Anomalous hydrodynamics and gravity

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Summary of the talk

- Hydrodynamics: an old theory, describing finite temperature systems
- The presence of anomaly modifies hydrodynamics
- We first learned about this modification from gauge/gravity duality.
- We know understand anomalous hydrodynamics more generally

Hydrodynamics as low-energy effective theory

Finite-temperature system: finite mean free path

$$\lambda_{\rm mfp}$$

Dynamics at large distances $\ell \gg \lambda_{\rm mfp}$

is simple: very few degrees of freedom matter

In particular: conserved densities (of energy, momentum, conserved charges)

Relativistic hydrodynamics

Conservation laws: $\partial_\mu T^{\mu\nu} = 0$

$\partial_\mu j^\mu = 0$ (one conserved charge)

Constitutive equations: local thermal equilibrium

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu}$$

$$j^\mu = n u^\mu$$

Total: 5 equations, 5 unknowns

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$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$

$$j^\mu = n u^\mu + \nu^\mu$$

Total: 5 equations, 5 unknowns

Dissipative terms: ~ first derivatives, involve kinetic coefficients (shear and bulk viscosities, diffusion constants)

Parity-odd terms?

- What happens if the conserved current is axial?
  - example: QCD with massless quarks: axial currents conserved in absence of external EM fields
- Parity invariance does not forbid

$$j^{5\mu} = n^5 u^\mu + \xi(T,\mu)\omega^\mu$$

$$\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta \qquad \text{vorticity}$$

- The same order in derivatives as dissipative terms (viscosity, diffusion)

New effect: chiral separation

- Rotating piece of quark matter
- Initially only vector charge density $\neq 0$
- Rotation: lead to j^5: chiral charge density develops
- Can be thought of as chiral separation: left- and right-handed quarks move differently in rotation fluid
- Similar effect in nonrelativistic fluids?

Chiral separation by rotation

Chiral separation by rotation

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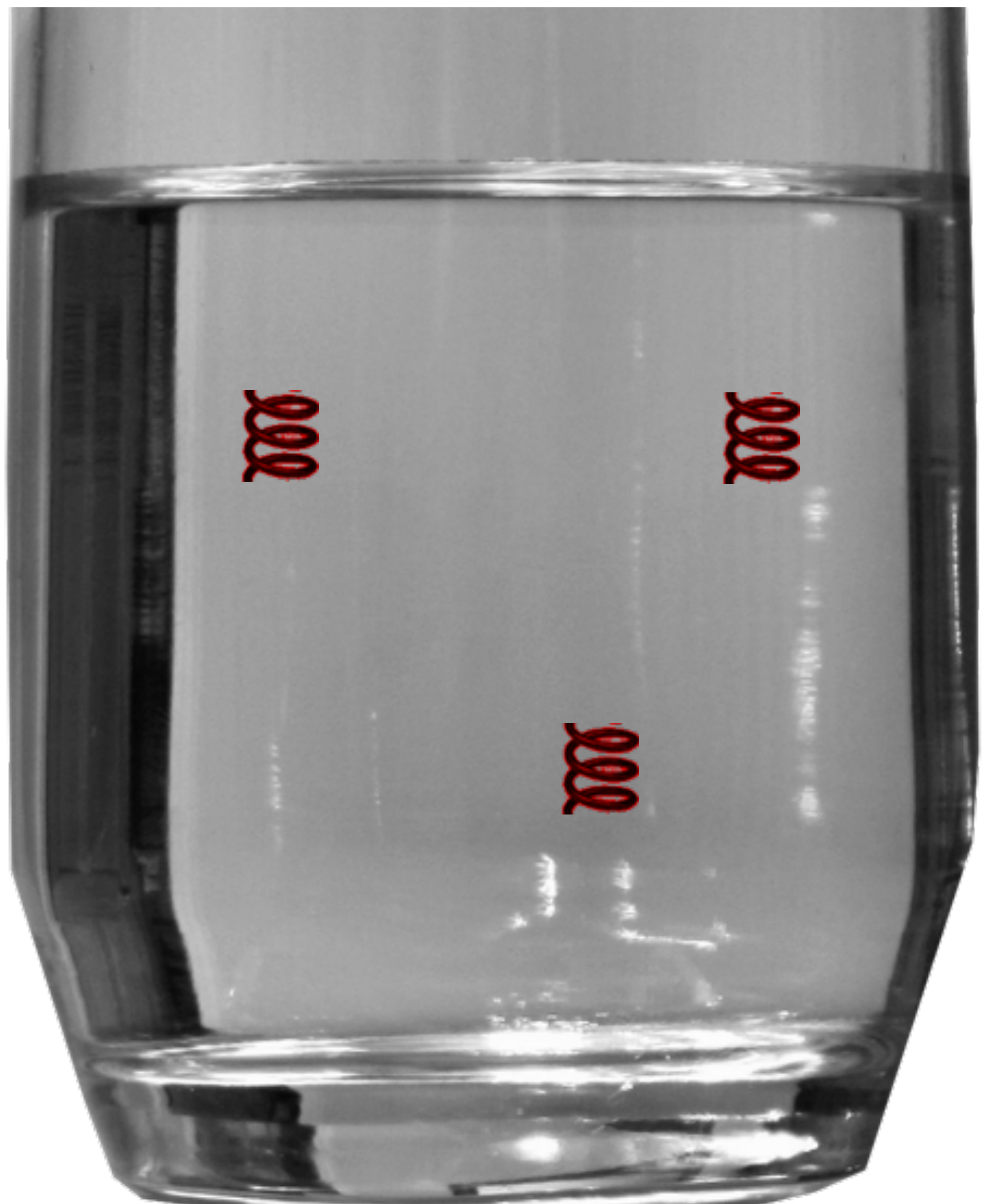
Chiral separation by rotation



Chiral separation by rotation

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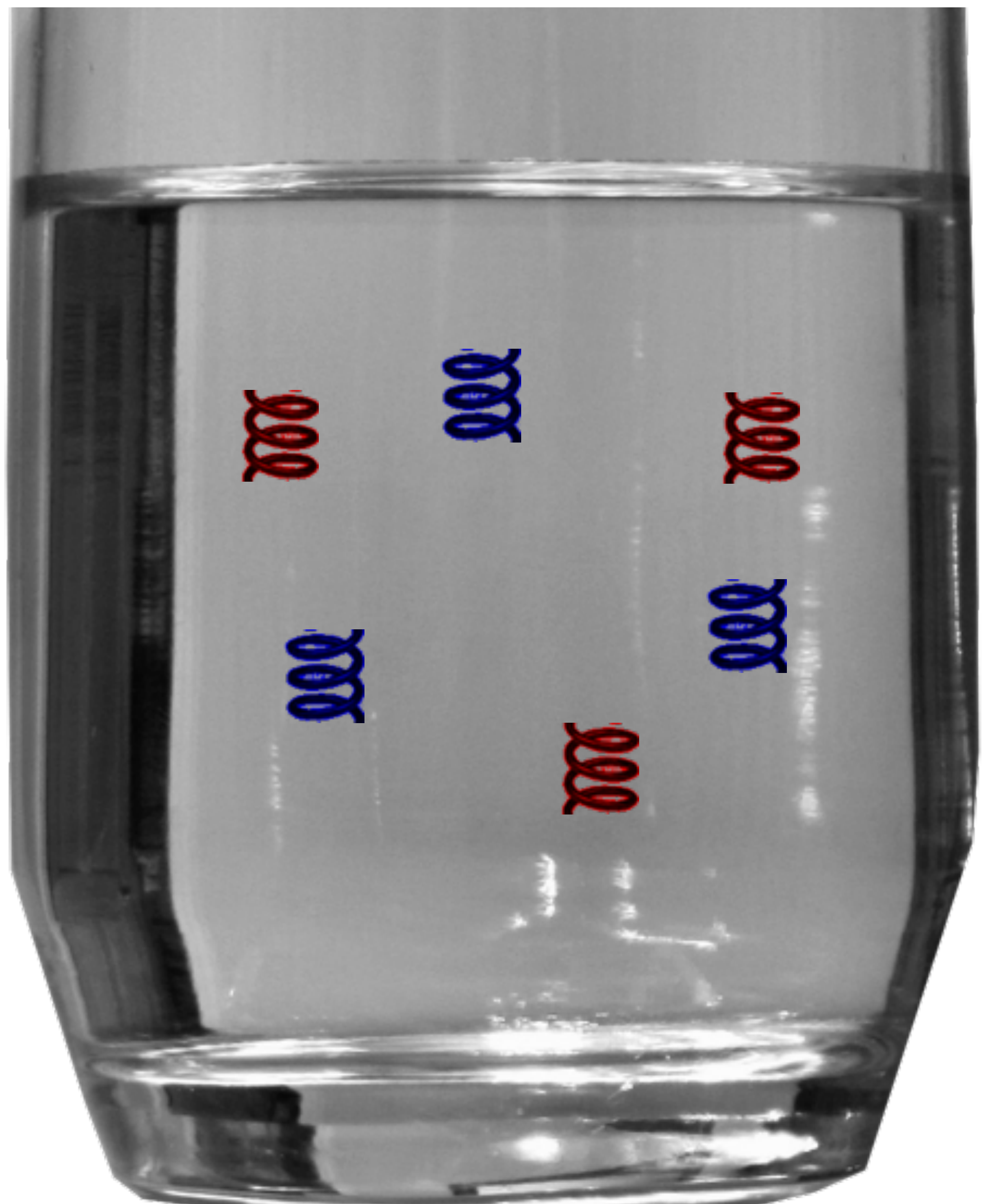

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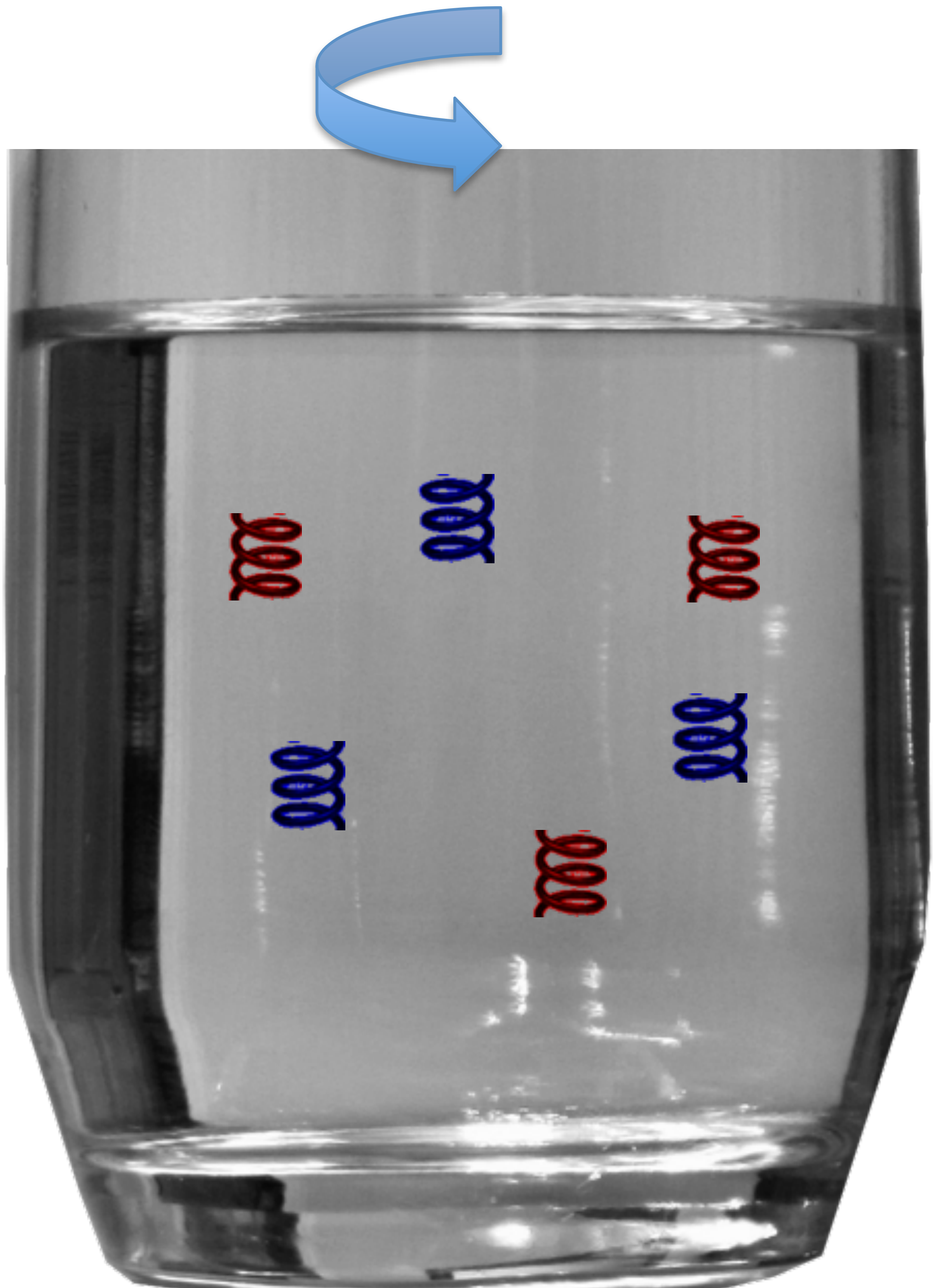

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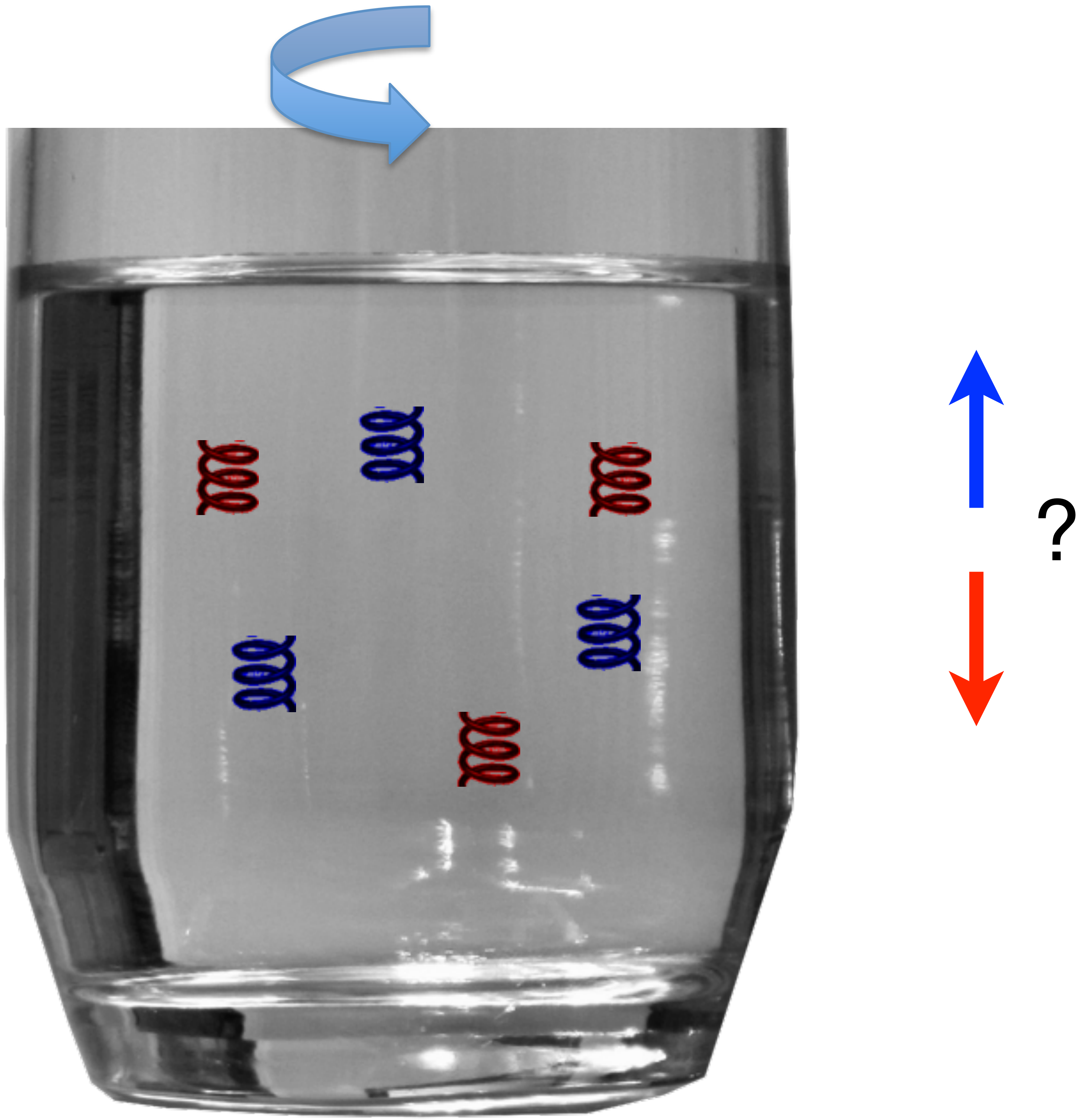


Can chiral separation occur in rigid rotation?

- If a chiral molecule rotates with respect to the liquid, it will moves
- In rigid rotation, molecules rotate with liquid
- $\Rightarrow$ no current in rigid rotation.

Relativistic theories are different

- There can be current ~ vorticity
- It is related to triangle anomalies

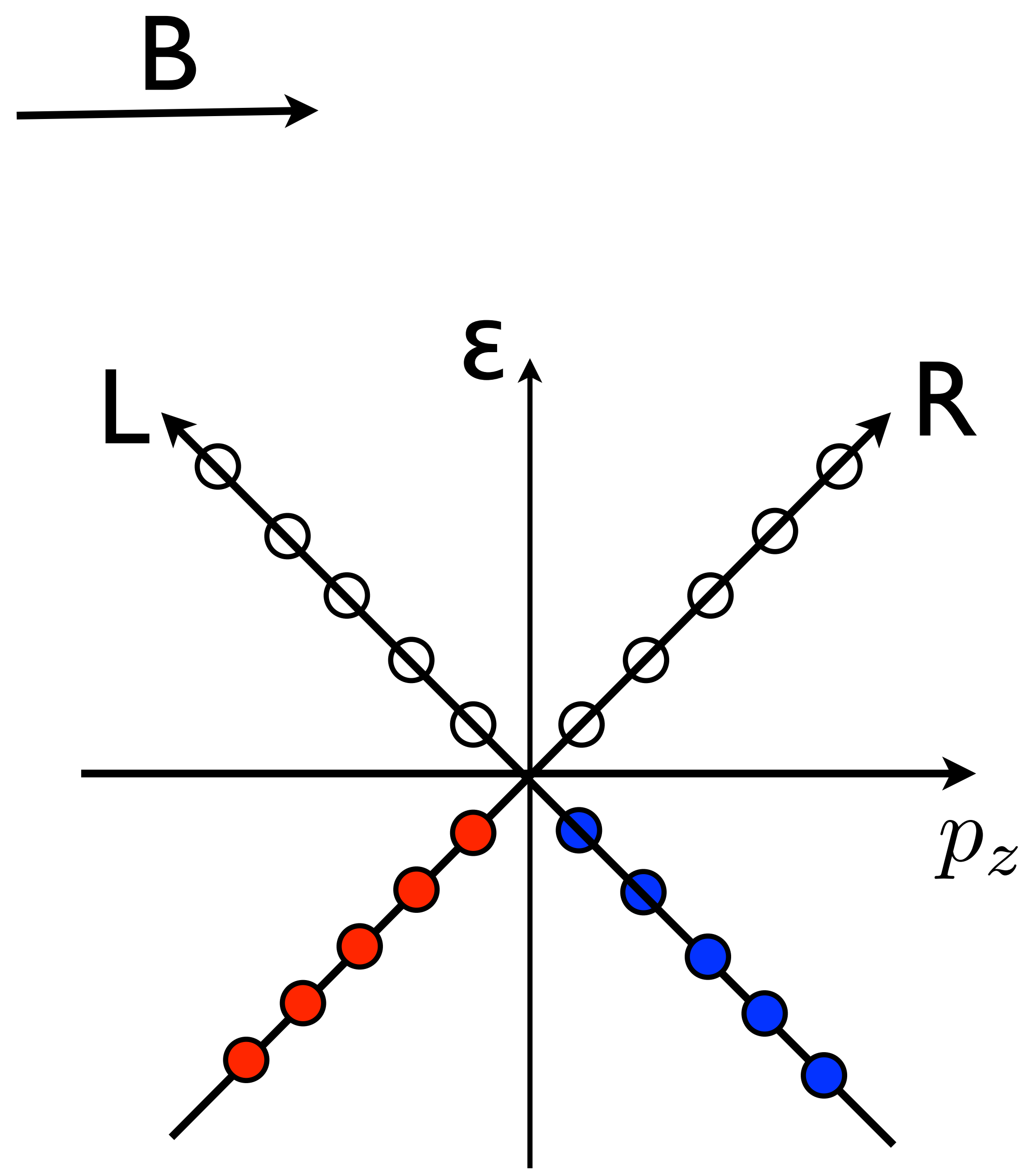
$$\partial_\mu j^{5\mu} = C\vec{E} \cdot \vec{B} \qquad C = 1/2\pi^2$$

but the effect is there even in the absence of external field

- The kinetic coefficient ξ is determined (almost) completely by anomalies and equation of state

Anomalies

Massless fermions: lowest Landau level is chiral



Anomalies

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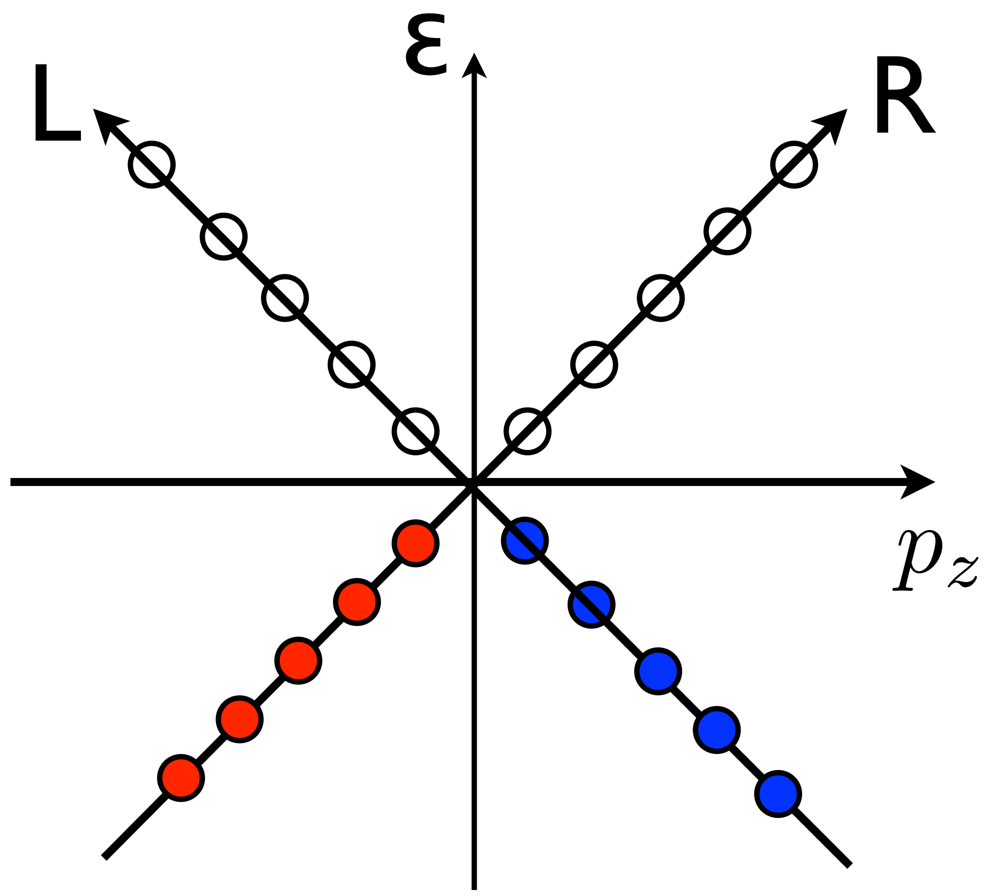

Anomalies

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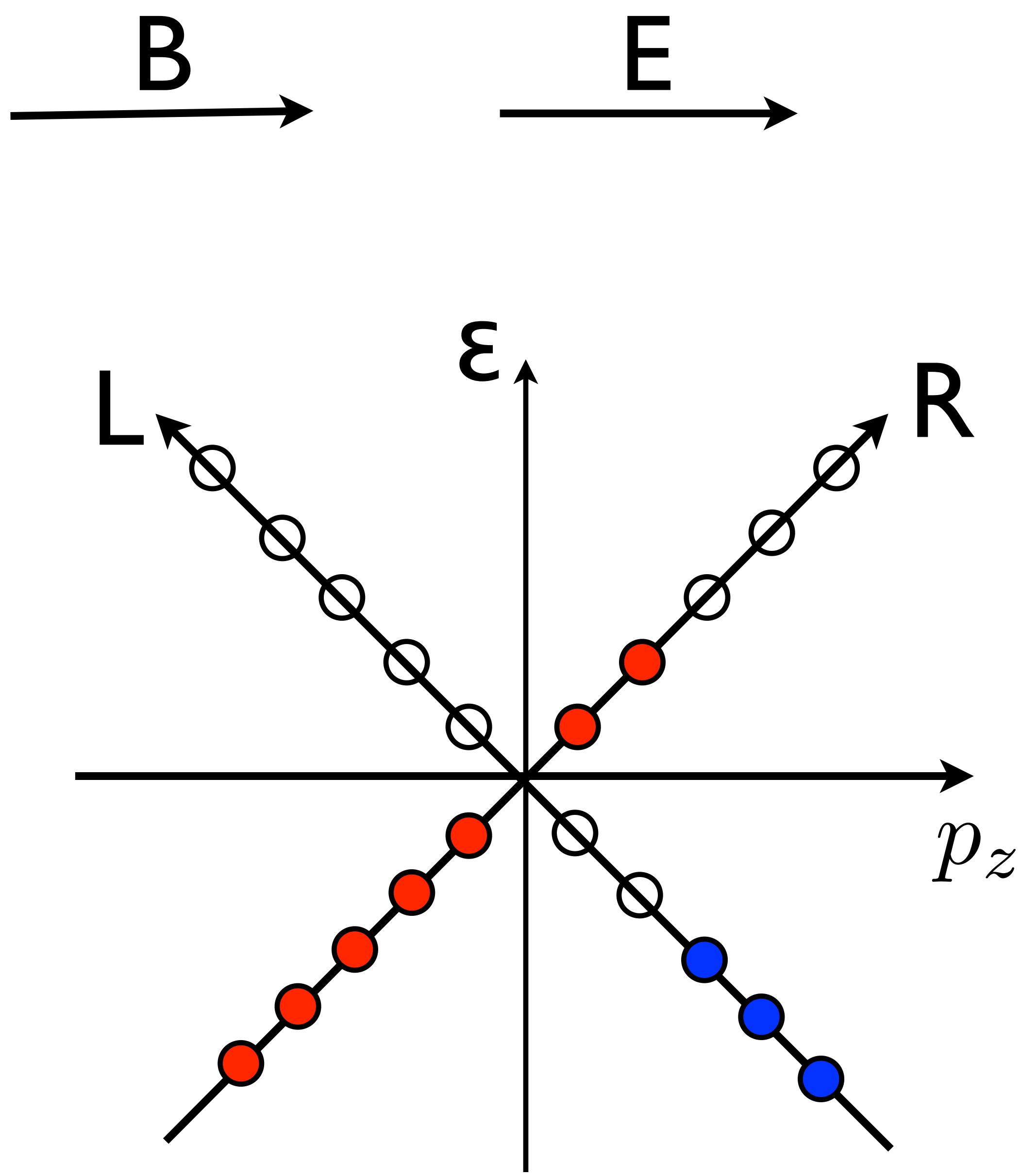

Anomalies

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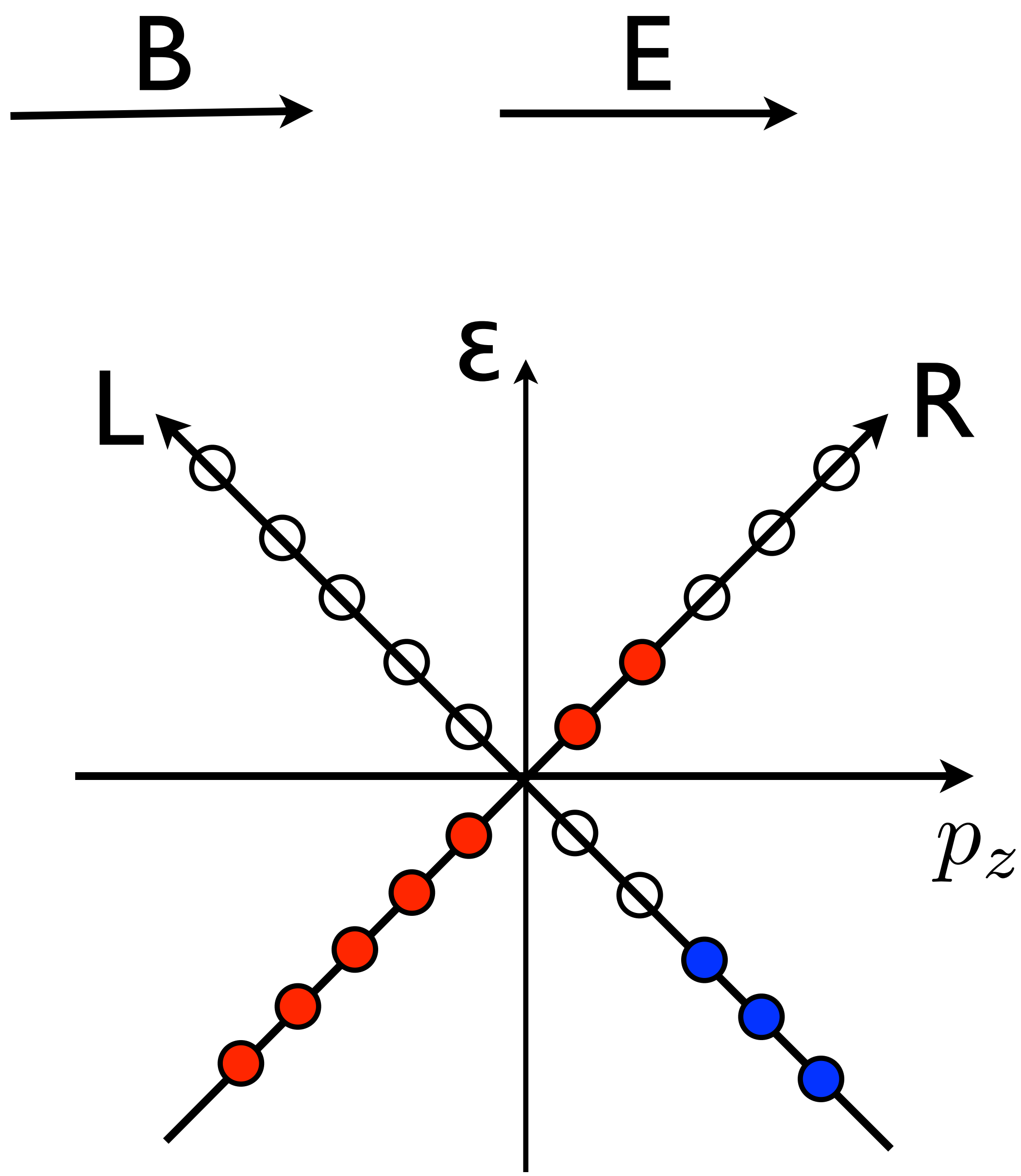


$$\frac{d}{dt}(N_R - N_L) \sim E \cdot B$$

Forbidden?

- it is usually thought that the parity odd terms are forbidden because of the 2nd law of thermodynamics

  hydrodynamic equations must be consistent with the existence of a non-decreasing entropy

- All textbook treatments do not include the parity odd term

Gauge gravity duality

- A method originating from string theory
- Maps some QFT (e.g., N=4 super-Yang-Mills) at strong coupling to gravity in higher dimensions
- Finite temperature plasma ↔ black hole in anti de-Sitter space

$$ds^2 = \frac{r^2}{R^2}(-f(r)dt^2 + d\vec{x}^2) + \frac{R^2}{r^2 f(r)} \qquad f(r) = 1 - \frac{R^4}{r^4}$$

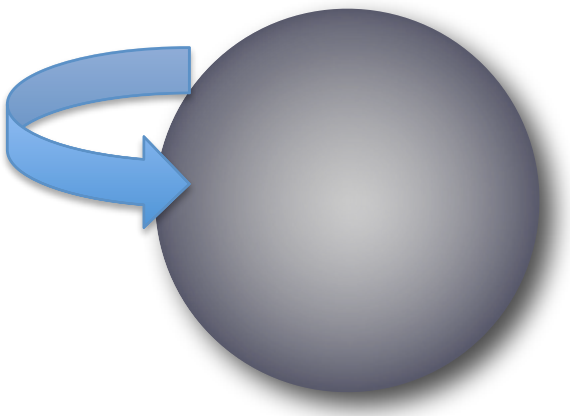
entropy of plasma ↔ black hole entropy

fluctuations of plasma ↔ fluctuations of black hole

Hydrodynamics is encoded in black hole dynamics

Holography

The first indication that standard hydrodynamic equations are not complete comes from considering



rotating 3-sphere of N=4 SYM plasma ↔ rotating BH

If the sphere is large: hydrodynamics should work

no shear flow: corrections ~ 1/R^2

Instead: corrections ~ 1/R   Bhattacharyya, Lahiri, Loganayagam, Minwalla

Holography (II)

Erdmenger et al., Banerjee et al., 2008

considered N=4 super Yang Mills at strong coupling
finite T and μ

should be described by a hydrodynamic theory

discovered that there is a current ~ vorticity

Found the kinetic coefficient ξ(T,μ)

$$\xi = \frac{N^2}{4\sqrt{3}\pi^2}\mu^2 \left(\sqrt{1+\frac{2}{3}\frac{\mu^2}{\pi^2T^2}}+1\right)\left(3\sqrt{1+\frac{2}{3}\frac{\mu^2}{\pi^2T^2}}-1\right)^{-1}$$

Anomalous hydrodynamics

- Closer consideration of the hydrodynamic theory reveals a unique way to modify hydrodynamics to be consistent with anomalies
- The value of ξ can be predicted from 2nd law of thermodynamics

for theory with U(1) charge with U(1)^3 anomaly:

$$\xi = C\left(\mu^2 - \frac{2}{3}\frac{n\mu^3}{\epsilon + P}\right)$$

DTS, Surowka 2009

anomaly coefficient ($1/4\pi^2$)

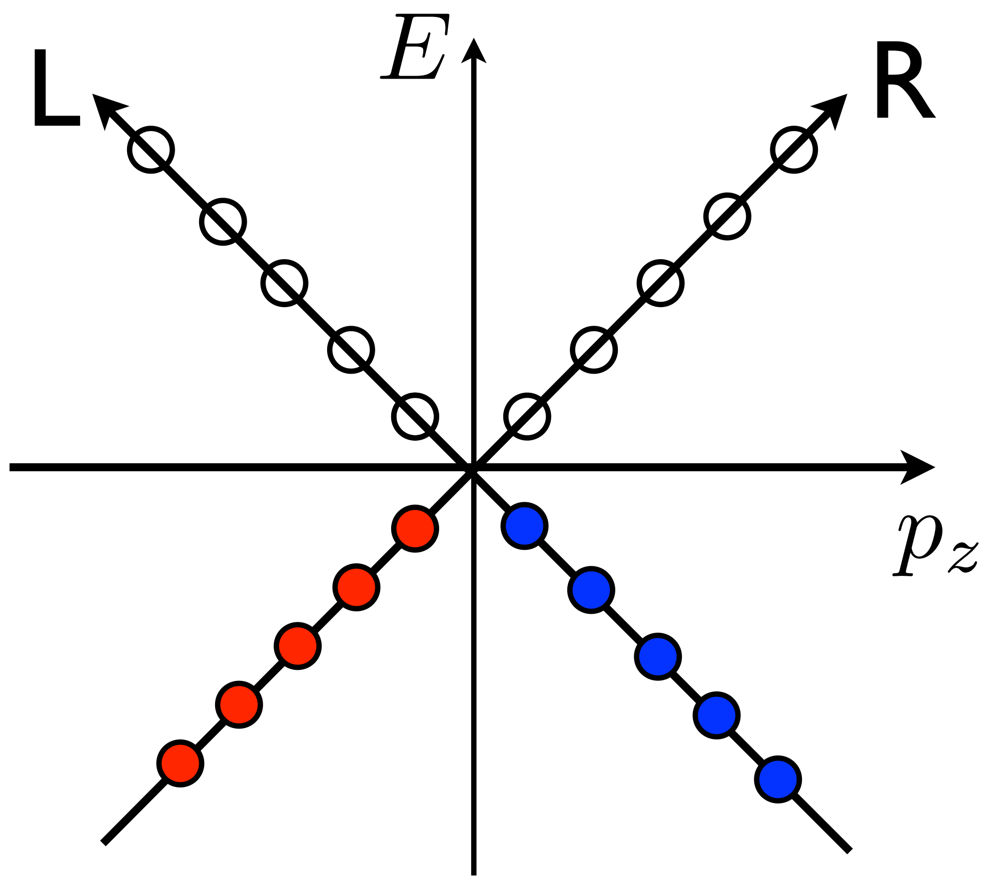
current ~ vorticity
similar effect: current ~ magnetic field $j^\mu = nu^\mu + \xi\omega^\mu + \xi_B B^\mu$

Current induced by magnetic field

Spectrum of Dirac operator:

$$E^2 = 2nB + p_z^2$$

All states LR degenerate except for n=0



$$j_{\mathrm{L}} \sim -C\mu B$$
$$j_{\mathrm{R}} \sim C\mu B$$

$$j_5 = j_R - j_L \sim C\mu B$$

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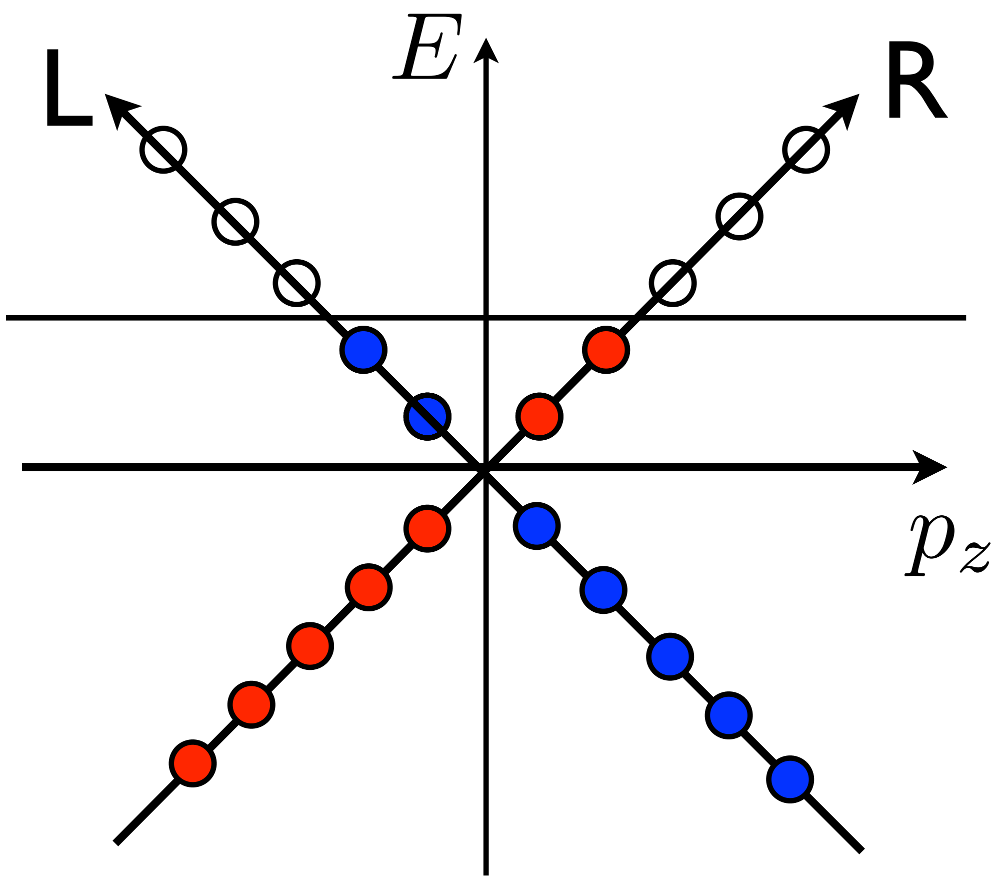

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$$j_{\mathrm{R}} \sim C\mu B$$

$$j_5 = j_R - j_L \sim C\mu B$$

Multiple charges (II)

Example: theory with one massless Dirac fermion

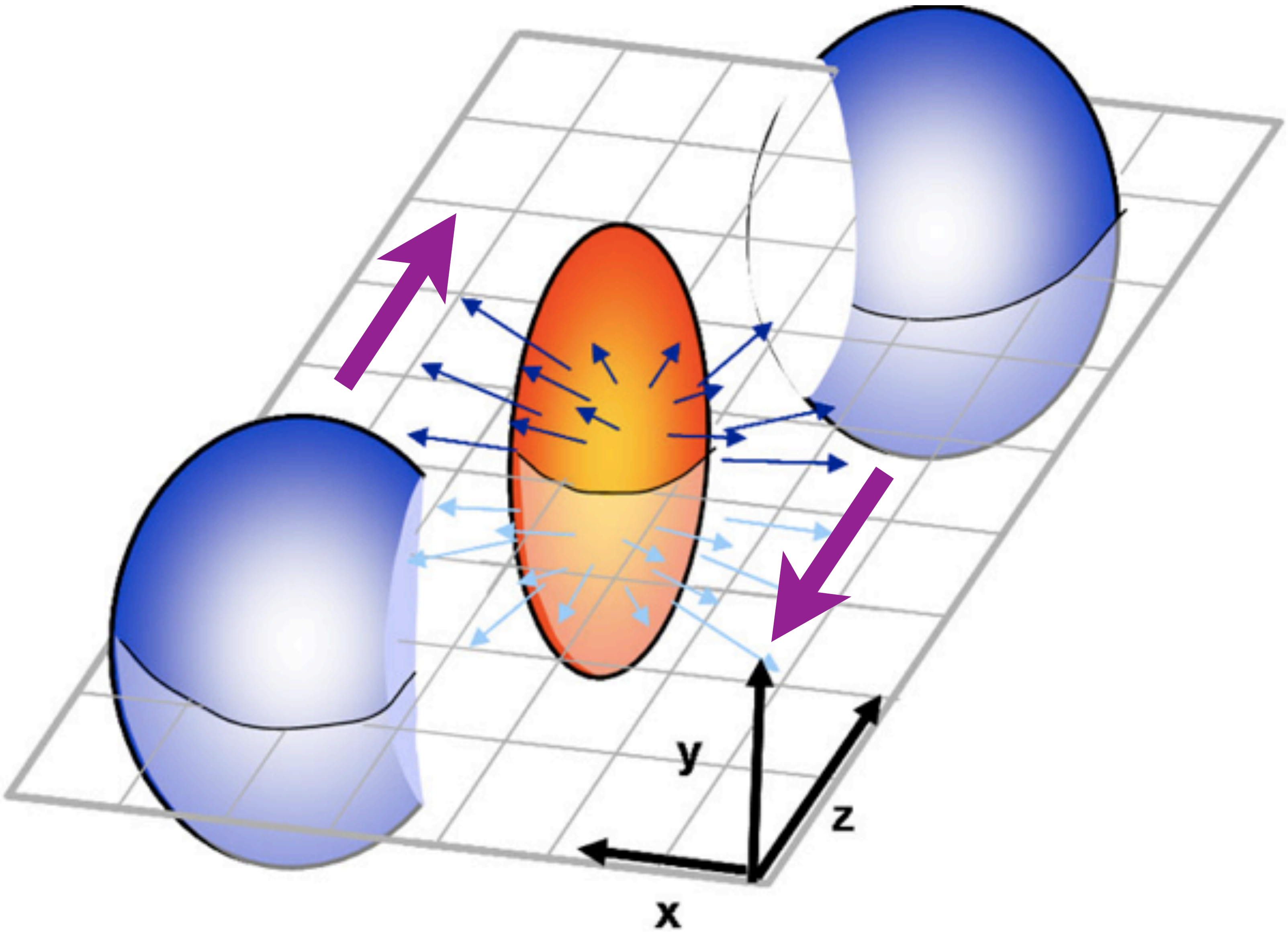
$$j^\mu = \frac{1}{2\pi^2}(2\mu\mu_5\omega^\mu + \mu_5 B^\mu + \mu B_5^\mu)$$

$$j_5^\mu = \frac{1}{2\pi^2}((\mu^2 + \mu_5^2)\omega^\mu + \mu B^\mu + \mu_5 B_5^\mu)$$

$$+C' T^2 \omega^\mu$$

not fixed by entropy argument
seems to be related to gravitation anomaly
(Landsteiner, Megias, Pena-Benitez 2011)

Observable effect on heavy-ion collsions?

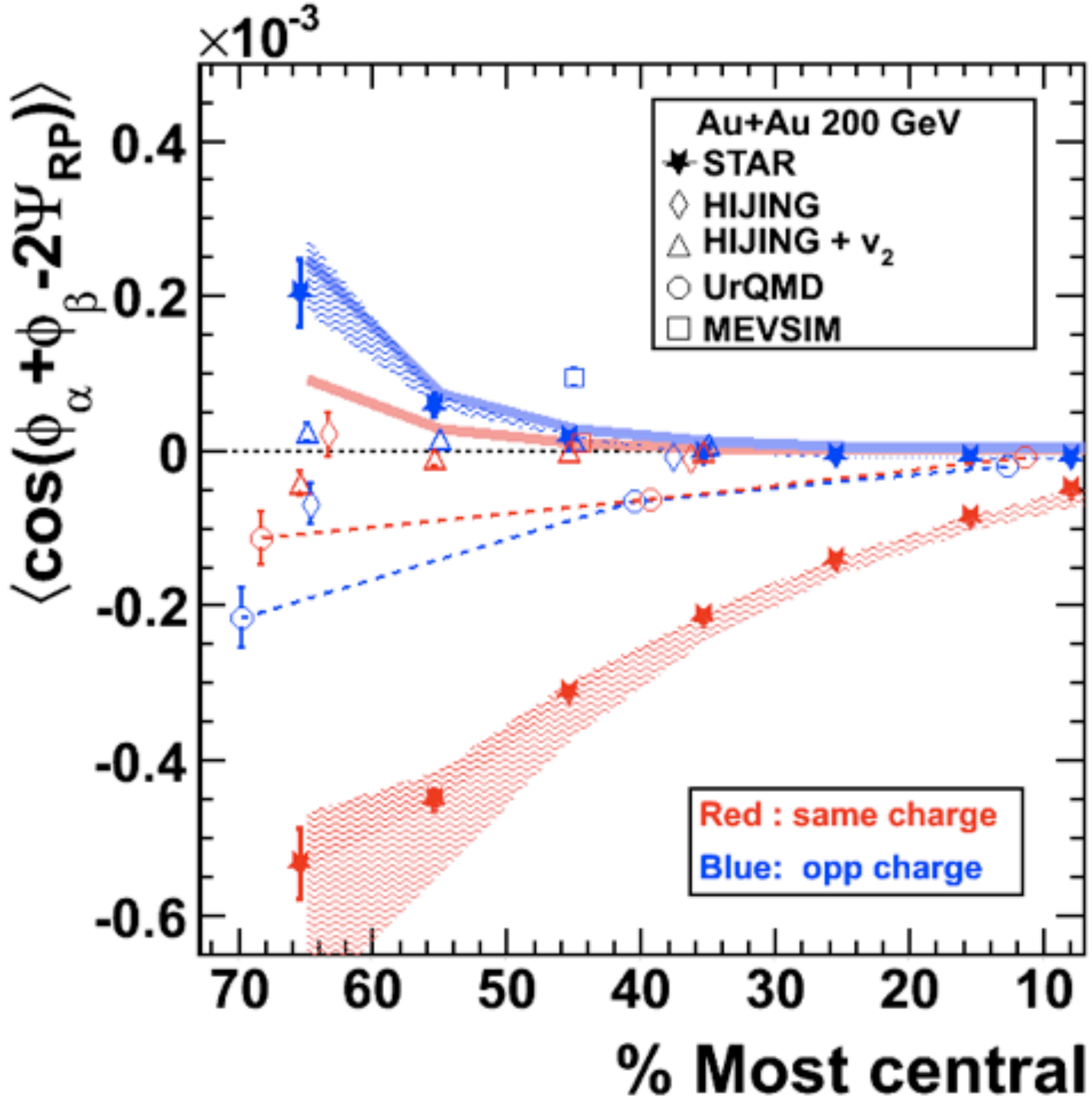


Chiral charges accumulate at the poles: what happens when they decay?

“Chiral magnetic effect”

- Large axial chemical potential μ_5 for some reason
- Leads to a vector current: charge separation
- π^+ and π^- would have anticorrelation in momenta
- Some experimental signal?
- Attempts to explain the signal by $j \sim \mu_5 B$ Kharzeev et al

STAR result



Abelev et al. PRL 2009 (arxiv:0909.1739)

Conclusions

- Anomalies affect hydrodynamic behavior of relativistic fluids
- First seen in holographic models, but can be found by reconciling anomalies and 2nd law of thermodynamics
- New terms in hydrodynamics (almost) completely fixed
- Indicate subtle effects in kinetic theory