Recent positivity constraints for spin observables and parton distributions

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- 3 - General positivity bounds in particle inclusive production
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Positivity constraints for spin observables

- Spin observables allow a deeper understanding of the underlying dynamics
- For example, spin sector of pQCD must be checked

Positivity constraints for spin observables

- Spin observables allow a deeper understanding of the underlying dynamics
- For example, spin sector of pQCD must be checked
- Some typical situations:

  - one or two observables are measured first

  Question: which new observable will provide the best improvement of knowledge ?

  - two or three observables are measured first

  Question: is it possible to test if they are compatible?

Xavier ARTRU, Mokhtar ELCHIKH, Jean-Marc RICHARD, J. S. and Oleg TERYAEV
Physics Reports, 470,1-92 (2009)

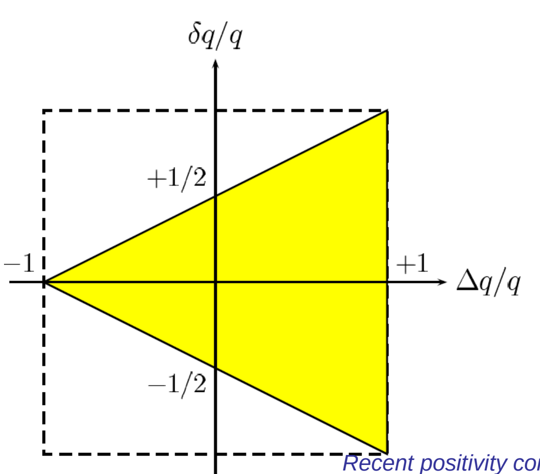
Quark Transversity Distribution $\delta q(x, Q^2)$

It was first mentioned by Ralston and Soper in 1979, in $pp \to \mu^+\mu^- X$ with transversely polarized protons, but forgotten until 1990, where it was realized that it completes the description of the quark distribution in a nucleon as a density matrix

$$\mathcal{Q}(x,Q^2) = q(x,Q^2) I \otimes I + \Delta q(x,Q^2)\sigma_3 \otimes \sigma_3 + \delta q(x,Q^2)(\sigma_+ \otimes \sigma_- + \sigma_- \otimes \sigma_+)$$

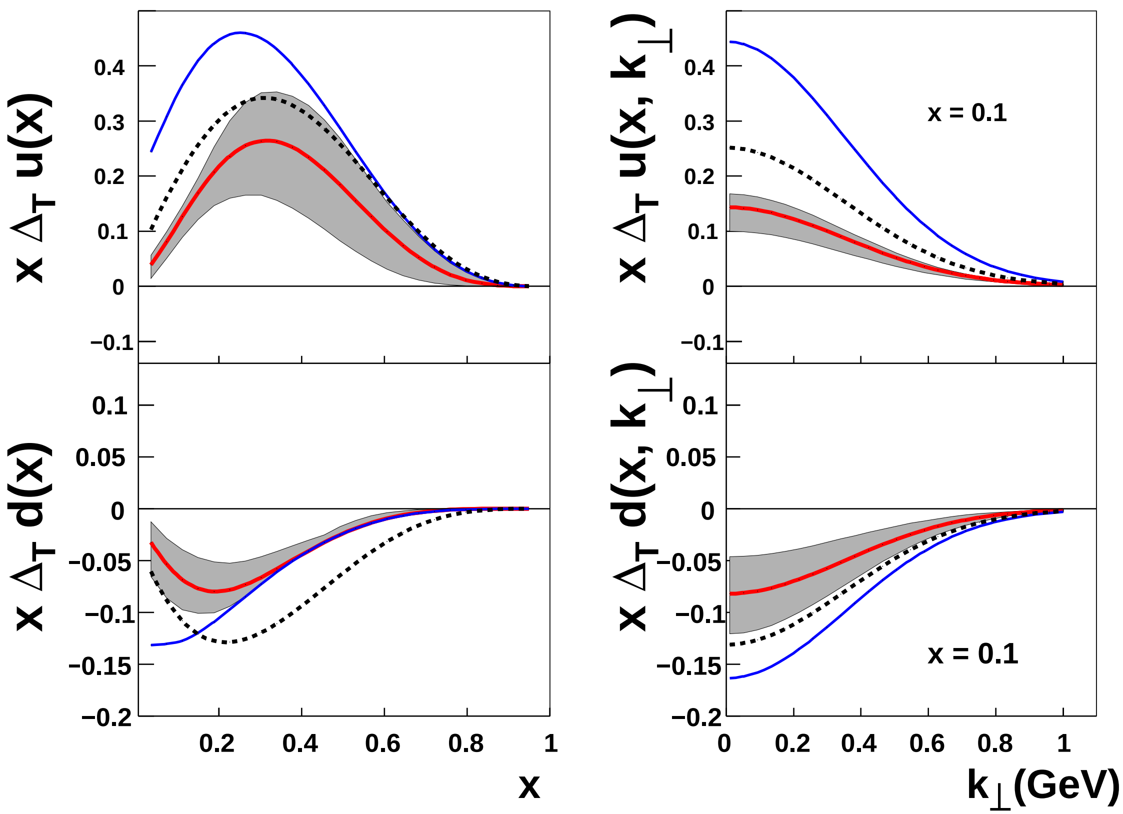
This new distribution function $\delta q(x, Q^2)$ is chiral odd, leading twist and decouples from DIS. Only recently, it has been extracted indirectly, for the first time.
There is a positivity bound (J.S., PRL 74,1292,1995) survives up to NLO corrections

$$q(x,Q^2) + \Delta q(x,Q^2) \geq 2|\delta q(x,Q^2)|$$



Current status on $\delta q(x, Q^2)$, Anselmino et al, arXiv: 0812.4366

Global analysis combining Collins effect from COMPASS and Collins FF from BELLE



General positivity bounds in particle inclusive production

Zhong-Bo Kang, J.S., PRD83 114020 (2011)

To start, we consider the inclusive reaction of the type $A(\text{spin 1/2}) + B(\text{spin 1/2}) \to C + X$, where both initial spin $1/2$ particles can be in any possible directions and no polarization is observed in the final state. The spin-dependent corresponding cross section $\sigma\left(P_a, P_b\right) = \mathrm{Tr}\left(M\rho\right)$, can be defined through the 4×4 cross section matrix M and the spin density matrix ρ, where P_a, P_b are the spin unit vectors of A and B, $\rho = \rho_a \otimes \rho_b$ is the spin density matrix with $\rho_a = (I_2 + P_a \cdot \vec{\sigma}_a)/2$, and similar for ρ_b. Here I_2 is the 2×2 unit matrix, and $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ stands for the 2×2 Pauli matrices.

The 4×4 cross section matrix M can be parametrized as

$$M=\sigma_0\left[I_4 + A_{aN}\sigma_{ay}\otimes I_2 + A_{bN}I_2\otimes\sigma_{by} + A_{NN}\sigma_{ay}\otimes\sigma_{by} + A_{LL}\sigma_{az}\otimes\sigma_{bz}\right.$$
$$\left.+A_{SS}\sigma_{ax}\otimes\sigma_{bx} + A_{LS}\sigma_{az}\otimes\sigma_{bx} + A_{SL}\sigma_{ax}\otimes\sigma_{bz}\right]+$$
$$\sigma_0\left[A_{aL}\sigma_{az}\otimes I_2 + A_{bL}I_2\otimes\sigma_{bz} + A_{aS}\sigma_{ax}\otimes I_2 + A_{bS}I_2\otimes\sigma_{bx}\right.$$
$$\left.+A_{LN}\sigma_{az}\otimes\sigma_{by} + A_{NL}\sigma_{ay}\otimes\sigma_{bz} + A_{NS}\sigma_{ay}\otimes\sigma_{bx} + A_{SN}\sigma_{ax}\otimes\sigma_{by}\right].$$

In blue the 8 parity-conserving asymmetries and in red the 8 parity-violating asymmetries

General positivity bounds in particle inclusive production

Zhong-Bo Kang, J.S., PRD 83,114020 (2011)

The crucial point is that M is a Hermitian and positive matrix and in order to derive the positivity conditions. In the transverse basis where σ_y is diagonal, we have found that the diagonal matrix elements M_{ii} are given by
$M_{11} = (1 + A_{NN}) + (A_{aN} + A_{bN})$, $M_{22} = (1 - A_{NN}) + (A_{aN} - A_{bN})$,
$M_{33} = (1 - A_{NN}) - (A_{aN} - A_{bN})$, $M_{44} = (1 + A_{NN}) - (A_{aN} + A_{bN})$.
Since one of the necessary conditions for a Hermitian matrix to be positive definite is that all the diagonal matrix elements has to be positive $M_{ii} \geq 0$, we thus derive

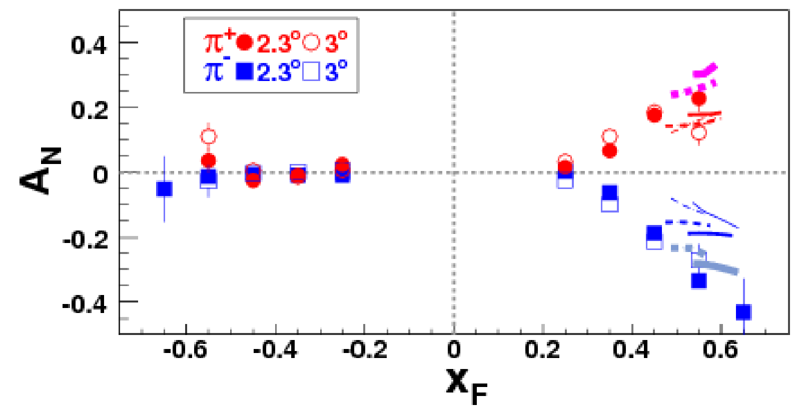
$$1 \pm A_{NN} \geq |A_{aN} \pm A_{bN}| \, .$$

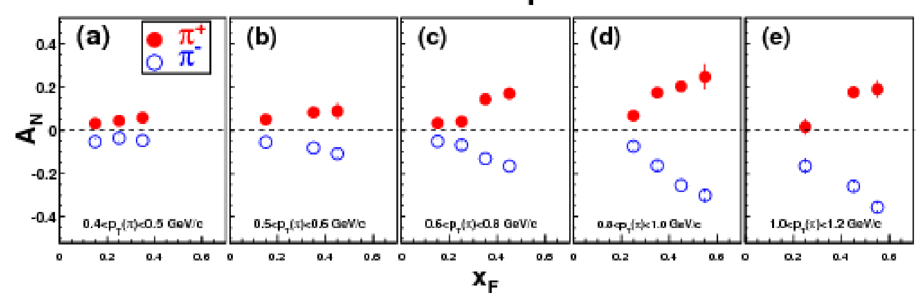
which is valid in full generality, for both parity-conserving and parity-violating processes. Back to the case for $p^\uparrow + p^\uparrow \to C + X$ where the initial particles are identical, we have $A_{aN}(y) = -A_{bN}(-y)$. Using this relation , one obtains,

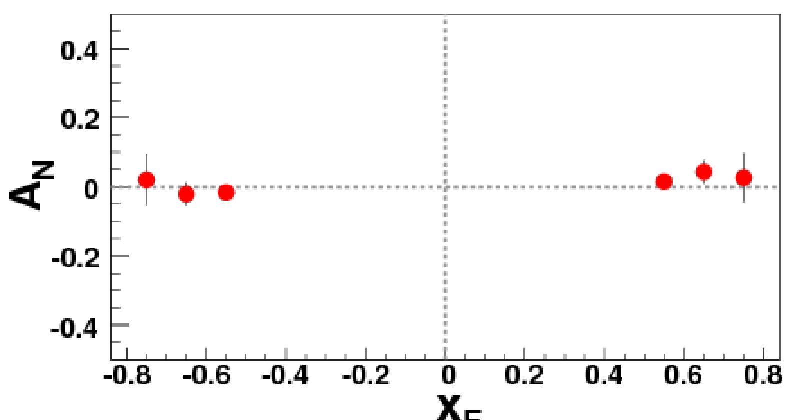
$$1 \pm A_{NN}(y) \geq |A_N(y) \pm A_N(-y)| \, .$$

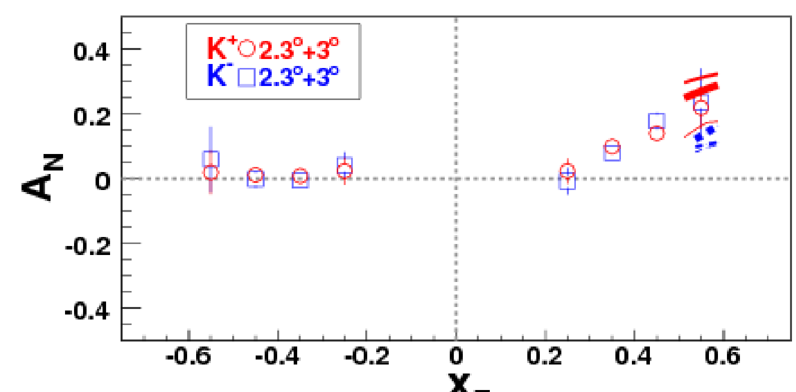
This is an interesting result which, can be used, in principle, with available data on A_N for $\pi^\pm$, $K^\pm$, π^0, η production, to put some non trivial contraints on $A_{NN}(y)$

BRAHMS results at BNL on A_N



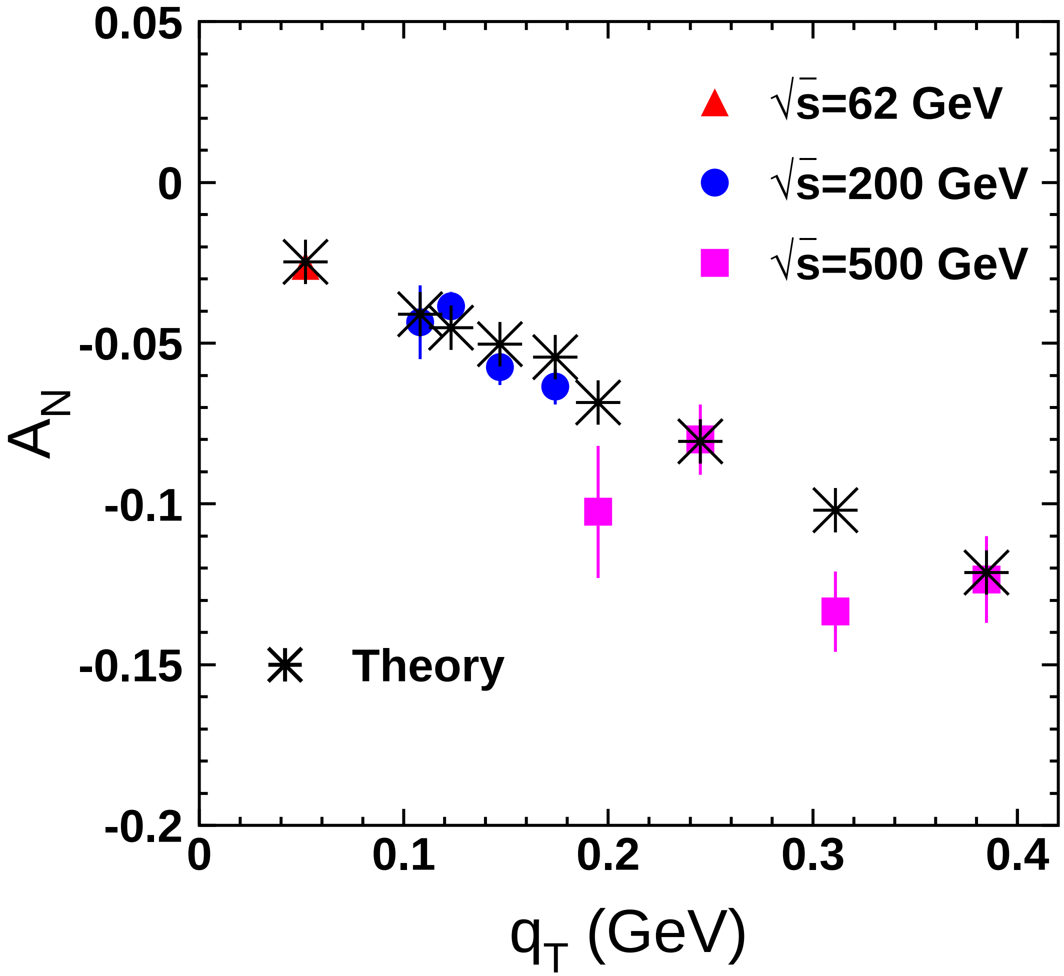






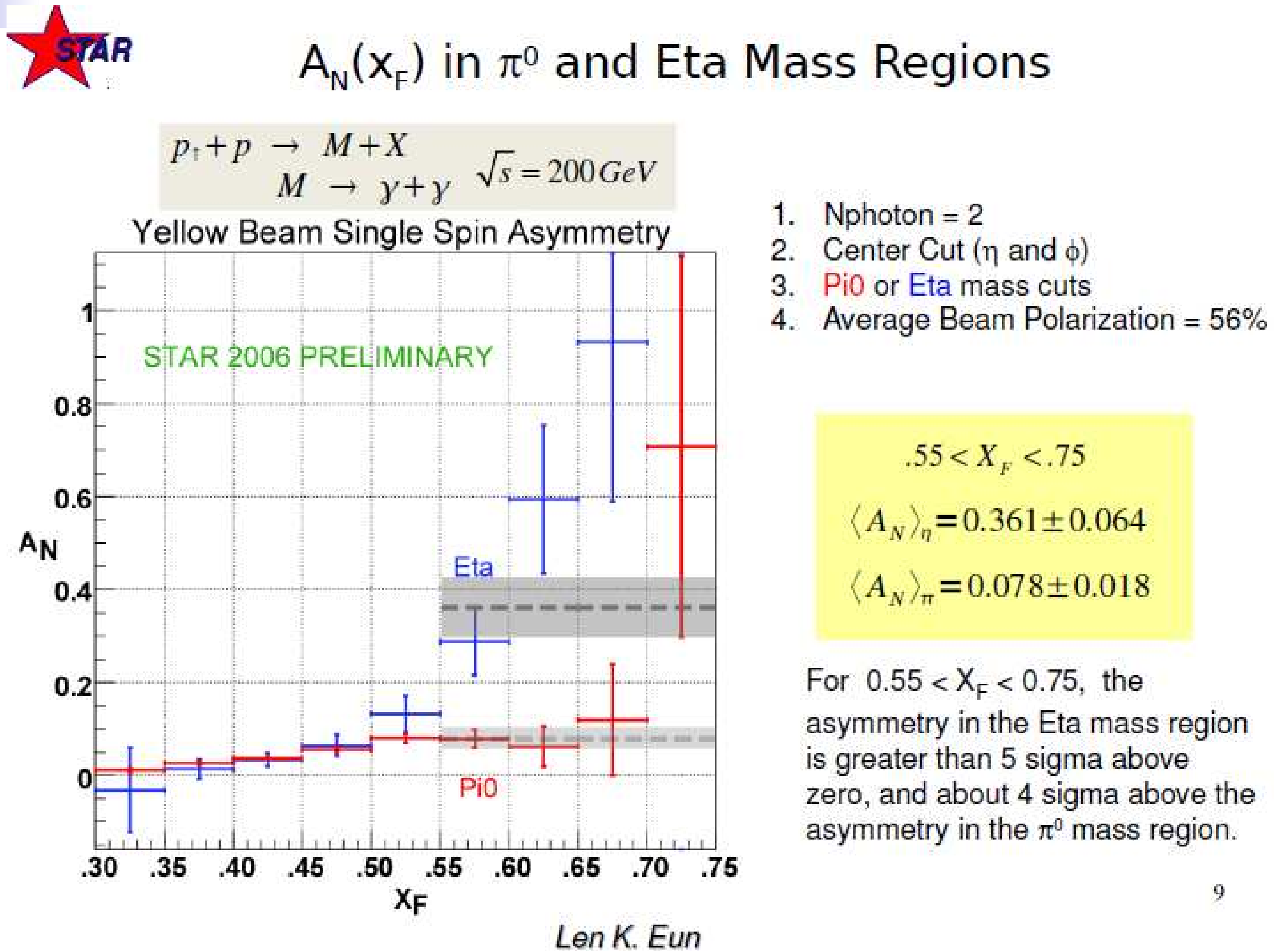
PHENIX results at BNL on A_N for $p^\uparrow p \to n + X$

(recently explained arXiv:1109.2500, to be published Phys. Rev. D)



For very forward neutrons big effect from interference π - a_1 Regge exchanges

STAR results on A_N at BNL

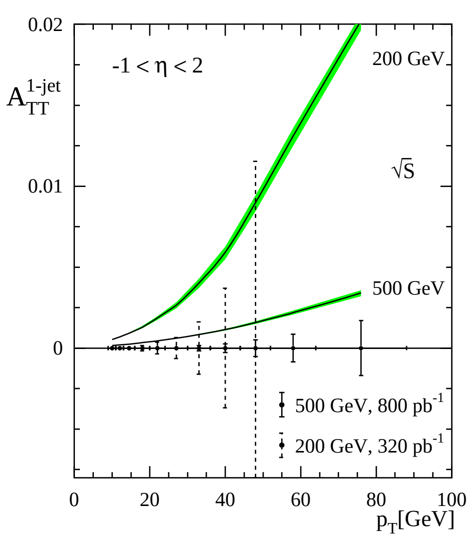


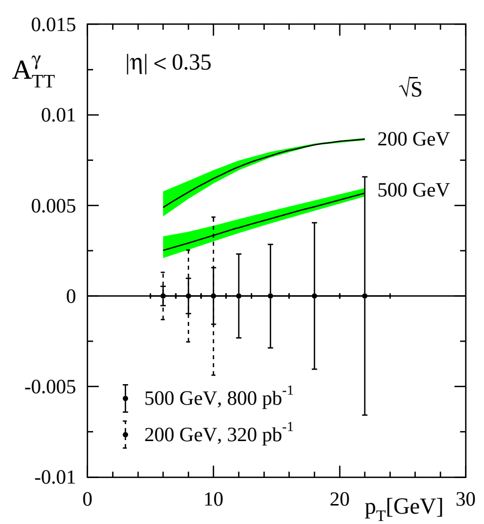
Upper bounds on A_{NN} in Jet and prompt photon productions

Theoretically A_{NN} is expected to be small. Its sensitivity to δq is only in the high p_T region, dominated by $qq \to qq$, because no transversity for the gluons. Use positivity bound on transversity. Expect $|A_{NN}| << |A_{LL}|$

It is important to verify this theoretical expectation

(J.S., M.Stratmann and W.Vogelsang, Phys. Rev. D65, 114024 (2002))





A_{NN} for Z production

$$A_{NN}(Z) = \frac{\sum_q (b_q^2 - a_q^2)\delta q(x_1, M^2)\delta\bar{q}(x_2, M^2) + (1 \leftrightarrow 2)}{\sum_q (b_q^2 + a_q^2) q(x_1, M^2)\bar{q}(x_2, M^2) + (1 \leftrightarrow 2)}$$

For W production, since W coupling is V-A, i.e. $a_q = b_q$, we expect $A_{NN}(W) = 0$

This must be checked at BNL

However single spin asymmetry $A_N(W^\pm)$ allows the flavor separation of Sivers functions for u ,$\bar{u}$, d and $\bar{d}$. (I. Schmidt and J.S., Phys. Lett. B 563, 179 (2003))

Actually, it is possible to use positivity to go one step forward

Positivity bounds for Sivers functions

Zhong-Bo Kang, J.S., Phys. Lett. B695, 275 (2011)

Now let us study the implication of $1 \pm A_{NN}(y) \geq |A_N(y) \pm A_N(-y)|$, in a parity-violating process $p^\uparrow + p^\uparrow \to W^\pm + X$. Thus we will assume $A_{NN} \approx 0$, and therefore for $y = 0$, the bound reduces to

$$1 \geq 2|A_N(y=0)|\,,$$

to be compared with the usual trivial bound $1 \geq |A_N(y=0)|$.
The TMD quark distribution in a transversely polarized hadron of spin $\vec{S}$, can be expanded as

$$f_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) \equiv f_{q/h}(x, k_\perp) + \frac{1}{2}\Delta^N f_{q/h^\uparrow}(x, k_\perp)\, \vec{S} \cdot \left(\hat{p} \times \hat{\mathbf{k}}_\perp\right)\ ,$$

where $\hat{p}$ and $\hat{\mathbf{k}}_\perp$ are the unit vectors of $\vec{p}$ and $\mathbf{k}_\perp$, respectively. $f_{q/h}(x, k_\perp)$ is the spin-averaged TMD distribution, and $\Delta^N f_{q/h^\uparrow}(x, k_\perp)$ is the Sivers function. There is a trivial positivity bound for the Sivers functions which reads

$$|\Delta^N f_{q/h^\uparrow}(x, k_\perp)| \leq 2 f_{q/h}(x, k_\perp).$$

Positivity bounds for Sivers functions

Zhong-Bo Kang, J.S., Phys. Lett. B695, 275 (2011)

A_N, is expressed by an integral in terms of the Sivers functions, and it is usually assumed in the phenomenological studies that the x and $k_\perp$ dependence of the TMD distributions can be further factorized as follows

$$\begin{aligned} f_{q/h}(x, k_\perp) &= f_q(x) g(k_\perp), \\ \Delta^N f_{q/h^\uparrow}(x, k_\perp) &= \Delta^N f_{q/h^\uparrow}(x)\, h(k_\perp) g(k_\perp). \end{aligned}$$

For the $k_\perp$ dependence, a Gaussian ansatz is usually introduced , which has the form

$$g(k_\perp) = \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2/\langle k_\perp^2 \rangle}$$

$$h(k_\perp) = \sqrt{2e}\, \frac{k_\perp}{M_1} e^{-k_\perp^2/M_1^2}$$

This choice of $h(k_\perp)$ satisfies $h(k_\perp) \leq 1$. Thus the previous positivity bound implies

$$\left| \Delta^N f_{q/h^\uparrow}(x) \right| \leq 2 f_q(x).$$

Positivity bounds for Sivers functions

Zhong-Bo Kang, J.S., Phys. Lett. B695, 275 (2011)

Then one can carry out the integration analytically in A_N to obtain

$$A_N(y=0) \quad = \quad H(q_\perp)\frac{\sum_{ab}|V_{ab}|^2\Delta^N f_{a/p^\uparrow}(x)\, f_b(x)}{\sum_{ab}|V_{ab}|^2\, f_a(x)\, f_b(x)},$$

where $x = M_W/\sqrt{s}$ for $y=0$ and $H(q_\perp)$ is given by

$$H(q_\perp) = \vec{S}_\perp \cdot (\hat{p}\times \mathbf{q}_\perp)\, \frac{\sqrt{2e}}{M_1}\, \frac{\langle k_s^2\rangle^2}{[\langle k_\perp^2\rangle + \langle k_s^2\rangle]^2}\, e^{-\left[\frac{\langle k_\perp^2\rangle - \langle k_s^2\rangle}{\langle k_\perp^2\rangle + \langle k_s^2\rangle}\right]\frac{\mathbf{q}_\perp^2}{2\langle k_\perp^2\rangle}},$$

where $\langle k_s^2\rangle = M_1^2\, \langle k_\perp^2\rangle/[M_1^2 + \langle k_\perp^2\rangle]$. The $q_\perp$-dependent function $H(q_\perp)$ reaches its maximum $H(q_\perp)_{\rm max}$ when $q_\perp^2 = \langle k_\perp^2\rangle(\langle k_\perp^2\rangle + \langle k_s^2\rangle)/(\langle k_\perp^2\rangle - \langle k_s^2\rangle)$, with $H(q_\perp)_{\rm max}$ given by

$$H(q_\perp)_{\rm max} = \frac{\langle k_s^2\rangle^2}{[\langle k_\perp^2\rangle + \langle k_s^2\rangle]^2}\left[\frac{2\langle k_\perp^2\rangle}{M_1^2}\, \frac{\langle k_\perp^2\rangle + \langle k_s^2\rangle}{\langle k_\perp^2\rangle - \langle k_s^2\rangle}\right]^{\frac{1}{2}}.$$

Positivity bounds for Sivers functions

Zhong-Bo Kang, J.S., Phys. Lett. B695, 275 (2011)

Using the fact that $1 \geq 2|A_N(y=0)|$ for any $q_\perp$ and $\sqrt{s}$, we thus derive a new bound for the Sivers functions

$$\frac{\left|\sum_{ab}|V_{ab}|^2 \Delta^N f_{a/A^\uparrow}(x)\, f_b(x)\right|}{\sum_{ab}|V_{ab}|^2\, f_a(x)\, f_b(x)} \leq \frac{1/2}{H(q_\perp)_{\max}}.$$

For W^+, it can be simplified as $\left|\frac{\Delta^N u(x)}{u(x)} + \frac{\Delta^N \bar{d}(x)}{\bar{d}(x)}\right| \leq \frac{1}{H(q_\perp)_{\max}}$.

For W^-, one obtains the following constraint $\left|\frac{\Delta^N d(x)}{d(x)} + \frac{\Delta^N \bar{u}(x)}{\bar{u}(x)}\right| \leq \frac{1}{H(q_\perp)_{\max}}$.

For $\langle k_\perp^2 \rangle = 0.25\ \mathrm{GeV}^2$ and $M_1^2 = 0.34^{+0.30}_{-0.16}\ \mathrm{GeV}^2$, $2.6 < 1/H(q_\perp)_{\max} < 4.75$
To be compared with the trivial bound which gives the number 4 on the r.h.s.
Not a spectacular result but these new bounds are useful for consistency checks

Can be also applied to the Sivers gluon function accessible in direct photon production
I. Schmidt, J.S. and J.J. Yang, Phys. Lett. B612,258 (2005)

Positivity bounds involving parity-violating asymmetries

Zhong-Bo Kang, J.S., Phys. Rev. D83, 114020 (2011)

In the helicity basis it is easy to obtain the explicit form of M and now from $M_{ii} \geq 0$, we have $1 \pm A_{LL} > |A_{aL} \pm A_{bL}|$. It is important to note that for identical initial particles scattering, one has $A_{aL}(y) = A_{bL}(-y)$, so one gets

$$1 \pm A_{LL}(y) > |A_L(y) \pm A_L(-y)|.$$

These bounds should be tested in RHIC experiments for $W^\pm$ or Z^0 production in longitudinal pp collisions, $\vec{p}\vec{p} \to W^\pm/Z^0 + X$. In perturbative QCD formalism, at leading-order and restricting to only up and down quarks, one has the following simple expressions for the single spin asymmetries

$$\begin{aligned}
A_L^{W^+}(y) &= \frac{-\Delta u(x_a)\bar{d}(x_b) + \Delta \bar{d}(x_a)u(x_b)}{u(x_a)\bar{d}(x_b) + \bar{d}(x_a)u(x_b)}\,, \\
A_L^{W^-}(y) &= \frac{-\Delta d(x_a)\bar{u}(x_b) + \Delta \bar{u}(x_a)d(x_b)}{d(x_a)\bar{u}(x_b) + \bar{u}(x_a)d(x_b)}\,, \\
A_L^{Z^0}(y) &= \frac{\sum_q (2v_q a_q)\left[-\Delta q(x_a)\bar{q}(x_b) + \Delta \bar{q}(x_a)q(x_b)\right]}{\sum_q (v_q^2 + a_q^2)\left[q(x_a)\bar{q}(x_b) + \bar{q}(x_a)q(x_b)\right]}\,,
\end{aligned}$$

Positivity bounds involving parity-violating asymmetries

Zhong-Bo Kang, J.S., Phys. Rev. D83, 114020 (2011)

and for the double spin asymmetries

$$A_{LL}^{W^+}(y) = -\frac{\Delta u(x_a)\Delta\bar{d}(x_b) + \Delta\bar{d}(x_a)\Delta u(x_b)}{u(x_a)\bar{d}(x_b) + \bar{d}(x_a)u(x_b)}\,,$$

$$A_{LL}^{W^-}(y) = -\frac{\Delta d(x_a)\Delta\bar{u}(x_b) + \Delta\bar{u}(x_a)\Delta d(x_b)}{d(x_a)\bar{u}(x_b) + \bar{u}(x_a)d(x_b)}\,,$$

$$A_{LL}^{Z^0}(y) = -\frac{\sum_q (v_q^2 + a_q^2)\left[\Delta q(x_a)\Delta\bar{q}(x_b) + \Delta\bar{q}(x_a)\Delta q(x_b)\right]}{\sum_q (v_q^2 + a_q^2)\left[q(x_a)\bar{q}(x_b) + \bar{q}(x_a)q(x_b)\right]}\,,$$

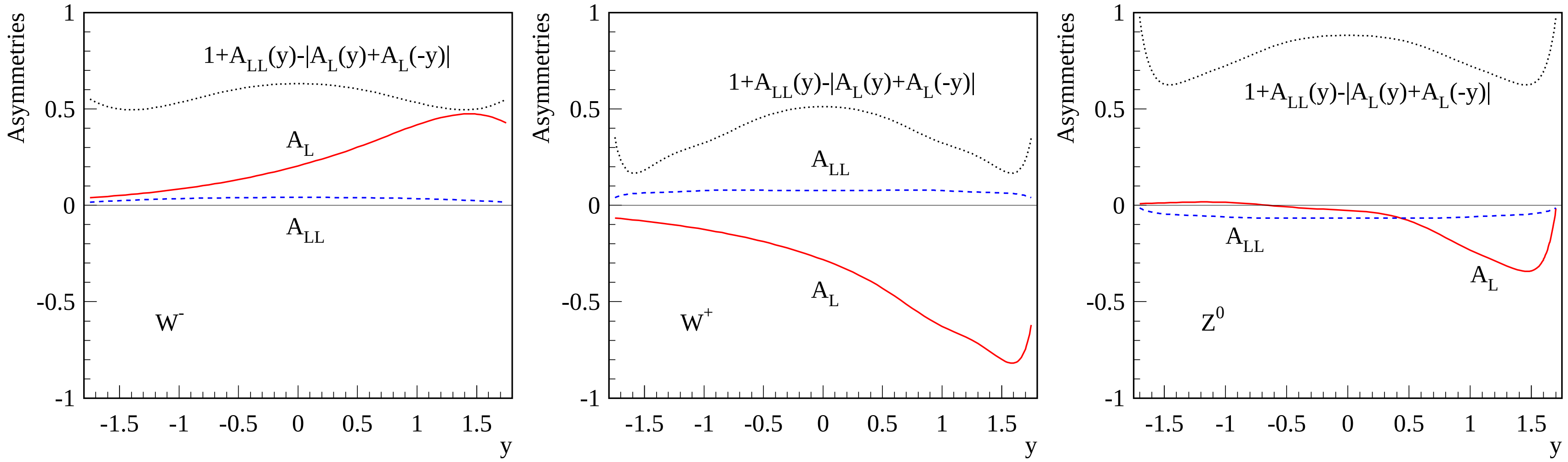
where $\Delta q(x)$ and $q(x)$ are the helicity distribution and unpolarized parton distribution function, respectively. v_q and a_q are the vector and axial couplings of the Z^0 boson to the quark. $x_{a,b}$ are the parton momentum fractions given by $x_a = m_Q/\sqrt{s}\,e^{y}$, $x_b = m_Q/\sqrt{s}\,e^{-y}$, with m_Q and y, the mass and rapidity of the W (or Z) boson and $\sqrt{s}$ the center-of-mass energy.

A_L and A_{LL} involve only helicity distributions

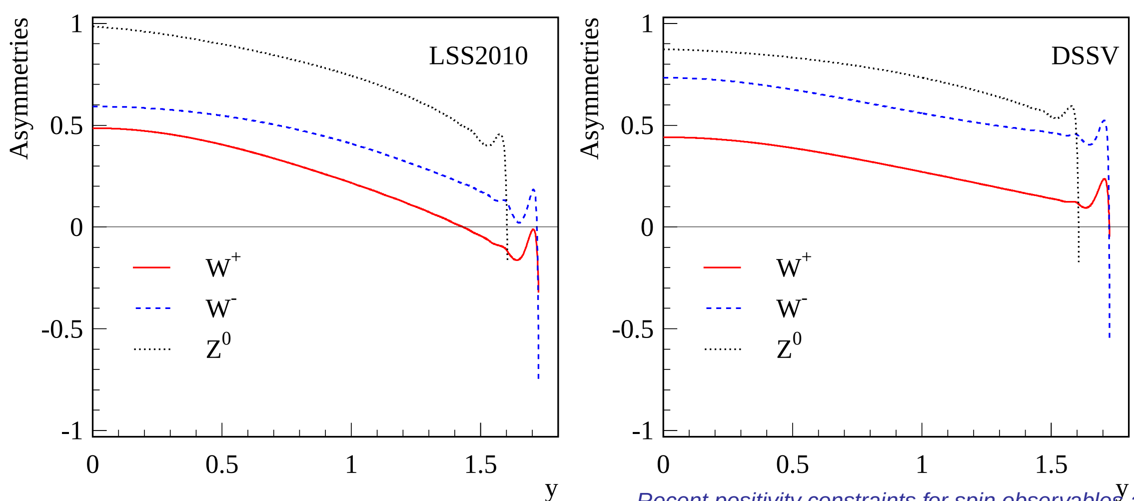
Positivity bounds involving parity-violating asymmetries

Zhong-Bo Kang, J.S., Phys. Rev. D83, 114020 (2011)

The statistical PDF satisfy the positivity bound

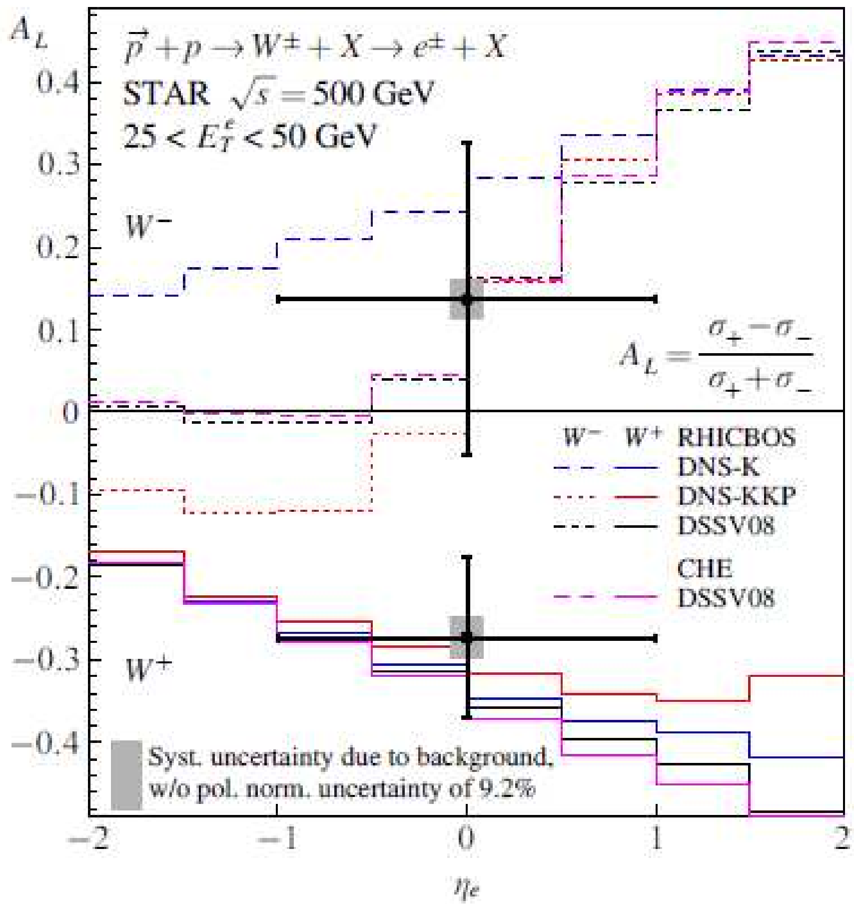


Two sets from global fits violate the bound at high $|y|$

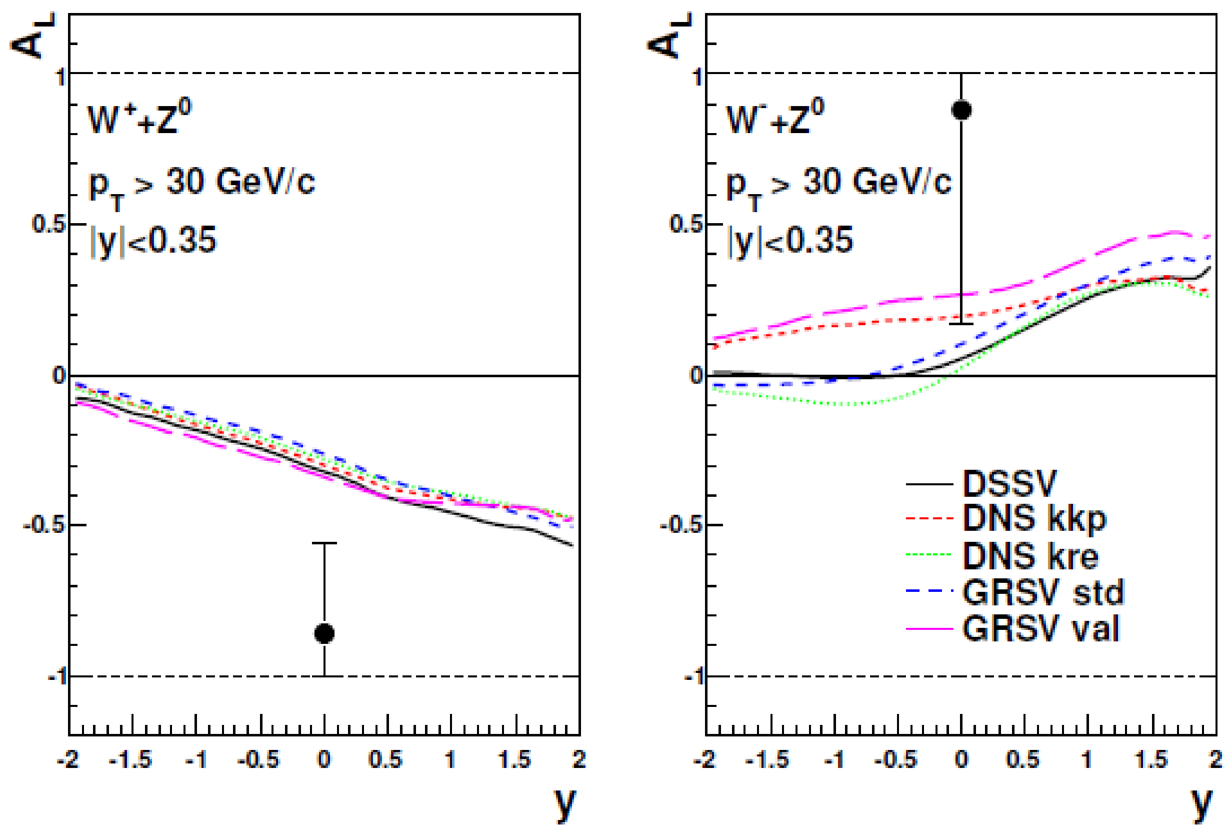


STAR results, M Aggarwal *et. al*., Phys. Rev. Lett. 106, 062002 (2011)

Our positivity bound implies also $|A_L(y=0)| \leq 1/2$



PHENIX results, A. Adare *et. al*., Phys. Rev. Lett. 106, 062001 (2011)



Concluding remarks

Many interesting cases were not presented here:

* Photoproduction of vector mesons

* Total cross sections

* Deep Inelastic Scattering

* Generalized parton distributions

* etc............

POSITIVITY PROVIDES VERY USEFUL NON-TRIVIAL BOUNDS ON SPIN OBSERVABLES

DON'T FORGET IT !!!

We all look forward to new exciting data!!

Thank you !