

# QCD resummation for jet substructures

Hsiang-nan Li

Academia Sinica, Tawan

Presented at EDS Blois 2011

Dec. 17, 2011

collaborated with Z. Li and CP Yuan

# Outlines

- Motivation
- Jet factorization
- Resummation
- Jet energy profiles
- Heavy-quark jet
- Summary

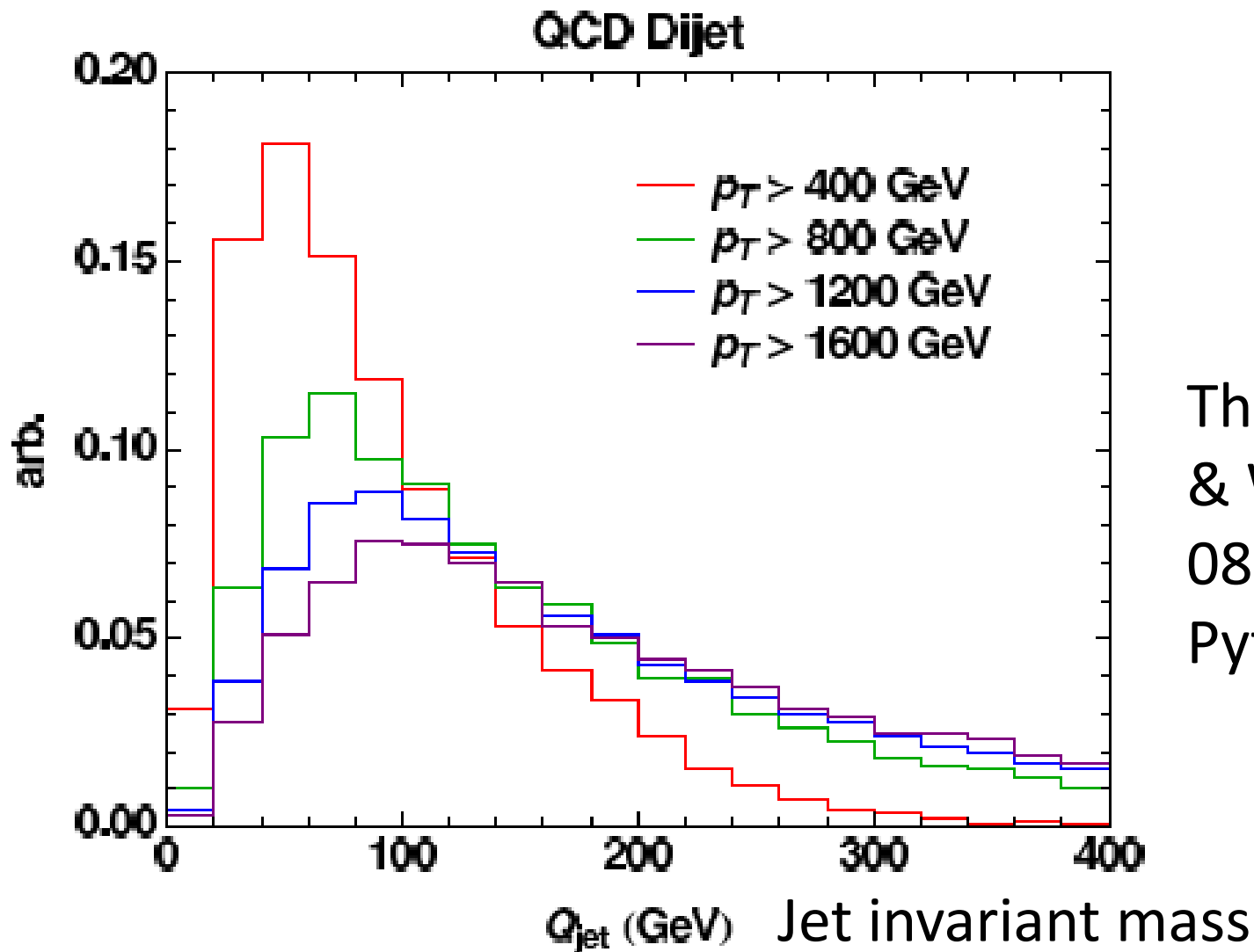
# Motivation

To propose a theoretical framework  
for study of jet physics based on  
perturbative QCD

# Boosted heavy particles

- Large Hadron Collider (LHC) provide a chance to search new physics
- New physics involve heavy particles decaying possibly through cascade to SM light particles
- New particles, if not too heavy, may be produced with sufficient boost -> a single jet
- How to differentiate heavy-particle jets from ordinary QCD jets?
- Similar challenge of identifying energetic top quark at LHC

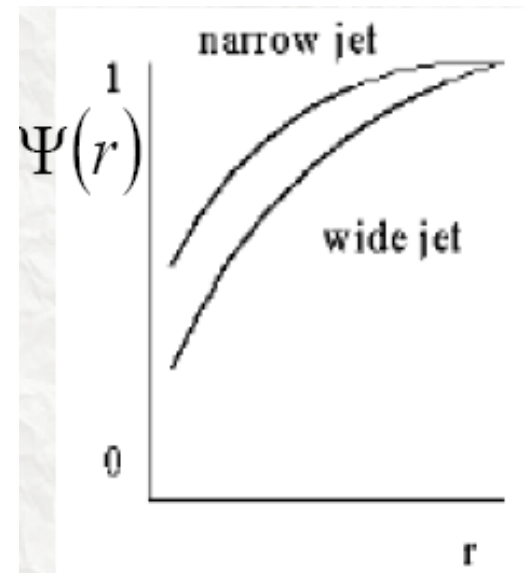
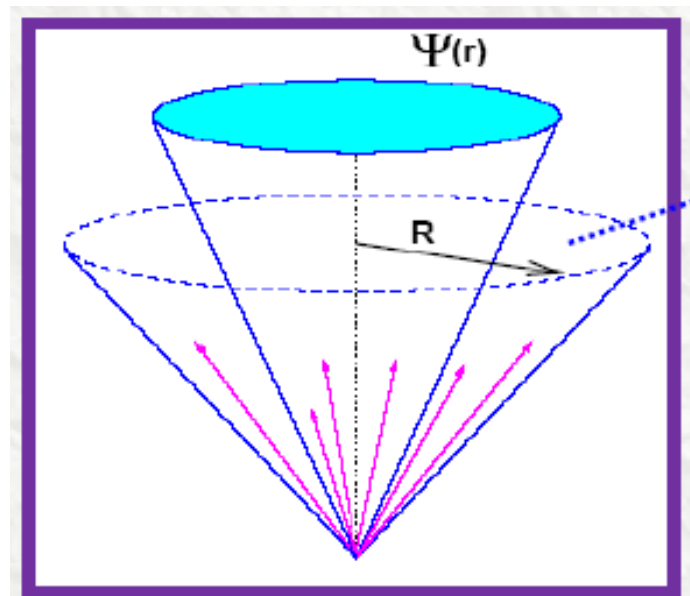
# Fat QCD jet fakes top jet at high $p_T$



Thaler  
& Wang  
0806.0023  
Pythia 8.108

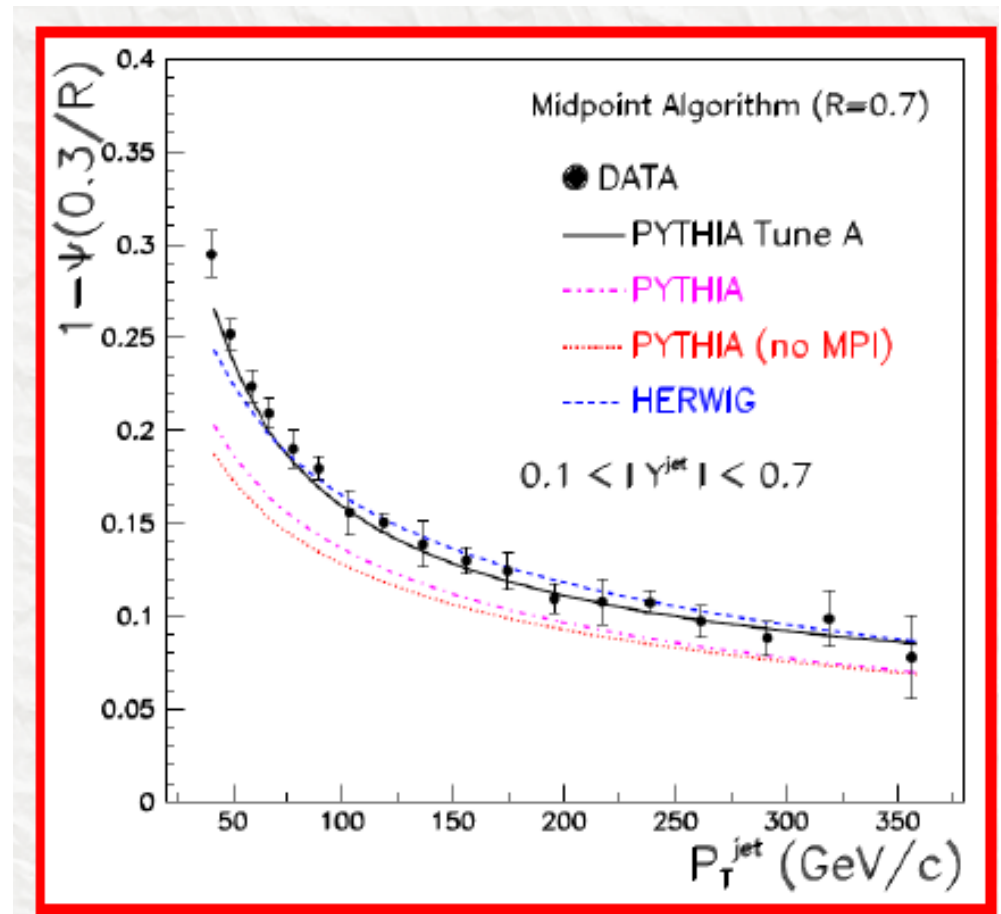
# Jet substructure

- Make use of jet internal structure in addition to standard event selection criteria
- Energy fraction in cone size of  $r$ ,  $\Psi(r)$ ,  $\Psi(R) = 1$
- Quark jet is narrower than gluon jet
- Heavy quark jet energy profile should be different



# Why resummation?

- Monte Carlo may have ambiguities from tuning scales for coupling constant
- NLO is not reliable at small jet mass
- Predictions from QCD resummation are necessary



Tevatron data vs MC predictions

N. Varelas 2009

# Jet factorization

Achieved by eikonalization and  
Ward identity



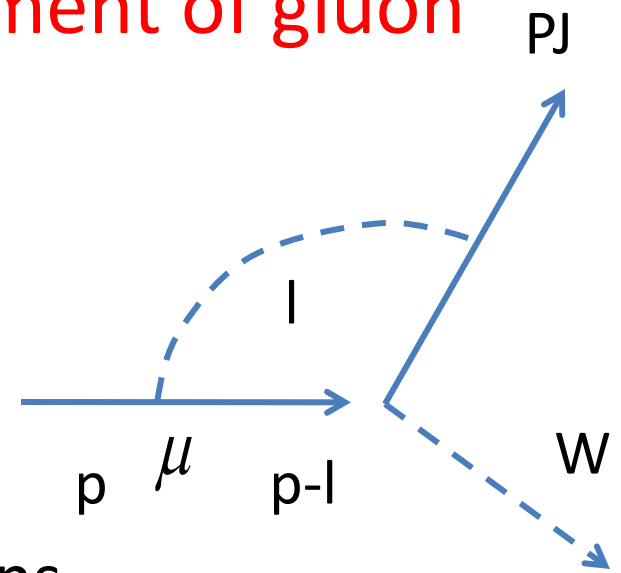
# Eikonalization

- Jet is dominated by collinear dynamics from loop momentum  $l$  to parallel jet momentum  $P_J^+$
- For attachment of collinear gluon, eikonalization holds  $\rightarrow$  detachment of gluon
- For  $l^- \ll l^+$

$$\frac{(p-l)_\alpha \gamma^\alpha + m_t}{(p-l)^2 - m_t^2} \gamma^\mu \approx \frac{\xi^\mu}{-\xi \cdot l}$$

eikonal vertex, eikonal propagator

$\rightarrow$  Wilson line, collect collinear gluons



$$\Phi_\xi^{(f)}(\infty, 0; 0) = \mathcal{P} \left\{ e^{-ig \int_0^\infty d\eta \xi \cdot A^{(f)}(\eta \xi^\mu)} \right\}$$

# Jet definitions Almeida et al. 08

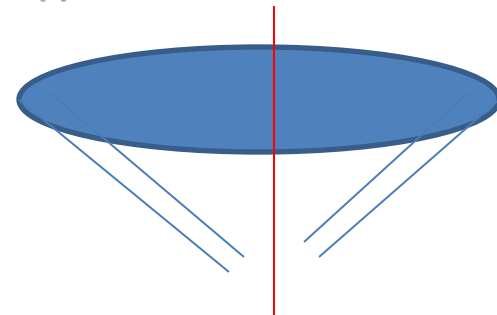
- Eikonalization leads to factorization

- Quark 
$$J_i^q(m_J^2, p_{0,J_i}, R) = \frac{(2\pi)^3}{2\sqrt{2}(p_{0,J_i})^2} \frac{\xi_\mu}{N_c} \sum_{N_{J_i}} \text{Tr} \left\{ \gamma^\mu \langle 0 | q(0) \Phi_\xi^{(\bar{q})\dagger}(\infty, 0) | N_{J_i} \rangle \right. \\ \left. \times \langle N_{J_i} | \Phi_\xi^{(\bar{q})}(\infty, 0) \bar{q}(0) | 0 \rangle \right\} \delta(m_J^2 - \tilde{m}_J^2(N_{J_i}, R)) \\ \times \delta^{(2)}(\hat{n} - \tilde{n}(N_{J_i})) \delta(p_{0,J_i} - \omega(N_{J_c}))$$

- Gluon 
$$J_i^g(m_J^2, p_{0,J_i}, R) = \frac{(2\pi)^3}{2(p_{0,J_i})^3} \sum_{N_{J_i}} \langle 0 | \xi_\sigma F^{\sigma\nu}(0) \Phi_\xi^{(g)\dagger}(0, \infty) | N_{J_i} \rangle \\ \times \langle N_{J_i} | \Phi_\xi^{(g)}(0, \infty) F_\nu^\rho(0) \xi_\rho | 0 \rangle \delta(m_J^2 - \tilde{m}_J^2(N_{J_i}, R)) \\ \times \delta^{(2)}(\hat{n} - \tilde{n}(N_{J_i})) \delta(p_{0,J_i} - \omega(N_{J_c}))$$

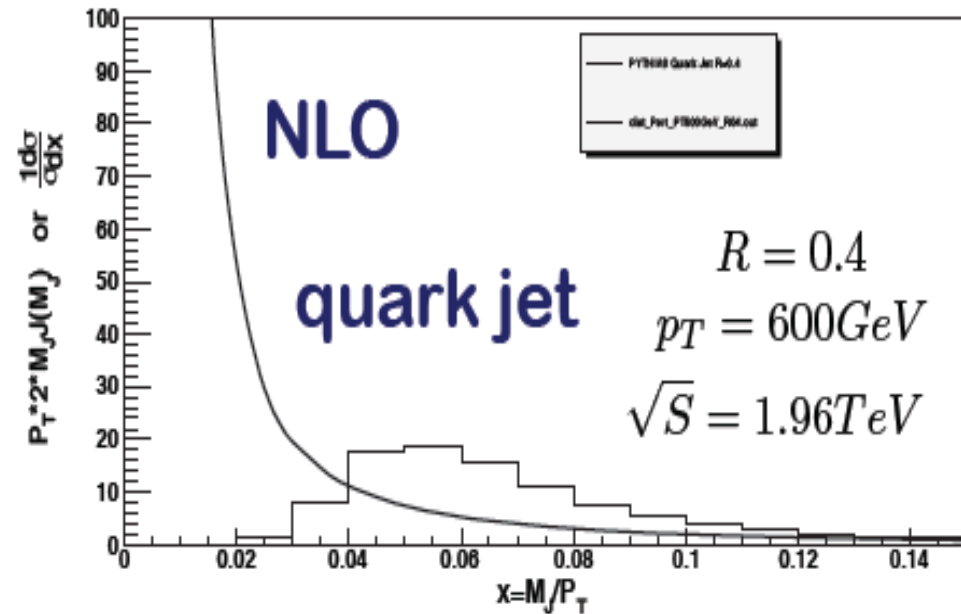
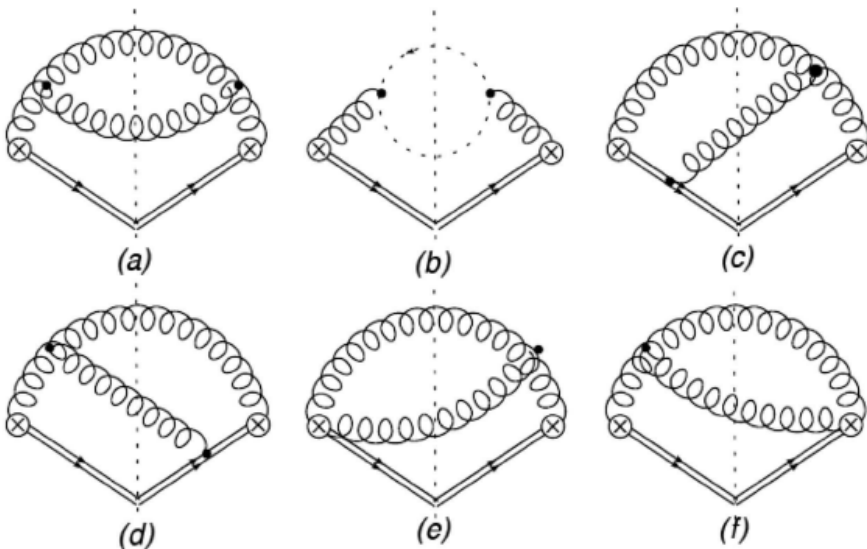
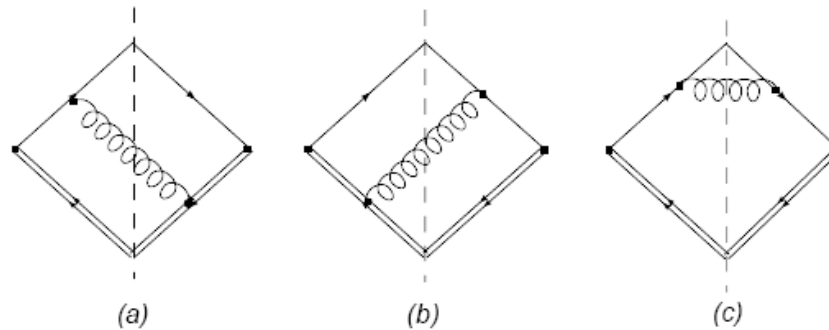
- LO jet

$$J_i^{(0)}(m_{J_i}^2, p_{0,J_i}, R) = \delta(m_{J_i}^2)$$



# NLO diagrams

- quark jet
- gluon jet



# Resummation

Technical part, ideas only

See alternative approach based on  
SCET, Ellis et al. 2010

# Key idea

- Vary Wilson line to arbitrary vector  $n$
- Collinear dynamics is independent of  $n$
- Variation effect does not contain collinear dynamics, and can be factorized from jet

- Study derivative  $-\frac{n^2}{P_J \cdot n} P_{J\alpha} \frac{d}{dn_\alpha} J$

special  
vertex

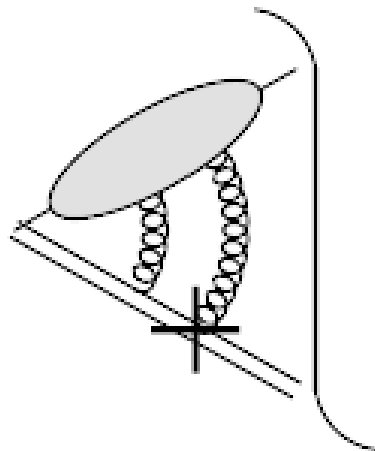
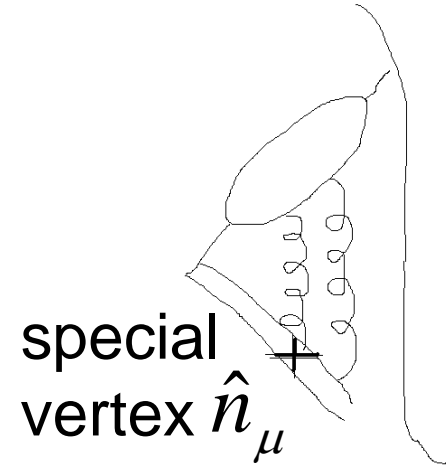
- Differentiation applies only to Wilson line

$$-\frac{n^2}{P_J \cdot n} P_{J\alpha} \frac{d}{dn_\alpha} \frac{n_\mu}{n \cdot l} = \frac{n^2}{P_J \cdot n} \left( \frac{P_J \cdot l}{n \cdot l} n_\mu - P_{J\mu} \right) \frac{1}{n \cdot l} = \frac{\hat{n}_\mu}{n \cdot l}$$

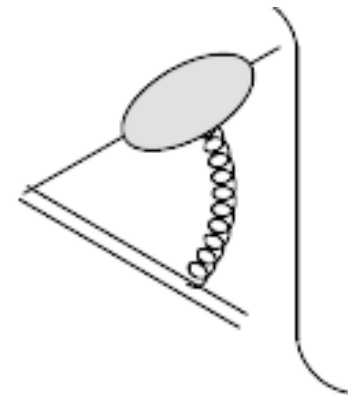
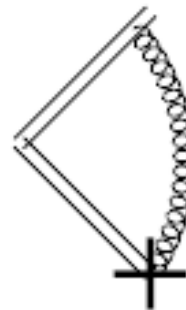
- Differentiated gluon gives hard or soft contribution

# Soft factorization (virtual)

- If differentiated gluon is soft, special vertex locates at outer end of Wilson line
- If it locates inside (see figure), both gluons are soft  
-> NLO soft kernel



$\sim$

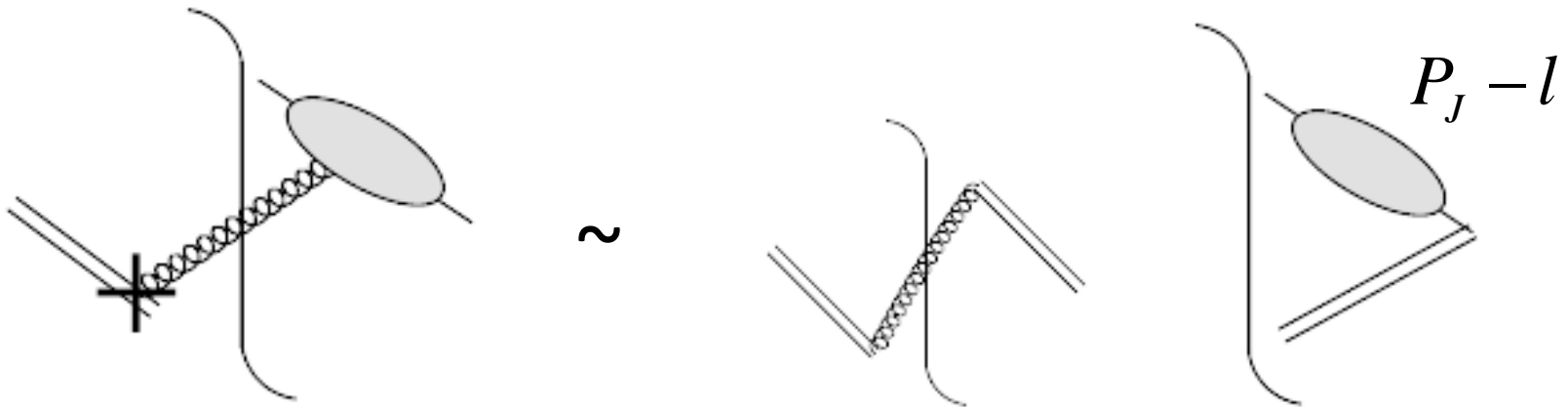


LO soft kernel  $K_v^{(1)}$

# Soft factorization (real)

- Similar argument applies to factorization of differentiated soft real gluon

LO soft kernel  $K_r^{(1)}$

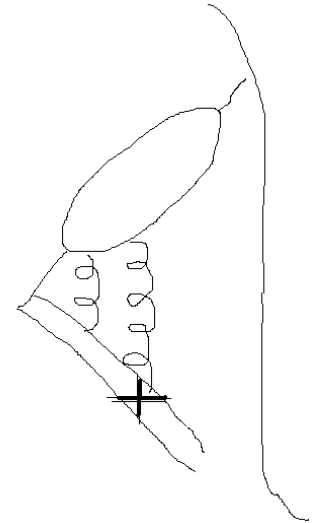


$$g^2 C_F \int \frac{d^4 l}{(2\pi)^4} \frac{\hat{n} \cdot P_J}{(n \cdot l + i\epsilon)(P_J \cdot l - i\epsilon)} 2\pi \delta(l^2 - a^2) J(m_J^2 - 2P_J \cdot l, P_J \cdot n, n^2, R)$$

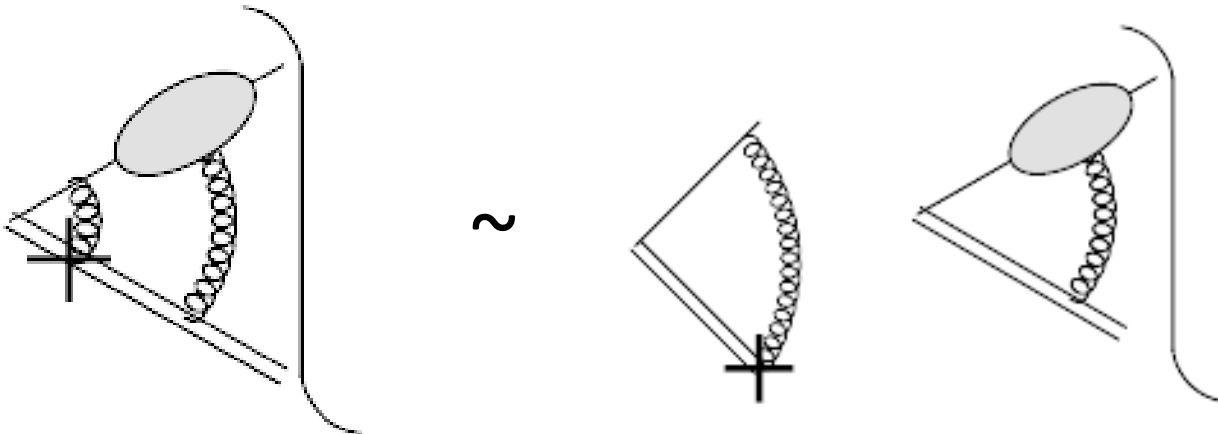
Jet invariant mass excluding soft momentum  $l$ ,  $(P_J - l)^2$

# Hard factorization

- If differentiated gluon is hard, special vertex locates at inner end of Wilson line
- If it locates at outside, both gluons are hard -> NLO hard kern



LO hard kernel  $G^{(1)}$





# Resummation equation

- Up to leading logs, resummation equation

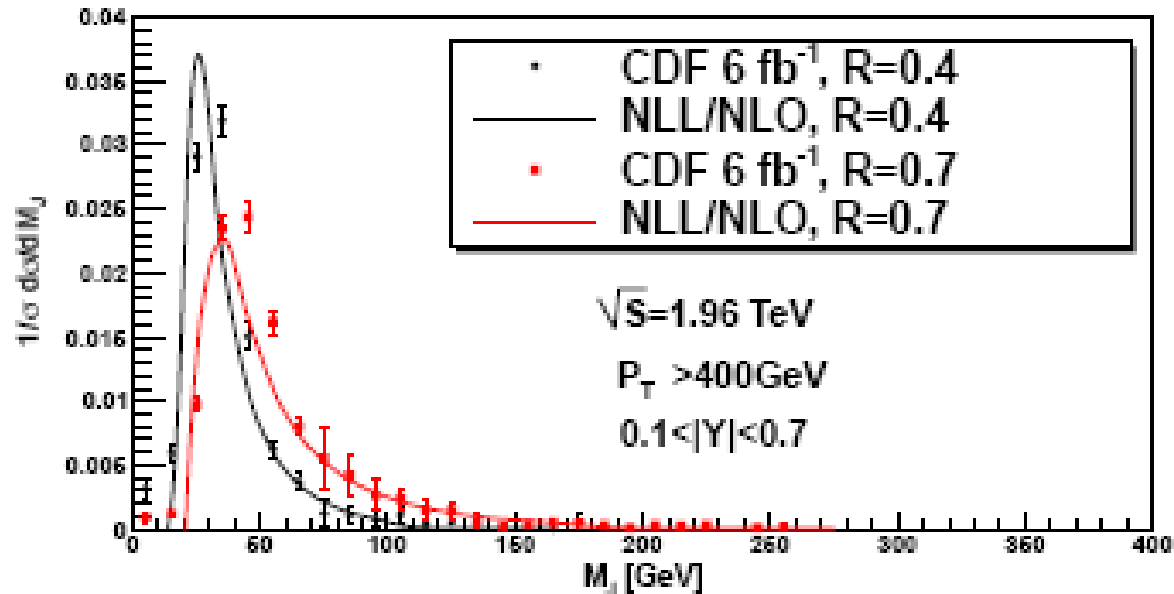
$$-\frac{n^2}{P_J \cdot n} P_J^\alpha \frac{d}{dn^\alpha} J = [G^{(1)} + K_v^{(1)} + K_r^{(1)}] \otimes J$$

- For next-to-leading-logarithm accuracy, G and K are evaluated to two loops
- Solve the equation in Mellin N space

$$\bar{J}_q(N, P_T, \nu^2, R, \mu^2) \equiv \int_0^1 dx (1-x)^{N-1} J_q(x, P_T, \nu^2, R, \mu^2),$$

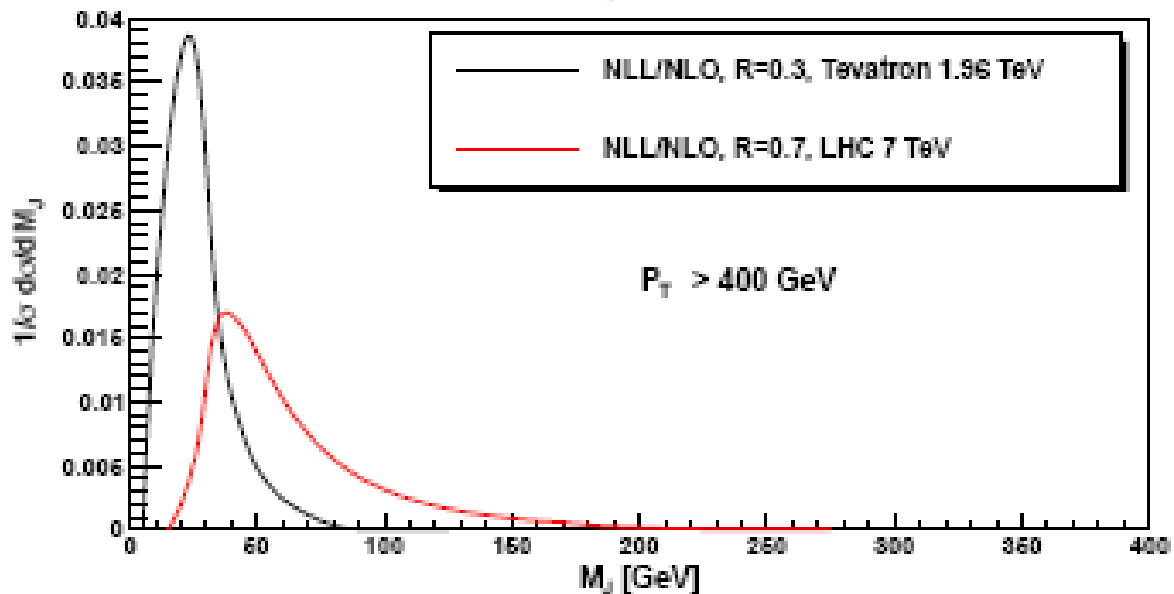
$$x \equiv m_J^2 / (RP_T)^2 \quad \nu^2 \equiv 4(v \cdot n)^2 / |n^2| \quad v = P_J^\perp / P_J^0$$

# Predictions for jet mass distribution



NLL in  
resummation  
NLO in  
initial condition

CTEQ6L PDFs



# Jet energy profiles

# Jet energy function

- Define jet energy function  $J^E(r)$  by associating  $k_i^0 \Theta(r - \theta_i)$  with each final-state particle  $i$  within jet cone  $r, r < R$

- Still vary Wilson direction  $n$ .

- Separation  $\sum_i k_i^0 \Theta(r - \theta_i) = \sum_i' k_i^0 \Theta(r - \theta_i) + l^0 \Theta(r - \theta)$

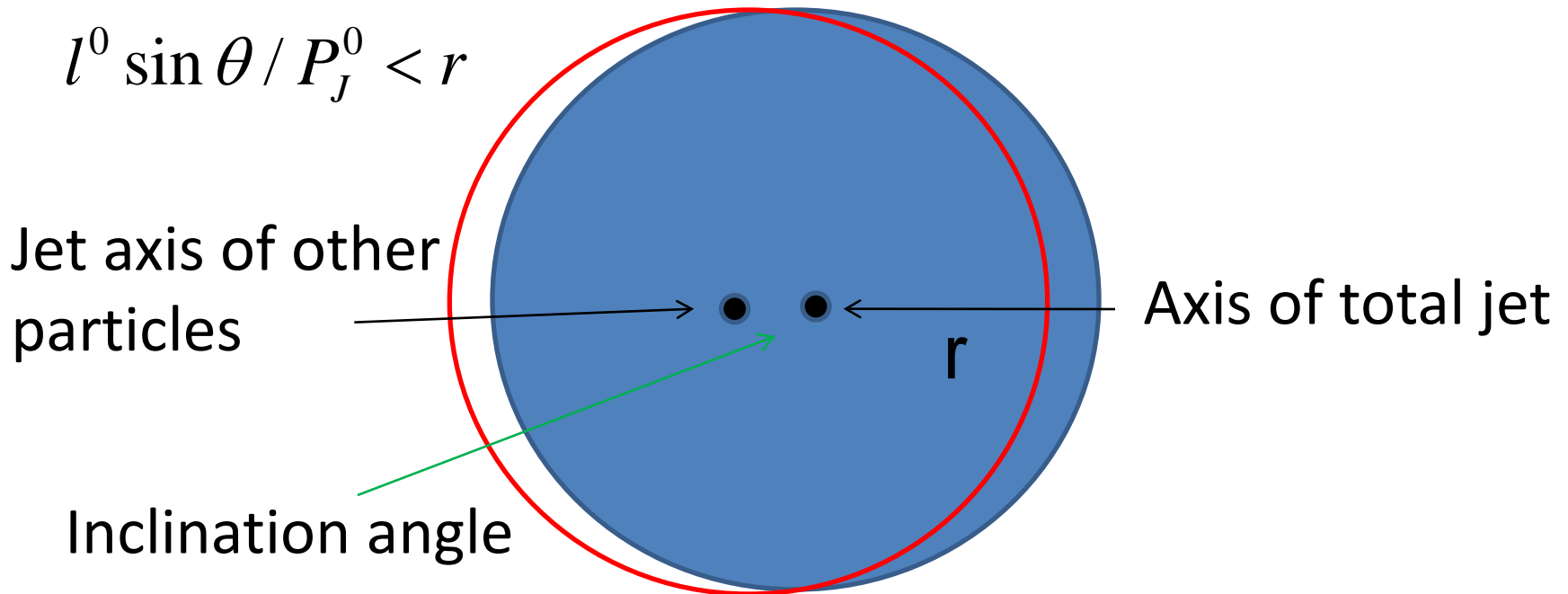
differentiated soft real gluon  
negligible in soft region

- First term gives

$$[G^{(1)} + K_v^{(1)} + K_r^{(1)}] \otimes J^E$$

# Soft gluon effect

- Differentiated soft real gluon renders jet axis of other particles inclined by small  $\arctan(l^0 \sin \theta / P_J^0)$
- This jet axis can not go outside of the subcone



# Resummation equation

- Resummation equation for jet profile

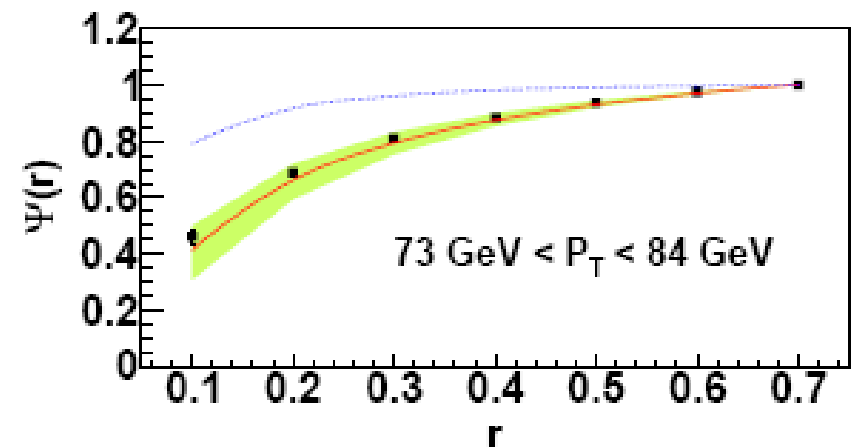
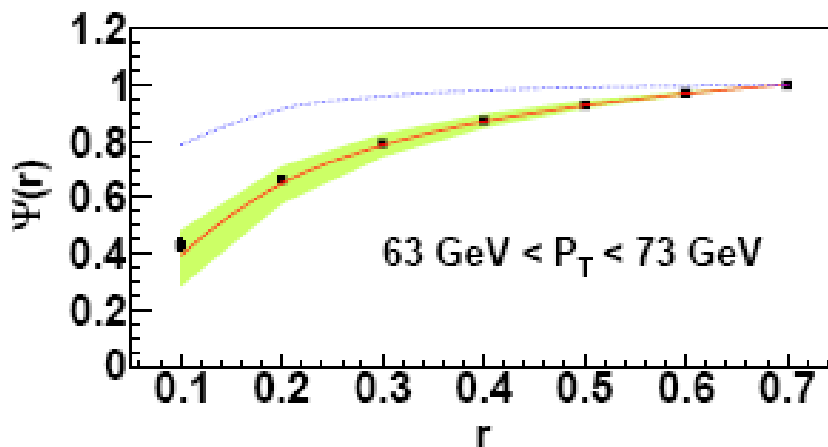
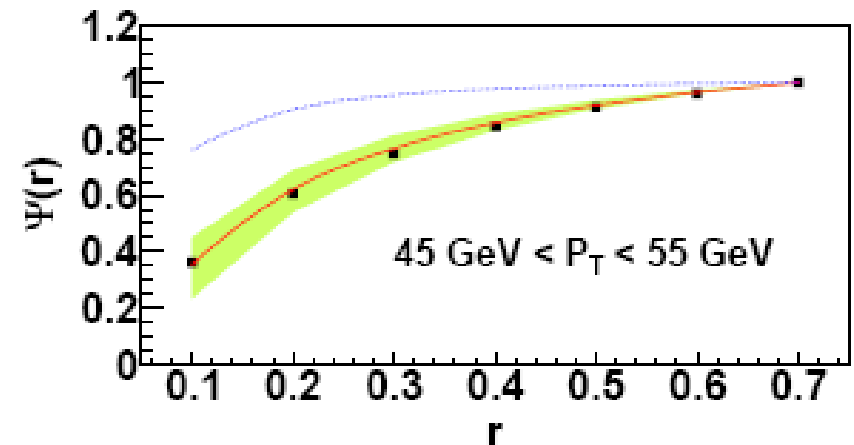
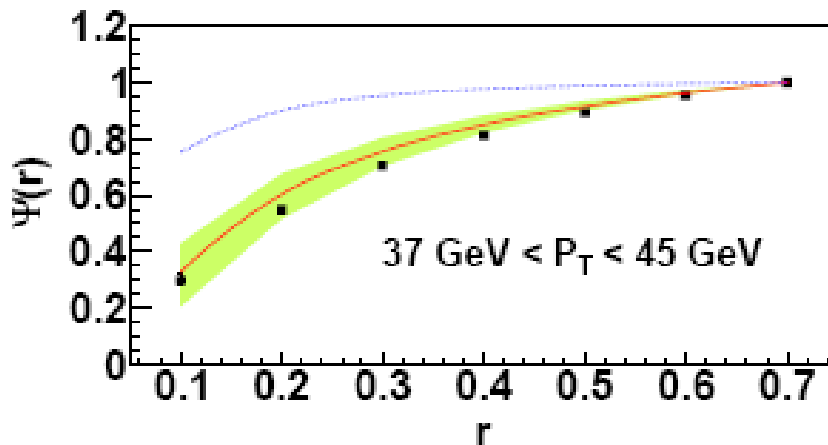
$$\begin{aligned} \bar{K}_r^{(1)}(1) &= g^2 C_F \int \frac{d^4 l}{(2\pi)^3} \frac{n^2}{(n \cdot l + i\epsilon)^2} \delta(l^2 - a^2) \Theta \left( r - \frac{|l| \sin \theta}{P_J^0} \right) \\ &\quad - \frac{n^2}{v \cdot n} v_\alpha \frac{d}{dn_\alpha} \bar{J}_q^E(1, P_T, \nu^2, R, r) \\ &= 2[G^{(1)} + K^{(1)}(1)] \bar{J}_q^E(1, P_T, \nu^2, R, r) \end{aligned}$$

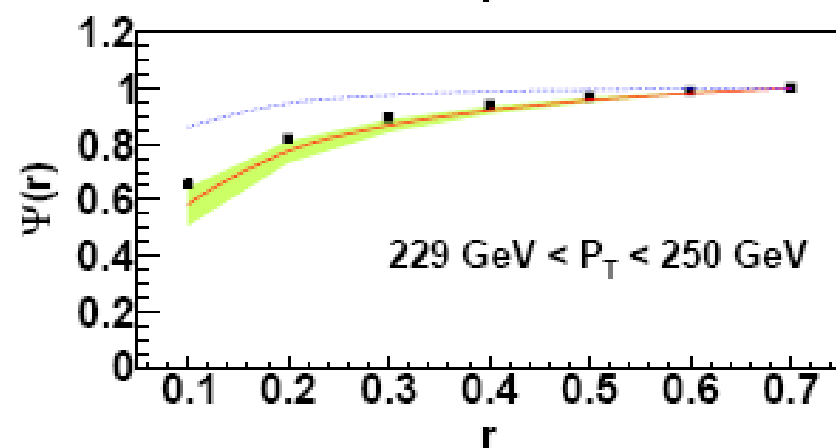
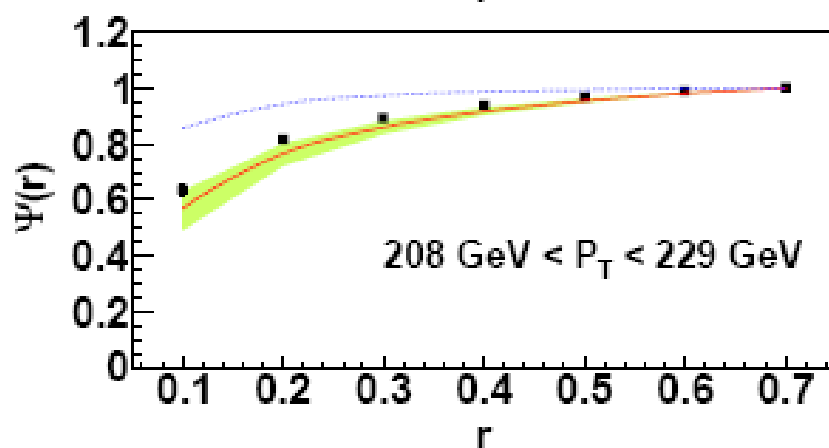
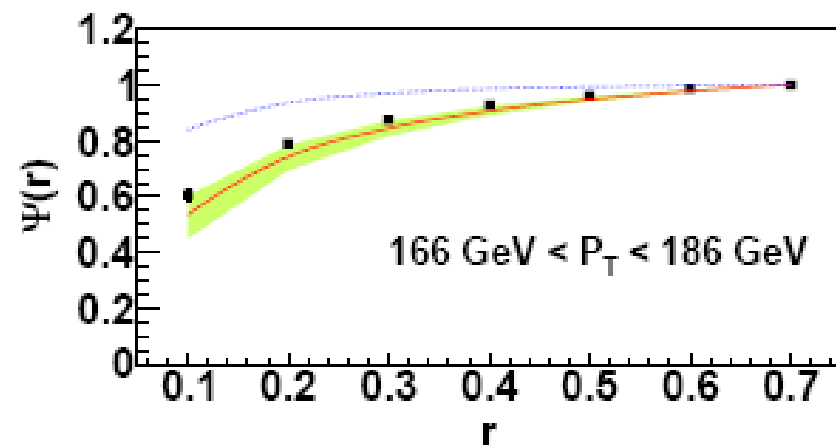
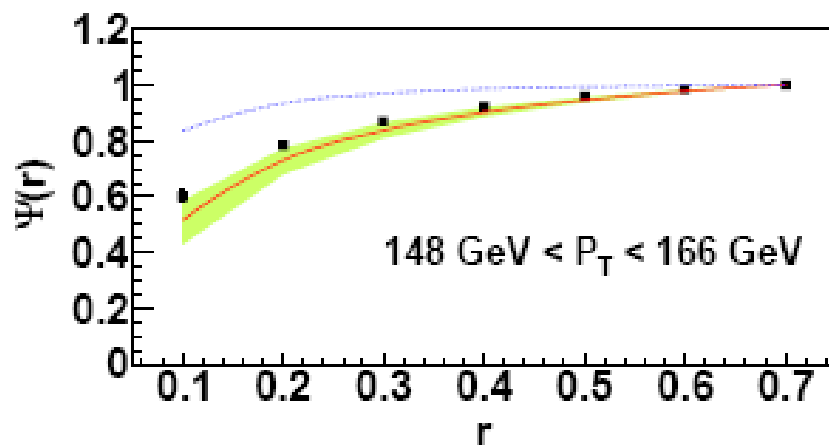
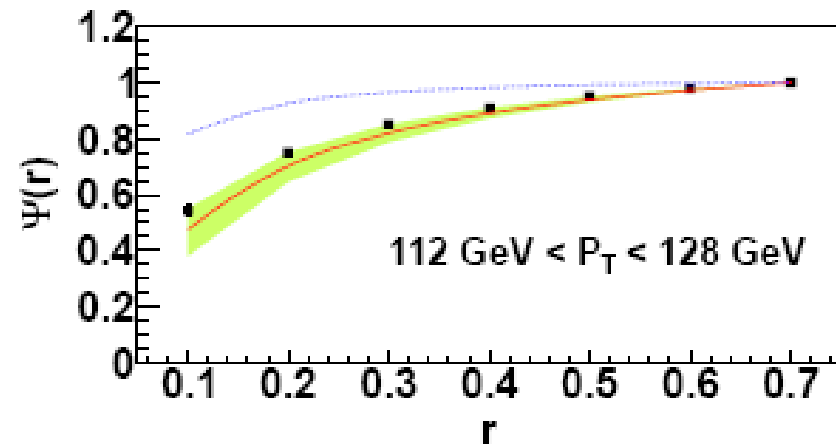
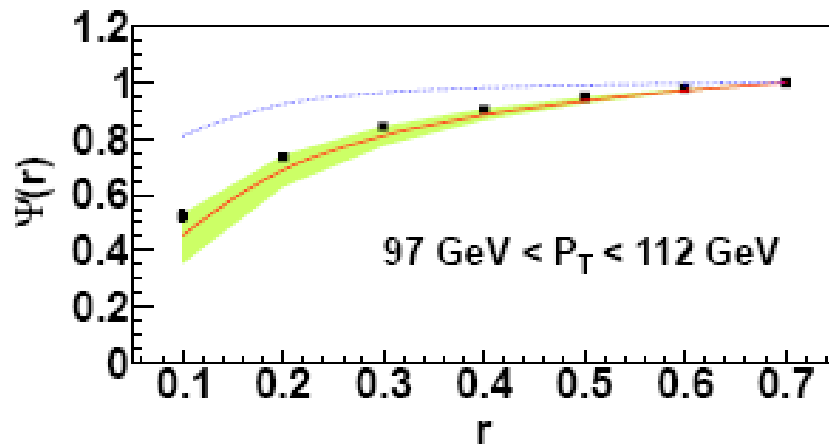
- Have considered N=1 here, corresponding to integration over jet mass (insensitive to nonperturbative physics)
- Resum  $\alpha_s \ln^2 r$ ,  $\alpha_s \ln r$  from phase space constraint for real gluons

# Comparison with CDF data

$$\Psi(r) = \frac{1}{N_{\text{jet}}} \sum_{\text{jets}} \frac{P_T(0, r)}{P_T(0, R)}, \quad 0 \leq r \leq R$$

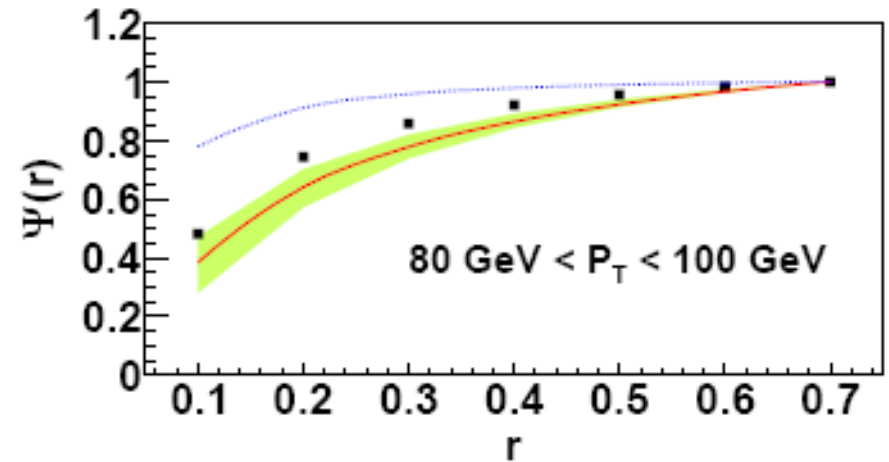
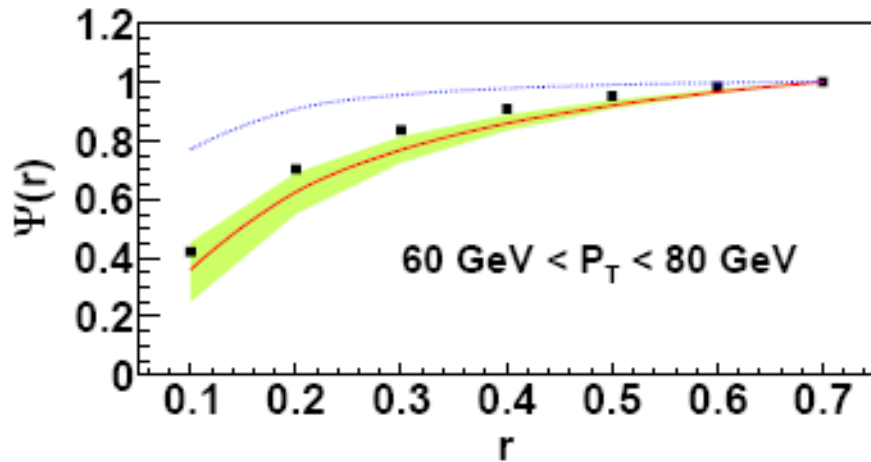
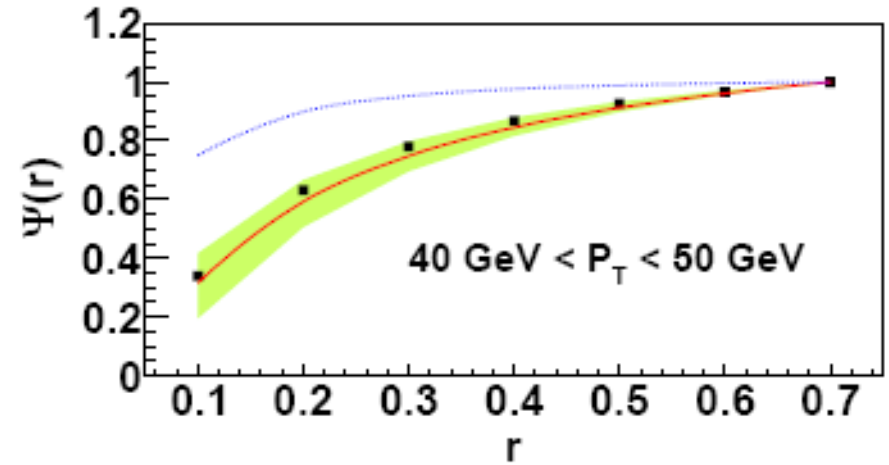
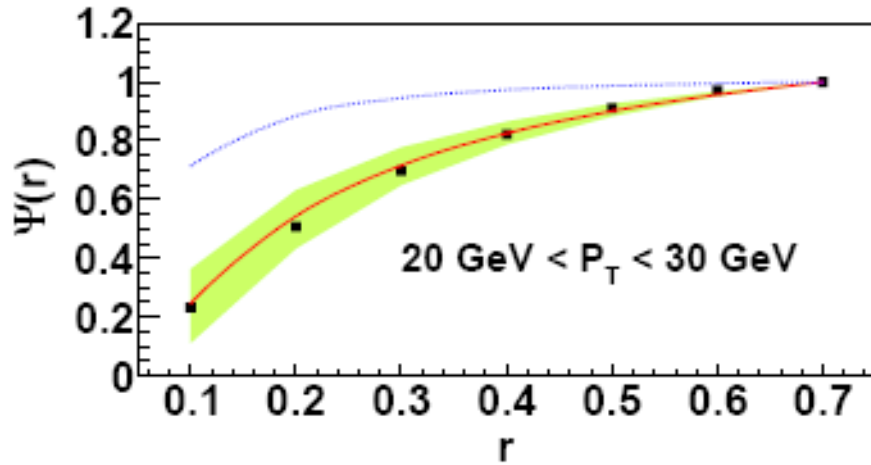
quark, gluon jets, convoluted with LO hard scattering, PDFs







# Comparison with CMS data

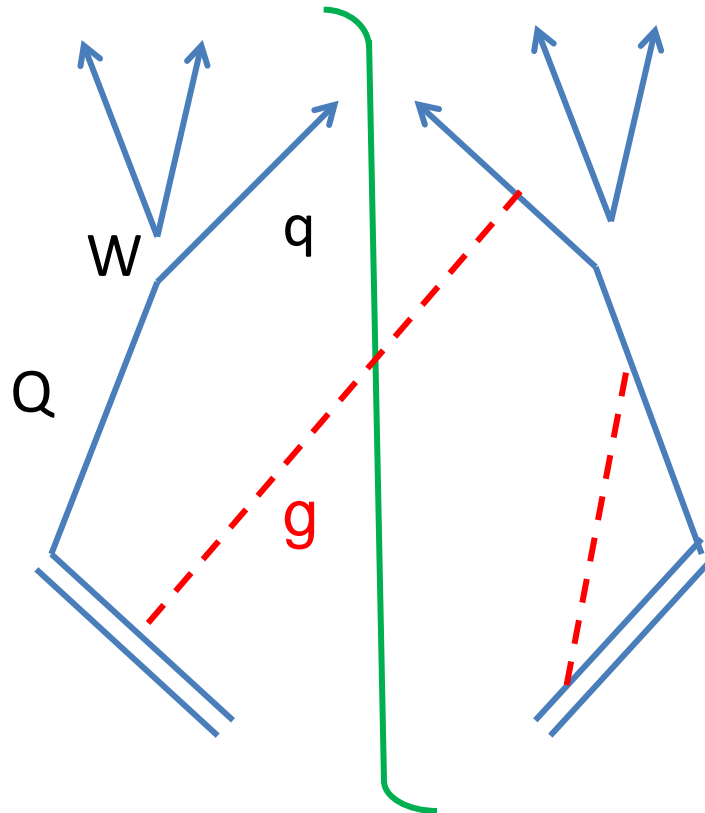


# Heavy-quark jet

Work in progress

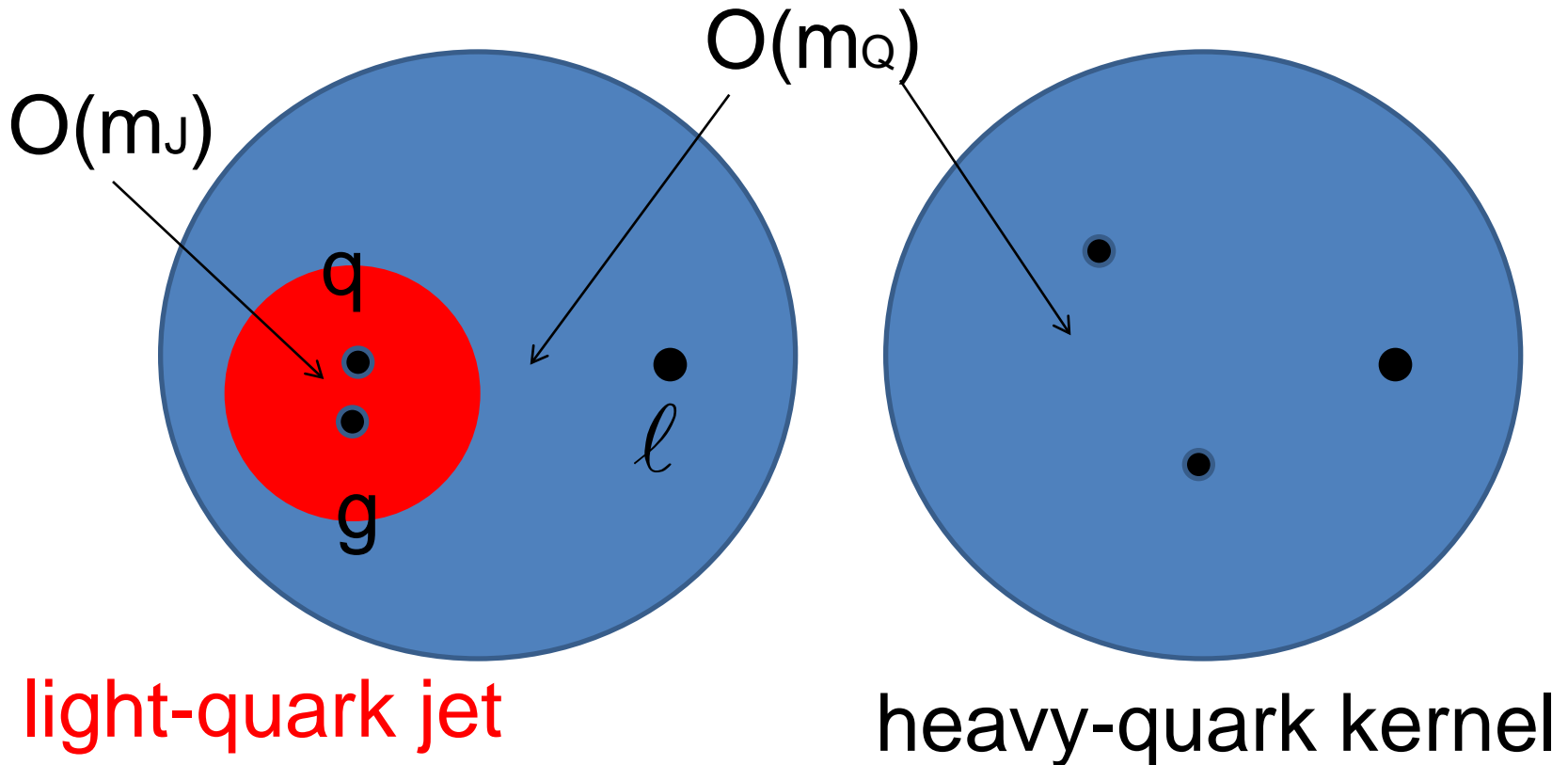
# Factorization (semileptonic)

- Factorize heavy quark-quark jet first at jet energy scale  $E_Q$ , which contains weak decay



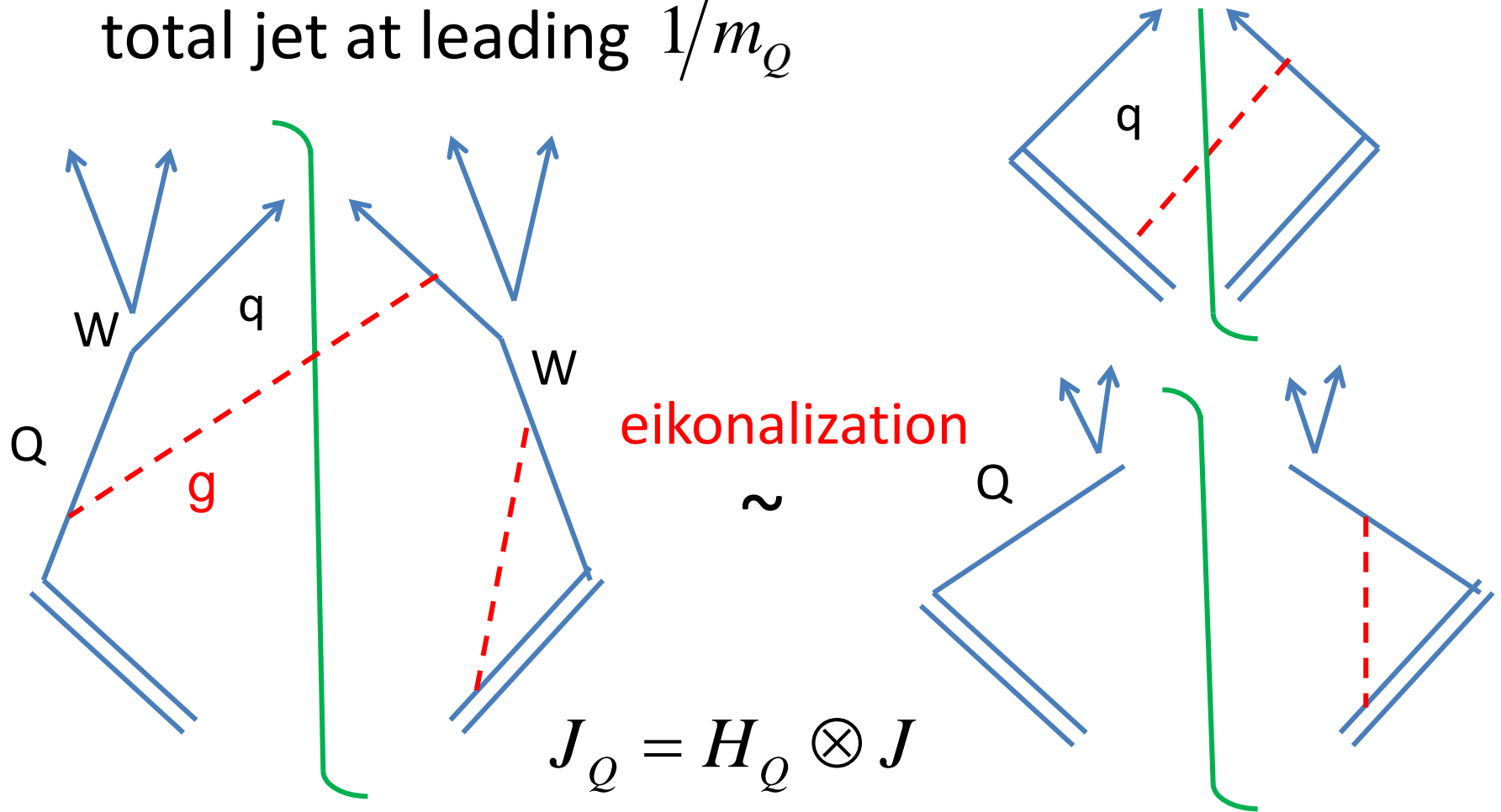
# Scale hierarchy $E_Q \gg m_Q \gg m_J$

- The two lower scales  $m_Q$  and  $m_J$  characterize different dynamics, which can be factorized



# Further factorization (HQET)

- Then factorize the light-quark jet from the total jet at leading  $1/m_Q$

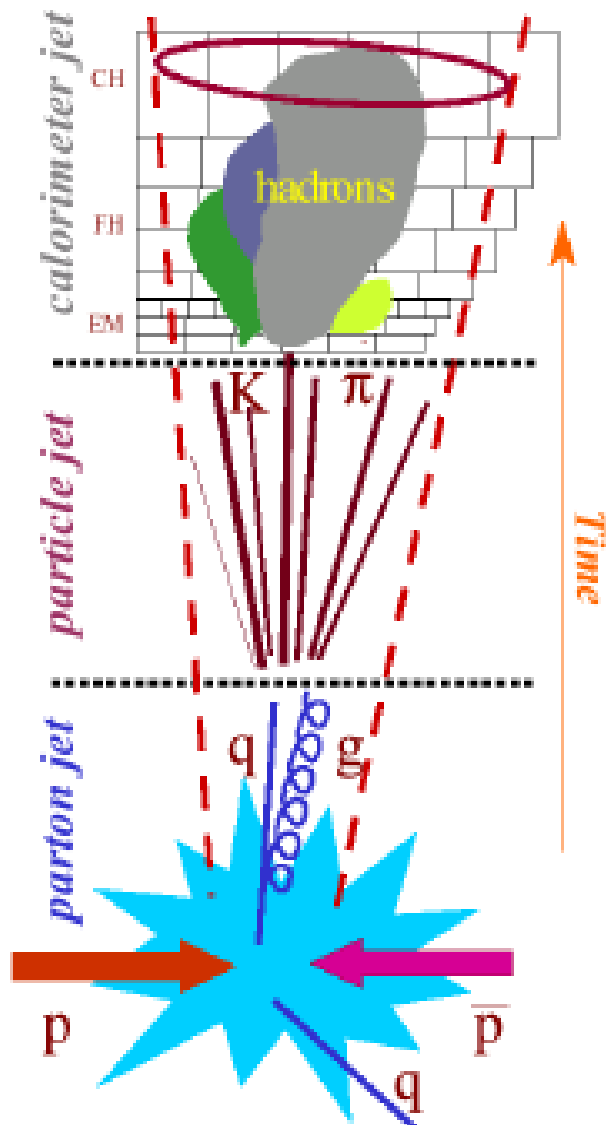


# Summary

- Jet substructure improves jet identification
- Perturbative calculation is not reliable in extreme kinematic region (e.g. small  $m_J$ )
- Event generators may have ambiguities (from tuning scales for coupling constant)
- QCD resummation provides reliable prediction, independent check, and alternative approach
- Analyzed jet function and profile for light particles. Results consistent with current data
- Numerical work on heavy-quark jet in progress

Back-up slides

# Jet Finding



## • Calorimeter jet (cone)

- ◆ jet is a collection of energy deposits with a given cone  $R$ :  $R = \sqrt{\Delta\phi^2 + \Delta\eta^2}$
- ◆ cone direction maximizes the total  $E_T$  of the jet
- ◆ various clustering algorithms

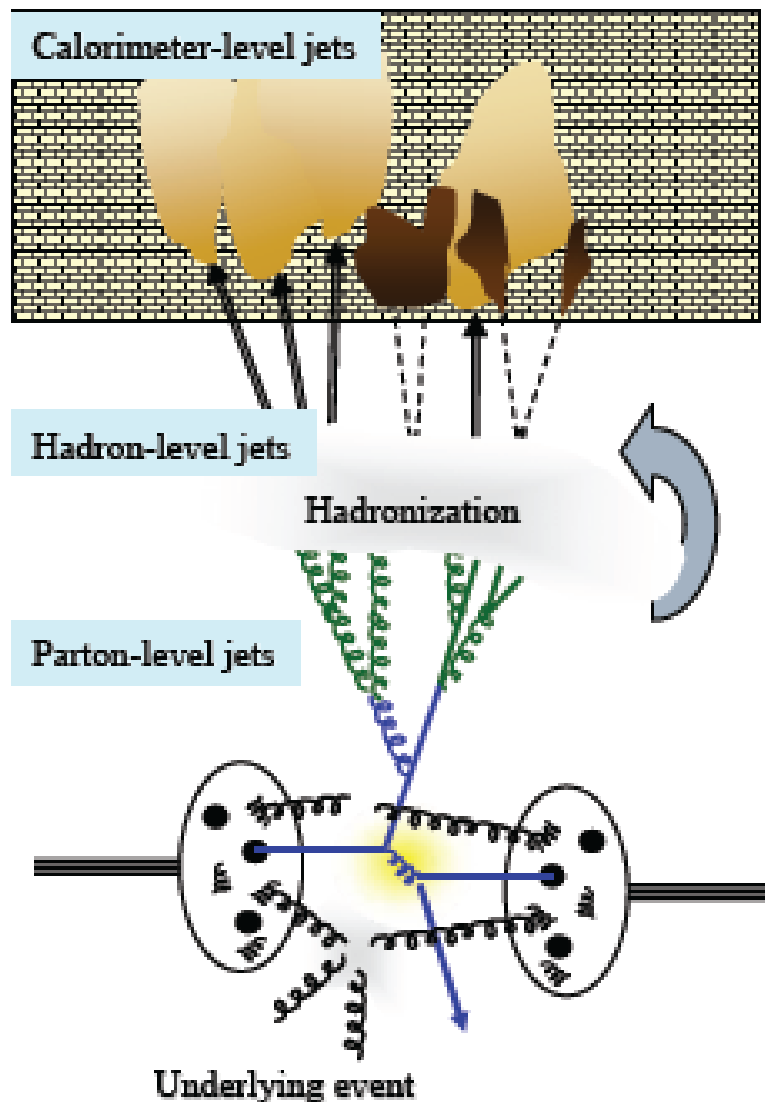
- correct for finite energy resolution
- subtract underlying event
- add out of cone energy

## • Particle jet

- ◆ a spread of particles running roughly in the same direction as the parton after hadronization

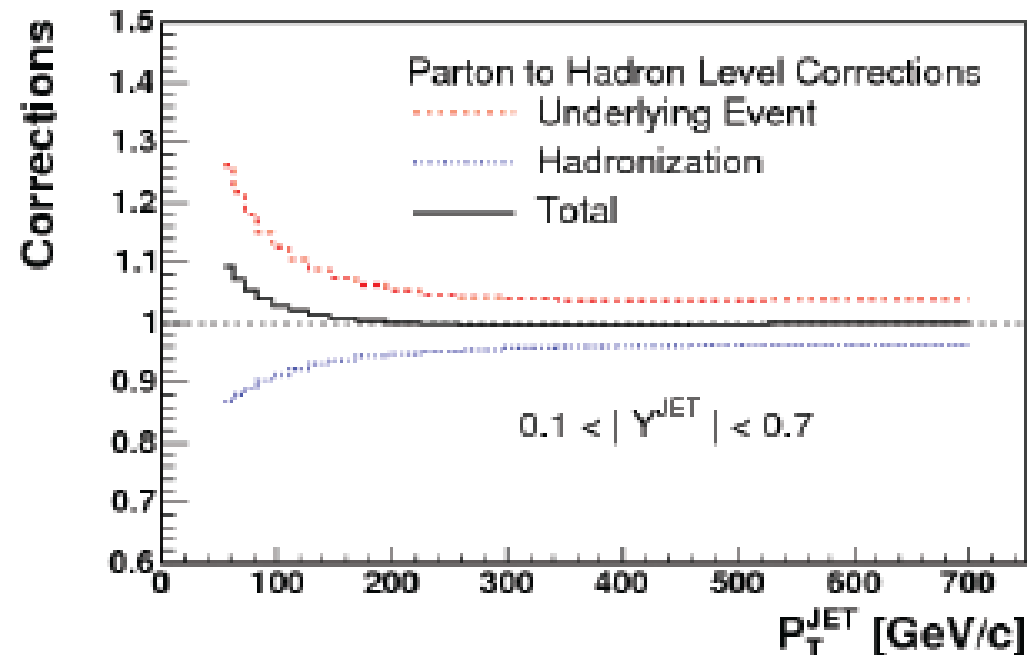


# Underlying Event & Hadronization Correction



- UE and hadronization effects are in the opposite directions

CDF Run-2



- With  $R=0.7$ , the UE effect is larger than the hadronization effects.

- ~10% in cross section at low jet  $P_T$