat{g}_1 + \hat{g}_2) A.$$

with $\hat{g}_1 + \hat{g}_2$ representing the two-particle "dipole" generators.

# They take the form:

$$\hat{p}^- = p_1^- + p_2^- = -\left(\frac{p_1^i p_1^i}{2p_1^+} + \frac{p_2^i p_2^i}{2p_2^+}\right)$$

$$\hat{m}^{-i} = x_1^- p_1^i + x_2^- p_2^i + x_1^i \frac{p_1^j p_1^j}{2p_1^+} + x_2^i \frac{p_2^j p_2^j}{2p_2^+}$$

$$\hat{d} = t\hat{p}^- + x_1^- p_1^+ + x_2^- p_2^+ + x_1^i p_1^i + x_2^i p_2^i + 2d_\phi$$

$$\hat{k}^+ = t^2 \hat{p}^- + t(x_1^i p_1^i + x_2^i p_2^i + 2d_\phi) - \frac{1}{2} x_1^i x_1^i p_1^+ - \frac{1}{2} x_2^i x_2^i p_2^+$$

SO(2,3) with 10 generators operating in the 5d dipole space:

$$(\tau; x_1^-, x_1; x_2^-, x_2)$$

# III. ONE-TO-ONE MAP:

- $CFT_3$: collective bi-local fields   $AdS_4$: higher spin fields

$$\Psi(x^+;(x_1^-,x_1),(x_2^-,x_2)) \Longleftrightarrow \Phi(x^+;x^-,x,z;\theta)$$

- Same number of dimensions

1+2+2 = 1+3+1

- AdS4 :Representation of the conformal group SO(2,3)

Clear from analysis of the two representations that one is not going to have a simple coordinate transformation

# Light-cone form of EOM

- Through the metric:

$$ds^2 = \frac{2dtdx^- + dx^2 + dz^2}{z^2}$$

- Light-cone equations of motion

$$\left( z^2(2\partial_+\partial_- + \partial_I^2) + \frac{1}{2}M_{ij}^2 - \frac{(d-4)(d-6)}{4} \right) |\phi\rangle = 0$$

- From which we deduce the generator

$$P^- = \partial_+ = -\frac{\partial_I^2}{2\partial_-} + \frac{1}{2z^2\partial_-}\left( -\frac{1}{2}M_{ij}^2 + \frac{(d-4)(d-6)}{4} \right)$$

- Other generators of the space-time SO(2,3) group can be evaluated similarly.

- In four dimensions, the only non-vanishing spin matrix is $M^{xz}$

Represents Spin (Helicity)

$$M^{xz} \longrightarrow \frac{\partial}{\partial \theta}$$

- The physical field

Extra AdS coordinate

$$\Phi(x^+, x^-, x, z; \theta)$$

HS coordinate

- Symmetry generators

$$G = \int dx^- dx dz d\theta \; \bar{\Phi} \hat{g} \Phi$$

# The transformation

- Identifying the generators of the dipole with the generators of HS: gives 10 equations of 2×4=8 canonical variables

$$x^- = \frac{x_1^- p_1^+ + x_2^- p_2^+}{p_1^+ + p_2^+},$$

$$p^+ = p_1^+ + p_2^+,$$

$$x = \frac{x_1 p_1^+ + x_2 p_2^+}{p_1^+ + p_2^+},$$

$$p^x = p_1 + p_2.$$

$$z = \frac{(x_1 - x_2)\sqrt{p_1^+ p_2^+}}{p_1^+ + p_2^+},$$

$$p^z = \sqrt{\frac{p_2^+}{p_1^+}} p_1 - \sqrt{\frac{p_1^+}{p_2^+}} p_2,$$

$$\theta = 2 \arctan \sqrt{\frac{p_2^+}{p_1^+}}.$$

$$p^\theta = \sqrt{p_1^+ p_2^+}(x_1^- - x_2^-) + \frac{x_1 - x_2}{2}\left(\sqrt{\frac{p_2^+}{p_1^+}} p_1 + \sqrt{\frac{p_1^+}{p_2^+}} p_2\right).$$

# From bi-local field to HS field:

- In the dual (momentum) space: ( $p_1^+, p_2^+, p_1, p_2$ ), one observes the transformation takes the form of a point transformation:

$$
\begin{aligned}
p^+ &= p_1^+ + p_2^+ , \\
p^x &= p_1 + p_2 , \\
p^z &= p_1 \sqrt{p_2^+/p_1^+} - p_2 \sqrt{p_1^+/p_2^+} , \\
\theta &= 2 \arctan \sqrt{p_2^+/p_1^+} ,
\end{aligned}
$$

- Consequently perform a Fourier transform:

$$
\tilde{\Phi}(p_1^+, p_2^+, p_1, p_2) = \int dx_1^- dx_2^- dx_1 dx_2 e^{-i(x_1^- p_1^+ + x_2^- p_2^+ + x_1 p_1 + x_2 p_2)} \Phi(x_1^-, x_2^-, x_1, x_2)
$$

# A Canonical Map

- We have the $AdS_4$ canonical variables in terms of the dipole canonical variables (of $CFT_3$).
- A consistency check on the correctness of the map: the Poisson brackets take the canonical form.
- Assuming

$$\{x_i^-, p_i^+\} = \{x_i, p_i^x\} = 1$$

we verify that

$$\{x^-, p^+\} = \{x, p^x\} = \{z, p^z\} = \{\theta, p^\theta\} = 1$$

- We have a reconstruction of AdS space-time from the bi-local one.

# The Transform:

- Changing to AdS variables ,plus an integral transform gives the AdS higher-spin field in terms of the bi-local one:

$$
\begin{aligned}
\Psi(x^-, x, z, \theta) \quad = \quad & \int dp^+ dp^x dp^z e^{i(x^- p^+ + xp^x + zp^z)} \\
& \int dp_1^+ dp_2^+ dp_1 dp_2 \delta(p_1^+ + p_2^+ - p^+) \delta(p_1 + p_2 - p^x) \\
& \delta\left(p_1 \sqrt{p_2^+/p_1^+} - p_2 \sqrt{p_1^+/p_2^+} - p^z\right) \\
& \delta(2\arctan\sqrt{p_2^+/p_1^+} - \theta)\, \tilde{\Phi}(p_1^+, p_2^+, p_1, p_2)
\end{aligned}
$$

# Checking the z=0 projection

- The bilocal field gives a strong off-shell operator re-construction of the HS field: $\Phi = W|_{\text{gauge}}$
- One check on our identification of the extra AdS coordinate z is the evaluation of the z=0 limit
- We expect that they reduce to the conformal operators
- At z=0:

$$\Phi(x^+, x^-, x, \theta) = \int dp_1^+ dp_2^+ e^{ix^-(p_1^+ + p_2^+)} \delta(\theta - 2\tan^{-1}\sqrt{p_2^+/p_1^+})\tilde{\psi}(p_1^+, p_2^+, x, x)$$

- Expanding the delta function in Fourier series, one has the binomial expansion

$$(\sqrt{p_1^+} - i\sqrt{p_2^+})^{2s} = \frac{(-1)^k (2s)!}{(2k)!(2s-2k)!}(p_1^+)^k (p_2^+)^{s-k}$$

- The conformal operators for a fixed spin s is

$$\mathcal{O}^s = \sum_{k=0}^{s} \frac{(-1)^k \Gamma(s+1/2)\Gamma(s+1/2)}{k!(s-k)!\Gamma(s-k+1/2)\Gamma(k+1/2)} (\partial_+)^k \varphi (\partial_+)^{s-k} \varphi$$

- The expansion coefficients agree up to an overall normalization

$$\frac{(2s)!}{(2k)!(2s-2k)!} = \frac{s!\ \Gamma(s+1/2)\Gamma(1/2)}{k!(s-k)!\ \Gamma(s-k+1/2)\Gamma(k+1/2)}$$

# conserved currents:

- Conserved currents:

Makeenko '81; Mikhailov '02
Braun, Korchemsky, Muller '03

$$J_{\mu_1\ldots\mu_s} = \sum_{k=0}^{s}(-1)^k \binom{s-1/2}{k}\binom{s-1/2}{s-k} \partial_{\mu_1}\cdots\partial_{\mu_k}\phi\, \partial_{\mu_{k+1}}\cdots\partial_{\mu_s}\phi - \text{traces}$$

Composite operators ,conformal,can be packed into a generating function

$$\mathcal{O}(\vec{x},\vec{\epsilon}) = \phi^i(x-\epsilon)\sum_{n=0}^{\infty}\frac{1}{(2n)!}(2\epsilon^2\overleftarrow{\partial_x}\cdot\overrightarrow{\partial_x} - 4(\epsilon\cdot\overleftarrow{\partial_x})(\epsilon\cdot\overrightarrow{\partial_x}))^n\phi^i(x+\epsilon)$$

Couple to Higher Spin Gauge Fields

Bekaert

# Sumarry

- there is no coordinate transformation between coordinates of $AdS_4$ and bi-local coordinates emerging from $CFT_3$,

$$\Phi(x^\mu, z; \alpha) \leftrightarrow \Psi(x^+; (x_1^-, x_1), (x_2^-, x_2))$$

- The relationship between field is slightly more non-trivial:

✓ From the canonical transformation we can deduce that there exists a kernel K (AdS/dipole):

$$\Phi(x^\mu, z; \alpha) = \int K(x^\mu, z; \alpha | x_\mu^1, x_\mu^2) \Psi_c(x_1^\mu, x_2^\mu) d^3x_1 d^3x_2$$

✓ Nonlocal Map : AdS/CFT Holography

# VI. More on Gauge theory of Higher Spins

- Infinite Sequence of higher spins:S=2,4,6….in AdS4

Fronsdal in1980's gave a formulation of free field equations

M.Vasiliev 1990 :Interactions

Symmetric, double-traceless tensor fields:

$$h_{\mu_1 \ldots \mu_s} \qquad g^{\mu_1 \mu_2} g^{\mu_3 \mu_4} h_{\mu_1 \mu_2 \mu_3 \mu_4 \ldots \mu_s} = 0$$

- Covariant gauge conditions:

- $$\nabla^{\mu} h_{\mu \mu_2 \ldots \mu_s} = 0 \qquad g^{\mu_1 \mu_2} h_{\mu_1 \mu_2 \ldots \mu_s} = 0$$

- Embedding $AdS_4$ into $R^5$:

$$x^\mu \to y^\alpha \qquad y^\alpha y_\alpha = -1, \quad \alpha = 0, 1, 2, 3, 5$$

- Higher-spin fields:

$$h_{\mu\ldots} = y^\alpha_\mu \cdots k_{\alpha\ldots} \qquad\qquad y^\alpha_\mu = \partial y^\alpha / \partial x^\mu$$

- Synthesis of all integer spins

$$K(y,z) = \sum_s z^{\alpha_1} \cdots z^{\alpha_s} k_{\alpha_1 \cdots \alpha_s}(y)$$

# Covariant gauge

- Summary of all conditions:

| | $k_{\alpha_1 \cdots \alpha_s}(y)$ | $K(y, z)$ |
|---|---|---|
| transversality | $y \cdot k = 0$ | $y \cdot \partial_z K = 0$ |
| gauge condition | $\partial \cdot k = 0$ | $\partial_y \cdot \partial_z K = 0$ |
| traceless | $k' = 0$ | $\partial_z^2 K = 0$ |
| ~~$y^2 = -1$~~ | $(y \cdot \partial_y + s + 1)k = 0$ | $(y \cdot \partial_y + z \cdot \partial_z + 1)K = 0$ |

# Mapping to bi-local representation:

$$(y \cdot \partial_y + z \cdot \partial_z + 1)K = 0 \qquad (p \cdot \partial_p + 1/2)\Phi = 0$$

$$y \cdot \partial_z K = 0 \qquad (q \cdot \partial_q + 1/2)\Phi = 0$$

EOM

$$\partial_y^2 K = 0 \qquad \partial_p^2 \Phi = 0$$

$$\partial_z^2 K = 0 \qquad \partial_q^2 \Phi = 0$$

$$\partial_y \cdot \partial_z K = 0 \qquad \partial_p \cdot \partial_q \Phi = 0$$

EOM

First-class constraints

This gives a 3+3 dimensional representation.

One can extend this to the nonlinear, interacting level.

# Vasiliev ('80-'92):

- Developed an extension to include interactions.
- Fields in the HS Lie algebra:

$$\Phi(x|y,\bar{y},z,\bar{z}) = \sum \phi_{(n,m,n',m')}(x)y^n\bar{y}^m z^{n'}\bar{z}^{m'}$$

- Star product:

$$f(y,z) * g(y,z) = \int d^2u d^2v e^{u^\alpha v_\alpha} f(y+u,z+u) g(y+v,z-v),$$

- EOMs:

$$\begin{aligned}
&(1) \qquad d_x W + W * W = 0,\\
&(2) \qquad d_Z W + d_x S + \{W,S\}_* = 0,\\
&(3) \qquad d_Z S + S * S = B * K dz^2 + B * \bar{K} d\bar{z}^2,\\
&(4) \qquad d_x B + W * B - B * \pi(W) = 0,\\
&(5) \qquad d_Z B + S * B - B * \pi(S) = 0.
\end{aligned}$$

# AdS/CFT DUALITY:

- Interacting Higher Spin Gravity is Equivalently described by Bi-local (Dipole) Field Theory( density of dipoles)

Effective action:

$$S_{eff} = \mathrm{Tr}[-(\partial_x^2 + \partial_y^2)\Phi(x,y) + m^2\Phi(x,y) + V] + \frac{N}{2}\mathrm{Tr}\ln\Phi$$

- Origin of the $\ln\Phi$ interaction: Jacobian

$$\int d\vec{\phi} e^{-S} \rightarrow \int d\Phi \det\left|\frac{\partial\phi^a(x)}{\partial\Phi(x_1,x_2)}\right| e^{-S}$$

space

momentum cutoff

# Expansion

- Equation of motion:

$$\frac{\partial S_c}{\partial \Phi} = 0 \Leftrightarrow -(\partial_x^2 + \partial_y^2)\delta(x-y) + \frac{\lambda}{2}\Phi(x,y) + N\frac{1}{\Phi(x,y)} = 0$$

- 1/N expansion parameter:

$$\Phi = \Phi_0 + \frac{1}{\sqrt{N}}\eta$$

$$S_c = S[\Phi_0] + \mathrm{Tr}[\Phi_0^{-1}\eta\Phi_0^{-1}\eta] + \frac{g}{4}\eta\eta$$

$$+\frac{1}{\sqrt{N}}\mathrm{Tr}[\Phi_0^{-1}\eta\Phi_0^{-1}\eta\Phi_0^{-1}\eta] + \sum_n N\Big(\frac{1}{\sqrt{N}}\Big)^n \mathrm{Tr}(B^n)$$

$$B = \Phi_0^{-1}\eta$$

- $S_c$-exact: Reproduces all invariant correlators

$$\langle \phi(x_1)\cdot\phi(y_1)\ \phi(x_2)\cdot\phi(y_2)\cdots\phi(x_n)\cdot\phi(y_n)\rangle$$

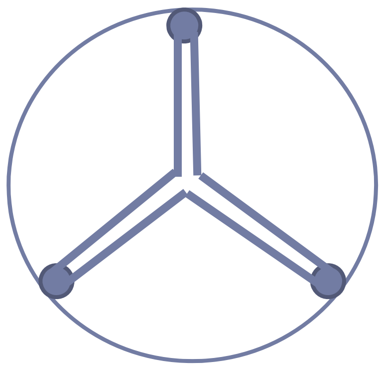
# Topology: Witten diagram! (bulk AdS)

$1$

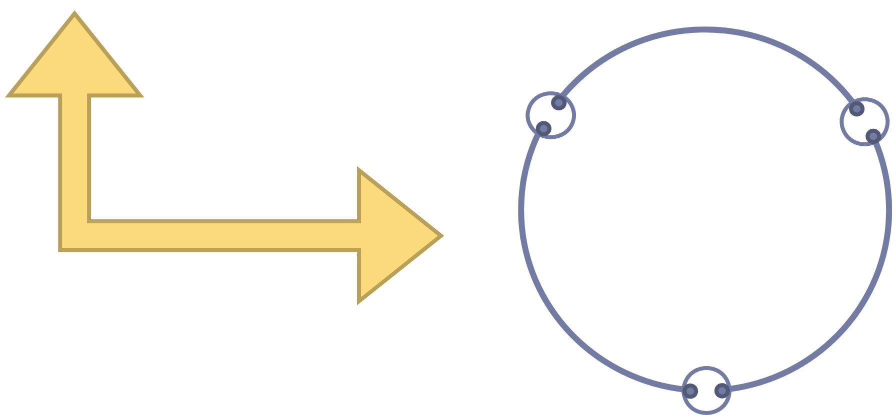
$$\langle \Phi(x_1, y_1) \Phi(x_2, y_2) \rangle$$

Propagator

$$\frac{1}{\sqrt{N}}$$



$$\langle \Phi(1) \Phi(2) \Phi(3) \rangle$$



3 Reggeon Vertex

# Concusion

- Collective Dipole Construction gives a
Construction of HS theory (from bi-local ):

$$\Phi = W\Big|_{\text{gauge}}$$

- Strong,operator level construction:
generating the $AdS_4$ space-time and

Spin s=2,4,6…(single Regge trajectory)

Emergent Gravity as bound state

Scheme for Anti-de-Sitter Feynman Rules for onium-onium Scattering [ A.J. + Q.Ye , to appear….]