

# Studying GPD in Gravity Dual

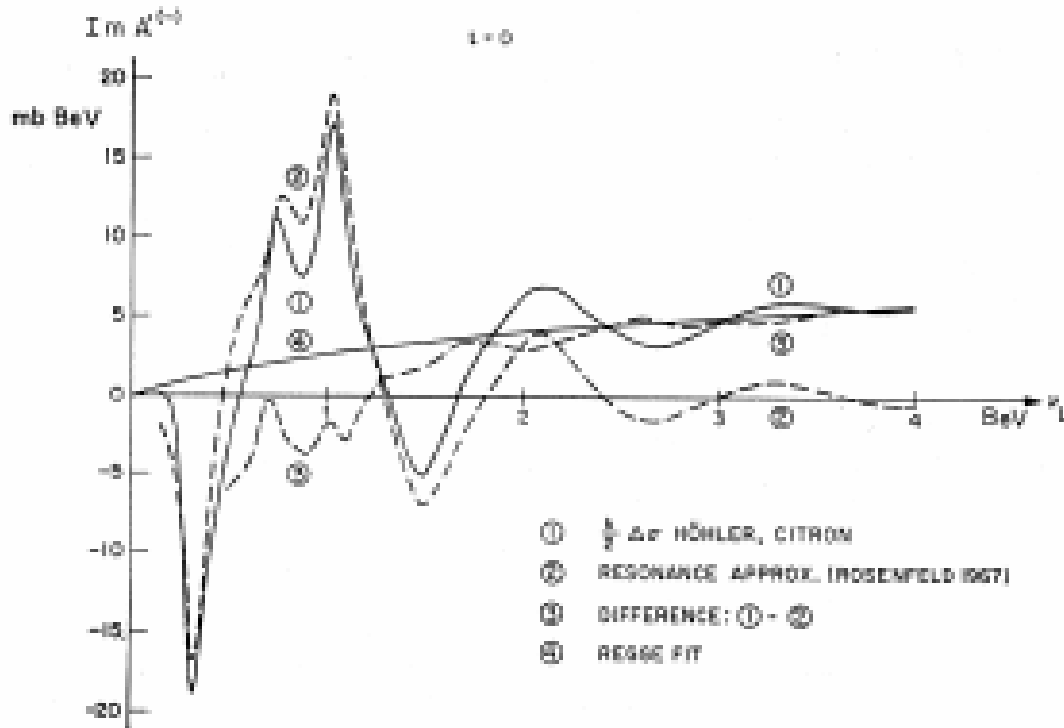
Elastic and Diffractive Scattering

Dec.17 (Sat) at Qui Nhon

Taizan Watari (IPMU)

based on [arXiv:1105.2907](https://arxiv.org/abs/1105.2907) and [1105.2999](https://arxiv.org/abs/1105.2999) [hep-ph]  
in collaboration with **Ryoichi Nishio** (IPMU)

# string theory for hadron scattering <sup>2/10</sup>



S-T duality in  
 $\pi - N$  elastic scattering

Large angle scattering

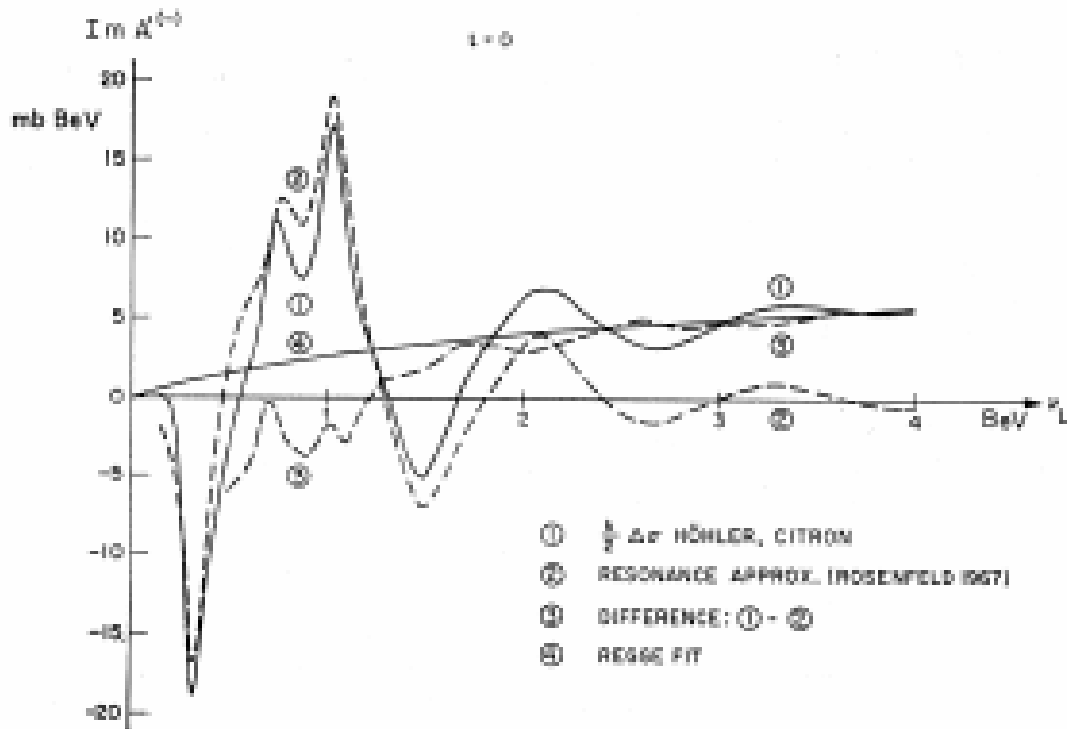
In string theory on **3+1-dim.**

$$e^{\alpha' t} \quad \text{exp. fall-off}$$

string

- hadron spectrum
- scattering

# string theory for hadron scattering 3/10



S-T duality in  $\pi - N$  elastic scattering

Large angle scattering

In string theory on **3+1-dim.**

$$e^{\alpha' t} \quad \text{exp. fall-off}$$

In string theory on **a warped spacetime**

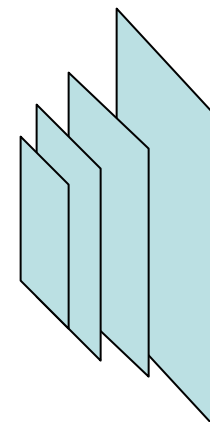
string on **warped spaces**

- hadron spectrum
- scattering

**exp. fall-off** at small  $|t|$



**power law** for large  $|t|$



Maldacena '98, Polchinski—Strassler '01

# limitation in the string approach

large  $Q^2$ , small  $x$  (large  $W$ )

$$\lambda \ll 1$$

perturbative QCD

but

small  $|t|$  ( $|t| \leq \Lambda_{QCD}^2$ )

real world QCD  
interpolates both.

$$1 \ll \lambda = \alpha_s N_c$$

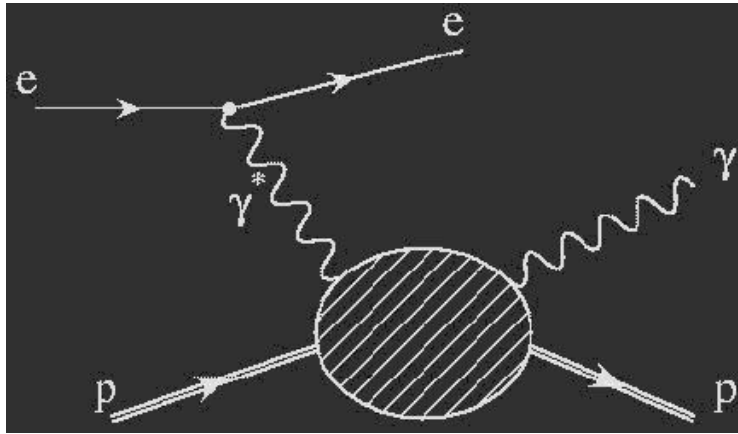
gravity dual (string theory)

experimental data

- qualitative understanding of data (if possible)
- provide ideas for fitting function
- not for quantitative predictions

# partons in strongly coupled theories?? <sup>5/10</sup>

$$\gamma^{(*)} - h \text{ scattering} \quad \int d^4x e^{iq_2 \cdot x} \langle h(p_2) | T \{ J^\mu(x) J^\nu(0) \} | h(p_1) \rangle$$



$$= iT^{\mu\nu}$$

$$= \sum_I C_I^{\mu\nu}(q) \langle h(p_2) | O_I(0) | h(p_1) \rangle.$$

OPE

partons (PDF/GPD)  $\longleftrightarrow$  twist-2 oprtrs.

Polchinski Strassler '02

Brower Polchinski Strassler Tan '06

$\longleftrightarrow$  leading trajectory in string theory

Gubser et.al. '02

BPST '06

$$A(s, t) \simeq \frac{c_s}{2\kappa_8^2} \frac{\pi}{2R^3} \int dz \sqrt{-g(z)} P(z) \int dz' \sqrt{-g(z')} P(z') K(s, t; z, z'),$$

$$K(s, t; z, z') = -8R\sqrt{\lambda} \int_0^\infty dv \frac{1}{2\pi i} \int_{C_1(\phi)} d\phi \frac{1 + e^{-i\phi}}{\sin \pi \phi} \frac{1}{\Gamma^2(\phi/2)}$$

$$\left( \frac{\alpha' \bar{s}}{4} \right)^j \frac{1}{j - \left( j_\nu + \frac{\delta_\nu}{2\sqrt{\lambda}} \right)} e^{-jA(z)} \Psi_{\underline{\nu}}^{(j)}(t, z) e^{-jA(z')} \Psi_{\underline{\nu}}^{(j)}(t, z').$$

pion cloud  
not included

# Basic Properties of PDF / GPD

Hatta Iancu Mueller '07

Brower Djurici Sarcevic Tan '10

Nishio TW '11

- parton contribution

$$A(s, t) \simeq \frac{c_s}{2\kappa_8^2} \frac{\pi}{2R^3} \int dz \sqrt{-g(z)} P(z) \int dz' \sqrt{-g(z')} P(z') \mathcal{K}(s, t; z, z'),$$

$$\mathcal{K}(s, t; z, z') = -8R\sqrt{\lambda} \int_0^\infty d\nu \frac{1}{2\pi i} \int_{C_1(\nu)} dj \frac{1 + e^{-i\pi j}}{\sin \pi j} \frac{1}{\Gamma^2(j/2)} \left(\frac{\alpha' \bar{s}}{4}\right)^j \frac{1}{j - \left(j_\nu + \frac{\delta_1}{2\sqrt{\lambda}}\right)} e^{-jA(z)} \Psi_{\bar{z}}^{(j)}(t, z) e^{-jA(z')} \Psi_{\bar{z}'}^{(j)}(t, z').$$

Polchinski Strassler '02

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# Basic Properties of PDF / GPD

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$$\text{Im } I_i \simeq \int dj f(j) \left( \frac{1}{\sqrt{\lambda x}} \right)^j \left( \frac{\Lambda}{q} \right)^{\gamma(j)} g_j(t).$$

$$\gamma(j) \simeq \sqrt{4 + 2\sqrt{\lambda}(j-2)} - j \quad \begin{array}{l} \text{anomalous dim.} \\ \text{(BPST '06)} \end{array}$$

- find a saddle point for large (but finite)  
 $\lambda$ ,  $N_c$ ,  $\ln(1/x)$  and  $\ln(q/\Lambda)$

$$\text{Im } I_i \simeq x^{-j^*} \left( \frac{\Lambda}{q} \right)^{\gamma(j^*)} g_{j^*}(t).$$

$$\left. \frac{\partial \gamma(j)}{\partial j} \right|_{j=j^*} = \frac{\ln(1/x)}{\ln(q/\Lambda)}$$

(in conformal theories)

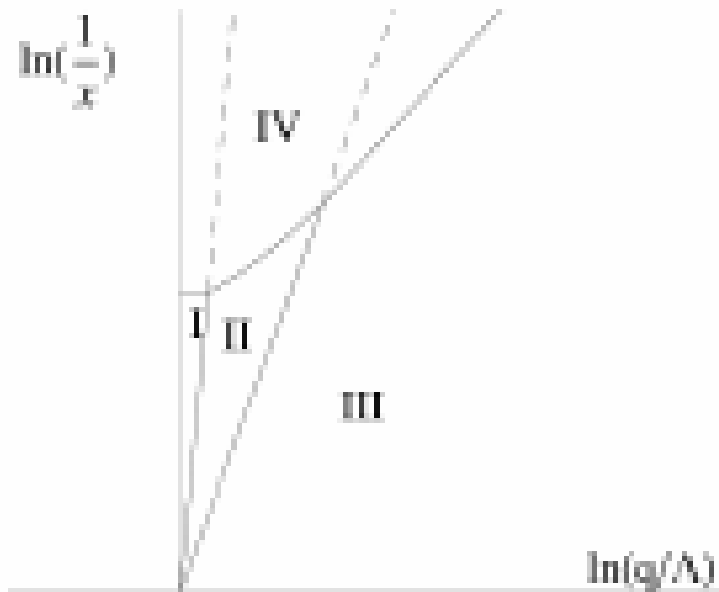
# Basic Properties of PDF / GPD

Hatta Iancu Mueller '07

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Nishio TW '11

## Phase Diagram (fixed $t$ )



In the II + III phase

## DGLAP evolution

$$\frac{\partial \ln \text{Im } I_i(q, x)}{\partial \ln(q/\Lambda)} = \gamma_{BPST}(j^*(q, x, t)).$$

## $\ln(1/x)$ evolution

$$\frac{\partial \ln \text{Im } I_i(q, x)}{\partial \ln(1/x)} = j^*(q, x, t) - 1.$$

## DVCS x-sect.

$$\frac{d\sigma}{dt} \sim \frac{\alpha_{QED}^2}{\Lambda^4} \left(\frac{W}{\Lambda}\right)^{4(j^*-1)} \left(\frac{\Lambda^2}{q^2}\right)^{\gamma(j^*)+2j^*}$$

IV: multi-Pomeron exchange important

I: low-energy regime vs II+III: high-energy regime

$$j^*(q, x, t) \prec \alpha_{IP,1}(t) \quad \text{or} \quad \alpha_{IP,1}(t) \prec j^*(q, x, t).$$



# GPD model and t-slope B

Nishio TW '11

$$\text{Im } I_i \simeq \int dj f(j) \left( \frac{1}{\sqrt{\lambda x}} \right)^j \left( \frac{\Lambda}{q} \right)^{\gamma(j)} g_j(t).$$

similar model D.Mueller '06

- analytic expression

$$g_j(t) \times (\varepsilon \Lambda)^{\gamma(j)}$$

$$= \frac{1}{K_5^2} \int dz \sqrt{-g} P_{hh} e^{-2jA} (R\Lambda)^j \left( \frac{\sqrt{-t}}{2\Lambda} \right)^{-j} \frac{2}{\Gamma(i\omega_j)} (\varepsilon \Lambda)^{\gamma(j)}$$

$$\times \left[ e^{(j-2)A} \left( K_{i\omega_j}(\sqrt{-t}z) - \frac{K_{i\omega_j}(\sqrt{-t}/\Lambda)}{I_{i\omega_j}(\sqrt{-t}/\Lambda)} I_{i\omega_j}(\sqrt{-t}z) \right) \right]$$

$$g_j(t) |_{\varepsilon=1/\Lambda} \sim \frac{(1-t/\Lambda^2)^{-n}}{j - \alpha_{IP}(t)}.$$

$$= \sum_n \frac{g_{nhh}(j) F_{j,n}(\varepsilon)}{t - m_{j,n}^2}.$$

- model in collinear factorization approach (dual parametrization)

**Kaluza-Klein tower of Pomeron trajectories**

Belitsky, Geyer, Mueller, Schafer, Polyakov Shuvaev, Kumericki '97--

# GPD model and t-slope B

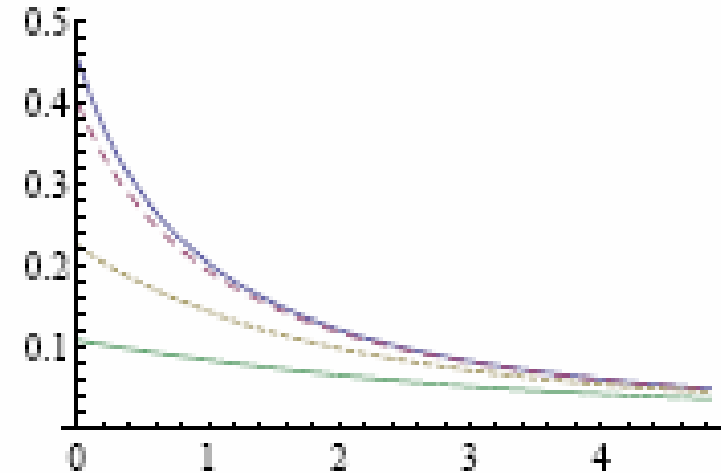
Nishio TW '11

$$\text{Im } I_i \simeq \int dj f(j) \left( \frac{1}{\sqrt{\lambda x}} \right)^j \left( \frac{\Lambda}{q} \right)^{\gamma(j)} g_j(t).$$

- analytic expression

$$g_j(t) \times (\varepsilon \Lambda)^{\gamma(j)}$$

$$= \frac{1}{K_S^2} \int dz \sqrt{-g} P_{\mu\nu} e^{-2iA} (R\Lambda)^j \left( \frac{\sqrt{-t}}{2\Lambda} \right)^{-j} \frac{2}{\Gamma(i\omega_j)} (\varepsilon \Lambda)^{\gamma(j)} \\ \times \left[ e^{(j-2)A} \left( K_{i\omega_j}(\sqrt{-tz}) - \frac{K_{i\omega_j}(\sqrt{-t}/\Lambda)}{I_{i\omega_j}(\sqrt{-t}/\Lambda)} I_{i\omega_j}(\sqrt{-tz}) \right) \right]$$



**B: decreasing fcn  
of  $j^*$  and of  $q^2$**

- t-slope

$$B = 2 \frac{\partial \ln g_j(t)}{\partial t}$$

depends on  $q$  and  $x$  ( $W$ )  
through  $j^*(q, x)$ .

**sensitive  
to  $q$  if  $x \ll 1$**

**consistent with H1  
DVCS data**