From theory to phenomenology in the CGC

José Guilherme Milhano

CENTRA-IST (Lisbon) & CERN PH-TH

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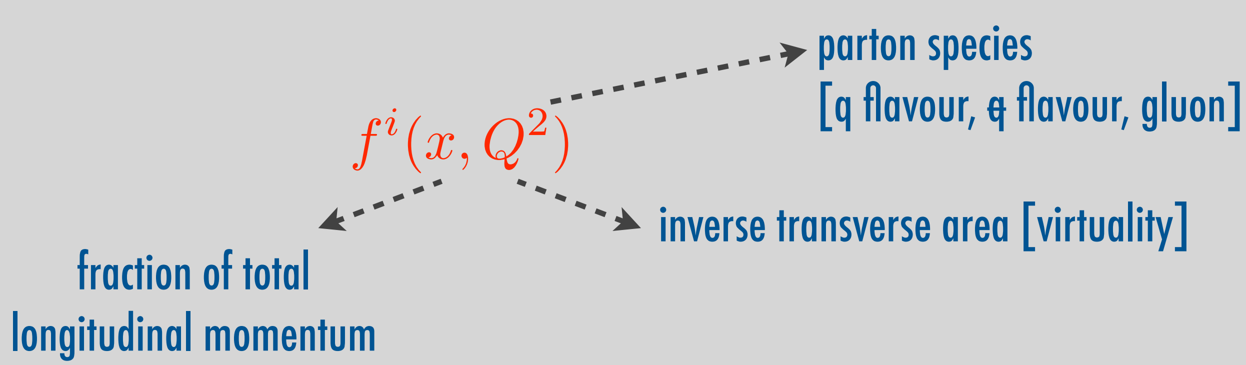
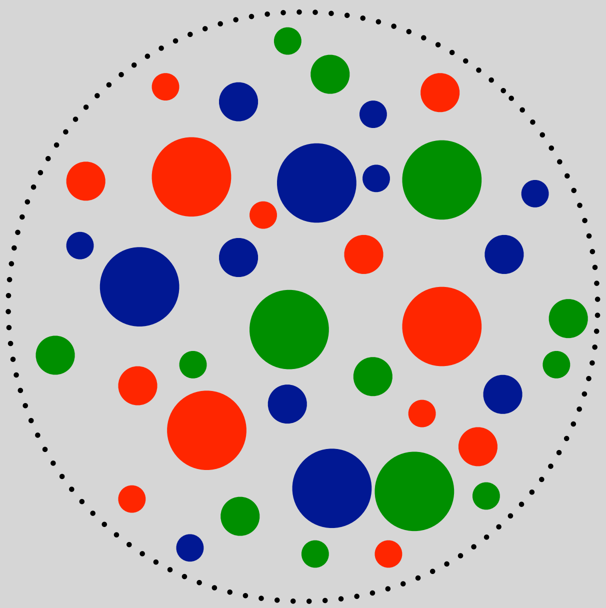
initial conditions in hadronic collisions

initial conditions ≡ knowledge of internal structure of colliding objects at all relevant scales [including unmeasured ones]

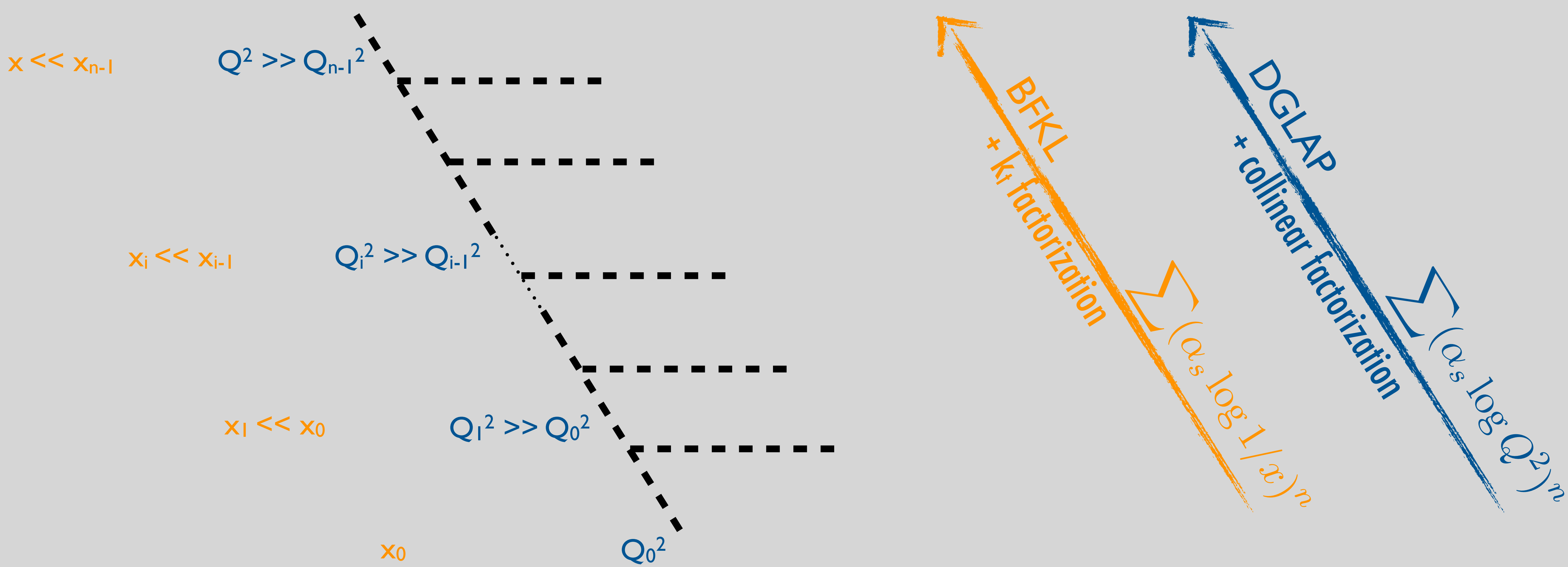
- structure at scales much smaller than typical hadronic scale
  - ↪ ensemble of partons
    - suitably written as parton distribution functions [pdfs]
- essential knowledge for
  - ↪ computation of physical observables and reliable identification of new physics
  - ↪ distinction between initial and final state effects and 'first principle' initialization of hydrodynamical evolution [in heavy ion collisions]
- a rich physics playground in its own right
- useful encoding of initial conditions necessarily universal [process independent]
  - ↪ reliant on some form of factorization





evolution

- partons with x and/or Q^2 far from typical hadronic ones (x_0, Q_0^2) come into being via a perturbative splitting chain
  - ↪ chain dominated by phase space log-enhanced contributions
  - ↪ resummation of chain ≡ evolution of pdfs from (x_0, Q_0^2) to (x, Q^2)
  - ↪ evolution reliably [perturbatively] computable given initial condition
    - initial condition intrinsically non-perturbative [phenomenological 'guess', lattice, global data fit]



linearity

both DGLAP and BFKL are linear approaches

evolution independent of ensemble

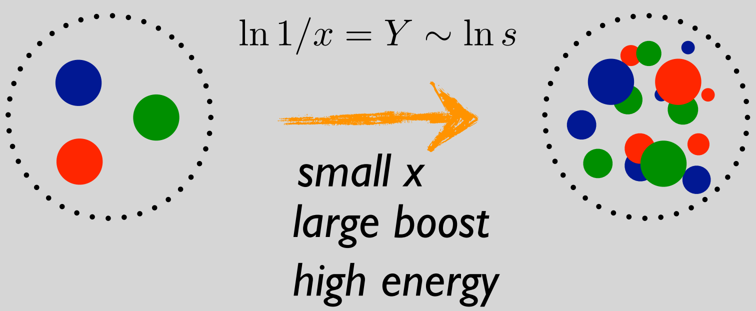
:: underlying assumption ::

- ensemble is dilute and remains so throughout evolution
- no collective behaviour in parton splitting

DGLAP

evolution [DGLAP] in Q^2 is intrinsically linear

BFKL



- evolution towards smaller x increases density
  - assumption of perpetual linearity violated
  - evolution in x should account for the ensemble

:: parton overlap, parton recombination, phase space reduction ::

evolution in x becomes naturally non-linear

neither approach is sufficient :: need non-linear generalization of BFKL

gluon dominance

- the infrared sensitivity of parton splitting favours the emission of soft [small-x] gluons :: at small-x the ensemble is gluon dominated
- both BFKL and DGLAP [DLA] predict a very steep rise of the gluon density
  - ↪ observed at HERA
  - ↪ if perpetuated leads unitarity violation [Froissart bound]

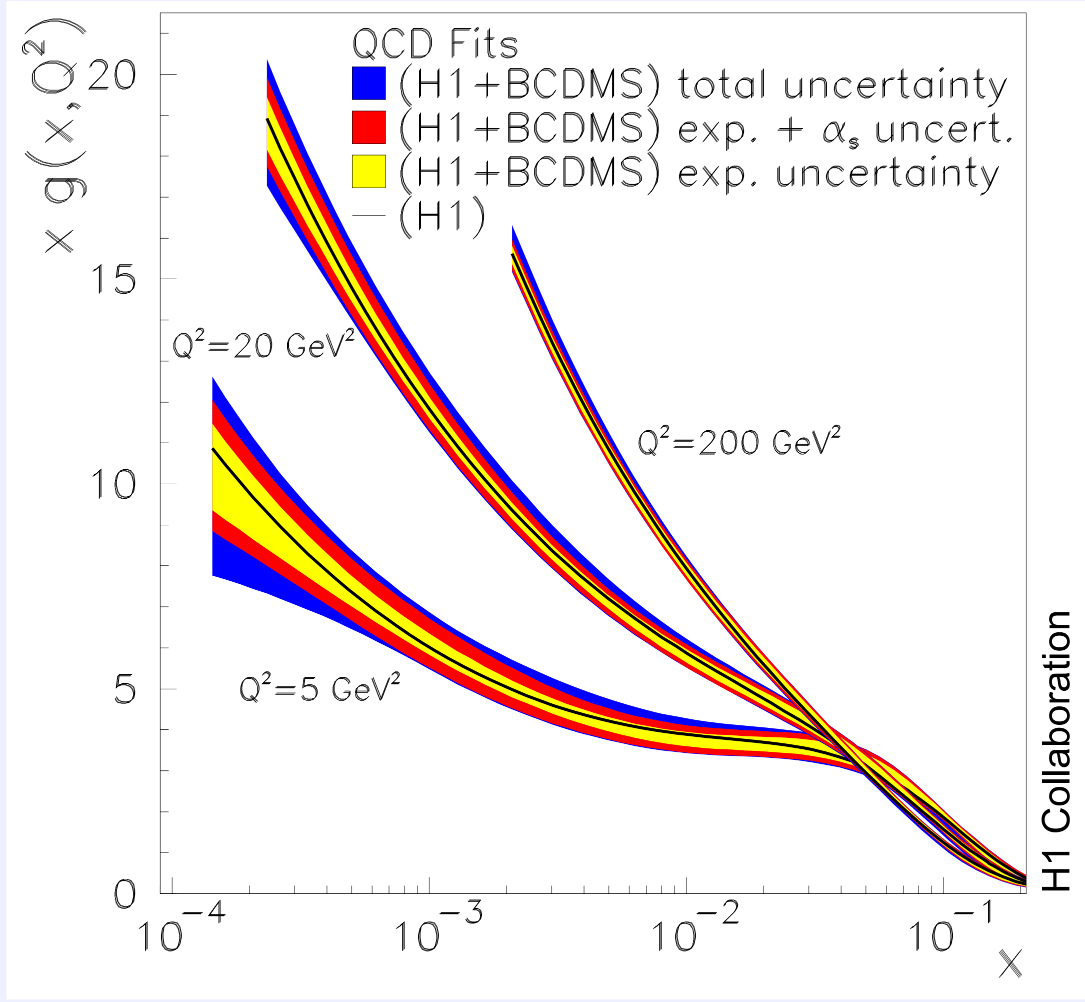
$$xg(x,Q^2) \sim x^{-12 \ln 2 \frac{\alpha_s}{\pi}} \sim x^{-0.5}$$

BFKL

$$xg(x,Q^2) \sim \exp\left\{\left(\frac{48}{11-\frac{2}{3}N_f} \ln \frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2} \ln 1/x\right)^{1/2}\right\}$$

DLA-DGLAP

the growth of partonic density should be tempered by non-linearities when the density becomes large



lore

- simple physical arguments require the inclusion of non-linearities in the evolution; the C(olour)G(lass)C(ondensate) is the correct framework in which to address small-x physics
  - ↪ how sizeable are the effects ?
  - ↪ what is the relevant kinematical domain ?
  - ↪ can observables be computed from ‘first principles’ ?

- DGLAP provides extremely accurate description of ALL available experimental data
  - ↪ can properties of the evolution be disentangled from ingenious choices of initial conditions ?
  - ↪ how uncertain are extrapolations into the unmeasured small-x region ?
  - ↪ can results from non-linear approaches be accommodated in the description by simply tuning initial conditions?

- can these questions be answered ?

the CGC and how to test it

- CGC [in this talk] is a 'first principle' effective theory for the description of the small-x glue
  - well established non-linear evolution equations [B-JIMWLK]
  - large Nc approximation [BK] for suitable observables
  - NOT [in this talk] phenomenological models encoding 'saturation physics'

- testing strategy
  - extract universal unintegrated gluon distribution from cleanest process [DIS]
  - use to compute observables in pp, pA and AA collisions [cf. talk by Itakura]
  - devise a set-up in which to compare DGLAP and CGC evolutions
    - find non-overlapping failing regions

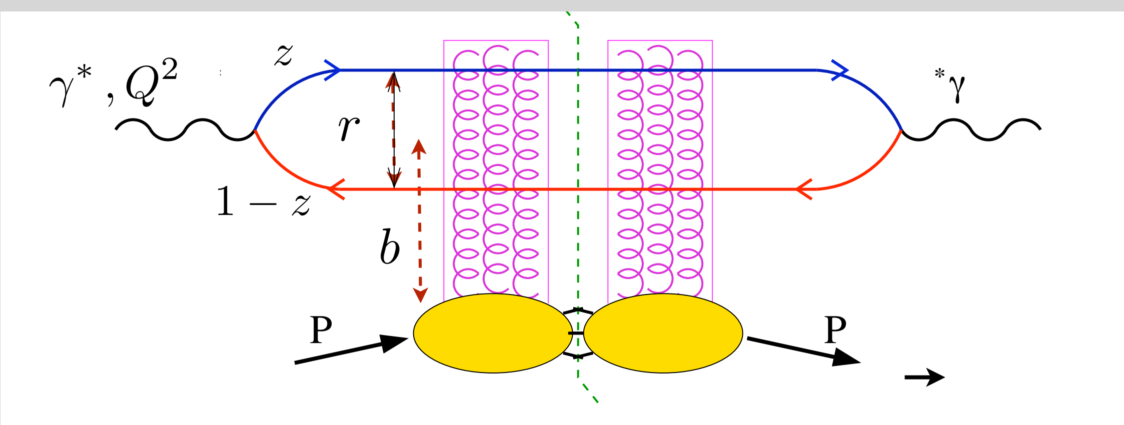
dipole formulation of QCD

- at high energy [x << 1] the coherence length of the virtual photon fluctuation

$$l_c \sim (2m_N x)^{-1} \simeq 0.1/x \text{ fm} \gg R_N$$

- total virtual photon-proton cross section can be factorized as

$$\sigma_{T,L}(x, Q^2) = 2 \sum_f \int_0^1 dz \int d\mathbf{b}\, d\mathbf{r}\, |\Psi^f_{T,L}(e_f, m_f, z, Q^2, \mathbf{r})|^2 \, \mathcal{N}(\mathbf{b}, \mathbf{r}, x)$$

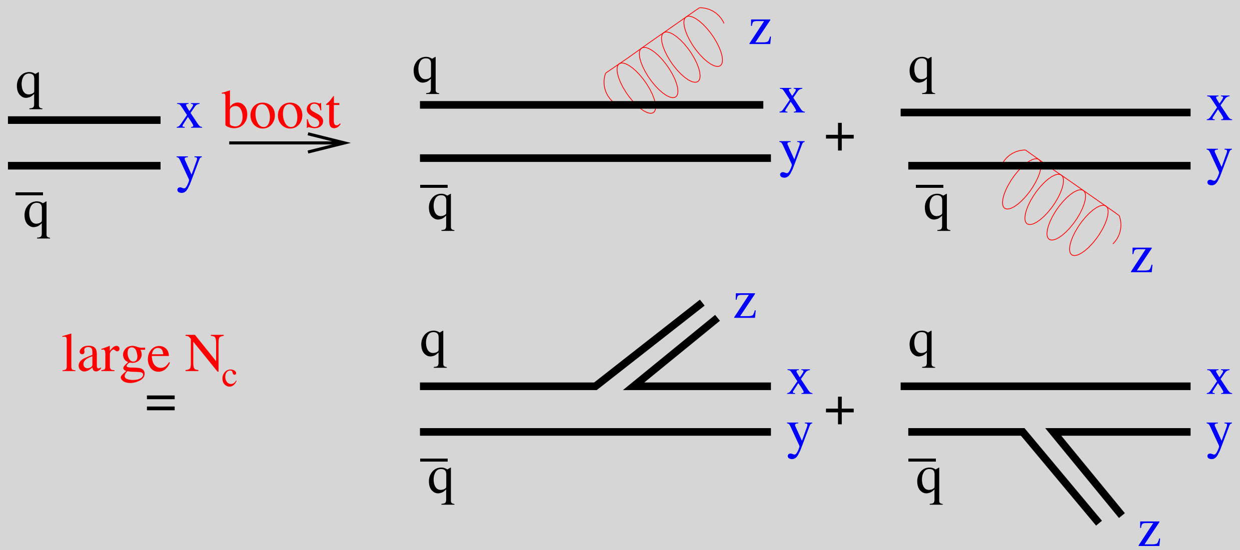


QED calculation

[imaginary part of]
dipole-target scattering amplitude
:: all QCD information
:: all x dependence
:: non-perturbative, but x-evolution computable from first principles [rcBK]

Balitsky-Kovchegov equation [LO]

[rapidity evolution of scattering probability $N(x, y; Y)$ of $q\bar{q}$ dipole with hadronic target]



$$\vec{r} = \vec{x} - \vec{y}$$
$$\vec{r}_1 = \vec{x} - \vec{z}$$
$$\vec{r}_2 = \vec{y} - \vec{z}$$

homogeneous target with radius much larger than any dipole size

$\hookrightarrow$ neglect impact parameter dependence (2-dim into 1-dim)
improved treatment very sensitive to gluon mass [Berger and Stasto]

non-linear effect [double scattering]

$$\frac{\partial N(r, Y)}{\partial Y} = \int \frac{d^2 z}{2\pi} K(\vec{r}, \vec{r}_1, \vec{r}_2) \Big[N(r_1, Y) + N(r_2, Y) - N(r, Y) - N(r_1, Y) N(r_2, Y) \Big]$$

$$K(\vec{r}, \vec{r}_1, \vec{r}_2) = \bar{\alpha}_s \frac{r^2}{r_1^2 r_2^2}, \qquad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

BFKL kernel:
probability of gluon (two dipoles) emission

NLO-BK

NLO-BK = [all orders in $\alpha_s N_f$]+ [other conformal]

[running coupling] + [subtraction]

:: numerically challenging ::

numerically demanding, but

contribution minimized in Balitsky's subtraction scheme

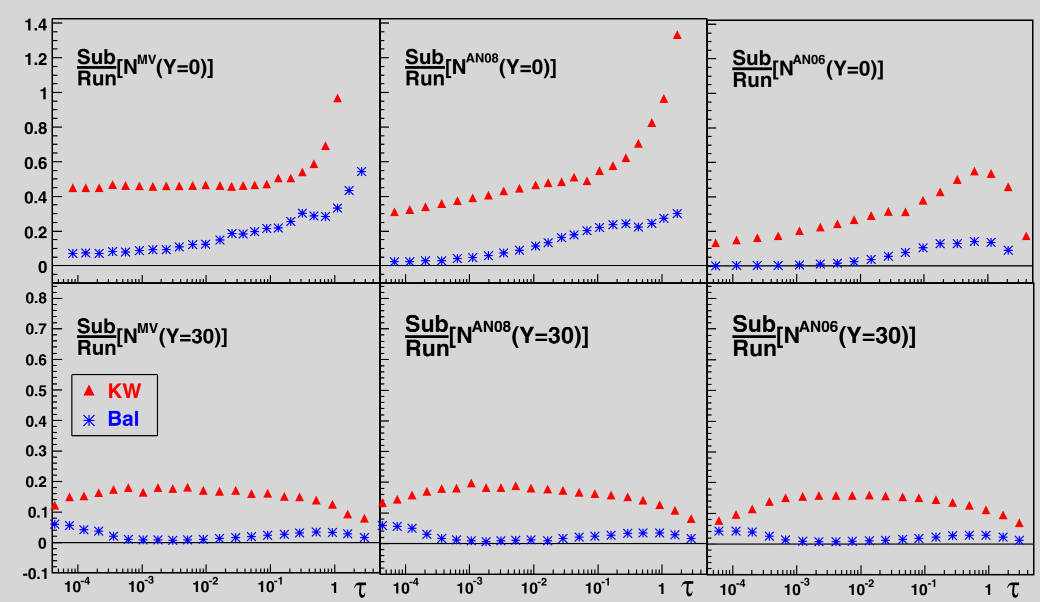
[rcBK]

AAMQ$_s$ implementation

+ energy conservation

$$\mathbf{K}^{\mathbf{run}}\left(1-\frac{\mathbf{d}}{\mathbf{dY}}\right)$$

[Kuokkanen, Rummukainen Weigert]



[Albacete, Kovchegov]

rcBK

- running coupling BK [rcBK]
  - fully compatible with DIS data
  - first principle, numerically implementable, incarnation of non-linear QCD

$$\frac{\partial \mathcal{N}(r,x)}{\partial \ln(x_0/x)} = \int d\mathbf{r_1}\, K^{\rm run}(\mathbf{r},\mathbf{r_1},\mathbf{r_2}) \; [\mathcal{N}(r_1,x)+\mathcal{N}(r_2,x)-\mathcal{N}(r,x)-\mathcal{N}(r_1,x)\,\mathcal{N}(r_2,x)]$$

same structure as LO-BK

modified kernel

$$K^{\rm run}(\mathbf{r},\mathbf{r_1},\mathbf{r_2}) = \frac{N_c\,\alpha_s(r^2)}{2\pi^2}\left[\frac{r^2}{r_1^2\,r_2^2}+\frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)}-1\right)+\frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)}-1\right)\right]$$

LO-BK :: BFKL kernel

$$K(\vec{r},\vec{r}_1,\vec{r}_2) = \bar{\alpha}_s\,\frac{r^2}{r_1^2 r_2^2}, \qquad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

AAMQ$_s$

installation [Spring 2009]

AAMQs setup

Albacete, Armesto, Milhano, Quiroga, Salgado

- LO impact factors [virtual photon-proton cross section]
- proton homogeneous in transverse plane
  - ↪ effective transverse area is a fit parameter
- different initial conditions
  - ↪ generalized GBW and MV forms

$$\mathcal{N}^{GBW}(r, x = x_0) = 1 - \exp\left[-\frac{\left(r^2\, Q_{s\,0}^2\right)^\gamma}{4}\right]$$

$$\mathcal{N}^{MV}(r, x = x_0) = 1 - \exp\left[-\frac{\left(r^2\, Q_{s\,0}^2\right)^\gamma}{4} \ln\left(\frac{1}{r\,\Lambda} + e\right)\right]$$

:: differ in UV behaviour ::

    - initial saturation scale and sharpness of edge fall-off are fit parameters
  - ↪ rescaled asymptotic solutions
    - no fit possible in $AAMQ_s$
    - Weigert et al. report excellent fits [once energy conservation included]
- also fits including heavy quarks [not shown] :: F_{2c} constrained

AAMQs setup

- running coupling in rcBK is 1-loop in coordinate space

$$\alpha_{s,n_f}(r^2) = \frac{4\pi}{\beta_{0,n_f} \ln\left(\frac{4C^2}{r^2\Lambda_{n_f}}\right)} \qquad \beta_{0,n_f} = 11 - \frac{2}{3}n_f$$

  - ↪ fixed number of flavours [N_f=3] for fits shown [variable for heavy quark case]
  - ↪ C [fit parameter] accounts for uncertainty in FT from momentum to coordinate space
  - ↪ Λ fixed by reference measured value of α_s [either Z^0 mass or τ mass]
  - ↪ IR regulated by freezing α_s, $r > r_{fr}$, $\alpha_s(r_{fr}^2) \equiv \alpha_{fr} = 0.7$ [or other suitable value]

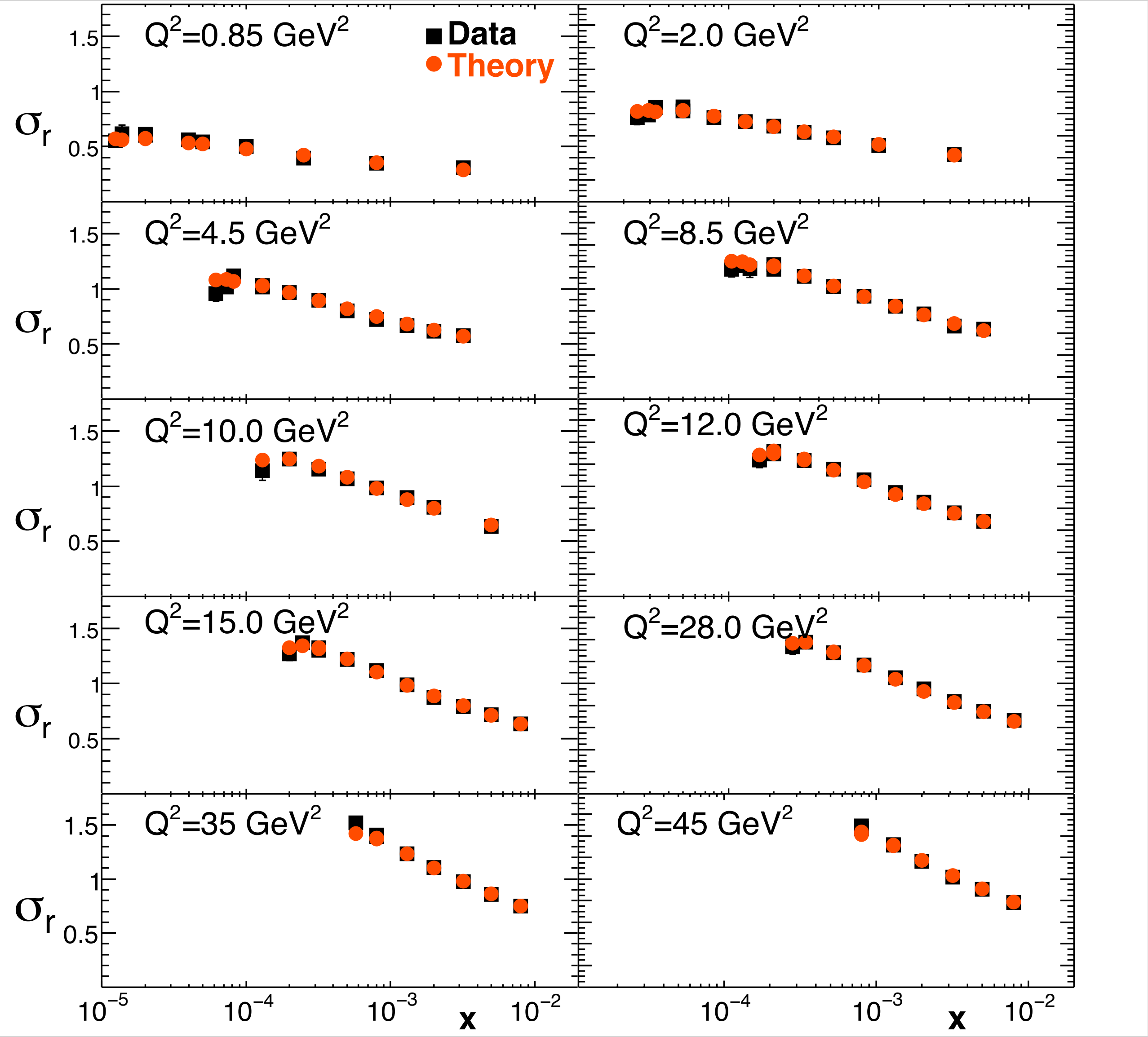
- H1-ZEUS combined set + non-HERA data [E665, NMC] with cuts

$$x \leq 10^{-2},\ Q^2 \leq 50\,\mathrm{GeV}^2$$

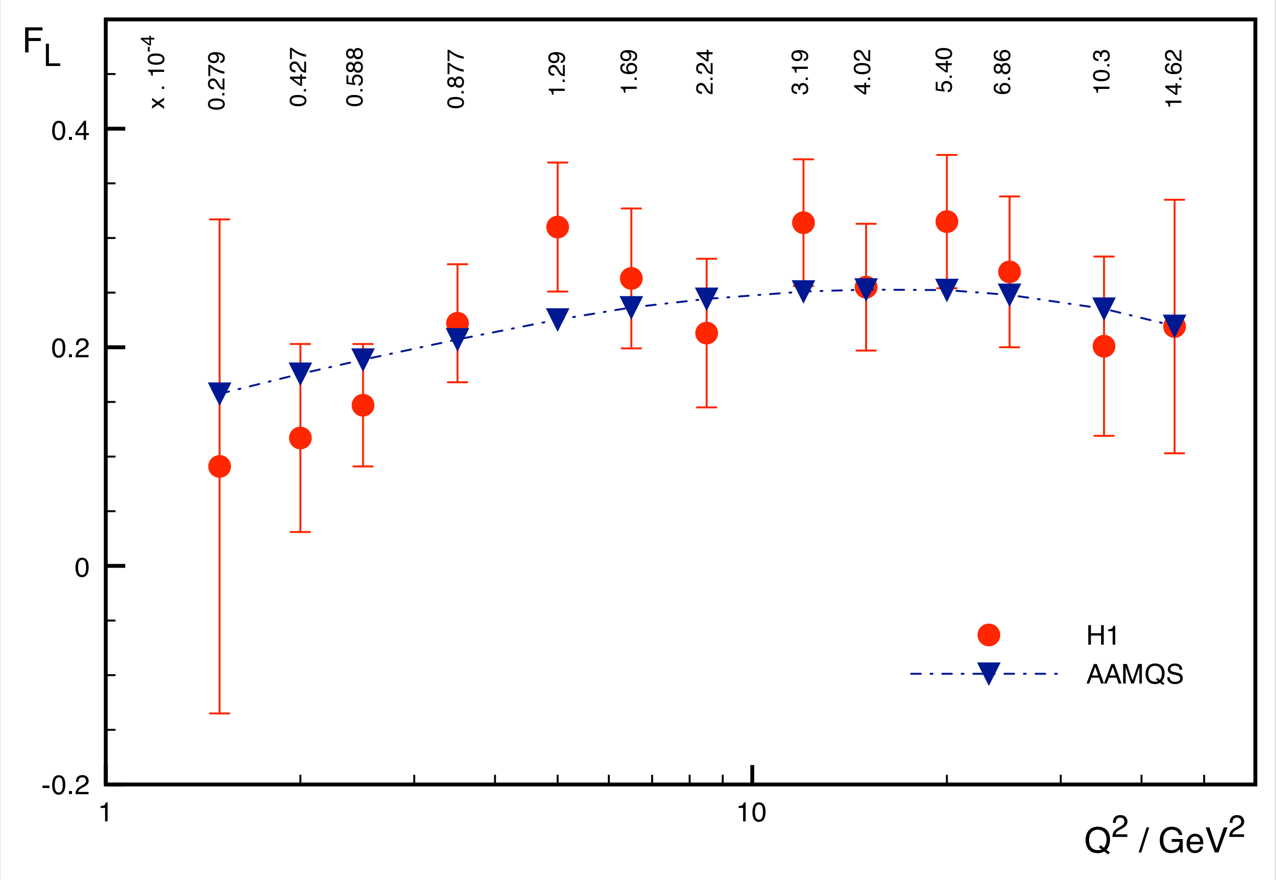
- kinematical redefinition of Bjorken-x

$$\tilde{x} = x\left(1 + \frac{4m_f^2}{Q^2}\right)$$

results

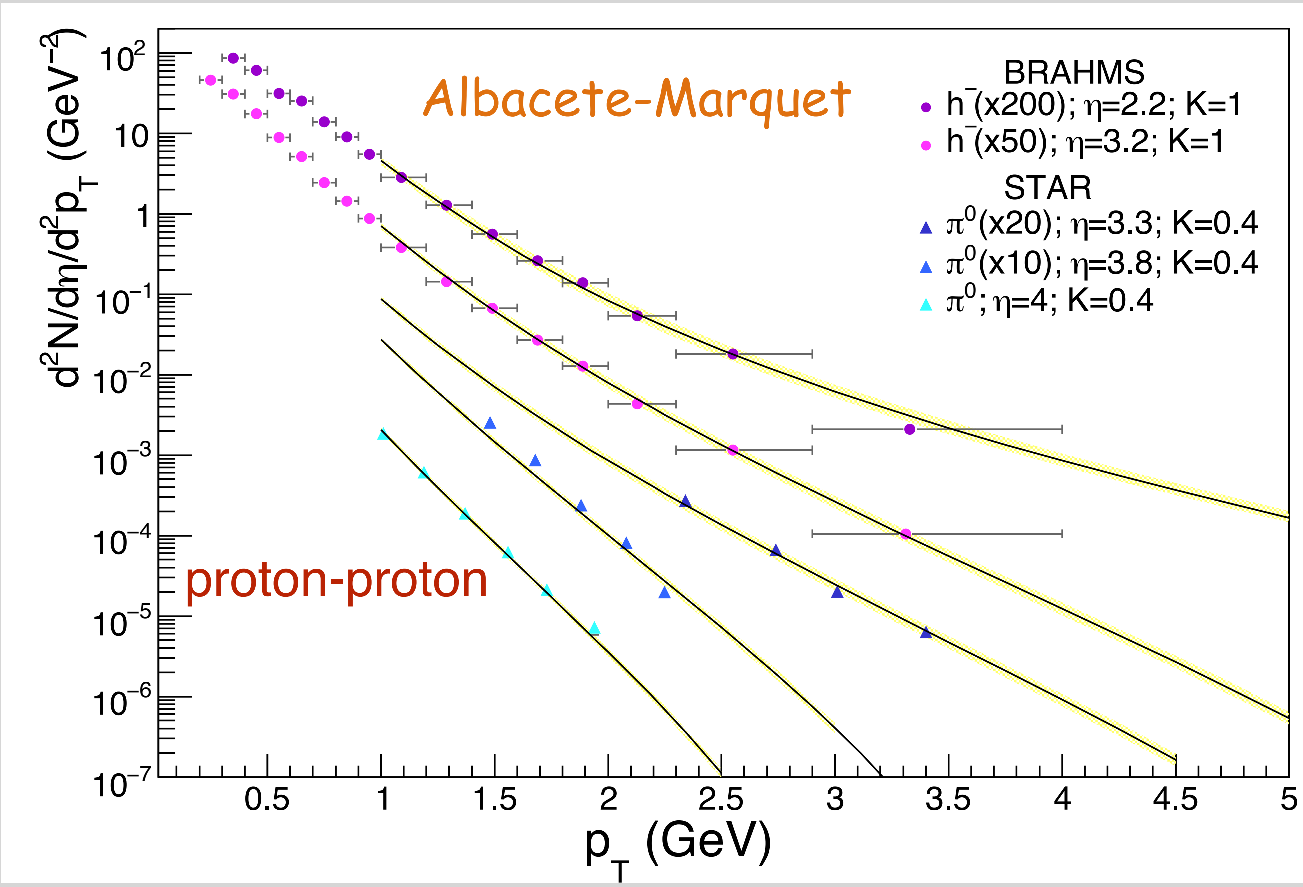


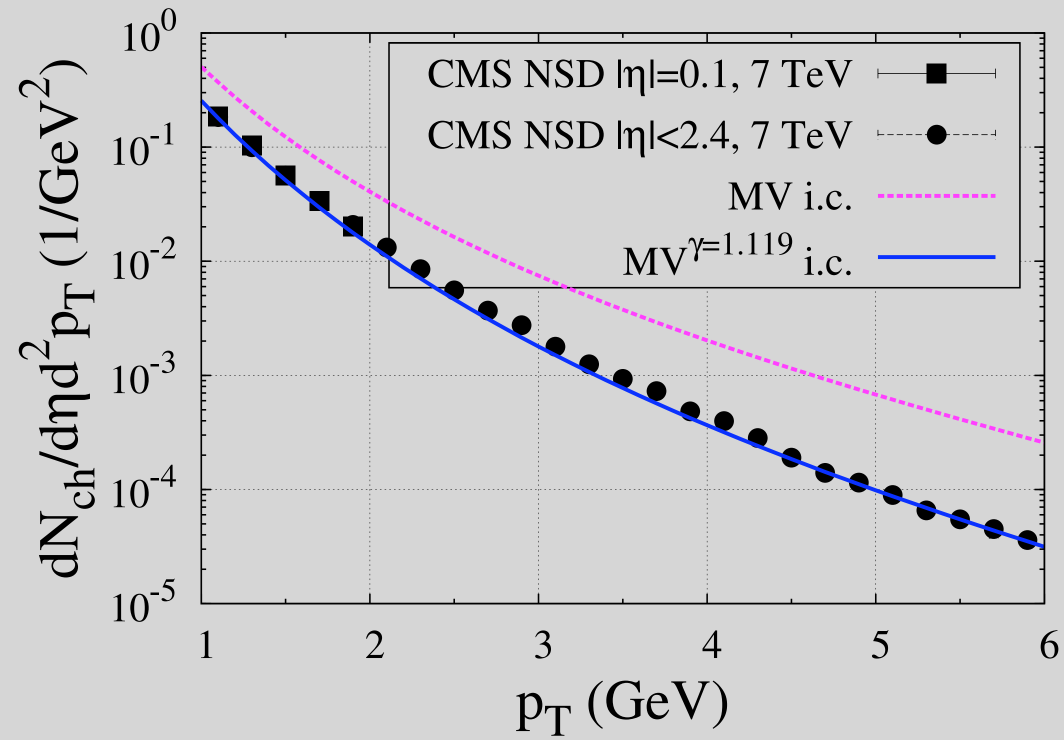
- excellent fit quality [chi2~1]
- fitted initial conditions are numerically 'essentially identical'
- good description of F_L [not fitted]



hadronic collisions

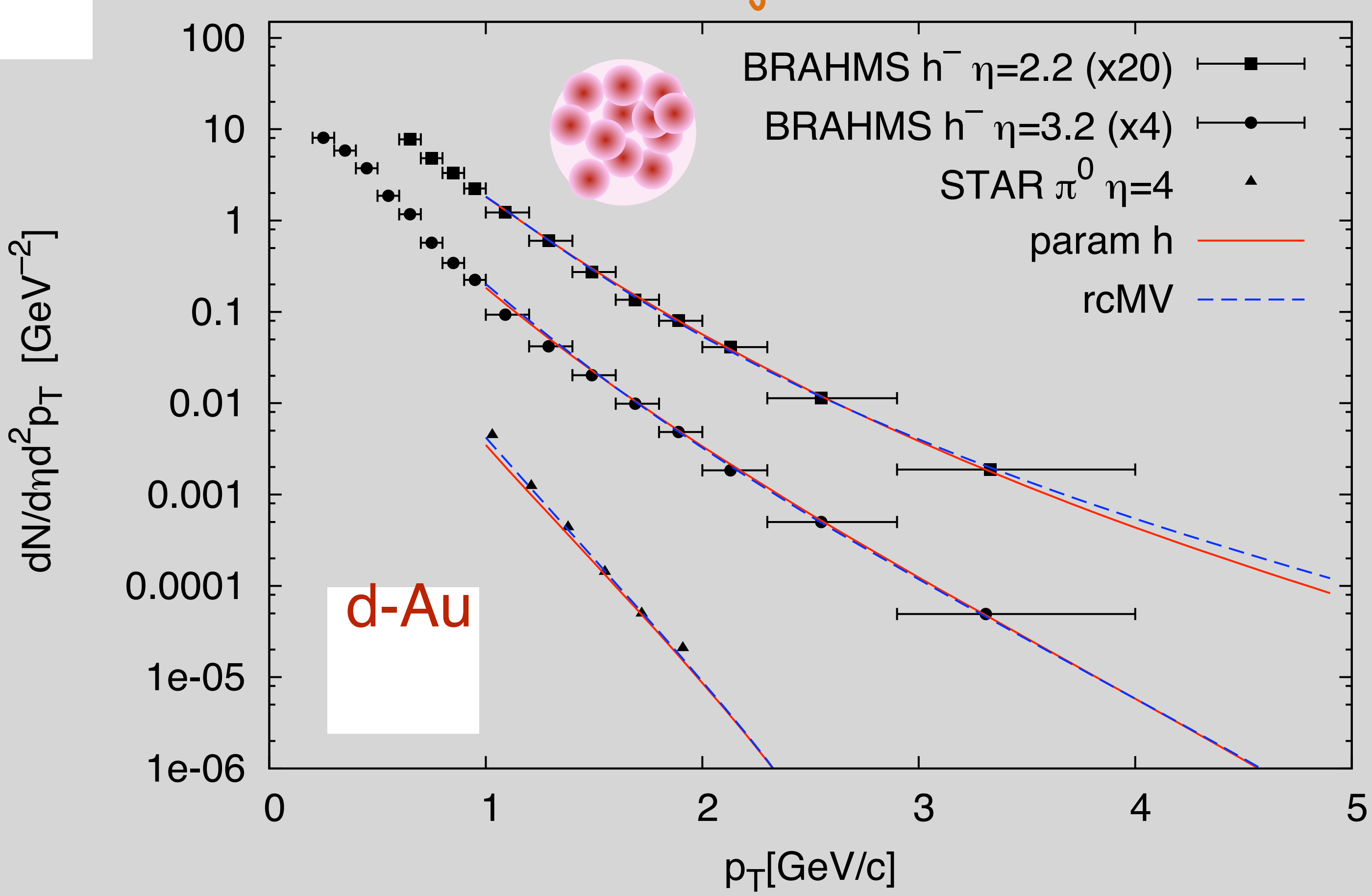
pp, pA and AA [see Itakura's talk]





- LO k_t factorization is major caveat
  - significant recent analytical progress
  - no numerical implementation yet

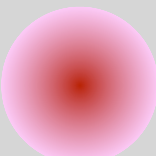
Fujii-Itakura-Kitadono-Nara



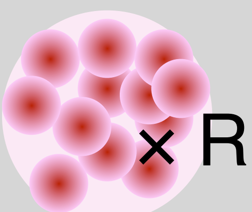
MC-rcBK [building nuclei from nucleons]

$$\phi^{\mathbf{A}}(\mathbf{x}, \mathbf{k_t}, \mathbf{B}) = \phi^{\mathbf{p}}(\mathbf{x}, \mathbf{k_t}, \mathbf{Q^2_{sp}} \to \mathbf{Q^2_{sA}}(\mathbf{B}))$$

1. Trivial: $\bar{\mathbf{Q}}_{\mathbf{s}}^{\mathbf{2},\mathbf{A}} \sim \mathbf{A}^{\mathbf{1/3}}\, \mathbf{Q}_{\mathbf{s}}^{\mathbf{2},\mathbf{N}}$



2. Mean field: $\mathbf{Q}_{\mathbf{s}}^{\mathbf{2},\mathbf{A}}(\mathbf{B}) \sim \mathbf{T_A}(\mathbf{B})\, \mathbf{Q}_{\mathbf{s}}^{\mathbf{2},\mathbf{N}}$



3. Monte Carlo (realistic i.c for heavy ion collisions)

a). Initial conditions for the evolution (x=0.01)

$$N(\mathbf{R}) = \sum_{i=1}^{A} \Theta\left(\sqrt{\frac{\sigma_0}{\pi}} - |\mathbf{R} - \mathbf{r_i}|\right) \longrightarrow Q^2_{s0}(\mathbf{R}) = N(\mathbf{R})\, Q^2_{s0,\,\mathrm{nucl}}$$

$$\downarrow$$

$$\varphi(x_0 = 0.01, k_t, R)$$

b) Solve **local** rcBK evolution at each transverse point

$\downarrow$ rcBK equation or KLN model

$$\varphi(x, k_t, R)$$

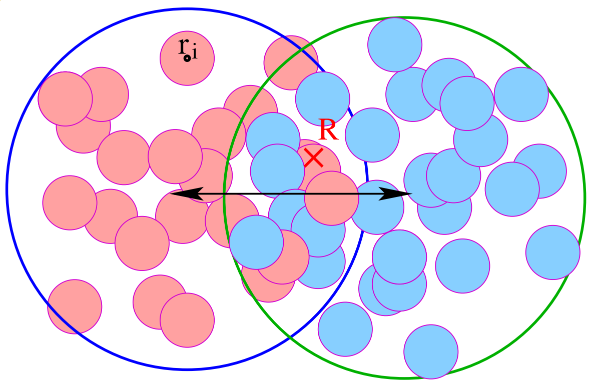
Nucleons can be regarded as disks ( ) or gaussian ( ) or ...

Is using the same functional form for proton and nuclei u.g.d a good idea?

Is diffusion in the transverse plane negligible?

MC-rcBK [building nuclei from nucleons]

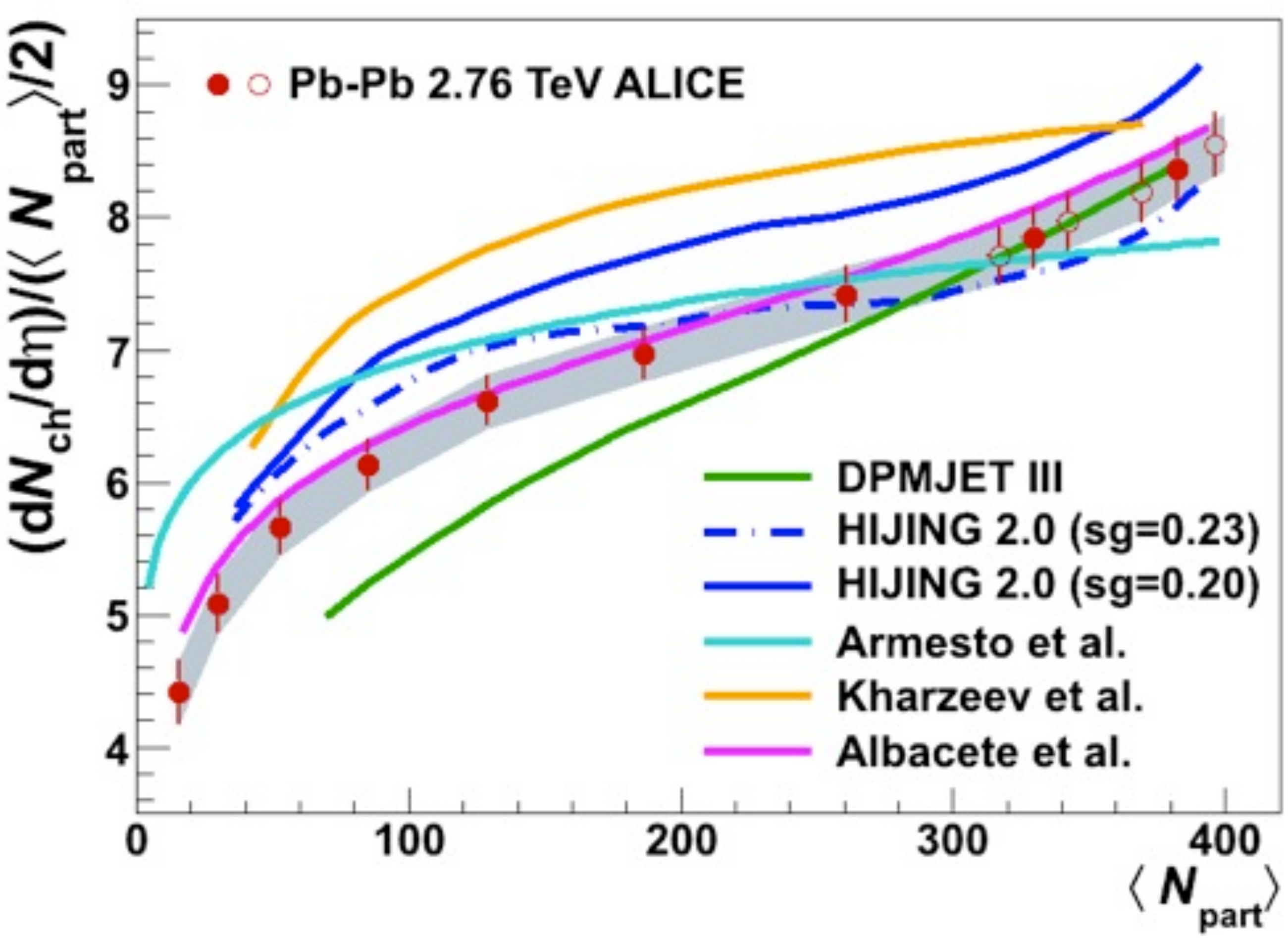
Albacete-Dumitru-Nara



- kt-factorization + running coupling BK evolution

$$\frac{d\sigma^{A+B\to g}}{dy\,d^2p_t\,d^2R} = \kappa\,\frac{2}{C_F}\frac{1}{p_t^2}\int^{p_t}\frac{d^2k_t}{4}\int d^2b\,\alpha_s(Q)\,\varphi(\frac{|p_t+k_t|}{2},x_1;b)\,\varphi(\frac{|p_t-k_t|}{2},x_2;R-b)$$

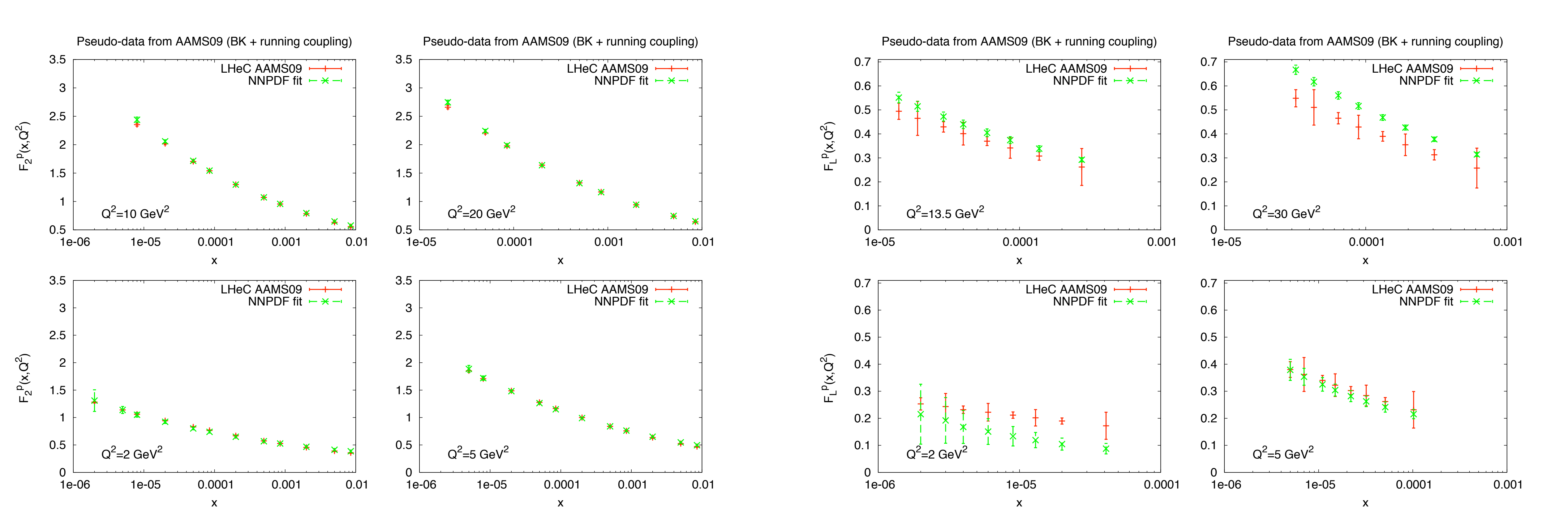
$$\frac{dN^{A+B\to g}}{dy\,d^2p_t\,d^2R} = \frac{1}{\sigma_s}\frac{d\sigma^{A+B\to g}}{dy\,d^2p_t\,d^2R}$$



Good description of Pb+Pb data

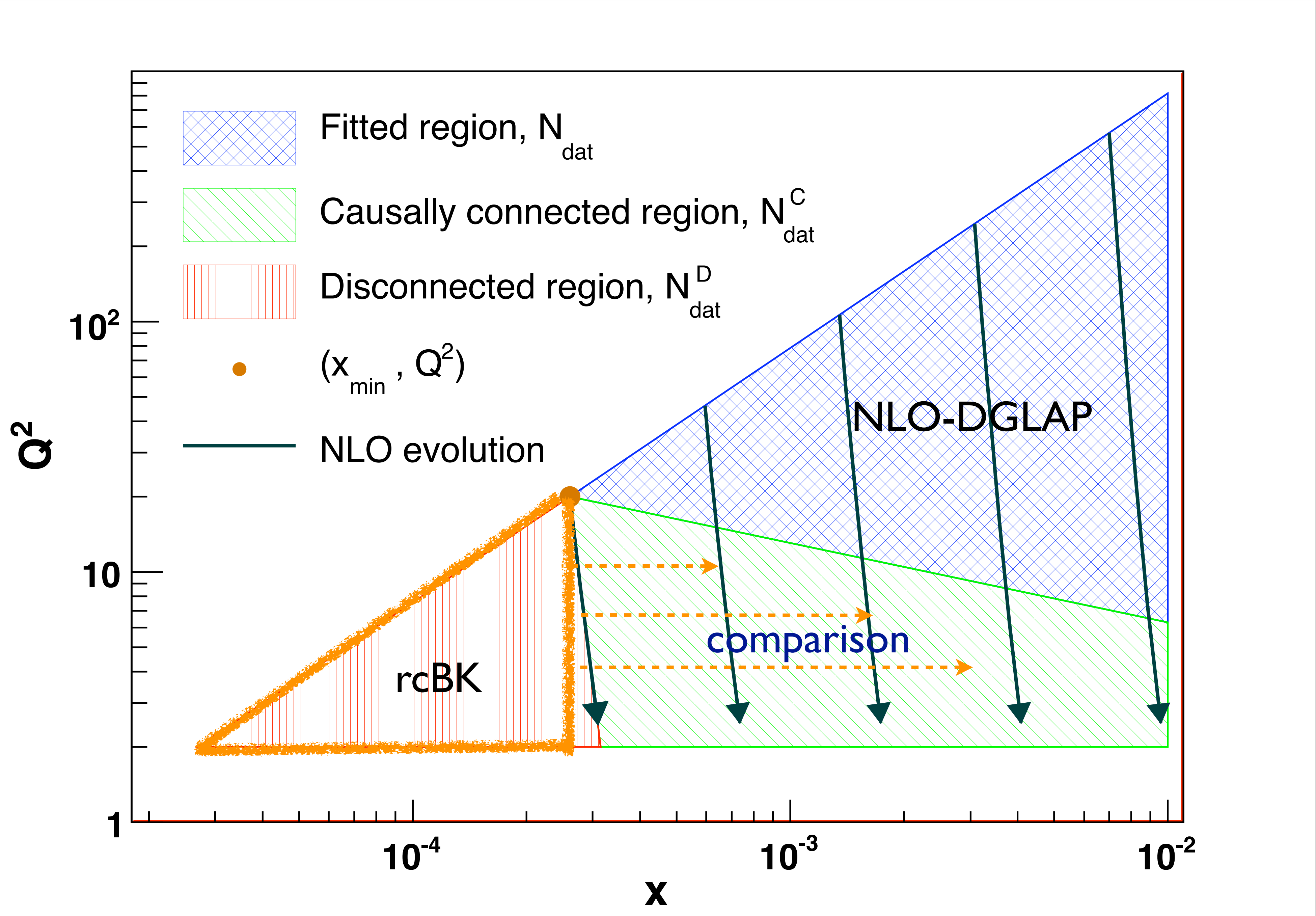
CGC models for multiplicities can also be tested in a p+Pb run

DGLAP comparison



comparison with DGLAP

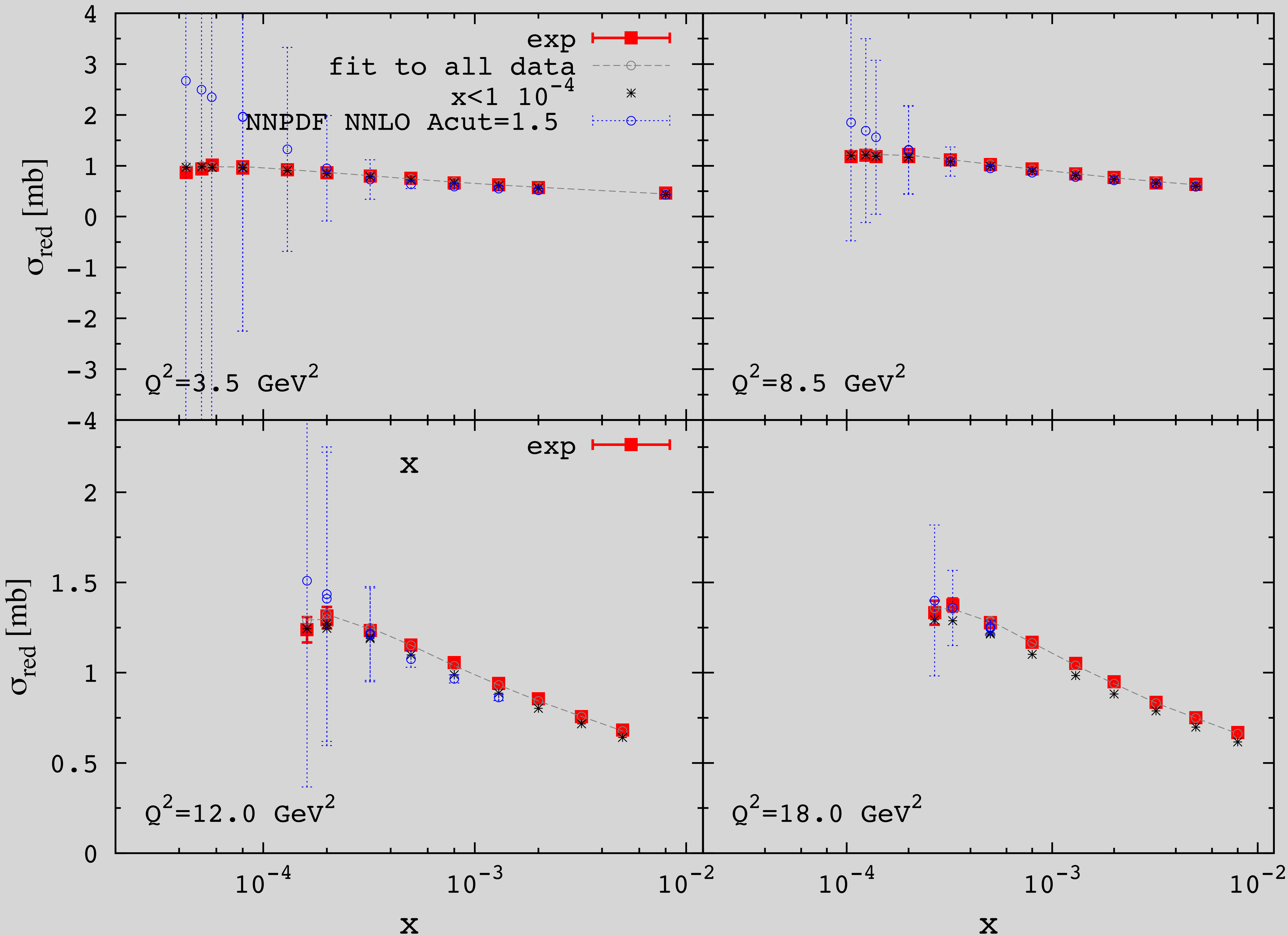
Albacete, Milhano, Quiroga [very soon]



test the evolution NOT the choice of initial conditions

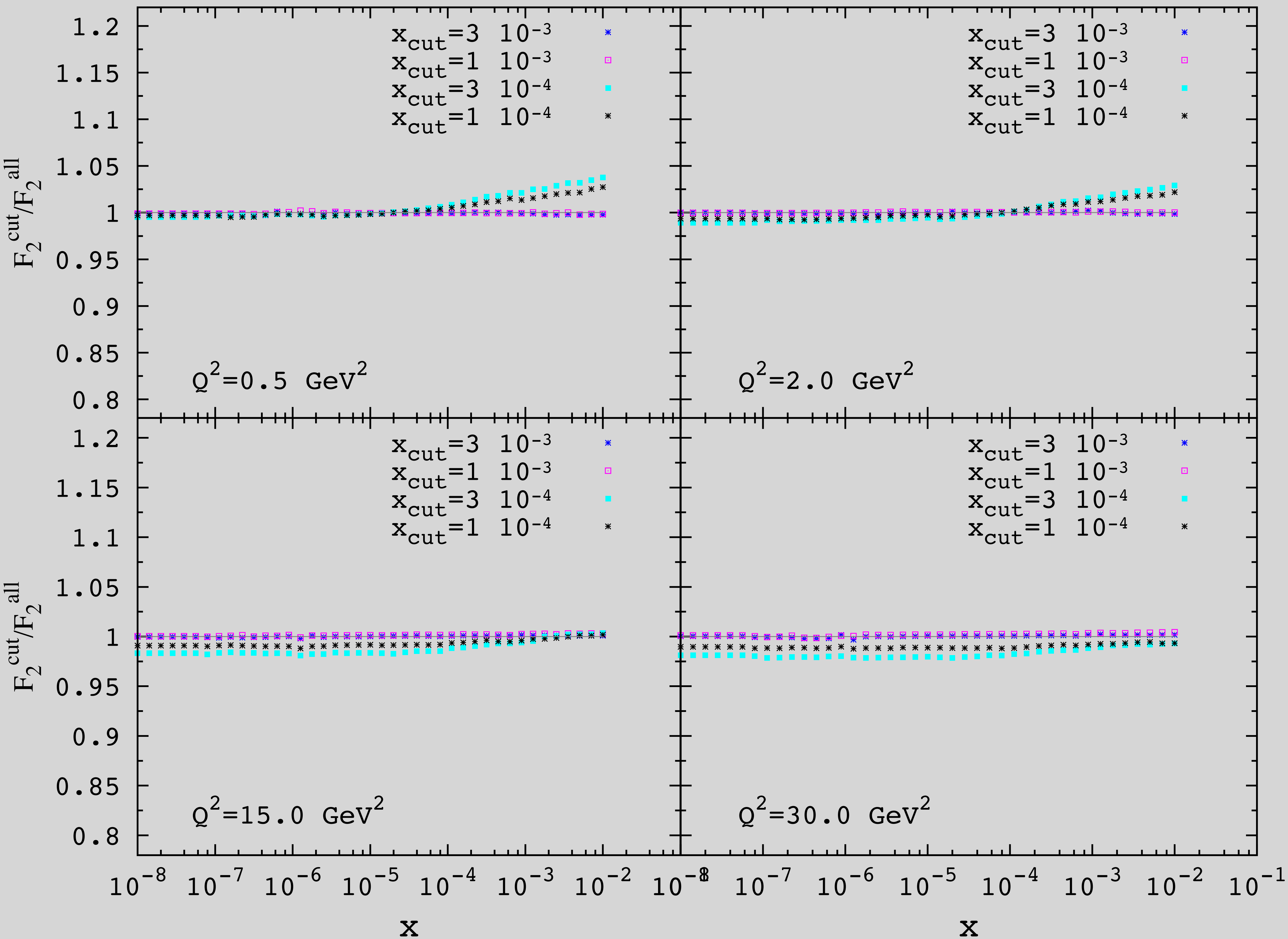
comparison with DGLAP

Albacete, Milhano, Quiroga [very soon]



predictive power at small-x

Albacete, Milhano, Quiroga [very soon]

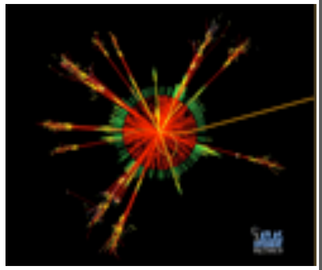


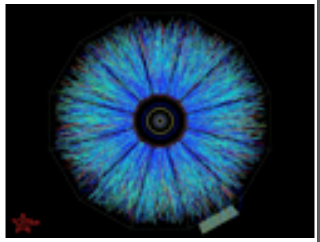
limitations and outlook

- large k_t behaviour unconstrained from DIS data alone
  - ↪ pp and mostly pA data needed
- impact parameter dependence for nucleus problematic
  - ↪ very sensitive to treatment of edge [gluon mass]
- NLO k_t factorized production badly needed for definite statements
  - ↪ great analytical progress, numerical work to be done
- less inclusive observables require evolution for higher order correlators [beyond BK]
  - ↪ great analytical progress, numerical work to be done
- insufficient eA data for direct constraint on nuclear case [eRHIC, LHeC]
  - ↪ pA data can help
- identification of kinematical ‘discovery’ regions for the CGC now possible via detailed comparision with DGLAP

AAMQ$_s$ 1.0

Dipole-proton cross section





The imaginary part of the dipole-proton scattering amplitude is available as a FORTRAN routine for public use. This quantity has been fitted to lepton-proton data using the Balitsky-Kovchegov evolution equations with running coupling. More details can be found at

J. L. Albacete, N. Armesto, J. G. Milhano, P. Quiroga Arias and C. A. Salgado, arXiv:1012.4408

Please refer to this publication when using the routine.

In order to compute the dipole cross section, simply multiply the output from the routine by the corresponding values in Table 1 of arXiv:1012.4408 (the actual values depend on the chosen set of parameters). These values are

For the fits with only light flavors (subroutine aamqs10l):

sigma0=32.357 mb for GBW initial conditions, set a
sigma0=32.895 mb for MV initial conditions, set e

For the fits with light+heavy flavors (subroutine aamqs10h):

sigma0=35.465 mb for GBW initial conditions, light, set b
sigma0=18.430 mb for GBW initial conditions, heavy, set b
sigma0=35.449 mb for MV initial conditions, light, set f
sigma0=19.066 mb for MV initial conditions, heavy, set f

Full instructions and explanations can also be found at the headers of the routines.

To download the code, please follow this link

The main novelties on these parametrizations with respect to our older one arXiv:0902.1112 are the use of the new (H1 and ZEUS combined) HERA data with much smaller error bars as well as the inclusion of heavy flavors in the fits.

If you find any problem, please, let us know

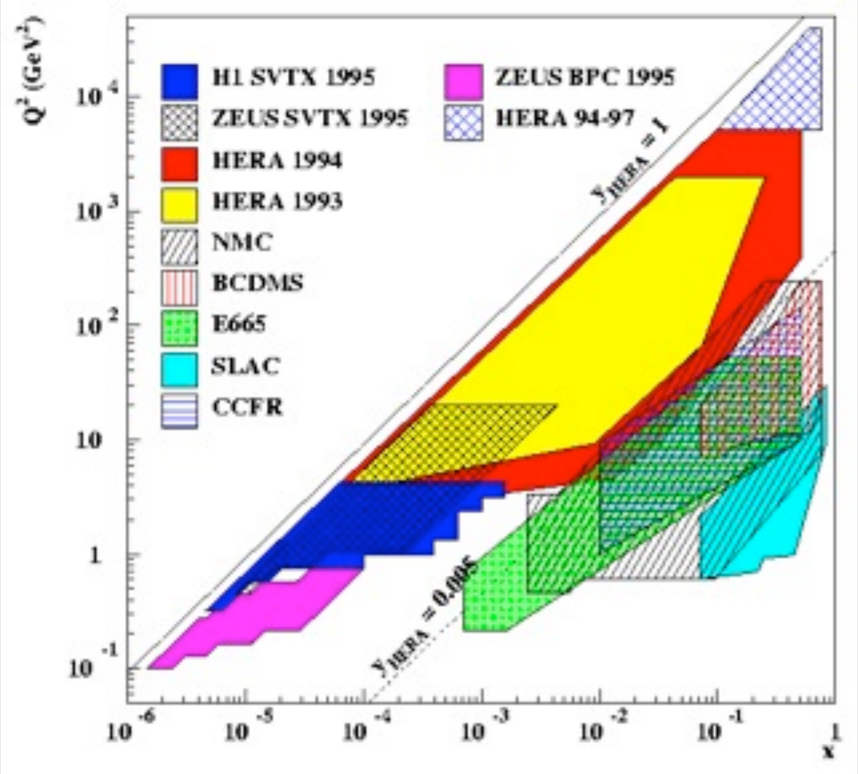
HTTP://WWW-FP.USC.ES/PHENOM/AAMQS/AAMQS.HTML

backups

proton vs. nuclear pdfs

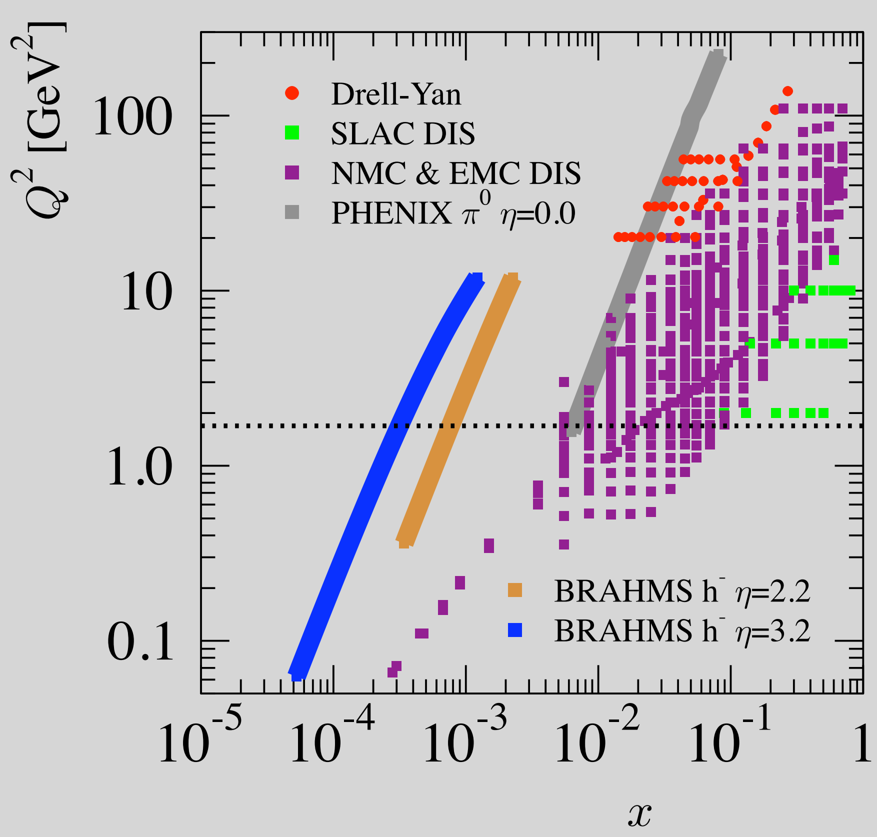
- *proton case*
  - collinear factorization theorems proven for some processes
  - wealth of data (DIS, DY, jets)
    - ↪ very reliable pdfs in 'data covered' kinematical range
  - large number of parameters in i.c.
    - ↪ very 'accommodating'
    - ↪ large uncertainty where data not available [small-x for moderate Q^2]

- but [see later] small x effects beyond collinear approach



⟳ proton vs. nuclear pdfs

- *nuclear case*
  - collinear factorizability is a working assumption
    - ↪ encoding of all nuclear effects in npdfs is a huge leap of faith
    - ↪ could be reliably tested in pA LHC collisions [will discuss later]
  - relatively scarce data
  - standardly encoded as nuclear modification of proton pdfs [inherits proton pdf uncertainties]



$$f_{i/A}(x,Q^2) = R_i^A(x,Q^2)\, f_{i/p}(x,Q^2)$$

- $f_{i/A}(x,Q^2)$: pdf of parton in nucleon inside nucleus
- $R_i^A(x,Q^2)$: nuclear modification
- $f_{i/p}(x,Q^2)$: pdf of parton in free nucleon

why does partonic density grow ?

- hadronic wave function [ensemble] can be described by a colour charge density ρ^a

  :: associated non-dynamical longitudinal Coulomb field

- when boosted, longitudinal field becomes transverse

  :: equivalent [Weizsacker-Williams] gluons

$$E^i(r) = \frac{g}{4\pi}\frac{r^i}{|r|^3} \quad \rightarrow E^i = \frac{g}{2\pi}\frac{X^i_\perp}{X^2_\perp}\delta(X^-)$$

- 'emerging' gluons

$$a(k) \sim g\frac{1}{\sqrt{k^+}}\frac{k^i_\perp}{k^2_\perp}$$

$$E^i(k) = i\sqrt{\omega(k)}[a_i(k) - a_i^\dagger(k)] = i\sqrt{k^+}[a_i(k) - a_i^\dagger(k)]$$

$$n(k_\perp) = \int dk^+ \langle a_i^\dagger(k) a_i(k)\rangle = \frac{\alpha_s}{k^2_\perp}\int\frac{dk^+}{k^+} = \frac{\alpha_s}{k^2_\perp}\eta$$

and these are colour charged
:: own WW field ::

more gluons

ditto, ditto ...

AAMQ$_s$ setup

- DIS reduced cross section

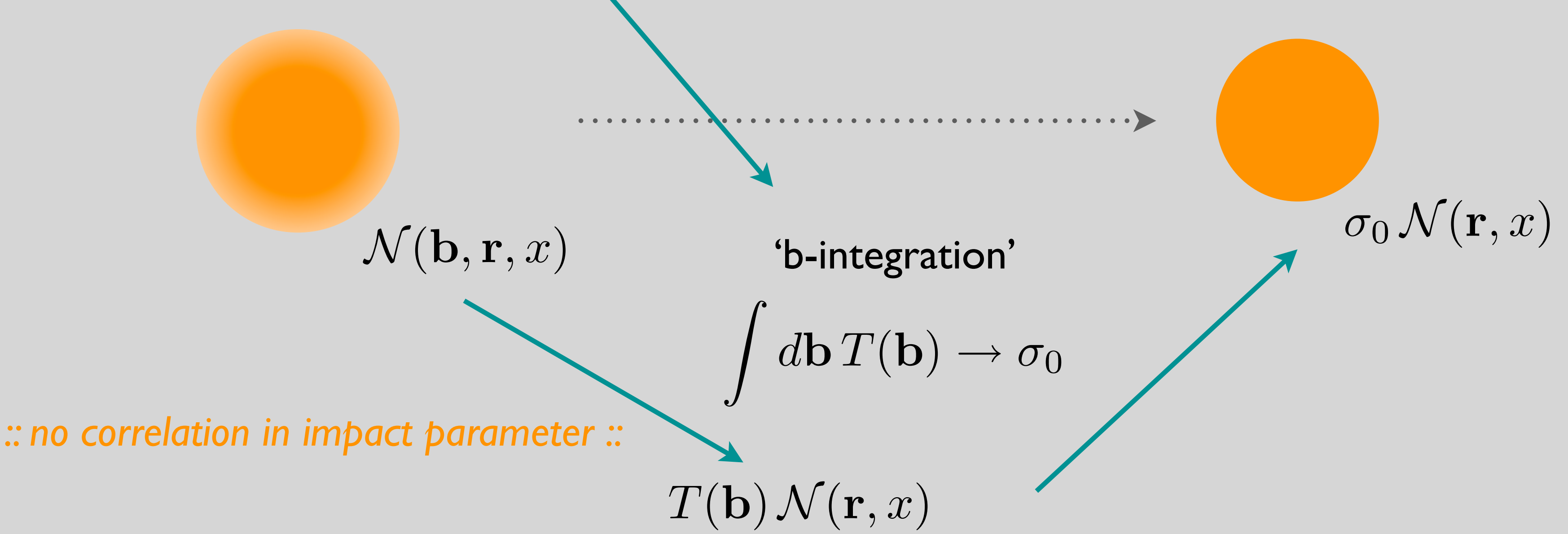
$$\sigma_r(x, y, Q^2) = \frac{Q^2}{4\,\pi^2\alpha_{em}}\left(\sigma_T + \frac{2(1-y)}{1+(1-y)^2}\sigma_L\right)$$

$$\sigma_{T,L}(x, Q^2) = 2\sum_f \int_0^1 dz \int d\mathbf{b}\, d\mathbf{r}\, |\Psi^f_{T,L}(e_f, m_f, z, Q^2, \mathbf{r})|^2\, \mathcal{N}(\mathbf{b}, \mathbf{r}, x)$$

- b-dependence governed by long-distance non-perturbative physics [extra model input]
- AAMQ$_s$ resorts to translational invariance approximation
  - proton homogeneous in transverse plane

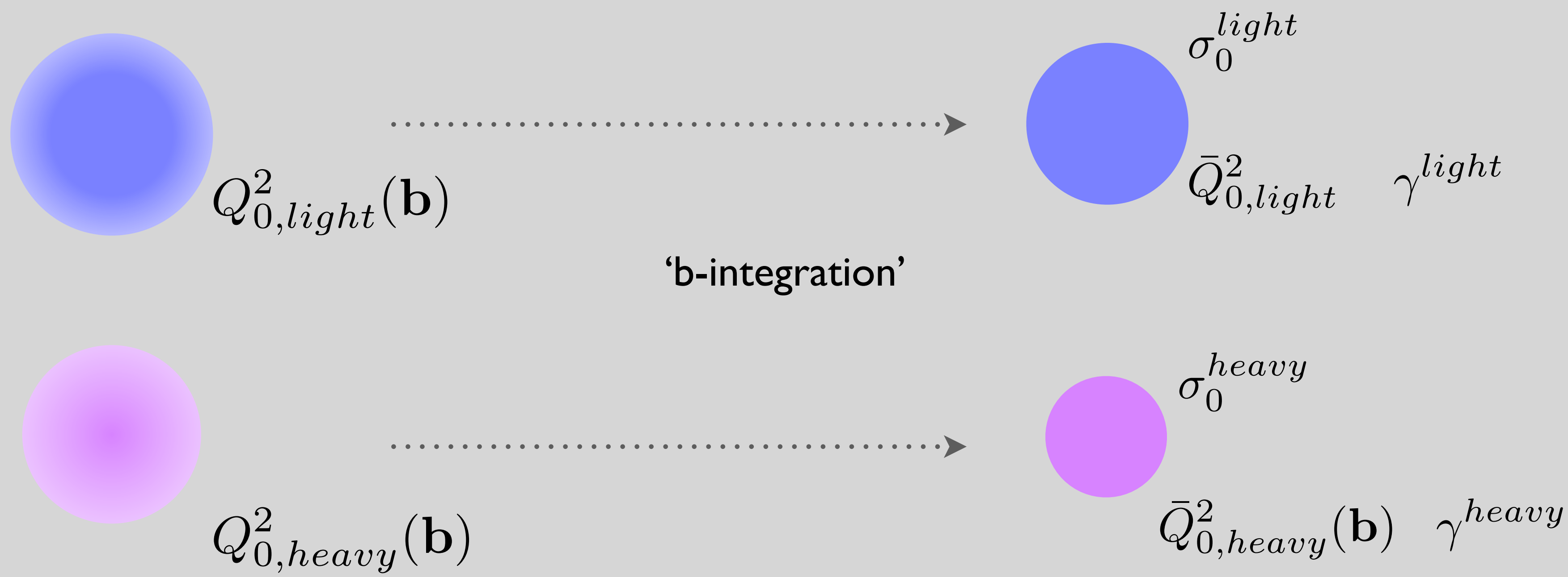
$$\sigma_{T,L}(x, Q^2) = \sigma_0 \int_0^1 dz \int d\mathbf{r}\left|\Psi_{T,L}(z, Q^2, \mathbf{r})\right|^2 \mathcal{N}(r, x)$$

only LO
[QED calculation]
NLO now available
[Balitsky & Chirilli]



AAMQ$_s$ setup :: including heavy quarks

- allow for independent light and heavy i.c.



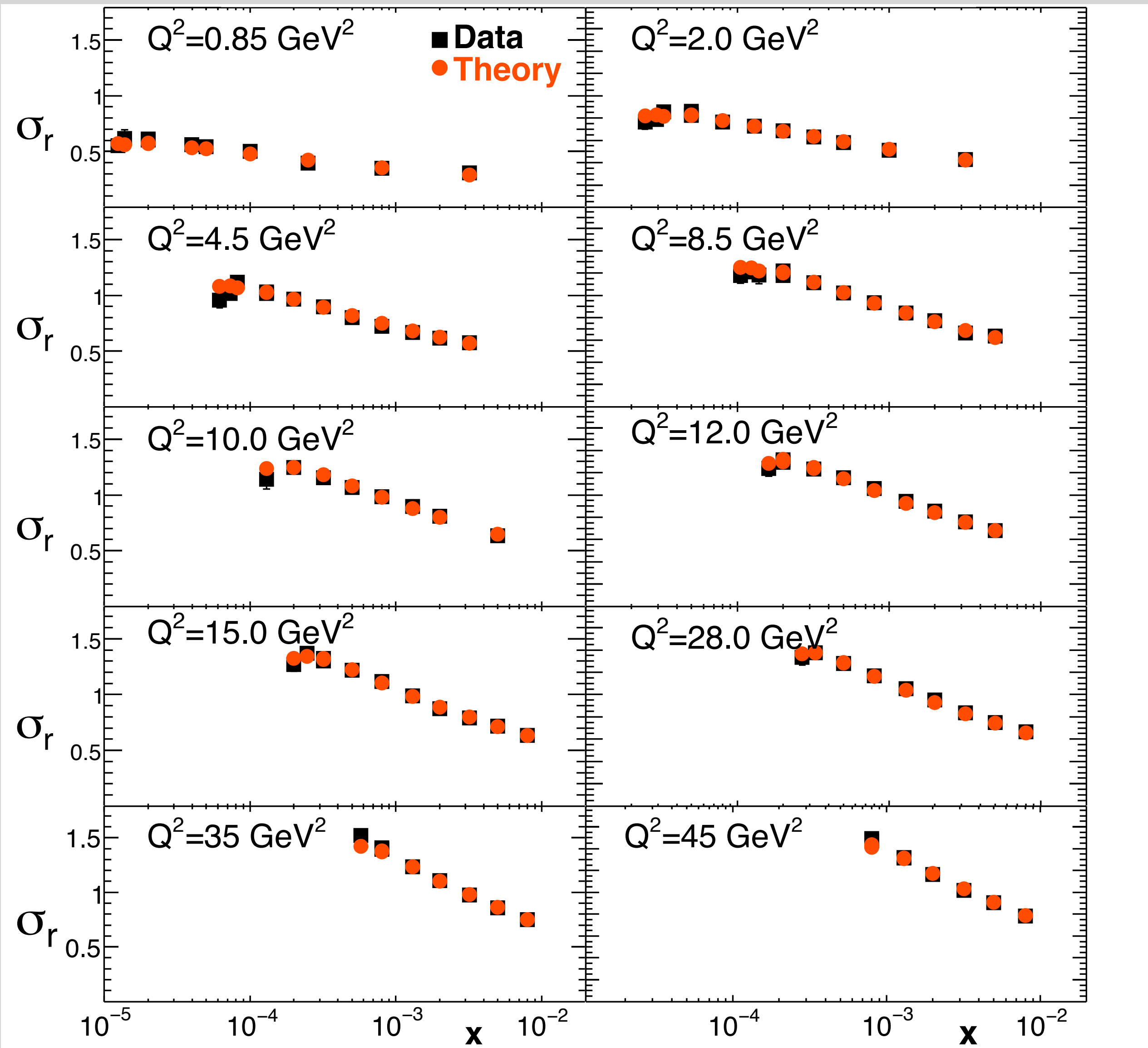
- should follow from 'better' treatment of b-dependence
  - heavy quark dipoles couple differently to target ...

- additional fit parameters ...

$$\sigma_{T,L}(x,Q^2) = \sigma_0 \sum_{f=u,d,s} \int_0^1 dz\, d\mathbf{r}\, |\Psi^f_{T,L}(e_f, m_f, z, Q^2, \mathbf{r})|^2\, \mathcal{N}^{light}(\mathbf{r}, x)$$

$$+\, \sigma_0^{heavy} \sum_{f=c,b} \int_0^1 dz\, d\mathbf{r}\, |\Psi^f_{T,L}(e_f, m_f, z, Q^2, \mathbf{r})|^2\, \mathcal{N}^{heavy}(\mathbf{r}, x)$$

results



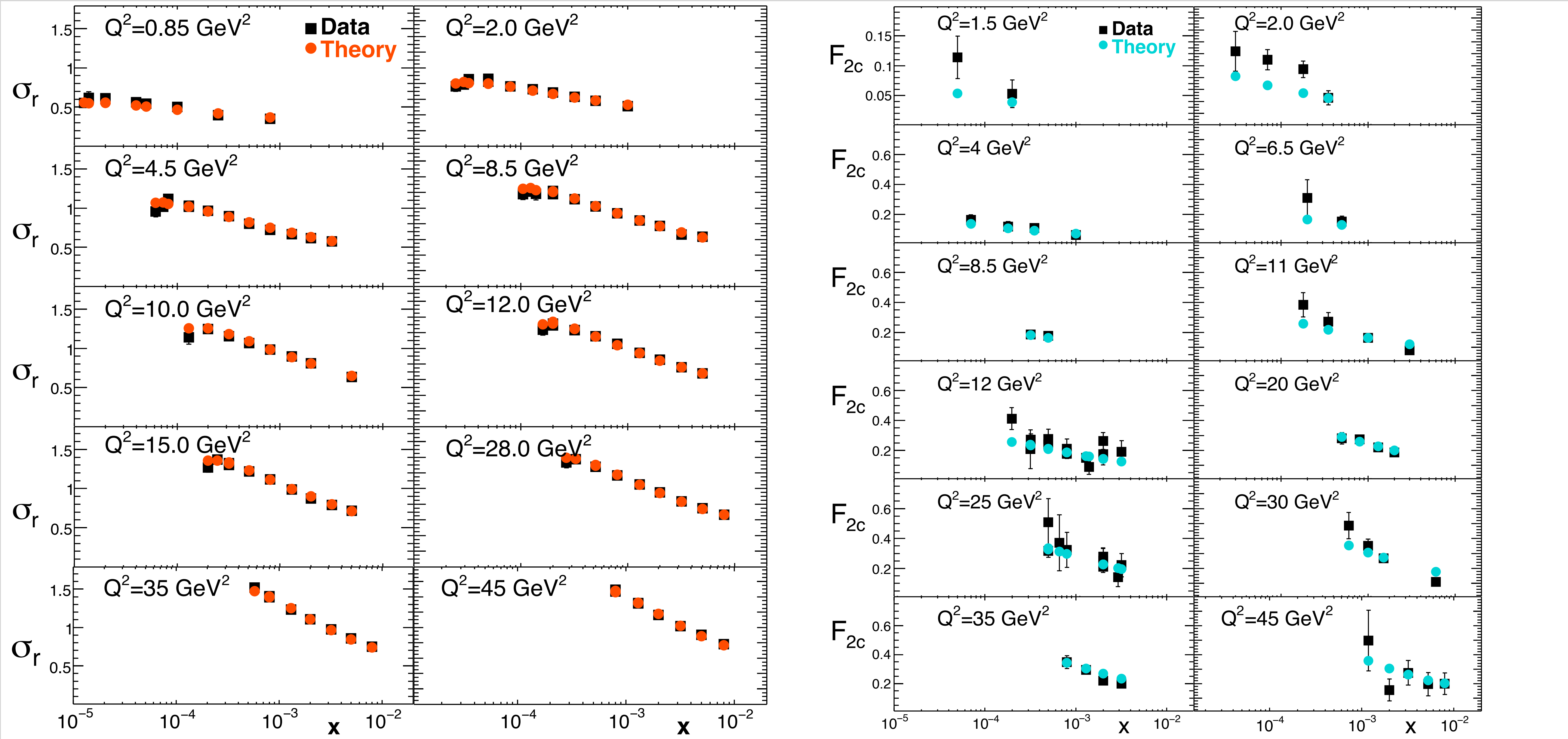
- fully consistent with results obtained with 'old' HERA data [AAMS]
  - very mild change of parameters
  - tension with high Q^2 data [and this is good] :: not shown
- fitted initial conditions are numerically 'essentially identical'
  - physically meaningful

fits to 'old' HERA data

Initial condition	σ_0 (mb)	Q^2_{s0} (GeV^2)	C^2	γ	χ^2/d.o.f.
GBW	31.59	0.24	5.3	1 (fixed)	916.3/844=1.086
MV	32.77	0.15	6.5	1.13	906.0/843=1.075

	fit	$\frac{\chi^2}{d.o.f}$	$Q^2_{S,0}$	σ_0	γ	C	m_l^2
	GBW						
a	$\alpha_f = 0.7$	1.226	0.241	32.357	0.971	2.46	fixed
a'	$\alpha_f = 0.7\ (\Lambda_{m_\tau})$	1.235	0.240	32.569	0.959	2.507	fixed
b	$\alpha_f = 0.7$	1.264	0.2633	30.325	0.968	2.246	1.74E-2
c	$\alpha_f = 1$	1.279	0.254	31.906	0.981	2.378	fixed
c'	$\alpha_f = 1\ (\Lambda_{m_\tau})$	1.244	0.2329	33.608	0.9612	2.451	fixed
d	$\alpha_f = 1$	1.248	0.239	33.761	0.980	2.656	2.212E-2
	MV						
e	$\alpha_f = 0.7$	1.171	0.165	32.895	1.135	2.52	fixed
f	$\alpha_f = 0.7$	1.161	0.164	32.324	1.123	2.48	1.823E-2
g	$\alpha_f = 1$	1.140	0.1557	33.696	1.113	2.56	fixed
h	$\alpha_f = 1$	1.117	0.1597	33.105	1.118	2.47	1.845E-2
h'	$\alpha_f = 1\ (\Lambda_{m_\tau})$	1.104	0.168	30.265	1.119	1.715	1.463E-2

results [light + heavy]



- excellent global σ_{red} / F_2 + F_2^c description χ^2/#dof ~ 1.3

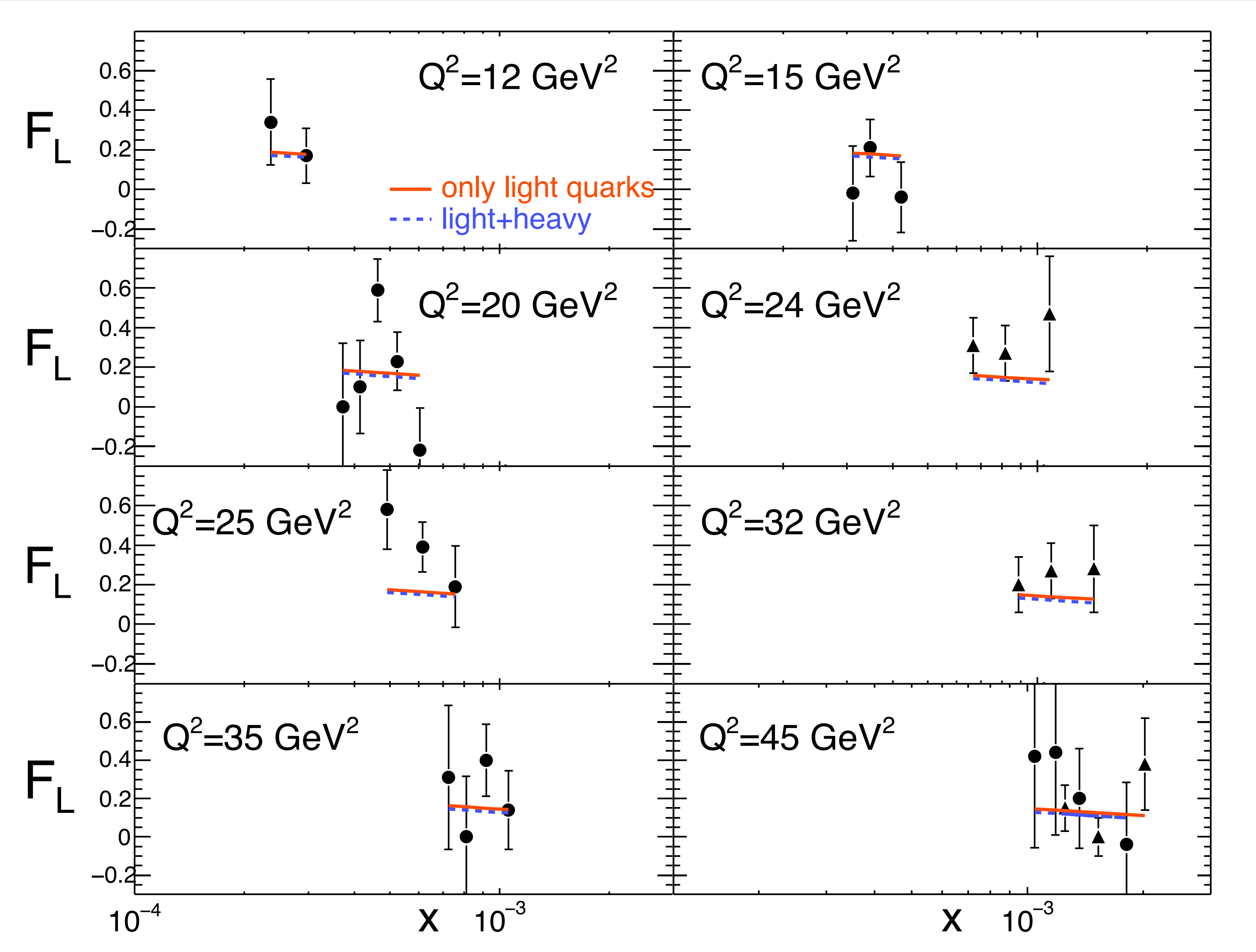
results [light + heavy]

	fit	$\frac{\chi^2}{d.o.f}$	$Q^2_{S,0}$	σ_0	γ	$Q^2_{S,0,c}$	$\sigma_{0,c}$	γ_c	C	m_l^2
	GBW									
a	$\alpha_f = 0.7$	1.269	0.2294	36.953	1.259	0.2289	18.962	0.881	4.363	fixed
a’	$\alpha_f = 0.7$ (Λ_{m_τ})	1.302	0.2341	36.362	1.241	0.2249	20.380	0.919	7.858	fixed
b	$\alpha_f = 0.7$	1.231	0.2386	35.465	1.263	0.2329	18.430	0.883	3.902	1.458E-2
c	$\alpha_f = 1$	1.356	0.2373	35.861	1.270	0.2360	13.717	0.789	2.442	fixed
d	$\alpha_f = 1$	1.221	0.2295	35.037	1.195	0.2274	20.262	0.924	3.725	1.351E-2
	MV									
e	$\alpha_f = 0.7$	1.395	0.1673	36.032	1.355	0.1650	18.740	1.099	3.813	fixed
f	$\alpha_f = 0.7$	1.244	0.1687	35.449	1.369	0.1417	19.066	1.035	4.079	1.445E-2
g	$\alpha_f = 1$	1.325	0.1481	40.216	1.362	0.1378	13.577	0.914	4.850	fixed
h	$\alpha_f = 1$	1.298	0.156	37.003	1.319	0.147	19.774	1.074	4.355	1.692E-2

- $\sigma_0^{charm} < \sigma_0^{light}$
  - also charm has a gentler fall-off in i.c [$\gamma^{charm} < \gamma^{light}$]

- F_2^c constrained

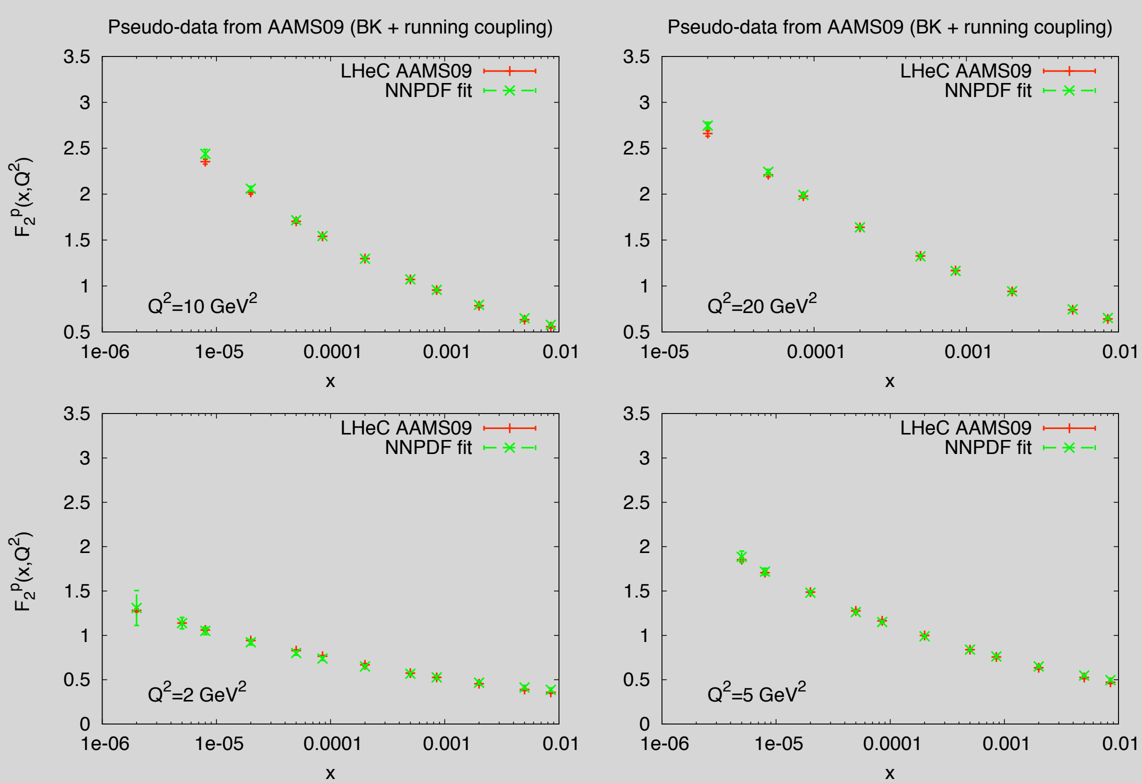
◌ efele

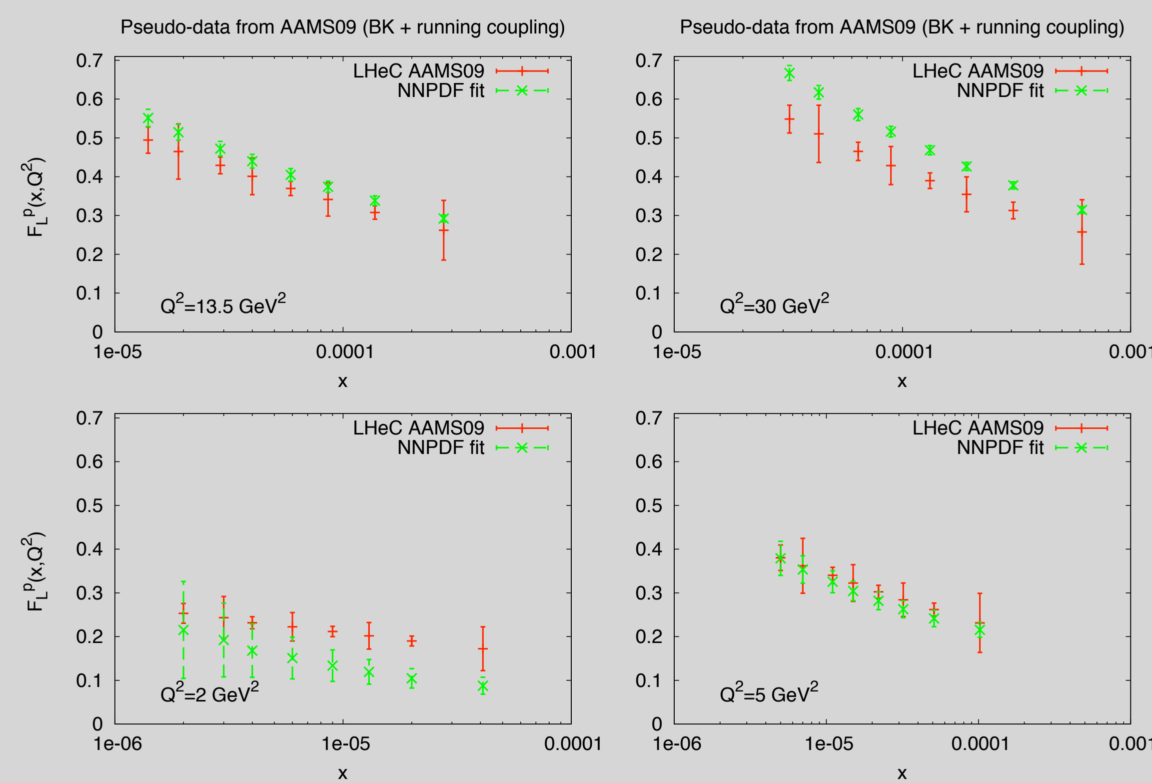


- H1 and ZEUS direct measurements
  - not included in the fit [independent test]

↺ vs. DGLAP

- AAMS 1.0 F_2 and F_L cannot be fitted by NLO-DGLAP [Rojo, LHeC working group]





- i.e., pseudo-data (for LHeC) generated from AAMS is inconsistent with NLO-DGLAP
- differences cannot be absorbed into initial condition [in which there are ~200 parameters]

Balitsky-Kovchegov (BK) equation

$$\frac{\partial N(r,Y)}{\partial Y} = \int \frac{d^2z}{2\pi} K(\vec{r},\vec{r}_1,\vec{r}_2)\Big[N(r_1,Y)+N(r_2,Y)-N(r,Y)-N(r_1,Y)N(r_2,Y)\Big]$$

- closed evolution for scattering probability
- is unitary [scattering probability cannot grow above 1]
- large Nc limit (mean field) of infinite hierarchy [equivalent to target evolution]

$$N(x,y) = \langle n(x,y)\rangle_{\text{target}} = 1/N_c\langle 1 - V(x)V^\dagger(y)\rangle_{\text{target}}$$

$$\frac{\partial}{\partial Y}\langle n(x,y)\rangle = \cdots \langle n(x,y)n(y,z)\rangle$$

$$\frac{\partial}{\partial Y}\langle n(x,y)n(y,z)\rangle = \cdots$$

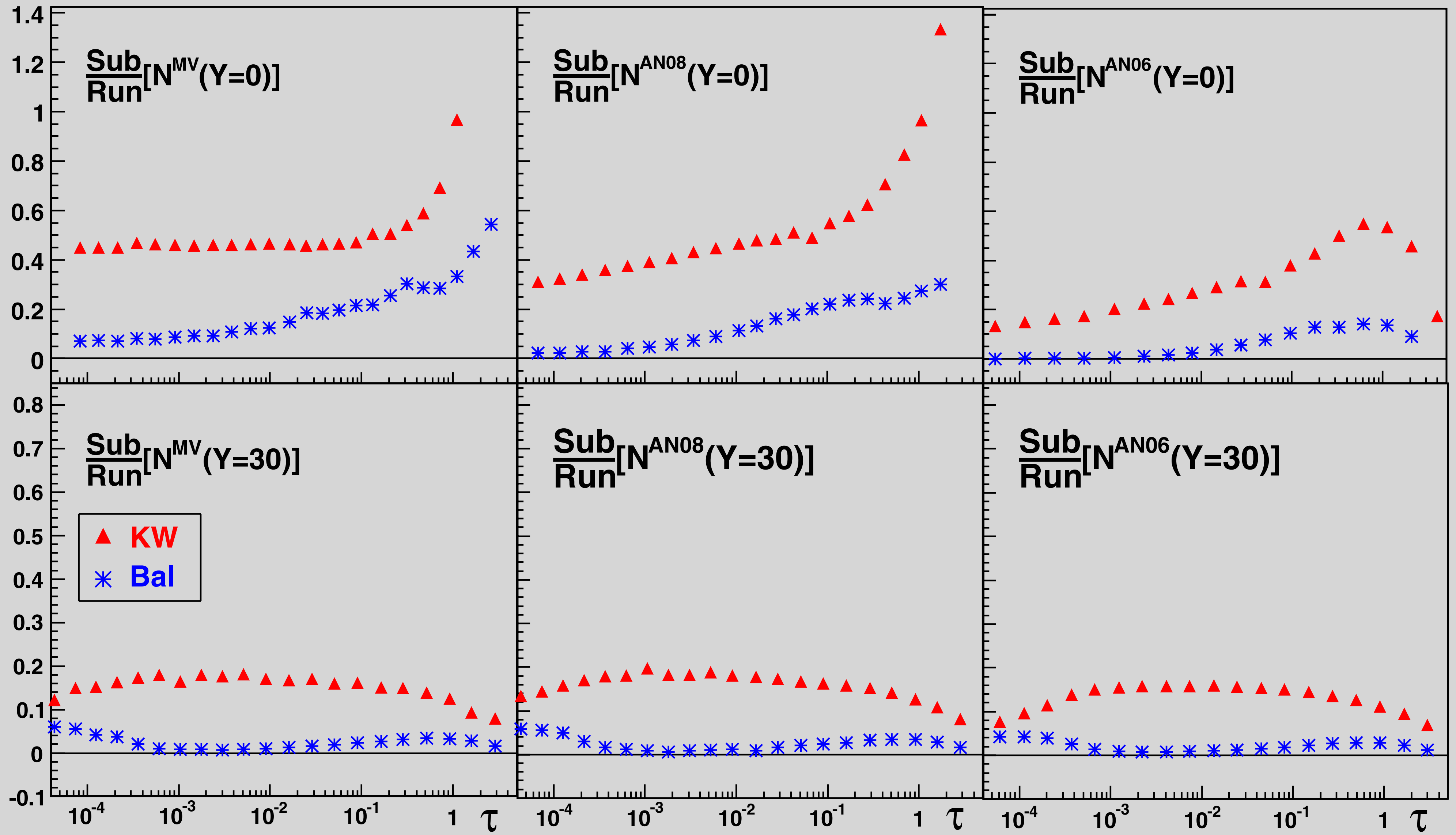
$$\cdots$$

large N_c limit

$$\langle n(x,y)n(y,z)\rangle = \langle n(x,y)\rangle\langle n(y,z)\rangle + o(1/N_c^2) = N(x,y)N(y,z) + o(1/N_c^2)$$

- no other consistent truncation possible
- numerical results for full hierarchy deviate 10% (0.1%) at most from full B-JIMWLK

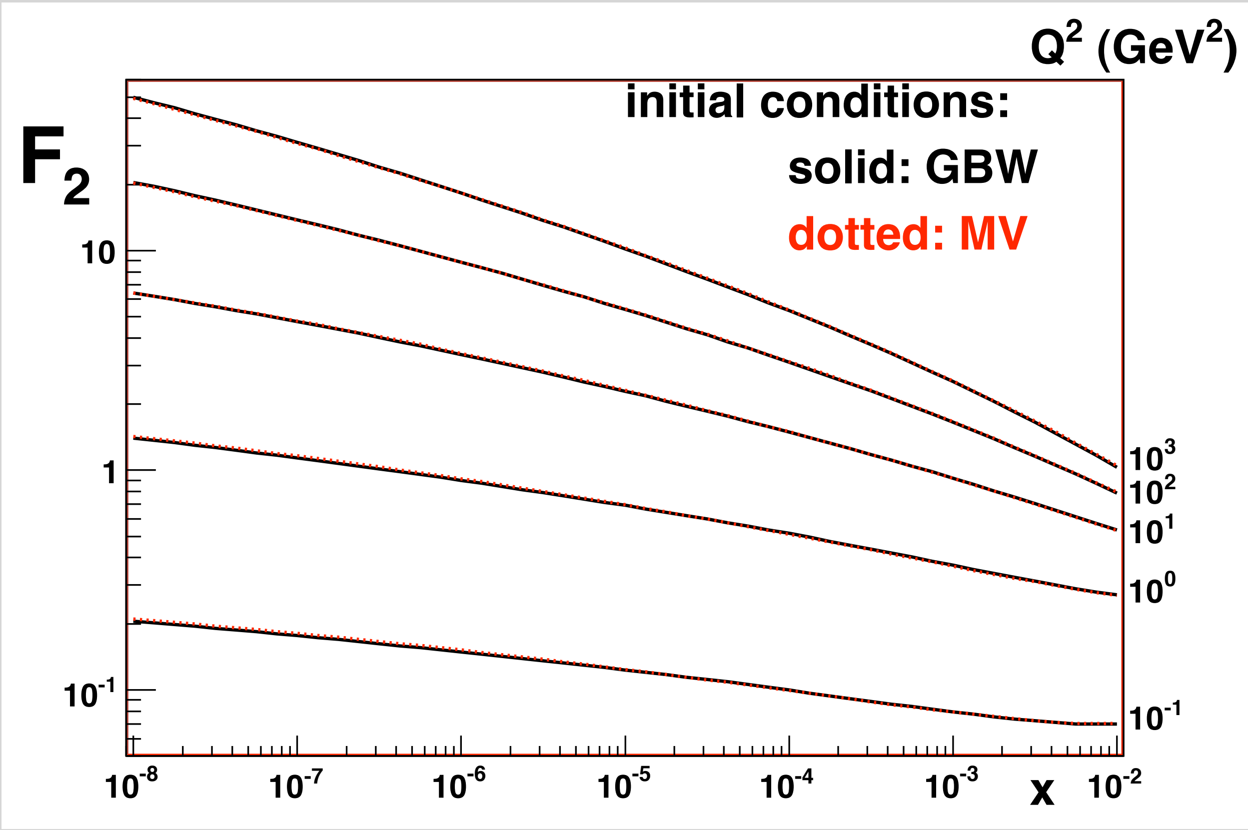
↻ on why B is B'

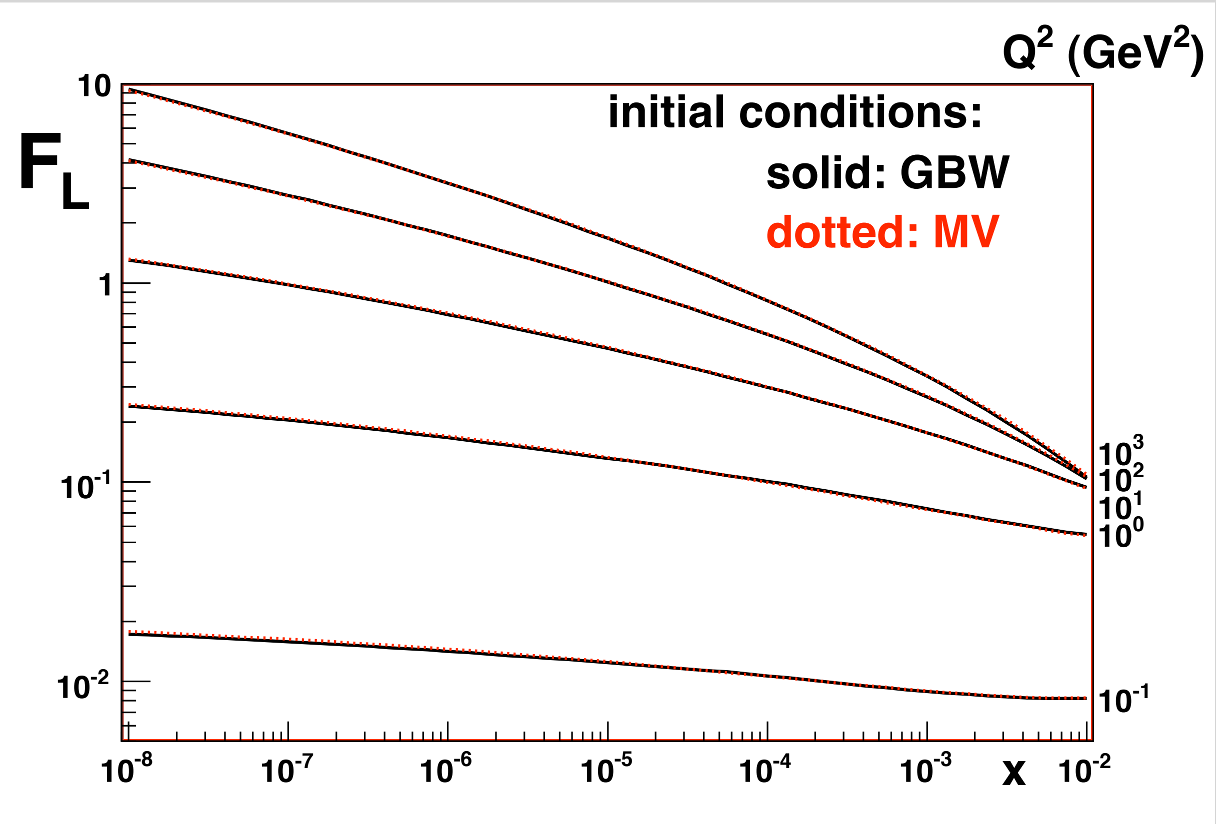


[Albacete, Kovchegov]

$$\frac{\partial \mathcal{N}(r,Y)}{\partial Y} = \mathcal{R}[\mathcal{N}] - \mathcal{S}[\mathcal{N}]$$

predictions

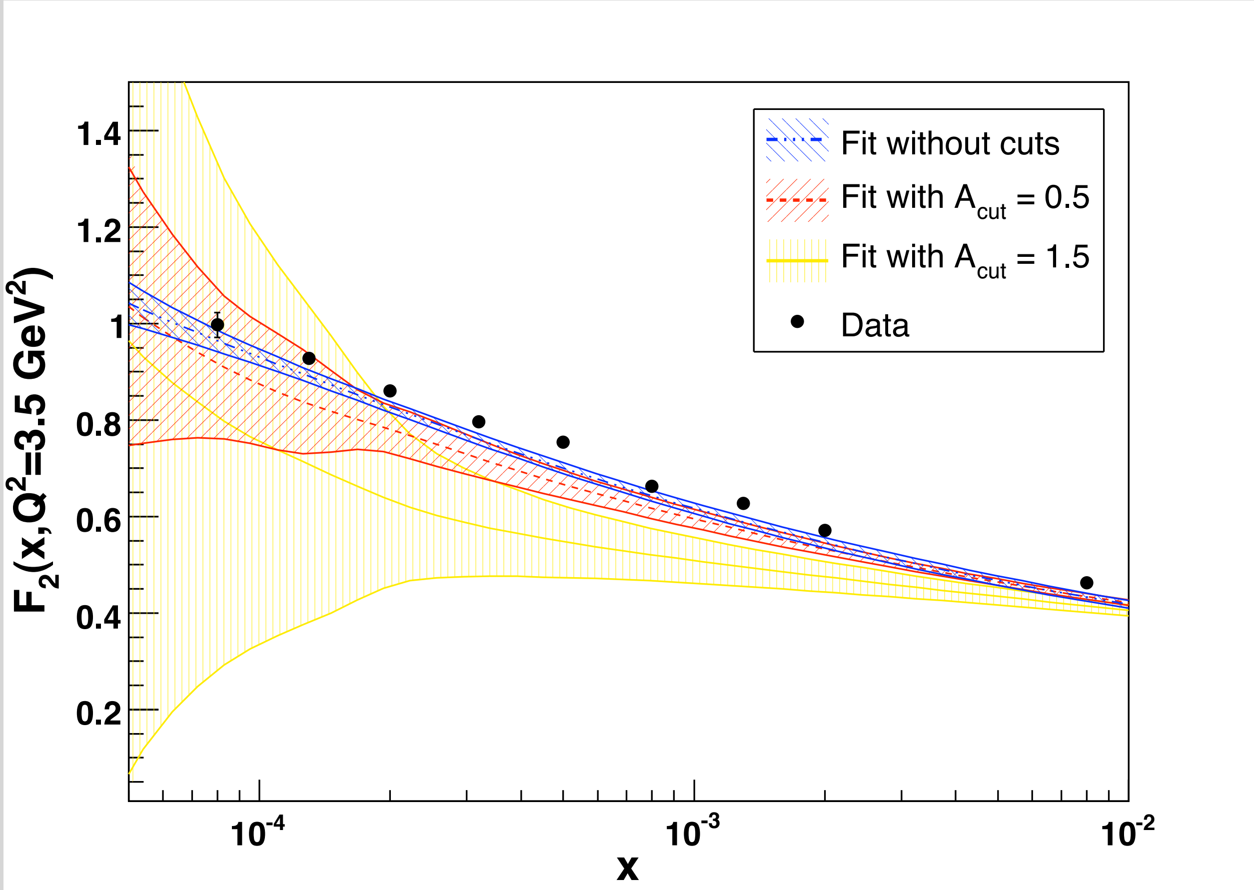




- F_2 and F_L extrapolated to LHeC and UHECR kinematical conditions
  - near independence on [tested] initial conditions
  - first principle approach allows for credible extrapolation
    - ↪ ‘all’ relevant physics included

DGLAP i.c. independent statements (ii)

Caola, Forte, Rojo



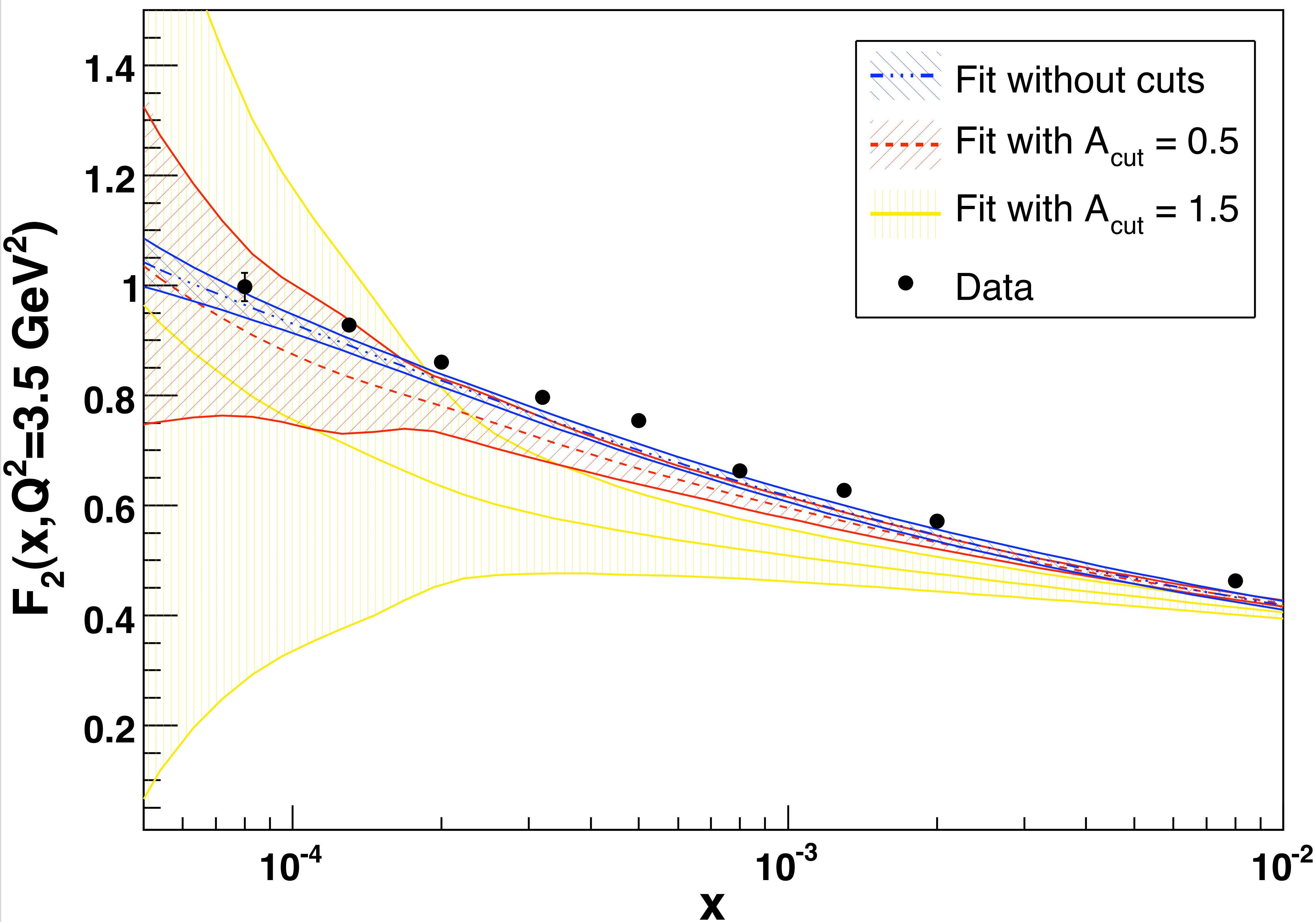
$$Q^2 \geq A_{\rm cut} \cdot x^{\lambda}$$

$$d_{\rm stat}(x,Q^2) \equiv \frac{F_{\rm data} - F_{\rm fit}}{\sqrt{\sigma^2_{\rm data} + \sigma^2_{\rm fit}}}$$

- small, but systematic, deviations from data found for F_2 [of all places...] HERA data
  - not NNLO [goes in the wrong direction]
  - not a mass effect [too small] :: should be settled by NNPDF 2.1
  - solvable by BFKL resummation in DGLAP kernel [interim fix...]

- predictive power rapidly degrades with 1/x

DGLAP dev



Fit without cuts
Fit with A_{cut} = 0.5
Fit with A_{cut} = 1.5
Data
$F_2(x,Q^2=3.5\ GeV^2)$
1.4
1.2
1
0.8
0.6
0.4
0.2
10^{-4}
10^{-3}
10^{-2}
x

AAMQs collaboration

installation [Spring 2009]

- 4 institutes
- 5 scientists
- ~200 cpu cores
- installation and commissioning in 2009
- online from early 2010

http://www-fp.usc.es/phenom/aamqs/aamqs.html