American Economic Journal: Microeconomics 2012, 4 (1): 32–69
http://dx.doi.org/10.1257/mic.4.1.32

Community Structure and Market Outcomes: A Repeated Games-in-Networks Approach†

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Consider a large market with asymmetric information, in which sellers have the option to "cheat" their buyers, and buyers decide whether to repurchase from different sellers. We model active trade relationships as links in a buyer-seller network and study repeated games in such networks. Endowing sellers with incomplete knowledge of the network, we derive conditions that determine whether a network is consistent with cooperation between every buyer and seller that are connected. Three network features reduce the minimal discount factor sufficient for cooperation: moderate and balanced competition, sparseness, and segregation. Incentive constraints are binding and rule out efficient networks. (*JEL* C73, D82, D85, Z13)

Economists have long noticed that it is difficult to sustain cooperation in large groups, especially if third party observability within the group is limited.[1] Nevertheless, even as markets grow and span across geographic and cultural borders, informal agreements continue to be an important part of markets' activity. A number of empirical studies document interesting patterns of trade within large groups. In particular, trade and trust are often concentrated in a subset of all possible relationships.[2] This paper suggests an explanation to the observed patterns of trade and trust.

We consider a market with asymmetric information. In every period, sellers with limited supply meet sequentially with buyers with limited demand and decide whether to cooperate or to defect and "cheat" a given buyer. Only the buyer cheated observes the seller's deviation. We model active relationships as links in a buyer-seller network and ask the following question: what structures of networks are consistent with an equilibrium in which every buyer and seller that are connected trade and cooperate with each other? The answer defines a set of networks in which a link

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[1] See also Kandori (1992), Greif (1993), Ellison (1994), and Araujo (2004).

[2] See also Hardle and Kirman (1995); Fafchamps (1996); Weisbuch, Kirman, and Herreiner (1996); McMillan and Woodruff (1999); Kirman and Vriend (2000); and Karlan et al. (2009). We review this literature in Section VI.

between seller s and buyer b implies that b can trust s to cooperate with him when they trade.

The scarcity of models of repeated games in networks is often attributed to the inherent intractability of the problem.[3] Our framework alleviates some of the difficulties and provides a simple expression that summarizes all the network information that seller s uses when deciding whether to cooperate with buyer b or cheat him. Consider a seller s and a buyer b that are connected. The immediate benefit for s from cheating buyer b is defined by the stage game and does not depend on the network. On the other hand, the cost of cheating depends on the entire network structure. As a starting point, consider the simple case that s deviates only in an interaction with b and cooperates with all other buyers that are connected to her, and that none of the other buyers can learn about the deviation of seller s. In this simple case, for every period that b "punishes" s by not purchasing from her, s loses her expected per-period future value from cooperation with b, which we denote by $FV_{s,b}$. If $FV_{s,b}$ is large, s does not find it profitable to deviate and lose the option to trade with b, even if her intertemporal discount factor is low and the immediate benefit from deviating is large. In Theorem 1, we establish conditions under which the following one-deviation-principle holds: $FV_{s,b}$ is a sufficient statistic for determining whether a *fully cooperative equilibrium* (an equilibrium in which every buyer and seller that are connected always cooperate with each other) exists.

Despite the simplification, $FV_{s,b}$ still depends on the entire network structure and can be difficult to calculate, especially in large networks. To evaluate $FV_{s,b}$, s asks the following question: "What is the probability that I will be able to sell a good to b and not be able to sell it to any other buyer?" The answer reflects the probability that *s needs b* in a given period, and depends on the network structure in two ways. First, the network structure determines the frequency of interactions between s and b; when their frequency of interaction rises, *s needs b* more, and values more their connection. Second, the network structure determines the outside options of s if she were not connected to b. When other buyers with whom s is connected are more likely to buy from s, seller *s needs b* less. For illustration, assume that in every period meetings between buyers and sellers occur in an order chosen uniformly at random, and let each seller produce one unit of a good and each buyer have demand for one unit of a good. Then, in a fully cooperative equilibrium, successful interactions between seller s and buyer b in Figure 1A are more frequent than in Figure 1B (in the latter there is a probability of $1/4$ that in a given period s does not sell at all). However, in Figure 1A s has a guaranteed outside option because buyer b' cannot transact with any other seller, whereas in Figure 1B, there is a positive probability that b is the only buyer who offers to buy from s, which raises the value of this connection for seller s. Focusing on Figure 1B, if we eliminate the link between s' and b, the connection between s and b becomes more valuable due to higher frequency of

[3] Recently, several researchers take on different approaches to modeling repeated games in networks (see Ali and Miller 2009; Mihm, Toth, and Lang 2009; Nava and Piccione 2011; Jackson, Rodriguez-Barraquer, and Tan 2011; and Lippert and Spagnolo 2011). As their approach and research questions differ significantly from ours, we defer the discussion of these papers to Section VI.

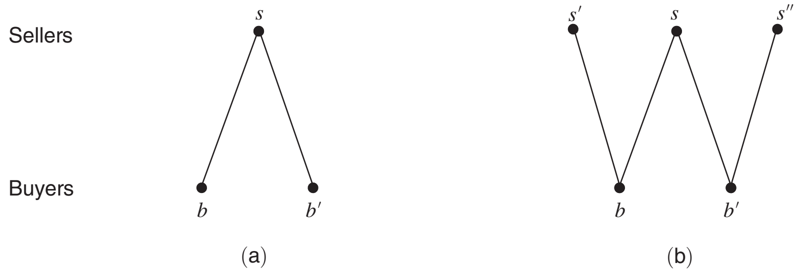


FIGURE 1.

interactions, and the connection between s and buyer b' becomes less valuable due to an improved outside option for s.

The expectations of seller s with respect to $FV_{s,b}$ depend on what s knows about the network. In our model, sellers and buyers know who they are connected to, and the number of connections (*degree*) of each of the buyers or sellers that they are connected to. Additionally, they know the number of buyers and sellers in the network (n_b and n_s respectively), as well as some aggregate information regarding the network structure, such as the degree distribution of buyers and sellers in the network, and the probability of sharing more than one neighbor with the same individual.

Focusing on a family of equilibria in simple strategies, which excludes equilibria that rely on community enforcement through contagion,[4] we find that in any (asymptotically) large network and for every seller s and buyer b that are connected, $FV_{s,b}$ can be summarized by a simple expression that captures $FV_{s,b}$ in a corresponding random tree. Using this insight, we show that three network features increase the values of links: (i) *moderate and balanced competition*: the degrees of all buyers and sellers are similar; (ii) *sparseness*: the degrees of sellers and buyers are small; and (iii) *segregation*: sellers who have one buyer in common, have connections to similar sets of buyers overall. For fixed intertemporal discount factors, our results describe systematic constraints on the structure of networks that can sustain cooperation. In contrast with much of the existing literature, the constraints are not due to exogenous costs of creating or sustaining links.

Ignoring the incentive constraints and assuming that sellers always cooperate, networks that maximize the expected volume of trade are dense—the exact opposite of (ii) above. This difference leads to inefficiencies and is especially robust in stochastic environments, in which sellers' supply is subject to exogenous fluctuations. Noting that previous theoretical results find endogeneously formed buyer-seller networks to be efficient in facilitating trade, highlights that the inefficiencies are due to the incentive constraints imposed by moral hazard.[5]

We consider three social and formal institutions: *Reputation Networks*, *Litigation*, and *Third-Party Evaluation Services*. The direct effect of each of these institutions

[4] Consider a seller s that cheats a buyer b. Contagion via the network requires buyer b to refuse to buy from competitors of seller s.

[5] For example, Kranton and Minehart (2001) offer a model in which buyers decide whether to costly connect to sellers before auctions take place and find that the resulting network facilitates the efficient outcome.

on cooperation is well studied. However, the integration of reduced-form models of these institutions into our framework highlights a new insight: in the presence of either of these institutions, denser networks can sustain cooperation (i.e., these institutions complement the network rather than substitute for it in enforcing cooperation).

Methodologically, we extend prior literature on games in networks in several ways. Most notably, with the exception of a few papers that are discussed in Section VI, most of the literature focuses on static games (for extensive surveys, see Goyal 2007 and Jackson 2008). In addition, the current literature focuses either on complete information of the network structure, or on incomplete information where an agent knows only her own degree and the degree distribution of others in the network.[6] We allow for incomplete yet richer knowledge of the network structure. By doing so, we achieve tractability in large networks, while maintaining the ability to analyze complex changes in the network structure.

Finally, most related to our paper is Fainmesser and Goldberg (2011)—hereafter FG—who analyze how the structure of an informational network between buyers affects the ability to sustain cooperation between buyers and sellers. FG show that the impact of the entire structure of the buyer-seller network on the incentives of a seller to cooperate can be approximated by focusing on the seller's local neighborhood—a small network that includes only buyers and sellers that are *close* to the seller. Furthermore, when sellers have a sufficiently high level of uncertainty with respect to the network structure, FG find that a seller expects her local neighborhood to look approximately like a random tree—a network that has no cycles and in which the degrees of buyers and sellers in the network are drawn independently at random from some degree distribution.[7] We make use of these graph-theoretic results in our characterization of large networks for which fully cooperative equilibria exist.

The paper is organized as follows. The following section offers two motivating examples. In Section II, we present the model, and in Section III we characterize the future value of links in large networks and derive conditions to determine whether a network admits a fully cooperative equilibrium. Section IV characterizes differences in the future values of links within and across networks and relates these differences to the constraints on the structure of networks that admit fully cooperative equilibria. Section V investigates the trade-off between sustaining cooperation and maximizing trade volumes. Section VI offers a discussion of related literature and empirical evidence, as well as several institutions that affect the ability to sustain cooperation. Section VII offers concluding remarks.

I. Examples

To motivate our analysis, we briefly describe two examples of relevant applications.

[6] See Jackson and Yariv (2007), and Galeotti et al. (2010).

[7] Also related is Campbell (2010) who applies percolation theory (physics) to the study of monopoly pricing in the presence of WOM. Notably, percolation theory relies on the close connection between random graphs and tree-like networks.

Example 1 (job recommendations):[8]
Consider a group of recommenders (mentors/past employers) that have workers (mentees/past employees) to recommend and a group of firms that are seeking to hire. A recommender receives a positive payoff from getting a job for her worker. A worker's ability can be either high or low, and is observed by the firm only after the worker is hired (the recommender knows the ability of the worker). Assuming that firms want to hire only high quality workers, a recommender who has a low ability worker can benefit from recommending the worker to the firm as having high ability. In a one-recommender-one-firm setup, it is easy to solve for the minimal firm's discount factor that will allow to support an equilibrium in which the firm provides accurate recommendations. This paper studies an environment with many recommenders and many firms, and allows each recommender to condition her recommendation on the targeted firm.

Example 2 (catering and food deliveries):
Consider a group of food suppliers (caterers/restaurants) and a group of repeated clients that order food frequently. Providing high quality service and food costs more than providing low quality. In the absence of sufficient future payoffs that are contingent on providing high quality, a food supplier may shirk and provide low quality. We study how the patterns of interactions in this market affect the level of future payoffs that are contingent on providing high quality.[9]

II. Model

Consider a market with a set $S = \{1, 2, \ldots, n_s\}$ of sellers (recommenders/caterers) and a set $B = \{1, 2, \ldots, n_b\}$ of buyers (firms/clients). Time is discrete. Sellers live forever and seller s has a discount factor δ_s. Periods are ex-ante identical. In every period, any seller s has the capacity to produce one unit: with probability μ seller s can choose whether to produce a high quality good at a cost of $c_s \geq 0$, or a low quality good at no cost, whereas with probability $(1 - \mu)$ seller s can only produce a low quality good (at no cost). μ is common knowledge and the realization of μ is i.i.d. across sellers and periods. Goods are nondurable and cannot be transferred across periods. Buyers live forever and have unit demand in every period. Each seller s has an *active relationship* with only a subset of buyers, denoted by B_s. We first define a buyer-seller network that captures the active relationships of all sellers and later provide the activity rules that define the notion of an active relationship.

Let $\mathbf{m} = \langle S, B, E \rangle$ be a network, where E is a set of seller-buyer pairs such that $(s, b) \in E$ if and only if there is an edge (or link) connecting seller s and buyer b. Let $B_s(\mathbf{m})$ be the set of buyers that are (directly) connected to seller s, and let

[8] The importance of social networks for getting jobs has been long recognized. Granovetter (1974) documents that more than half of (white-collar) workers use personal connections to obtain jobs. Bewley (1999) summarizes 24 other US studies that point to similar results. Fainmesser (2011) shows that transmission of information over social networks can affect the timing of hiring in entry-level labor markets.

[9] Admittedly, eating is a social experience, so one might expect that clients share among themselves some information about past experiences. In Section VIB, we follow FG and allow for information sharing between buyers and consider its effect on market structure and cooperation.

$S_b(\mathbf{m})$ be the set of sellers that are connected to buyer b. The *degree* of seller s, $d_s = d_s(\mathbf{m}) = |B_s(\mathbf{m})|$ is the number of buyers that are connected to s; and the degree of buyer b, d_b, is the number of sellers that are connected to b. A *node* is an agent (buyer or seller) in the network. A *path* between node x and node x' in network $\mathbf{m}$ is a sequence of nodes $(x = x_0, x_1, x_2, \dots, x_n = x')$ such that for every $i \in \{1, 2, \dots, n\}$, $(x_{i-1}, x_i) \subset E$. The *length* of a path is the number of edges along the path. The *distance* between two nodes is the length of the shortest path between the two nodes. A path $(x = x_0, x_1, x_2, \dots, x_n = x')$ is also a *cycle* if $x = x'$. A *tree* is a network that has no cycles. A *rooted tree* is a tree in which one node is marked as the root. A node in a tree is called a *leaf* if its degree equals 1. The *depth* of a rooted tree is the largest distance between the root and any of the leafs in the tree. A network (tree) $\mathbf{m}'$ is a subnetwork (subtree) of $\mathbf{m}$ if $\mathbf{m}' \subset \mathbf{m}$. The network (tree) $\mathbf{m}'$ is a strict subnetwork (subtree) of $\mathbf{m}$ if $\mathbf{m}' \subsetneq \mathbf{m}$. A node x is called a *child* of a node x' in a rooted tree $\mathbf{m}$ if x and x' are connected AND x is at a larger distance from the root than x'.

The degree distribution in a network specifies for any d the fraction of buyers with degree d and the fraction of sellers with degree d. We use the degree distribution to express sellers' expectations with respect to the degrees of the buyers connected to them, sellers connected to the buyers connected to them, and so forth. Thus, for several of our results it is more convenient to denote the degree distribution in the following way: let $\mathbf{g} = \langle g^S, g^B \rangle$ be a pair of probability distributions such that if we choose a link $(s, b) \in E$ uniformly at random (u.a.r.), $g^B(d)$ is the probability that buyer b has degree d, and $g^S(d)$ is the (unconditional) probability that seller s has degree d.[10] We say that a probability distribution $\mathbf{g}$ is **admissible** if (i) for any d, $g^S(d)$, and $g^B(d)$ are *rational numbers*, and (ii) $\mathbf{g}$ has a *finite support*.

In every period, connected buyers and sellers meet at a random sequencing (i.i.d. across periods), or until their demand (if buyers) or supply (if sellers) has been exhausted. Formally, in every period, all of the links in E are ordered u.a.r. and then the links are chosen one by one according to that order. When a link (s, b) is chosen, s and b meet and get an opportunity to engage in trade unless either s or b has already traded (with anyone else) in the same period. Buyers and sellers observe only their own meetings, i.e., they do not observe the order in which links are chosen and meetings by other buyers and sellers occur. Hence, thinking of a period as a segment of time (e.g., a day) the random component in the order of meeting captures the idea that buyers and/or sellers do not know exactly when during a given segment of time they are going to meet and who their partner is going to meet before their meeting.[11]

REMARK 1: *We remain agnostic with respect to the formation of the network and treat the network as exogenous. It will become clear that by allowing sellers and buyers not to cooperate with each other we essentially allow them to eliminate links. Not allowing for the creation of new links captures the idea that*

[10] Conditional on n_b and n_s, there is a one-to-one mapping between g and the aforementioned fractions.

[11] The idea that interactions in markets have a random component is not new and is formalized in many models of market activity. As our focus is on the network structure and not on transient and irregular frictions in markets, we follow much of the networks literature and take the random component as exogenous (see also Bala and Goyal 2000; Pongou and Serrano 2011; and Manea 2011).

the formation of new relationships is a longer term process than the decision not to cooperate in a given period. Notably, we consider in our analysis also the complete buyer-seller network, so we do not a priori restrict the cooperation relationships that might persist. A suggested interpretation for our results is that given any physical or social underlying network, observed patterns of repeated trade are expected to take the form of a network that is consistent with trade and cooperation, e.g., an active trade network may be a strict subnetwork of a physical network that captures trade opportunities.

A. *Trade*

When seller s meets buyer b, seller s decides whether to invest in producing high quality (if possible) and whether to tell b that the good is of high quality or of low quality.[12] Buyer b decides whether to purchase the good from s or not. If b purchases the good, seller s receives a payoff of π (minus any production costs). Buyer b receives a positive net payoff if the good is of high quality, and a negative net payoff otherwise.[13] Payoffs are realized at the end of the period, and buyers and sellers who do not manage to trade in a given period have utility 0. Payoffs (and interaction outcomes) are privately observed and cannot be credibly communicated to a third party.[14]

DEFINITION 1: *We say that buyer b and seller s* ***cooperate*** *if when they meet:*

(1) *If s does not have high-quality capacity, she truthfully conveys that to b, and if s has high-quality capacity she invests in producing high quality (if b purchases the good).*

(2) *Buyer b chooses to purchase the good if and only if s claims to have high quality capacity.*

Note that the profit for seller s from not cooperating depends on the application through μ and c_s. Let $\overline{\Pi}^s$ be the maximal additional payoff that s can ever gain from deviation. In the adverse selection problem in Example 1, seller (recommender) s cannot choose the quality of the good and deviates by saying that a worker is of

[12] For some applications it is more natural to assume that a seller makes her quality decision at the beginning of a period, rather than upon meeting a buyer. This limits further the strategy space of sellers. Thus, for given discount factors, any network that supports cooperation in our setup does so in this alternative setup as well. The reverse claim is not correct. However, our results do not change much qualitatively.

[13] Our results can be extended to a setup in which there is also a market for low quality goods for a price that is lower than π as long as the difference between buyers' valuations for the high and low quality goods is greater than the difference in the production costs of the goods. One possible construction is by setting prices to leave buyers indifferent between purchasing a high quality good for π_H and a low quality good for π_L. If seller s ever sells a low quality good to buyer b for π_H, then buyer b agrees to pay only π_L to seller s in any of their future transactions.

Moreover, the model and all of the results extend immediately to games in which both parties have incentives to deviate (e.g., the standard prisoner's dilemma) as well as to stochastic games in which payoffs vary across periods.

[14] Section VIB considers credible communication between buyers so that more than one buyer can learn about a seller's deviation.

high quality when she is not.[15] As a result, s receives benefits of trade that would not have occurred had she told the truth, and $\overline{\Pi}^s = \pi$. In the moral hazard problem in Example 2, a deviation by a seller (caterer) is saving on effort costs, and $\overline{\Pi}^s = c_s$.

REMARK 2: *We assume that the payoff for a seller from a single transaction* (π) *does not depend on the network structure. Introducing endogenous bargaining increases the complexity significantly.*[16] *However, we note that* (i) *if we allow* π *to depend on* d_s, *any model in which the slope of* $\pi(d_s, \cdot)$ *is not too steep* (*at least for a large enough* d_s) *preserves the main insights of this paper; and* (ii) *in a bargaining procedure in which sellers make take-it-or-leave-it offers, it is straightforward to construct equilibria for which our analysis goes through without changes.*

B. *Large Networks and the Knowledge of the Network*

Our goal is to provide a framework for the analysis of large markets. This has proven to be a difficult task even in the study of static games, and especially when agents have complete knowledge of the network structure. Several authors suggest studying environments in which agents have incomplete information of the network structure. In particular, Jackson and Yariv (2007), and Galeotti et al. (2010) focus on static network games in which agents know only their own degree and the degree distribution in the network. We introduce an approach that is similar yet less restrictive, and derive conditions under which this approach simplifies the analysis of repeated games in networks. Assumption 1 is illustrated in Figure 2.

ASSUMPTION 1: *Seller s* (*buyer b*) *knows:* (i) *her own degree* d_s (d_b), (ii) *the degrees of all buyers* (*sellers*) *connected directly to her* $\{d_{b'}\}_{b' \in B_s}$ $(\{d_{s'}\}_{s' \in S_b})$, (iii) *the number of buyers and sellers in the network* (n_b *and* n_s *respectively*), *and* (iv) *the degree distribution* $\mathbf{g}$. *We denote by* $\mathbf{K}_s \triangleq \langle d_s, \{d_{b'}\}_{b' \in B_s}, n_s, n_b, \mathbf{g} \rangle$ $(\mathbf{K}_b \triangleq \langle d_b, \{d_{s'}\}_{s' \in S_b}, n_s, n_b, \mathbf{g} \rangle)$ *the knowledge that seller s* (*buyer b*) *has of the network structure.*

While stylized, Assumption 1 captures the idea that participants in the market have some knowledge of alternative trading opportunities of their trading partners. Restricting further the knowledge of the sellers and buyers does not change our analysis.[17] However, our results indicate that outside opportunities of trading partners have a first order effect on the incentives to cooperate. Extending the knowledge of sellers and buyers beyond $\mathbf{K}_s$ and $\mathbf{K}_b$ is an interesting exercise that we leave for future research.

To capture the idea that $\mathbf{K}_s$ and $\mathbf{K}_b$ contain all of the information that sellers and buyers have with respect to the network structure, we make the following assumption.

[15] Formally, suppose that $c_s = 0$ for all sellers. Then, a seller with high quality capacity always chooses to produce high quality and might benefit from a deviation only by misrepresenting her capacity to produce high quality.

[16] For models of bargaining in networks see Corominas-Bosch (2004); Abreu and Manea (2011); Elliott (2011a); Manea (2011); Nava (2010); and Gofman (2011).

[17] Our analysis holds for the informational assumptions used by Jackson and Yariv (2007), and Galeotti et al. (2010), as well as for intermediate levels of knowledge, in which buyers and sellers have imperfect knowledge of the degrees of sellers and buyers that are connected to them.

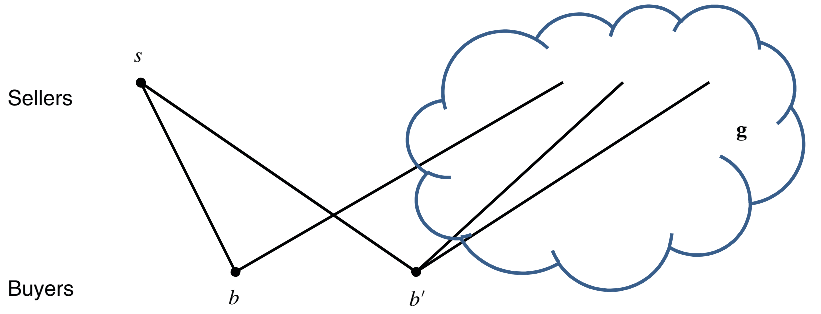


FIGURE 2.

Note: The network from the point of view of seller s who is connected to buyers b and b'.

ASSUMPTION 2: *At any period t, seller s (buyer b) attaches identical probability to the network being any of the possible networks conditional on $\mathbf{K}_s$ ($\mathbf{K}_b$).*[18]

REMARK 3: *The knowledge that individuals are expected to have in a repeated games setup deserves further discussion. Clearly, repeated interactions provide sellers and buyers with opportunities to learn about their environment. However, even excluding purely behavioral considerations as well as complexity issues, there are several reasons for market participants not to be able to learn beyond their close local network and some aggregate characteristics of the global environment. In Appendix C, we provide further discussion of our informational assumptions as well as an example of an environment in which our specification of incomplete knowledge of the network structure persists even if buyers and sellers use their own patterns of past interactions to learn the network structure.*

Large networks: To facilitate the study of large markets, consider the following notion of an increasing sequence of networks.[19]

DEFINITION 2: *Consider an admissible degree distribution* $\mathbf{g}$, *and let $m(n_b, \mathbf{g})$ be a network with n_b buyers and a degree distribution* $\mathbf{g}$. *We say that $\{m(n_b^i, \mathbf{g})\}_{i=1}^{\infty}$ is* ***an increasing sequence of networks*** *if for every $j > i$, $n_b^j > n_b^i$.*

For some $(n_b, \mathbf{g})$ a network $m(n_b, \mathbf{g})$ may not exist. In particular, for $m(n_b, \mathbf{g})$ to exist two conditions must be satisfied: (i) n_b must be such that g^B can be induced by some vector $(d_b^1, d_b^2, \ldots, d_b^{n_b})$; and (ii) there must exist some n_s and a vector $(d_s^1, d_s^2, \ldots, d_s^{n_s})$ such that $(d_s^1, d_s^2, \ldots, d_s^{n_s})$ is consistent with $\mathbf{g}$ and $\sum_{i=1}^{n_s} d_s^i = \sum_{i=1}^{n_b} d_b^i$. However, for every admissible $\mathbf{g}$, and starting from some n_b, there exists an increasing sequence

[18] Assumptions 1 and 2 are consistent with a seller (buyer) having a uniform prior over the set of all networks given n_s and n_b and updating her prior using K_s (K_b).

[19] See Golub and Jackson (2010), and Ozsoylev and Walden (2011) for a similar formulation of large networks in the context of information diffusion in networks.

as required.[20] In fact, given n_b and $\mathbf{g}$, n_s is uniquely determined (and is an increasing function of n_b given $\mathbf{g}$).

III. Equilibrium

In this section, we define a notion of a per-period future value (FV) of a connection that correspond to the following "simple-minded" calculation: assume that in all networks all buyers and sellers always cooperate and let the future value of the connection (s,b) in network $\mathbf{m}$ be the difference between the expected payoff of seller s in network $\mathbf{m}$ and her expected payoff in network $\mathbf{m}\backslash(s,b)$. Theorem 1 establishes conditions under which: (i) the naively calculated future values of links are sufficient statistics for determining whether a fully cooperative equilibrium exists, and (ii) the future values of links in a network $\mathbf{m}$ can be calculated as if $\mathbf{m}$ is a random tree. The following example demonstrates the simple conditions for cooperation in our model in a market with a single seller and a single buyer.

Example 3 (a market with one seller and one buyer):
Consider a single seller s who has unit capacity with probability μ and a single buyer b. Conditional on cooperation, with probability μ, s needs b in order to trade with a payoff $\pi - c_s$. Note that $\left(\delta_s/(1-\delta_s)\right) \cdot \mu \cdot (\pi - c_s)$ equals the maximal punishment that b can inflict on s (by not purchasing goods from s in subsequent periods). Therefore, an equilibrium in which seller s and buyer b cooperate exists if and only if $\left(\delta_s/(1-\delta_s)\right) \cdot \mu \cdot (\pi - c_s) \geq \overline{\Pi}^s$.

In networked markets with multiple sellers and buyers, the analysis is no longer straightforward. The maximal effective punishment that could be imposed on a seller s by a given buyer b depends on: (i) the outside option of the seller, and (ii) the frequency of interaction between s and b. Both (i) and (ii) depend on the entire network structure as well as on the strategies of all of the buyers and seller in the market. To acheive tractability without directly constraining the set of networks considered, we restrict attention to equilibria in which buyers and sellers use "trigger strategies".

DEFINITION 3: *We say that buyer b and seller s that are connected in the network use trigger strategies if there exists $T \in \mathbb{Z}^+$ such that when s and b meet they cooperate as long as neither deviated in an interaction with the other in the last T periods, and deviate otherwise.*[21]

DEFINITION 4: *A strict trigger Nash equilibrium (STNE) is a strict Nash equilibrium in which all buyers and sellers employ trigger strategies.*[22]

[20] This follows from the Gale-Reyser Theorem (see Krause 1996), and (in our particular setting) Theorem 1.3 of Greenhill, McKay, and Wang (2006).

[21] If $T = \infty$, a deviation effectively leads to the elimination of a link. By allowing T to be infinite we avoid imposing any constraint on the memory of buyers and sellers. Our results remain the same if we restrict T to be finite.

[22] In a strict Nash equilibrium, all players play a strict best response.

In the remainder of the paper, we focus on STNE unless stated otherwise. Limiting attention to trigger strategies rules out equilibria involving two families of strategies: (i) strategies in which a buyer b who is cheated by seller s responds by modifying her behavior in meetings with other sellers; and (ii) strategies in which a seller or a buyer respond to punishment spells between other buyers or sellers. As a result, our analysis does not consider contagion, which is not realistic in the markets motivating this paper.[23] Extending the analysis to a corresponding version of subgame perfect equilibrium (SPE) complicates the analysis significantly, but does not alter our results.[24]

Note that by definition, in any STNE, every buyer and seller that are connected cooperate with each other. Thus, on the equilibrium path, periods are ex-ante identical. For each seller s and buyer b, let $I^t_{\mathbf{m}}(s,b)$ denote the indicator of the event that s sold a good to b in period t in a STNE in network $\mathbf{m}$. We note that (i) $I^t_{\mathbf{m}}(s,b)$ is fully determined by the realizations of who of the sellers are active in period t and of the order of meetings in period t; and (ii) ex-ante $\Pr\left(I^t_{\mathbf{m}}(s,b)\right)$ is independent of t. A "simple-minded" calculation of the per-period future value of the link (s,b) for seller s is based on the difference in the probability of seller s selling a good in period t in network $\mathbf{m}$ and in the network $\mathbf{m}\backslash(s,b)$.

$$(1)\quad FV_{s,b}(\mathbf{m}) \triangleq \left[\sum_{b'\in B_s(\mathbf{m})} \Pr\left(I^t_{\mathbf{m}}(s,b')\right) - \sum_{b'\in B_s(\mathbf{m})\backslash b} \Pr\left(I^t_{\mathbf{m}\backslash(s,b)}(s,b')\right)\right] \cdot (\pi - c_s).$$

It is not at all obvious that we can extend the logic of Example 3 to claim that seller s cooperates with buyer b as long as $\left(\delta_s/(1-\delta_s)\right) \cdot FV_{s,b}(\mathbf{m}) > \overline{\Pi}^s$. For example, it is not clear that the best strategy of a seller s after deviating in an interaction with buyer b is to always cooperate with all other buyers in $B_s\backslash b$. Moreover, even if $FV_{s,b}(\mathbf{m})$ is a sufficient statistic for the existence of a STNE, $\Pr\left(I^t_{\mathbf{m}}(s,b)\right)$ is a complex mathematical object, making it costly to compute and analyze $FV_{s,b}(\mathbf{m})$, both for the modeler and for an expected utility maximizing seller s. In fact, given the information set $\mathbf{K}_s$, a direct calculation requires the computation of $\Pr\left(I^t_{\mathbf{m}}(s,b)\right)$ for each network $\mathbf{m}$ that is possible given the information of seller s, and then calculating the average over all such networks. Theorem 1 resolves this issue.

Consider a network $\mathbf{m}$ with a degree distribution $\mathbf{g}$, and a seller s with degree d_s. Let $\overline{\mathbf{b}}_s \in (\mathbb{Z}^+)^{d_s}$ be a sorted vector of the degrees of all buyers in $B_s(\mathbf{m})$. Now, let $T^d(\mathbf{m},s)$ denote the random depth $-d$ tree such that the root r has degree d_s, the sorted vector of degrees of the children of r is $\overline{\mathbf{b}}_s$, all subsequent nonleaf nodes at an even depth have a degree drawn i.i.d. according to g^S, all subsequent nonleaf nodes at an odd depth have a degree drawn i.i.d. according to g^B. Let $FV_{s,b}\left(T^\infty(\mathbf{m},s)\right) \triangleq \lim_{d\to\infty} FV_{s,b}\left(T^d(\mathbf{m},s)\right)$. Theorem 1 establishes that $\left\{FV_{s,b}\left(T^\infty(\mathbf{m},s)\right)\right\}_{(s,b)\in E}$ exist

[23] Section VIB discusses the implications of relaxing the assumption that payoffs are privately observed and allowing for community enforcement.

[24] Extending the analysis to a corresponding version of SPE requires an explicit assumption on whether buyers and sellers can detect punishment spells between other buyers and sellers. The analysis goes through with SPE if we assume that sellers and buyers do not update from the pattern of interaction about punishment spells in other parts of the network. This assumption is especially reasonable in large networks, and given incomplete knowledge of the network, private information of the payoffs, and random and unobserved order of interactions.

and are sufficient statistics to determine whether there exists an STNE with a large network $\mathbf{m}$.

THEOREM 1: *For any network* $\mathbf{m}$, $\{FV_{s,b}(T^{\infty}(\mathbf{m},s))\}_{(s,b)\in E}$ *exist. Moreover, let* $\mathbf{g}$ *be any admissible degree distribution. Then, for any increasing sequence of networks* $\{m(n_b^i,\mathbf{g})\}_{i=1}^{\infty}$ *there exists* $\bar{i}$ *such that for any* $i > \bar{i}$ *a STNE with network* $m(n_b^i,\mathbf{g})$ *exists if and only if for every seller s and buyer b that are connected in* $m(n_b^i,\mathbf{g})$,

$$\frac{\delta_s}{1 - \delta_s} \cdot FV_{s,b}(T^{\infty}(m(n_b^i,\mathbf{g}),s)) > \overline{\Pi}^s. \tag{2}$$

Theorem 1 implies that we can approximate the analysis of an STNE in any large network by focusing on a simple auxiliary network—a random tree. The proof consists of three main parts:

(i) In **any network m**, $FV_{s,b}(\mathbf{m})$ can be approximated by $FV_{s,b}(m^{\Delta}(s,\mathbf{m}))$ where $m^{\Delta}(s,\mathbf{m})$ is the subnetwork of $\mathbf{m}$ that consists of the links that are at a distance of no more than some constant Δ (independent of the size of the network $\mathbf{m}$) from seller s in $\mathbf{m}$ (and only those links). This step holds for any network structure and is therefore independent of our informational assumptions (Assumptions 1 and 2). Intuitively, the random order of meetings in every period implies that links that are "far" from seller s in the network have only a small impact on the probability that seller s trades with any buyer b. This is because a link (s',b') can influence the trade between s and b only if (i) (s',b') is chosen before (s,b), and (ii) there exists at least one path connecting (s',b') and (s,b) such that all of the links along the path are chosen before (s,b) and after (s',b'). The probability of (ii) is decreasing in the length of the aforementioned path.

(ii) Consider a large network $\mathbf{m}$ that is chosen u.a.r. conditional on an admissible degree distribution. Then (asymptotically on the size of the network) for any fixed Δ, the distribution ruling the shape of the local neighborhood of any seller s $(m^{\Delta}(s,\mathbf{m}))$ converges to the underlying distribution of a random tree of the same depth (Δ) as the local neighborhood considered. This step relies on the requirement that the degree distribution has finite support.

(iii) In any tree (not necessarily random), eliminating one of a seller's links weakly increases the future values of her remaining links. Consider a tree $\mathbf{T}$ and a seller s who is connected to buyers b and b' (and maybe some additional buyers). Then, $FV_{s,b}(\mathbf{T}) \leq FV_{s,b}(\mathbf{T}\backslash(s,b'))$. Intuitively, the tree structure implies that the only connections between two links (s,b) and (s,b') is via seller s. As a result, having less links affects the future values of remaining links only by decreasing the outside option of the seller.

Combining (i) and (ii) allows us to focus in our analysis on random trees. Given the focus on a tree structure, (iii) implies a one-deviation principle. The complete proof of Theorem 1 makes use of recent results by FG and is deferred to Appendix A.

We now take a closer look at the implications of Theorem 1. Consider a network $\mathbf{m}$ with a degree distribution $\mathbf{g}$, and a link $(s,b) \in E$. Let $T^d(\mathbf{m},s,b)$ denote the random depth$-d$ tree such that the root r has degree 1, the degree of the only child of r is d_b, all subsequent nonleaf nodes at an even depth have a degree drawn i.i.d. from g^S, all subsequent nonleaf nodes at an odd depth have a degree drawn i.i.d. from g^B. In words, $T^d(\mathbf{m},s,b)$ is constructed in the same way as the subtree of $T^d(\mathbf{m},s)$ that results from disconnecting all buyers (except from b) from seller s. In the context of the bigger network $T^d(\mathbf{m},s)$, $\Pr\big(I^t_{T^d(\mathbf{m},s,b)}(s,b)\big)$ captures the probability that seller s has the ability to produce a high quality good, and buyer b does not purchase a good before meeting seller s. Then, the future value of a link in a random tree $T^d(\mathbf{m},s)$ can be rewritten as

$$(3) \qquad FV_{s,b}\big(T^d(\mathbf{m},s)\big) = (\pi - c_s) \cdot \Pr\big(I^t_{T^d(\mathbf{m},s,b)}(s,b)\big) \cdot \Pi_{b' \in B_s \setminus b}\Big[1 - \frac{1}{\mu} \cdot \Pr\big(I^t_{T^d(\mathbf{m},s,b')}(s,b')\big)\Big].$$

With respect to $T^d(\mathbf{m},s)$, the expression

$$(4) \qquad \Pr\big(I^t_{T^d(\mathbf{m},s,b)}(s,b)\big) \cdot \Pi_{b' \in B_s \setminus b}\Big[1 - \frac{1}{\mu} \cdot \Pr\big(I^t_{T^d(\mathbf{m},s,b')}(s,b')\big)\Big]$$

captures the probability that in period t, seller s has the ability to produce high quality good, AND buyer b has demand for a good when he meets seller s, AND no other buyer $b' \in B_s \setminus b$ has demand when their link with seller s is chosen. Thus, seller s sells a good if she is connected to b, but would not have been able to sell a good had she not been connected to buyer b. The simple expression is due to the tree structure that guarantees the independence of $\big\{I^t_{T^d(\mathbf{m},s,b)}(s,b)\big\}_{b \in B_s}$ of each other. Moreover, for every seller s and buyer $b' \in B_s$, the tree structure and the independence of the degrees across subtrees guarantee that comparative statics over $\Pr\big(I^t_{T^d(\mathbf{m},s,b)}(s,b)\big)$ are governed by the following simple graph-theoretic rule.

LEMMA 1: *Suppose that for all $d \geq 1$, the random tree $\mathbf{T}^d_2 = T^d(\mathbf{m},s,b)$ can be constructed (on the same probability space) from the random tree $\mathbf{T}^d_1 = T^d(\mathbf{m}',s',b')$ by performing only the two operations:* 1. *appending (as children) subtrees to seller nodes in an arbitrary way, and* 2. *removing (as children) subtrees from buyer nodes in an arbitrary way. Then the probability that seller s sells to buyer b in a given period in $\mathbf{T}^d_2$ is at least as big as the equivalent probability in $\mathbf{T}^d_1$ $\big(\Pr\big(I^t_{\mathbf{T}^d_2}(s,b)\big) \geq \Pr\big(I^t_{\mathbf{T}^d_1}(s,b)\big)\big)$.*

Note that any change to the degree of a node in the network can be captured by appending or removing subtrees from the corresponding random tree. E.g., appending (as children) subtrees to seller nodes in the corresponding random tree can capture: (i) adding a link between s and some buyer; and/or (ii) increasing the degree distribution of sellers in the network as a whole. Lemma 1 shows that the effects of (i) and (ii) on $\Pr\big(I^t_{T^d(\mathbf{m},s,b)}(s,b)\big)$ are qualitatively the same. Given Theorem 1 and equation (3) this simplifies the analysis of the effect of the same changes to the networks structure on sellers' incentives to cooperate.

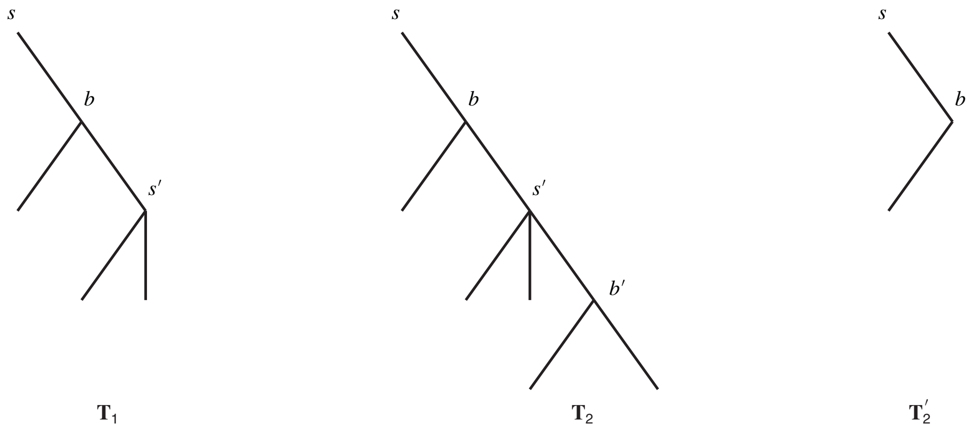


FIGURE 3.

Notes: The tree $\mathbf{T}_2$ can be constructed from $\mathbf{T}_1$ by appending (as a child) the subtree rooted at b' to the seller node s'. Similarly, $\mathbf{T}_2'$ can be constructed from $\mathbf{T}_1$ (or from $\mathbf{T}_2$) by removing (as a child) the subtree rooted at s' from the buyer node b. It is easy to verify that the probability that seller s sells to buyer b in a given period is larger in $\mathbf{T}_2'$ than in $\mathbf{T}_2$ and larger in $\mathbf{T}_2$ than in $\mathbf{T}_1$, i.e., $\Pr\big(I^t_{\mathbf{T}_2'}(s,b)\big) > \Pr\big(I^t_{\mathbf{T}_2}(s,b)\big) > \Pr\big(I^t_{\mathbf{T}_1}(s,b)\big)$.

Consider a seller s that is connected only to one buyer b and is at the root of a tree $\mathbf{T}'$. Lemma 1 implies that eliminating any seller $s' \neq s$ from the tree weakly increases the probability that s sells a good in any period, and that eliminating any buyer b from the tree weakly decreases the probability that s sells a good in any period. Figure 3 provides an example.

In the following section, we rely on Theorem 1 and Lemma 1 to characterize the set of networks for which STNE exist in terms of economically meaningful network parameters such as degree distribution and segregation.

IV. Network Structure and Cooperation

We now examine the relationship between the structure of network $\mathbf{m}$ and $FV_{s,b}\big(T^\infty(\mathbf{m},s)\big)$. A higher $FV_{s,b}\big(T^\infty(\mathbf{m},s)\big)$ implies more cooperation in a large network $\mathbf{m}$ in two ways: (i) holding the immediate payoff from deviation $\big(\overline{\Pi}^s\big)$ fixed, a higher $FV_{s,b}\big(T^\infty(\mathbf{m},s)\big)$ means that a lower discount factor (δ_s) is sufficient to support sustained cooperation on the link (s,b), and (ii) holding δ_s fixed, a higher $FV_{s,b}\big(T^\infty(\mathbf{m},s)\big)$ implies that cooperation can be sustained over the link (s,b) even if $\overline{\Pi}^s$ is higher. We relate our results to the level of competition between sellers in $\mathbf{m}$, the density of $\mathbf{m}$, and the level of segregation exhibited by $\mathbf{m}$. All proofs are deferred to Appendix A.

A. *Competition: The Relative Degrees of Buyers and Sellers*

Competition is imbalanced if many sellers with low degrees are connected to buyers with high degrees (fierce and imbalanced competition), or if sellers with high degrees are connected to many buyers with low degrees (weak and imbalanced competition). Competition is *moderate* and *balanced* if buyers and sellers have degrees that are similar and not too large. We find that the future values of links are highest in networks that exihibit moderate and balanced competition.

To see why, first note that in a given network $\mathbf{m}$, $FV_{s,b}(T^{\infty}(\mathbf{m},s))$ is lower for links in which the buyer (seller) has a high degree than for links in which the buyer (seller) has a low degree, i.e., each link is more valuable the fewer outside options both sides of the link have. For example, in Figure 1B, sellers s and s' have identical information sets with the only exception that seller s' is connected to one less buyer. Then, $FV_{s',b}(T^{\infty}(\mathbf{m},s')) > FV_{s,b}(T^{\infty}(\mathbf{m},s))$. Intuitively, seller s has connections to buyers with the same degrees as the buyers that s' is connected to and is also connected to an additional buyer. As a result, s has a better outside option in the case that buyer b does not purchase the good from her (compared with the outside option of seller s' in case that buyer b does not purchase the good from her). Similarly, consider a seller s who is connected to two buyers. Seller s has a higher value for the link with the buyer that has the lower degree of the two. This is because seller s expects fewer periods with demand from the buyer with the higher degree than periods with demand from the buyer with the lower degree (see Proposition 1 in Appendix B for details).

On the other hand, if the degrees of buyers in $B_s \backslash b$ are large, s is more likely to *need* buyer b in order to make a sale in period t. For example, in Figure 1A, if we add a connection between buyer b and some seller s' that we add to the figure, the connection (s,b) becomes less valuable, whereas the connection (s,b') becomes more valuable. This example, which is generalized in Proposition 2 in Appendix B, captures the positive externality of links: if $d_{b_1'} > d_{b_1}$ seller s' expects less periods with demand from b_1' than seller s expects periods with demand from b_1. As a result, s' (more than s) is likely to need her other connections in order to sell the good.

Theorem 1 and Lemma 1 also allow us to evaluate the effect of differences in the degree distribution across networks. Consider the minimal value of any link of seller s as defined by

$$\underline{FV}_s(\mathbf{m}) = \min_{b \in B_s}\{FV_{s,b}(T^{\infty}(\mathbf{m},s))\}. \tag{5}$$

Recall that if d_b is large, $FV_{s,b}(T^{\infty}(\mathbf{m},s))$ is small. This is mitigated if for every $b' \in B_s \backslash b$, $d_{b'}$ is also very large. Thus, networks in which buyers have "similar" degrees have a larger $\{\underline{FV}_s(\mathbf{m})\}_{s\in S}$.

More generally, consider two networks $\mathbf{m}$ and $\hat{\mathbf{m}}$ with degree distributions $\mathbf{g}$ and $\hat{\mathbf{g}}$ respectively. Suppose that $\widehat{g^B}$ first order stocastically dominates (FOSD) g^B and g^S FOSD $\widehat{g^S}$, and consider two sellers $s \in \mathbf{m}$ and $\hat{s} \in \hat{\mathbf{m}}$ with identical local neighborhoods, so that the only difference between their information sets ($\mathbf{K}_s$ and $\mathbf{K}_{\hat{s}}$) is the difference in the degree distributions $\mathbf{g}$ and $\hat{\mathbf{g}}$. Theorem 2 shows that (i) if s and $\hat{s}$ are connected to many buyers (large d_s and $d_{\hat{s}}$) then the fact that sellers are overall more connected, and buyers are overall less connected in $\mathbf{m}$ relative to $\hat{\mathbf{m}}$, implies that seller $\hat{s}$ has higher incentives to cooperate relative to seller s; and (ii) if s and $\hat{s}$ are connected to a small number of buyers (small d_s and $d_{\hat{s}}$), then the same difference in degree distributions implies that seller $\hat{s}$ has lower incentives to cooperate relative to seller s. The differences in degree distribution that are analyzed in Theorem 2 are illustrated in Figure 4.

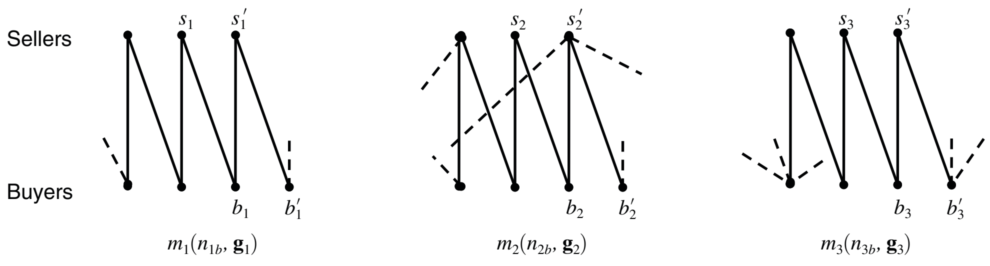


FIGURE 4.

Notes: In networks $\mathbf{m}_1$, $\mathbf{m}_2$, and $\mathbf{m}_3$ above, the broken lines represent links to buyers and sellers that are not in the diagram. Counting only the buyers and sellers in the figure, g_2^S FOSD g_1^S and g_3^B FOSD g_1^B. At the same time, sellers s_1, s_2, and s_3 have identical local neighborhoods.

THEOREM 2: *Let* $\widehat{g^{\hat{B}}}$ *FOSD* g^B, *and* g^S *FOSD* $\widehat{g^{\hat{S}}}$, *and let* $\mathbf{m} = \langle S, B, E \rangle$ *and* $\hat{\mathbf{m}} = \langle \hat{S}, \hat{B}, \hat{E} \rangle$ *be two networks with degree distributions* $\mathbf{g}$ *and* $\hat{\mathbf{g}}$ *respectively. Consider sellers* $s \in S$ *and* $\hat{s} \in \hat{S}$ *with identical local neighborhoods (equal degrees,* $d_{\hat{s}} = d_s$, *and identical vectors of neighbors' degrees,* $\{d_{\hat{b}}\}_{\hat{b} \in B_{\hat{s}}} \equiv \{d_b\}_{b \in B_s}$*). Then, there exist thresholds* $\overline{d}_s(\mathbf{m}, \hat{\mathbf{m}})$ *and* $\underline{d}_s(\mathbf{m}, \hat{\mathbf{m}})$, *such that*

(1) *if the degrees of s and* $\hat{s}$ *are below* $\underline{d}_s$, *then* $\underline{FV}_{\hat{s}}(T^{\infty}(\hat{\mathbf{m}}, \hat{s})) \leq \underline{FV}_s(T^{\infty}(\mathbf{m}, s))$, *and*

(2) *if the degrees of s and* $\hat{s}$ *are above* $\overline{d}_s$, *then* $\underline{FV}_{\hat{s}}(T^{\infty}(\hat{\mathbf{m}}, \hat{s})) \geq \underline{FV}_s(T^{\infty}(\mathbf{m}, s))$.

If g^S FOSD $\widehat{g^{\hat{S}}}$, the aggregate demand per seller in network $\mathbf{m}$ is larger than in $\hat{\mathbf{m}}$. This difference in effective demand affects the difference between the probability that $\hat{s}$ gets an opportunity to sell to $\hat{b}$ $\left(\Pr\left(I^t_{T^d(\hat{\mathbf{m}}, \hat{s}, \hat{b})}(\hat{s}, \hat{b})\right)\right)$, and the probability that s gets an opportunity to sell to b $\left(\Pr\left(I^t_{T^d(\mathbf{m}, s, b)}(s, b)\right)\right)$. In particular, $\Pr\left(I^t_{T^d(\mathbf{m}, s, b)}(s, b)\right) \geq \Pr\left(I^t_{T^d(\hat{\mathbf{m}}, \hat{s}, \hat{b})}(\hat{s}, \hat{b})\right)$, and generically the inequality is strict. This is true even if the local environments around (s, b) and around $(\hat{s}, \hat{b})$ are identical.

The difference between $\Pr\left(I^t_{T^d(\mathbf{m}, s, b)}(s, b)\right)$ and $\Pr\left(I^t_{T^d(\hat{\mathbf{m}}, \hat{s}, \hat{b})}(\hat{s}, \hat{b})\right)$ affects the value of links in two ways: (i) s has a better outside option than $\hat{s}$ in case one of her links is lost; and (ii) s has higher frequency of interactions with each one of the buyers connected to her. If sellers s and $\hat{s}$ are connected to many buyers, outside options are affected strongly by the difference in degree distributions, and (i) dominates. Consequently, seller s has lower values of links because she is very likely to sell even if she had less connections. On the other hand, if sellers s and $\hat{s}$ have only few connections (e.g., suppose that each is connected to only one buyer), the outside option is hardly affected by the degree distribution. However, the frequency of interactions is affected and (ii) dominates. The impact of differences in buyers' degree distributions follow a similar logic.

Summarizing our results so far, a network admits a STNE if: (i) buyers have degrees that are similar enough, (ii) sellers have degrees that are similar enough, and (iii) buyers' degrees are not too small or too large relative to those of the sellers that are connected to them. An immediate implication is that there exists a "bliss point"

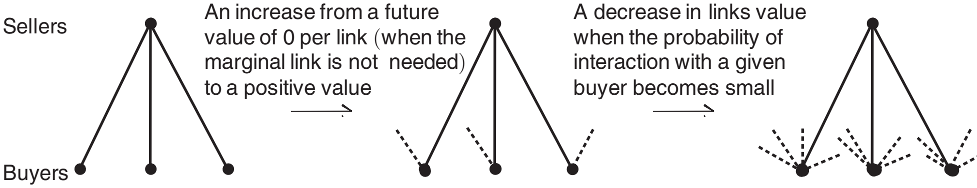


FIGURE 5.

Notes: When d^B is low (e.g., the leftmost network), each of the buyers connected to a seller is likely to have demand when meeting the seller, and the seller is likely to sell even if she has fewer connections. Raising d^B a little decreases the probability of a sale and the seller needs more connections. However, a drastic increase in d^B reduces the frequency with which a seller interacts with each buyer and the value of each link decreases.

to the ratio of buyers to sellers in any small neighborhood in the network, as well as in the network as a whole. We interpret our results in this section as suggesting that networks that exhibit *moderate and balanced competition* support a STNE for a large range of discount factors. The following example illustrates our interpretation by considering simple networks in which all sellers have the same degrees and production costs, and all buyers have the same degrees. In this special case, if buyers have degrees that are very small, or very large, relative to the degrees of sellers, the future values of links are low. Example 4 is generalized in Proposition 3 in Appendix B.

Example 4 (semi-regular networks):
Let $c_s = c$ for every $s \in S$, and consider a network $\mathbf{m}$ in which all buyers have degree d^B and all sellers have degree d^S. Thus, the values of all of the links in $\mathbf{m}$ are identical (i.e., for every $(s,b),(s',b') \in E$, $FV_{s,b}(T^{\infty}(\mathbf{m},s)) = FV_{s',b'}(T^{\infty}(\mathbf{m},s'))$). Denote this value by $FV^T(d^B,d^S)$. As illustrated in Figure 5, for every sellers' degree d^S there is a closed interval of values of d^B that maximizes $FV^T(d^B,d^S)$ and supports cooperation for the lowest feasible discount factor given d^S.

This section shows that networks which facilitate moderate and balanced competition are better in sustaining cooperation.[25] Consequently, one might expect to find moderate and balanced competition in networked markets that manage to rely on bilateral cooperation. In the following section, we show that the need to enforce cooperation may also constrain (or be constrained by) the overall connectivity in a network.

B. *Connectivity: Network Density*

Changes in observed patterns of trade are often attributed to corresponding changes in trade opportunities, which are in turn influenced by processes of modernization that reduce the costs of communication and transportation.[26] We now

[25] In a related work on competition and seller's reputation in an environment with price competition and no network, Bar-Isaac (2005) finds that competition can both aid and hinder reputation for quality.
[26] See Watts (2003) for a nontechnical survey.

evaluate whether such changes are consistent with sustaining cooperation. A negative result will imply that in an environment with moral hazard one should expect to observe such changes to a lesser extent in a network plotted based on data of the patterns of trade compared with a network plotted based on data of patterns of accessibility or acquaintances.

We consider two changes suggested in the literature: first, an increase in the degrees of agents in the network, often attributed to a reduction in the costs of creating and sustaining (trade) relationships. Second, a decrease in segregation, often attributed to a decline in costs of sustaining (trade) relationships across geographically distant regions.[27]

The effect of the first change is straightforward: if the degrees of buyers and sellers are "too" large, cooperation becomes impossible to sustain, even with moderate and balanced competition. Intuitively, the pivotal probability that seller s manages to sell to a specific buyer b, but would not have managed to sell to any other buyer, is negligible when sellers and buyers have many connections.

THEOREM 3: *Let* $m(\alpha, D)$ *be some network in which* $\min_b\{d_b\} = D$ *and* $\min_s\{d_s\} = \alpha \cdot D$. *For every* α, μ, *and* $\overline{FV} > 0$ *there exist* $\overline{D}(\alpha, \mu)$ *such that if* $D > \overline{D}$ *then*

$$\min_{(s,b)\in E}\left\{FV_{s,b}\left(T^{\infty}\left(m(\alpha, D), s\right)\right)\right\} < \overline{FV}. \tag{6}$$

As Theorem 3 reveals, a major value-creating role of the network is to provide coordination and specify who cooperates with whom. This necessary coordination is lost when sellers and buyers have many links. When anyone can potentially cooperate with everyone else, the value of a cooperating partner goes down, as each partner has only a small influence on outcomes.

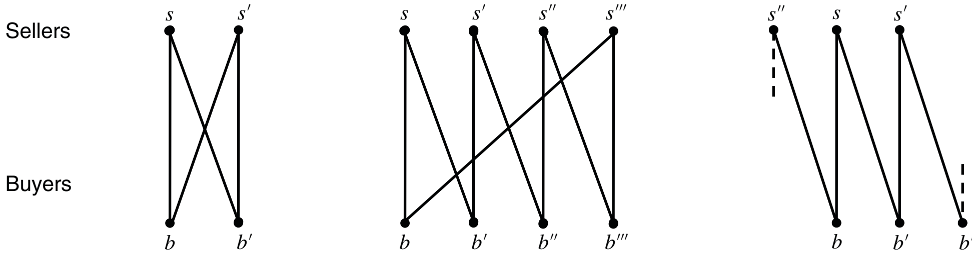
C. *Beyond the Degrees: Community Size and Segregation*

We now show that, holding all else equal, the ability to define small communities according to real (e.g., geographic) or artificial boundaries may increase the future values of links and improve the ability to sustain cooperation. Deriving this result requires extending our model to allow sellers and buyers to know more about their environment. Clearly, if sellers and buyers know that the network is divided to small communities their beliefs might be such that their incentives to cooperated cannot be approximated by the analysis of the corresponding random tree. Fortunately, we can compare the values of links in segregated versus in tree-like networks. Given our approximation result this maps to a comparison between networks that are known to be segregated and networks that are not.

For simplicity, let $c_s = c$ and let $d_s = d^S$, and $d_b = d^B$ for every seller s and buyer b throughout this section.[28] We extend our analysis to consider networks that are

[27] See also Rosenblat and Mobius (2004).

[28] Considering more general degree distributions requires putting additional structure in order to define segregation properly.



$\Psi = 1, d^S = d^B = 2$ $\Psi = 2, d^S = d^B = 2$ $\Psi \to \infty, d^S = d^B = 2$

Figure 6.

Notes: If seller s is informed that $\Psi = 1$, she knows that some seller s' is connected both to buyer b and to b'. Thus, seller s knows that there is perfect overlap between B_s and B_s' and between S_b and S_b'. If seller s is informed that $\Psi \to \infty$, Theorem 1 applies. Thus, seller s behaves as if she knows that apart from herself there is no other seller to whom both buyer b and b' are connected.

divided into islands, such that there are no links between buyers and sellers from different islands. In each island there are $\Psi \cdot d^B$ sellers, and $\Psi \cdot d^S$ buyers. $\Psi \in \mathbb{N}$ represents the size of each "island community." When comparing across networks with different Ψ, we keep d^B and d^S constant. This allows us to discriminate between the effects of differences in community sizes, and the effects of differences in the degrees of sellers and buyers. Figure 6 provides an illustration of a sample of networks with $d^S = d^B = 2$ and different values of Ψ.

Varying Ψ continuously raises technical difficulties and is beyond the scope of this paper. Instead, we focus on two interesting limit cases.

DEFINITION 5: *We say that a network is* ***segregated*** *if it is divided to small islands in which each of a group of d^S buyers is connected to each of a group of d^B sellers* $(\Psi = 1)$. *We say that a network is* ***global*** *if it is chosen u.a.r. conditional on d^B, d^S, n_s, and n_b, and without restrictions on Ψ.*

We are interested in the following question: When does the existence of a STNE in a segregated network imply that a STNE exists in the corresponding global network and vice versa? Given that sellers' actions are driven by their expectations of future trade, allowing for different network architectures makes a difference for the existence of a STNE only if sellers are aware of the differences.

ASSUMPTION 3: *Sellers know whether the network is segregated or global.*[29]

Consider a segregated network $\mathbf{m}_{seg}$. It is still true that a STNE exists if and only if for every seller s and buyer b that are connected $\left(\delta_s/(1-\delta_s)\right) \cdot FV_{s,b}(\mathbf{m}_{seg}) > \overline{\Pi}^S$. Consequently, Theorem 4(1a) suggests that in *sparse* networks that exhibit *moderate*

[29] A segregated network's topology is unique (up to permutations on the names of buyers and sellers). Consequently, when the network is segregated sellers put probability 1 on the correct network structure and our incomplete information environment coincides with one of complete information.

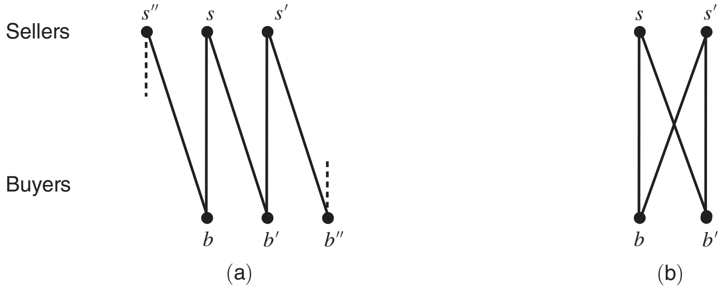


FIGURE 7.

and balanced competition, the following claim is true: if there exists a STNE in a global network, a STNE also exists in a segregated network with the same degree distribution.

THEOREM 4: *Let* $m_{seg}(d^B, d^S)$ *be a segregated network with degrees* d^B, d^S, *and let* $FV^{seg}(d^B, d^S) \triangleq FV_{s,b}(m_{seg}(d^B, d^S), s)$.[30] *Then,*

(1) *There exists* $\overline{d^S} > 1$ *such that for every* $d^S \leq \overline{d^S}$:

  (a) (*Moderate and balanced competition*) *There exist* $\underline{d^B} > 1$ *such that* $d^S \leq d^B \leq \underline{d^B}$ *implies that* $FV^{seg}(d^B, d^S) \geq FV^T(d^B, d^S)$.

  (b) (*Fierce competition*) *For every* $0 << \mu < 1/2$ *there exists* $\overline{d^B}(\mu)$ *such that* $d^B \geq \overline{d^B}$ *implies that* $FV^{seg}(d^B, d^S) \leq FV^T(d^B, d^S)$.

(2) (*Weak competition*) *If* $d^S > d^B$ *then* $FV^{seg}(d^B, d^S) \leq FV^T(d^B, d^S)$.

Sketch of the Proof: Theorem 4 is driven by two countervailing forces that can be demonstrated in Figure 7. On the one hand, *without* the link (s, b) in both networks, the segregated network in Figure 7B provides s with a higher probability of trading than the global network in Figure 7A. This is because in Figure 7B seller s' does not face any competition for selling to b, whereas in Figure 7A s' faces competition for selling to buyer b''. Therefore, s' is more likely not to sell to b' in Figure 7B. This causes the value of (s, b) in the segregated network (Figure 7B) to be *lower* than in the global network (Figure 7A).

Now consider "adding back" (s, b) to both networks. In the global network in Figure 7A, the opportunity for seller s to sell the good to buyer b is *independent* of her opportunity to sell the good to buyer b'. On the other hand, in Figure 7B, the opportunity of seller s to sell the good to b is *negatively correlated* with her opportunity to sell the good to b'. In fact, in the segregated network in Figure 7B, s is guaranteed to trade if she has a link to b. This causes the value of (s, b) in the segregated network (Figure 7B) to be higher than in the global network (Figure 7A). In the specific

[30] Due to the symmetry across sellers, buyers, and islands, the future value of a link in a segregated network does not depend on the size of the network (n_s and n_b) and is identical across links in the network.

networks in Figure 7, the second force dominates and the value of the link between s and b is higher in the segregated network.

If d^B is large, the negative correlation is weak; not being able to trade with b implies only that s has at most one less competitor for trading with any buyer $b' \in B_s \backslash b$. However, it is still the case that a seller with a missing link has higher probability of trading in the segregated network.

The second part of Theorem 4 is straightforward. In a segregated network with more buyers than sellers, a seller is guaranteed to trade with or without her marginal link. Consequently, the value of each link is zero. This is not true in a global network.

V. Welfare

In this section, we pose the following questions. When can cooperation that is supported only by repeated interaction in the network achieve the social optimum? What networks maximize aggregate welfare? What networks maximize constrained aggregate welfare when maximal welfare is not attainable? We express the answers to these questions in terms of solutions for two design problems. Let $\Delta\mathbf{m}$ be any probability distribution over network structures. In the *unconstrained network design problem*, a planner chooses $\Delta\mathbf{m}$ and *compels* all sellers (and buyers) to always cooperate. In the *cooperation constrained network design problem*, the planner chooses $\Delta\mathbf{m}$ and *recommends* that all sellers (and buyers) always cooperate. Sellers and buyers are then informed of $\Delta\mathbf{m}$ (as well as of their own degrees and the degrees of buyers and sellers that are connected to them) and follow the planner's recommendation if and only if $\Delta\mathbf{m}$ admits a STNE.

For a given network $\mathbf{m}$, let $E[V(\mathbf{m})] = E\big[\sum_{s\in S}\sum_{b\in B_s(\mathbf{m})}\Pr\big(I^t_{\mathbf{m}}(s,b)\big)\big]$ denote the expected volume of trade (number of transactions) in high quality goods that is achieved in a given period if all sellers (and buyers) always cooperate. Denote by $E[V(\Delta\mathbf{m})]$ the corresponding value given a probability distribution $\Delta\mathbf{m}$ over networks. Let $N^{uc}(B,S,\mu,\{c_s,\delta_s\}_{s\in B},\pi)$ $\big(N^{c}(B,S,\mu,\{c_s,\delta_s\}_{s\in B},\pi)\big)$ be the solution to the unconstrained (constrained) network design problem. Then,

$$N^{uc}(\cdot) \;=\; \arg\max_{\Delta\mathbf{m}} E[V(\Delta\mathbf{m})] \tag{7}$$

and

$$N^{c}(\cdot) \;=\; \arg\max_{\Delta\mathbf{m}\,|\,\Delta\mathbf{m}\text{ admits a STNE}} E[V(\Delta\mathbf{m})]. \tag{8}$$

Recall that transactions in high-quality goods are mutually beneficial. Thus, the proportion of the per-period welfare loss due to the constraints on the structure of networks that support STNE is

$$L(B,S,\mu,\{c_s,\delta_s\}_{s\in B},\pi) \;=\; 1 \;-\; \frac{E\big[V\big(N^{c}(\cdot)\big)\big]}{E\big[V\big(N^{uc}(\cdot)\big)\big]}\,. \tag{9}$$

If $L(\cdot) = 0$, then repeated interactions support the social optimum in every period.[31] The following definition is useful for interpreting our results.

DEFINITION 6: *An environment is* **constantly over-** (**under-**) **demanded** *if there are weakly more* (*less*) *buyers than sellers with high quality unit capacity in every period. We call an environment* **stationary** *if it is constantly over-* (*under-*) *demanded and* **stochastic** *otherwise.*

Recall that each buyer has unit demand in every period. Therefore, an environment is stationary if either there are less sellers than buyers ($n_s \leq n_b$), or if there are more sellers than buyers ($n_s > n_b$) *and* each seller has high quality capacity in every period ($\mu = 1$). In a stochastic environment there are more sellers than buyers ($n_s > n_b$) *and* each seller's high quality capacity is determined randomly in every period ($\mu < 1$). Theorem 5 shows that there exist a trade-off between sustaining cooperation and maximizing the volume of trade in stochastic environments, but not in stationary ones.[32]

THEOREM 5:

(1) *Assume that for every* $s \in S$, $\left(\delta_s/(1-\delta_s)\right) \cdot \mu \cdot (\pi - c_s) > \overline{\Pi}^s$. *Then, if the environment is stationary, there is no welfare loss imposed by the need to sustain cooperation via repeated interactions* (*i.e.,* $L(\cdot \mid \mu = 1) = 0$ *and* $L(\cdot \mid n_s \leq n_b) = 0$).

(2) *Consider a stochastic environment and assume that for every seller* $s \in S$, $\delta_s \in (0,1)$. *Then,*

(a) *there exists* $\overline{n}_b \in \mathbb{Z}^+$ *such that for all* $n_b > \overline{n}_b$, *the proportion of welfare loss is positive* ($L(\cdot \mid n_b, n_s, \mu) > 0$); *and*

(b) *for any* $n_b \in \mathbb{Z}^+$ *there exists* $\overline{n}_s \in \mathbb{Z}^+$ *such that for all* $n_s > \overline{n}_s$, *the proportion of welfare loss is positive.*

If $\left(\delta_s/(1-\delta_s)\right) \cdot \mu \cdot (\pi - c_s) < \overline{\Pi}^s$, no network supports a STNE. Now suppose that $\left(\delta_s/(1-\delta_s)\right) \cdot \mu \cdot (\pi - c_s) > \overline{\Pi}^s$. Then, a network that consists of pairs of buyers and sellers that are connected only to each other supports a STNE. In stationary environments, such a "pairs network" is optimal—it guarantees the maximal expected volume of trade. On the other hand, in stochastic environments, only the complete network (in which each seller is connected to all buyers) provides

[31] As long as buyers' discount factors are not too high relative to sellers' discount factors, supporting the per-period optimum maximizes also the long term optimum. If buyers' discount factors are significantly higher, one could conceive a program in which sellers deviate in the first few periods and cooperate thereafter. Such a program may increase the long term welfare. See also Lehrer and Pauzner (1999).

[32] In related work, Lee and Schwarz (2009) analyze interviewing decisions in labor markets. In their setup, the decision to interview is made after knowing that workers are of at least some minimal quality. Thus, there are no demand and supply fluctuations. Lee and Schwarz find that complete overlap in the interviewing decisions among groups of firms maximizes the number of positions filled.

the maximal expected volume of trade. However, the complete network is dense and potentially imbalanced, and may not support a STNE. In such circumstances, a constrained efficient network is sparse and balanced enough to support a STNE, yet dense enough to facilitate volumes of trade that are higher than the expected in the aforementioned "pairs network."

Existing literature on buyer-seller networks finds that in the absence of moral hazard, buyers and sellers form efficient trade networks (e.g., Kranton and Minehart 2001). Theorem 5 shows that moral hazard can prevent efficient networks from being created and sustained even when links are costless. In Section VIB we discuss institutions that complement networks in facilitating efficient trade.

VI. Discussion

In this section, we summarize the predictions of the model, review evidence, and discuss further the relation to existing literature.

A. *Community Structure and Cooperation*

Our model predicts that networks that are especially good in sustaining cooperation are: (i) *balanced and moderately competitive*: the degrees of a buyer and a seller that are connected are similar; (ii) *sparse*: the degrees of sellers and buyers in the network are small; and (iii) *segregated*: sellers who have one buyer in common, have connections to similar sets of buyers overall.

The result that networks facilitate cooperation when they are sparse is in contrast with existing theoretical literature. In Mihm, Toth, and Lang (2009), and Lippert and Spagnolo (2011) increasing the number of links increases the number of bilateral games that a player plays in every period. Consequently, adding links improves the ability to sustain cooperation over previously existing links. Kinateder (2008) offers a model in which all of the players play a common multiplayer game, and a network is used for transferring information about deviations. As before, adding links helps to sustain cooperation. Our focus is different. First, we separate between the network structure and the capacity of any seller or buyer. Second, we focus on buyer-seller networks in which contagion equilibria are not realistic nor feasible. As a result, our framework allows links to be substitutes or complements and additional links can improve or harm cooperation. This allows us to explain why cooperation is limited to sparse networks without assuming exogenous costs of creating and maintaining links.

A slightly more familiar result is that networks facilitate cooperation when they are segregated. However, the explanation suggested in this paper is new. Ali and Miller (2009), and Jackson, Rodriguez-Barraquer, and Tan (2011), highlight the role of cliques in providing the proper incentives for a sufficient number of agents to punish, whereas Kinateder (2008) focuses on the idea that small cycles shorten the delay between a deviation and the community punishment that follows. Haag and Lagunoff (2006) consider a different model in which an agent is restricted to take the same action with all of her neighbors and find that cliques facilitate cooperation when agents have similar (and sufficiently high) discount factors. In our model an agent can take different actions in interactions with different neighbors and contagion is ruled out. Thus, segregation is driven only by the negative correlation between the demands of different buyers in segregated networks—a correlation that is absent from models that do not separate between the network structure and the capacities of agents.

The empirical literature provides ample evidence on the role of networks in markets. Hardle and Kirman (1995); Weisbuch, Kirman, and Herreiner (1996); and Kirman and Vriend (2000) document a network of consistent loyalty and preferential treatment between buyers and sellers within the fish market in Marseille. Kirman and Vriend (2000) assert that the standard asymmetric information model "*seems a too loose application of the textbook argument*." They explain that this is because there is a fixed population of buyers and sellers in this market and "*every buyer* (*loyal or not*) *is a potential repeat buyer*" so "*a seller would have an incentive to deliver good quality to every single buyer*." Notably, the selective supply of high quality by sellers to only a subset of the population of buyers is consistent with our model—sellers do not have the incentives to maintain reputation with all of the buyers, even if all are potential repeated customers. Research in many markets in developing and transition economies provides further evidence consistent with the prevalence of networks of cooperation, and the requirement that they should be sparse.[33]

B. *Institutions and Networks*

Technological progress and decreasing communication and transportation costs are often considered the drivers of the growth and "globalization" of trade networks. Reinforcing this view, existing literature on buyer-seller networks finds that in the absence of moral hazard, buyers and sellers form efficient trade networks (e.g., Kranton and Minehart 2001). However, our results indicate that moral hazard may constrain the efficiency of trade networks.

Focusing on three well-studied institutions, we now demonstrate that institutions that help to sustain bilateral cooperation also increase the set of networks that facilitate STNE. This simple result implies that markets that lack trust-facilitating institutions suffer from a disadvantage because they are forced to compromise on the volume of trade in order to improve the enforcement of informal contracts.[34]

Community Based Institutions–Reputation Networks.—In addition to any buyer-seller network $\mathbf{m}$, *FG consider a network* $\mathcal{R}$ *that connects different buyers in B.*

[33] Fafchamps (1996) surveys manufacturing and trading firms in Ghana and finds that firms rely on repeated bilateral interactions to enforce contracts. McMillan and Woodruff (1999) study trading networks in Vietnam and find that a firm trusts its customer enough to offer credit only when the customer has difficulty locating alternative suppliers. Chaudhury and Matin (2002) and McIntosh and Wydick (2005) find that strategic default occurs when many lending institutions offer loans to the same borrowers and cannot condition the loan on repayment to other lenders.

[34] The prediction that trust-enhancing institutions allow for denser networks to sustain STNE is consistent with empirical evidence. Fafchamps (1996) finds that the absence of reputation mechanisms limits the economic reach of manufacturing and trading firms in Ghana, and Johnson, McMillan, and Woodruff (2002) show that the main effect of belief in the court system is to encourage the formation of new relationships. The complementarity of networks and institutions in the context of the transition to market economies in Eastern Europe is also documented in Woodruff (2002).

Buyers that are connected in $\mathcal{R}$ share with each other information about their past interactions with sellers. For simplicity, assume that a seller s knows whether any two buyers $b, b' \in B_s$ are connected in $\mathcal{R}$. When we add links to $\mathcal{R}$, seller s can lose more than the future value of one link after deviating. As a result, adding a sufficient number of links to $\mathcal{R}$ expands the set of buyer-seller networks for which a STNE exists (see also Proposition 4 in Appendix B). Interestingly, FG show also that not all buyer-seller networks admit a STNE even when the reputation network is complete.

External Institutions–Litigation and Third-Party Evaluation Services.—Litigation allows buyers that were cheated to prosecute the deviating seller. Third-party evaluation services inspect the goods *before* trade occurs. Let β^L be the probability that a buyer who was harmed by a deviation succeeds in prosecuting the deviating seller and receives a compensation λ (without loss of generality, λ is also the penalty for the seller). Let β^E be the probability that a third-party evaluation service detects that a low quality good is of low quality, in which case, the deviation of the seller is exposed even though trade does not occur, and buyers can punish the deviating seller.[35] Thus, an increase in the effectiveness of either institution (an increase in β^L, λ, or β^E) increases the set of networks for which a STNE exists (the increase is in the sense of set inclusion—see also Proposition 5 in Appendix B).

VII. Conclusion

This paper presents a framework that greatly simplifies the analysis of repeated games in networks and provides intuition relevant in many markets.

In contrast with previous literature on networks and markets (see Kranton 1996 and references therein), we do not analyze markets and networks as two mutually exclusive and competing ways to conduct the same activity. We rather focus on markets that are networked. We find that even when every agent in the market can potentially approach any other agent, the need to trust one's partners constrains the trade in the market and allows only certain networks to sustain long term cooperation. We are motivated by evidence that networks of trust and cooperation are present in many markets and suggest that understanding their role improves our understanding of these markets.

Our results show that network structure matters. On one hand, dense and global networks have the potential to maximize trade. On the other hand, these same networks cannot sustain cooperation in environments with asymmetric information and moral hazard. Without cooperation in these environments, there is a risk that no trade will take place at all (see Akerlof 1970).

Consistent with existing evidence, we show that welfare is maximized when proper institutions are in place, and that improving transportation and communication technologies is not enough to promote markets in the absence of trust-enhancing institutions.

[35] For simplicity, assume that an evaluation service never mistakes a good product to be of low quality.

Appendix A: Proofs

PROOF OF THEOREM 1:

We note that if there exists any STNE in network $\mathbf{m}$, there also exists an STNE in network $\mathbf{m}$ in which buyers employ grim trigger strategies (after being cheated, buyers do not buy from the cheating seller ever again). Thus, for the reminder of the proof, we focus on the case that buyers and sellers employ grim trigger strategies. Furthermore, since buyers never have incentives to deviate unilaterally when all sellers employ trigger strategies, we are left to prove the conditions for sellers only.

Consider a network $\mathbf{m}$ and assume that all buyers and sellers employ grim trigger strategies. Consider a seller s who considers whether to deviate or cooperate with all of the buyers that are connected to her, or to deviate in interactions with a subgroup of the buyers connected to her $\hat{B}_s \subseteq B_s$, starting with some buyer $b \in \hat{B}_s$. Let $\overline{u}_s(\mathbf{m})$ be the expected utility of seller s from using her best response given her knowledge and belief as implied by the network $\mathbf{m}$ and assumptions 1 and 2. Let $u_s^c(\mathbf{m})$ be the expected utility of seller s from using the strategy "always cooperate" given her knowledge and belief as implied by the network $\mathbf{m}$ and assumptions 1 and 2.

Let seller s meet with buyer b in network $\mathbf{m}$. If seller s deviates in her interaction with buyer b, her expected utility is $\overline{\Pi}^s + \delta_s \overline{u}_s(\mathbf{m}\backslash(s,b)) - \delta_s u_s^c(\mathbf{m})$. Thus, the strict best response of seller s is always cooperate with all buyers connected to her if and only if,

$$\text{(A1)} \qquad IC_s(\mathbf{m}) \triangleq \min_{b \in B_s} \left\{ \delta_s \left(u_s^c(\mathbf{m}) - \overline{u}_s(\mathbf{m}\backslash(s,b)) \right) - \overline{\Pi}^s \right\} > 0.$$

Let $\{m(n_b^i, \mathbf{g}) \,|\, s, d, \{d_j\}_{j=1}^d\}_{i=1}^{\infty}$ be an increasing sequence of networks such that seller s belongs to all networks in $m(n_b^i, \mathbf{g})$ and for any i, the degree of seller s in $m(n_b^i, \mathbf{g})$ is d and the degrees of all of the buyers that are connected to seller s are captured by $\{d_j\}_{j=1}^d$. Then we note the following observation.

LEMMA 2: (*FG*) *Let* $\mathbf{g}$ *be any admissible degree distribution. Then, for any increasing sequence of networks* $\{m(n_b^i, \mathbf{g}) \,|\, s, d, \{\overline{d}_b\}_{b \in \underline{B}_s}\}_{i=1}^{\infty}$ *and for any* l, $\lim_{d \to \infty} IC_s(T^d(m(n_b^l, \mathbf{g})))$, *and* $\lim_{i \to \infty} IC_s(\{m(n_b^i, \mathbf{g}) \,|\, s, d, \{\overline{d}_b\}_{b \in B_s}\})$ *both exist, and equal one another.*

Note that when $\mathbf{g}$ has a finite support there is only a finite combination of d_s and $\{d_b\}_{b \in B_s}$ feasible under $\mathbf{g}$. Therefore, by Lemma 2, there exists $\bar{i}$ such that for any $i > \bar{i}$ a STNE with network $m(n_b^i, \mathbf{g})$ exists if and only if for every seller s and buyer b that are connected in $m(n_b^i, \mathbf{g})$,

$$\text{(A2)} \qquad IC_s(T^{\infty}(m(n_b^i, \mathbf{g}), s) > 0.$$

In the final step of the proof we show that for any network $\mathbf{m}$,

$$\text{(A3)} \quad sign\left\{ \min_{s,b} \frac{\delta_s}{1 - \delta_s} \cdot FV_{s,b}(T^{\infty}(\mathbf{m}, s)) - \overline{\Pi}^s \right\} = sign\left\{ \min_s IC_s(T^{\infty}(\mathbf{m}, s) \right\}.$$

Noting that

$$\text{(A4)} \qquad u_s^c(\mathbf{m}) = \frac{1}{1-\delta_s} \sum_{b \in B_s} \Pr\big(I_{\mathbf{m}}^t(s,b)\big) .$$

We can rewrite (1) in the following way

$$\text{(A5)} \qquad FV_{s,b}(\mathbf{m}) = \delta_s\big(u_s^c(\mathbf{m}) - u_s^c(\mathbf{m} \backslash (s,b))\big) .$$

It follows immediately from (A1) and (A5) that for any $s \in S$, $\min_{s,b}\big(\delta_s/(1-\delta_s)\big) \cdot FV_{s,b}\big(T^{\infty}(\mathbf{m},s)\big) - \overline{\Pi}^s < 0$ implies that $\min_s IC_s\big(T^{\infty}\big(m(n_b^i,\mathbf{g}),s\big) < 0$. We next prove that $\min_s IC_s\big(T^{\infty}\big(m(n_b^i,\mathbf{g}),s\big) < 0$ implies that

$$\text{(A6)} \qquad \min_{s,b} \frac{\delta_s}{1-\delta_s} \cdot FV_{s,b}\big(T^{\infty}(\mathbf{m},s)\big) - \overline{\Pi}^s < 0.$$

Assume by contradiction that there exists a seller s such that $IC_s\big(T^{\infty}\big(m(n_b^i,\mathbf{g}),s\big) < 0$ and $\min_{b \in B_s}\big(\delta_s/(1-\delta_s)\big) \cdot FV_{s,b}\big(T^{\infty}(\mathbf{m},s)\big) - \overline{\Pi}^s \geq 0$. If the optimal strategy of seller s involves a deviation in an interaction with a single buyer b and cooperation with anyone else thereafter then $IC_s\big(T^{\infty}\big(m(n_b^i,\mathbf{g}),s\big) = \min_{b \in B_s}\big(\delta_s/(1-\delta_s)\big) \cdot FV_{s,b}\big(T^{\infty}(\mathbf{m},s)\big) - \overline{\Pi}^s < 0$ and we're done. Otherwise, consider a sequence of deviation with all buyers in $\hat{B}_s \subseteq B_s$, and let $\hat{b} \in \hat{B}_s$ be the last buyer such that seller s deviates in an interaction with $\hat{b}$ and cooperates with anyone else thereafter. Thus,

$$\text{(A7)} \qquad \min_{b} \frac{\delta_s}{1-\delta_s} \cdot FV_{s,b}\big(T^{\infty}(\mathbf{m} \backslash (\hat{B}_s \backslash \hat{b}),s)\big) - \overline{\Pi}^s < 0.$$

Substituting in (3) yields that for the buyer b that solves the minimization problem, $FV_{s,b}\big(T^{\infty}(\mathbf{m} \backslash (\hat{B}_s \backslash \hat{b}),s)\big) > FV_{s,b}\big(T^{\infty}(\mathbf{m},s)\big)$ which completes the proof.

PROOF OF LEMMA 1:

Consider the following algorithm for matching buyers and sellers in a network $\mathbf{m} = \langle S,B,E \rangle$. First, choose an ordering σ of E u.a.r. from all of the $|E|!$ possible orderings of E. Second, repeat the following action iteratively. Examine the link (s',b') that was chosen first in the ordering among the links that have not been removed in a previous step. If s' is active, match s' to b' and remove from the ordering all the links (s,b') and (s',b) for all $s \in S_b$, $b \in B_s$. We note that in any STNE, in any period t, the algorithm above can be coupled with the market activity, such that (s',b') are matched if and only if they trade with each other in period t. Consequently, we can interpret $\Pr\big(I_{\mathbf{m}}^t(s,b)\big)$ as the probability that edge (s,b) is selected by the appropriate randomized matching algorithm.

Following this interpretation, Lemma 1 follows immediately from Proposition 1 of Gamarnik and Goldberg (2010) who study randomized greedy algorithms for matchings in a graph, and the relationship between the local and global properties of the set of matchings of a graph.[36]

PROOF OF THEOREM 2:

If g^S FOSD $\widehat{g^{\hat{S}}}$ then by Lemma 1, for every $b \in B_s(\mathbf{m})$ and $\hat{b} \in B_{\hat{s}}(\hat{\mathbf{m}})$ such that $d_{\hat{b}} = d_b$, $\Pr\big(I^t_{T^d(\mathbf{m},s,b)}(s,b)\big) > \Pr\big(I^t_{T^d(\hat{\mathbf{m}},\hat{s},\hat{b})}(\hat{s},\hat{b})\big)$. If $d_s = 1$ then $FV_{\hat{s},\hat{b}}\big(T^\infty(\hat{\mathbf{m}},\hat{s})\big) \leq FV_{s,b}\big(T^\infty(\mathbf{m},s)\big)$ is immediate from Theorem 1 and equation (3). On the other hand,

$$\text{(A8)} \qquad \lim_{d_s \,=\, d_{\hat{s}} \to \infty} \frac{\big\{\Pi_{b' \in B_s(\mathbf{m}) \backslash b}\big[1 \,-\, \frac{1}{\mu} \cdot \Pr\big(I^t_{T^d(\mathbf{m},s,b')}(s,b')\big)\big]\big\}}{\big\{\Pi_{\hat{b}' \in B_{\hat{s}}(\hat{\mathbf{m}}) \backslash \hat{b}}\big[1 \,-\, \frac{1}{\mu} \cdot \Pr\big(I^t_{T^d(\hat{\mathbf{m}},\hat{s},\hat{b}')}(s,\hat{b}')\big)\big]\big\}} \,=\, 0,$$

and at the same time

$$\text{(A9)} \qquad \Pr\big(I^t_{T^d(\mathbf{m},s,b)}(s,b)\big) / \Pr\big(I^t_{T^d(\hat{\mathbf{m}},\hat{s},\hat{b})}(s,\hat{b})\big)$$

is independent of d_s and $d_{\hat{s}}$. Combining (A8), (A9), and (3), and evaluating $FV_{s,b}/FV_{\hat{s},\hat{b}}$ for any $b \in B_s$ and $\hat{b} \in B_{\hat{s}}$ such that $d_b = d_{\hat{b}}$, yields the result that $\underline{FV}_{\hat{s}}\big(T^\infty(\hat{\mathbf{m}},\hat{s})\big) \geq \underline{FV}_s\big(T^\infty(\mathbf{m},s)\big)$.

PROOF OF THEOREM 3:

Let $d_b^{\max}$ be the maximal degree of any buyer in network $\mathbf{m}$ (so $d_b^{\max} > D$), and let

$$\text{(A10)} \quad I^{\min}_{T^d(\mathbf{m},s,b)} \,=\, \min_{s \in S, b \in B} \Pr\big(I^t_{T^d(\mathbf{m},s,b)}(s,b)\big) \,=\, \big(\Pr\big(I^t_{T^d(\mathbf{m},s,b)}(s,b)\big) \,|\, d_b \,=\, d_b^{\max}\big).$$

Consider a seller s' that is connected to a buyer b', such that $d_{b'} = d_b^{\max}$. Then, substituting (A10) and $d_s = \alpha \cdot D + k$ in (3) yields

$$\text{(A11)} \quad FV_{s,b}\big(T^\infty(m(\alpha,D),s)\big) \,\leq\, (\pi \,-\, c) \,\cdot\, I^{\min}_{T^d(\mathbf{m},s,b)} \,\cdot\, \big[1 \,-\, I^{\min}_{T^d(\mathbf{m},s,b)}\big]^{\alpha \cdot D + k - 1}.$$

Because $0 \leq I^{\min}_{T^d(\mathbf{m},s,b)} \leq 1$ we have that $\lim_{D \to \infty} FV_{s,b}\big(T^\infty(m(\alpha,D),s)\big) \leq 0$ for any $k \in \mathbb{Z}^+$.

PROOF OF THEOREM 4:

Part 1a: Consider the case where $d^S = 2$ and $d^B = 2$ that is captured in Figure 7.

We start by analyzing $FV^{seg}(2,2)$ in Figure 7B. Assume that both b and b' are willing to purchase from s conditional on having demand when they meet. Then, (conditional on having a unit supply) s sells in period t with probability 1. Now assume that b is unwilling to purchase from s, and that b' is willing to purchase from s conditional on having demand when they meet. The probability that b' has demand when he meets s is $(1 - (1/3)\mu)$. To see why, note that b' has demand when meeting s, unless the link (s', b') is the first one to be chosen among $\{(s,b'),(s',b'),(s',b)\}$. Therefore

[36] I thank David Goldberg for suggesting this proof. An earlier and much longer proof that introduces an algorithm for approximating $\Pr\big(I^t_{\mathbf{m}}(s,b)\big)$ in large networks is available from the author.

(A12) $$FV^{seg}(2,2) = \left[1 - \left(1 - \frac{1}{3}\mu\right)\right](\pi - c_s) = \frac{1}{3}\mu \cdot (\pi - c_s).$$

We now turn to consider $FV^T(2,2)$ in Figure 7A. Let x be the probability that when s' and b' meet, b' has demand. More generally, for any seller and buyer that are connected, x is the probability that the buyer has demand when they meet. Then

(A13) $$FV^T(2,2) = x(1 - x)(\pi - c_s).$$

Focusing on the link (s', b') we note that $1 - x$ can be rewritten as the union of the two following mutually exclusive events:

(i) The event that s produces in period t; when s and b meet, b does not have demand; and s and b' meet before s' and b' meet.

(ii) The event that s produces in period t; when s and b meet, b has demand; s and b' meet before s and b meet; and s and b' meet before s' and b' meet.

The probability of the former is $(1/2)\mu(1 - x)$ whereas the probability of the latter is $\mu(1/3 - \varepsilon)x$ for some $\varepsilon > 0$. The addition of ε accounts for the fact that b has demand when he meets s indicates that (s, b) is more likely to have been chosen early. Thus,

(A14) $$1 - x = \frac{1}{2}\mu(1 - x) + \mu\left(\frac{1}{3} - \varepsilon\right)x$$

and

(A15) $$FV^T(2,2) = \frac{6 - 3\mu}{6 - \mu - 6\varepsilon\mu}\left(1 - \frac{6 - 3\mu}{6 - \mu - 6\varepsilon\mu}\right)(\pi - c_s).$$

We conclude that $FV^T(2,2) < FV^{seg}(2,2)$ if

(A16) $$\frac{6 - 3\mu}{6 - \mu - 6\varepsilon\mu}\left(1 - \frac{6 - 3\mu}{6 - \mu - 6\varepsilon\mu}\right) < \frac{1}{3}\mu,$$

which holds for every $0 \leq \mu$ and $\varepsilon \leq 1$. To complete the proof of part 1, note that $FV^T(1,2) = FV^{seg}(1,2)$ and $FV^T(1,1) = FV^{seg}(1,1)$ because when $d^B = 1$ the global and segregated networks are identical.

Part 1b: Consider the case where $d^S = 2$ and $d^B \to \infty$. Let the definition of x carry over from the proof of Part 1.

Consider a seller s that is connected to buyers b and b'. In the segregated network, the probability that b' does not have demand when meeting s and b has demand when meeting s is the probability that: (i) s is the first to meet b and not the first to meet b'; or that (ii) s is the second to meet b and seller s' who met b before s was the first to meet b'. When $d^B \to \infty$ this can be shown to equal

$$\left(1 - \frac{1}{\mu \cdot (d^B - 1)}\right) \cdot \frac{1}{\mu \cdot (d^B - 2)}. \tag{A17}$$

In the global network (in the limit when $d^B \to \infty$) a seller has $\mu \cdot (d^B - 1) \cdot \left[(1 - x) + x \cdot \left((1/2) - \varepsilon\right)\right]$ distinct *competitors* for selling to each of the buyers she is connected to. A competitor is an active seller who is connected to the same buyer and that cannot sell to their other connected buyer. Therefore,

$$x = \frac{1}{\mu \cdot (d^B - 1) \cdot \left[1 - \frac{1}{2}x - \varepsilon x\right]}. \tag{A18}$$

Again, $\varepsilon > 0$ because the fact that a competitor's other link was useful, implies that it was chosen early, so the probability that the relevant link was chosen before is less than $1/2$. Therefore,

$$FV^T(d^B, 2) = \frac{1}{\mu \cdot (d^B - 1) \cdot \left[1 - \frac{1}{2}x - \varepsilon x\right]} \cdot \left(1 - \frac{1}{\mu \cdot (d^B - 1) \cdot \left[1 - \frac{1}{2}x - \varepsilon x\right]}\right)(\pi - c_s). \tag{A19}$$

As $\mu \nrightarrow 0$ and $d^B \to \infty$, x is small (and in particular $x < 1/2$), so a lower bound on x provides a lower bound on $x(1 - x)$ and we can focus on demonstrating that

$$x(1 - x) \geq \left(1 - \frac{1}{\mu \cdot (d^B - 1)}\right) \cdot \frac{1}{\mu \cdot (d^B - 2)} \tag{A20}$$

for $0 << \mu < 1/2$.

From $x = 1/\left(\mu \cdot (d^B - 1) \cdot [1 - (1/2)x - \varepsilon x]\right)$ we get that $1 + \varepsilon x^2 \cdot \mu \cdot (d^B - 1) = x \cdot \mu \cdot (d^B - 1) - (1/2)x^2 \cdot \mu \cdot (d^B - 1)$ and $\varepsilon = 0$ provides a lower bound on x. Denote this lower bound as $\underline{x}$ such that $1/\left(\mu \cdot (d^B - 1)\right) = \underline{x} - (1/2)\underline{x}^2$ and $\underline{x} = 1/\left(\mu \cdot (d^B - 1)\right) + (1/2)\underline{x}^2 \geq 1/\left(\mu \cdot (d^B - 1)\right)$. Consequently, $\underline{x} \geq 1/\left(\mu \cdot (d^B - 1)\right) + (1/2)\left(1/\left(\mu \cdot (d^B - 1)\right)\right)^2$. Plugging $\underline{\underline{x}} = 1/\left(\mu \cdot (d^B - 1)\right) + (1/2)\left(1/\left(\mu \cdot (d^B - 1)\right)\right)^2$ into $x(1 - x)$ yields that

$$(1 - x)x \geq \left[1 - \left(\frac{1}{\mu \cdot (d^B - 1)} + \frac{1}{2}\left(\frac{1}{\mu \cdot (d^B - 1)}\right)^2\right)\right] \cdot \left(\frac{1}{\mu \cdot (d^B - 1)} + \frac{1}{2}\left(\frac{1}{\mu \cdot (d^B - 1)}\right)^2\right) \tag{A21}$$

and it is sufficient to show that

$$\text{(A22)} \quad \left[1 - \left(\frac{1}{\mu \cdot (d^B - 1)} + \frac{1}{2}\left(\frac{1}{\mu \cdot (d^B - 1)}\right)^2\right)\right] \cdot \left(\frac{1}{\mu \cdot (d^B - 1)} + \frac{1}{2}\left(\frac{1}{\mu \cdot (d^B - 1)}\right)^2\right) \geq \left(1 - \frac{1}{\mu \cdot (d^B - 1)}\right) \cdot \frac{1}{\mu \cdot (d^B - 2)}$$

for every $\mu < 1/2$.

With some algebra, this becomes

$$\text{(A23)} \quad \mu + \frac{1}{2} \cdot \frac{(d^B - 2)}{(d^B - 1)} + \frac{1}{\mu} \cdot \frac{(d^B - 2)}{(d^B - 1)^2} + \frac{1}{4\mu^2} \cdot \frac{(d^B - 2)}{(d^B - 1)^3} \leq 1.$$

Recalling that $\mu > 0$ and $d \to \infty$ this is simplified to $\mu + 1/2 + 0 + 0 \leq 1$ which hold for every $\mu < 1/2$.

Part 2: In a segregated network with more buyers than sellers, a seller is guaranteed to trade with or without her marginal link. Consequently, the value of each link is zero. This is not true for a global network.

PROOF OF THEOREM 5:

Part 1: consider a network $\mathbf{m}_1$ in which all agents on the short side of the market have degree one and the maximal degree of any agent in the network is 1 (e.g., if $n_s > n_b$ than all buyers have degree 1, n_b sellers have degree one, and $n_s - n_b$ sellers have degree 0). By definition, for every $(s,b) \in E$, $\big(\delta_s/(1 - \delta_s)\big) \cdot FV_{s,b}(\mathbf{m}_1) = \big(\delta_s/(1 - \delta_s)\big) \cdot \mu \cdot (\pi - c_s) > \overline{\Pi}^s$. Applying Theorem 1 and noting that $E[V(\mathbf{m}_1 \,|\, \mu = 1)] = \min\{n_b, n_s\}$ and $E[V(\mathbf{m}_1 \,|\, n_s < n_b)] = \mu \cdot n_s$ completes the proof.

Part 2a: Assume by contradiction that for any $\overline{n}_b$ there exists $n_b > \overline{n}_b$ and $n_s > n_b$ such that $L(\cdot \,|\, n_b, n_s, \mu) = 0$. Let n_s^t be the number of sellers that are able to produce in period t. The contradiction assumption implies that there exists $\Delta\mathbf{m}$ such that in every period $\min\{n_b, n_s^t\}$ trades take place and that $\min_{s \in S}\big[\big(\delta_s/(1 - \delta_s)\big) \cdot FV_{s,b}(\Delta\mathbf{m}) - \overline{\Pi}^s\big] > 0$. In particular, in every period t such that $n_s^t < n_b$ the number of trades need be n_s^t, independent of which are the sellers that produce. The only network $\mathbf{m}$ that guarantees that n_s^t take place is the complete network in which for every s, $d_s = n_b$, and for every b, $d_b = n_s$. Consider such a network. When $n_b \to \infty$, the probability that in period t a seller s sells if all of the buyers are willing to buy from her, and does not sell if all but one of the buyers is ready to buy from her is bounded above by

$$\text{(A24)} \quad \min\left\{\frac{n_b}{\mu \cdot n_s}, 1\right\} - \min\left\{\frac{n_b - 1}{\mu \cdot n_s}, 1\right\}.$$

To see why (A24) is an upper bound, recall that in network $\mathbf{m}$ all seller are symmetric and note that $\min\{n_b/(\mu \cdot n_s), 1\}$ is the probability that any seller manages to sell in a network $\mathbf{m}$ when $\mu \cdot n_s$ produce. We claim that $\min\{(n_b - 1)/(\mu \cdot n_s), 1\}$ is smaller than the probability that s sells in period t if only $n_b - 1$ of the buyers are willing to buy from her. This is because there is a positive probability that some seller s' sells to b before meeting any other buyer. Conditional on that event, the probability that seller s sells in period t is $\min\{(n_b - 1)/(\mu \cdot n_s - 1), 1\} > \min\{(n_b - 1)/(\mu \cdot n_s), 1\}$.

To conclude the proof, let $n_s = f(n_b)$. Then, for any function $f: \mathbb{Z}^+ \to \mathbb{Z}^+$ such that $f(k) > k$ for all $k \in \mathbb{Z}^+$,

$$\text{(A25)} \quad \lim_{n_b \to \infty} FV_{s,b}(\mathbf{m}) = \lim_{n_b \to \infty} \left\{ \frac{\delta_s}{1 - \delta_s} \cdot (\pi - c_s) \cdot \left[\min\left\{ \frac{n_b}{\mu \cdot n_s}, 1 \right\} - \min\left\{ \frac{n_b - 1}{\mu \cdot n_s}, 1 \right\} \right] \right\} = 0,$$

which contradicts the assumption that $\min_{s \in S}\left[(\delta_s/(1 - \delta_s)) \cdot FV_{s,b}(\Delta\mathbf{m}) - \overline{\Pi}^s \right] > 0$.

Part 2b: Fix n_b and assume by contradiction that for any $\overline{n}_s$ there exists $n_s > \overline{n}_s$ such that $L(\cdot | n_b, \overline{n}_s, \mu) = 0$. Let n_s^t be the number of sellers that are able to produce in period t. The contradiction assumption implies that there exists $\Delta\mathbf{m}$ such that in every period $\min\{n_b, n_s^t\}$ trades take place and that $\min_{s \in S}\left[(\delta_s/(1 - \delta_s)) FV_{s,b}(\Delta\mathbf{m}) - \overline{\Pi}^s \right] > 0$. However, given that $\mu < 1$, to satisfy that in every period $\min\{n_b, n_s^t\}$ trades take place, $\Delta\mathbf{m}$ must provide each seller with a positive probability of selling in every period that she produces. Thus,

$$\text{(A26)} \quad \min_{s \in S}\left[\frac{\delta_s}{1 - \delta_s} FV_{s,b}(\Delta\mathbf{m}) - \overline{\Pi}^s \right] < \min_{s \in S}\left[\frac{\delta_s}{1 - \delta_s} \cdot \frac{n_b}{n_s} \cdot (\pi - c_s) - \overline{\Pi}^s \right] < 0$$

and for $n_s/n_b > \max_{s \in S}[\delta_s(\pi - c_s)]/[(1 - \delta_s) \cdot \overline{\Pi}^s]$, we have that

$$\text{(A27)} \quad \min_{s \in S}\left[\frac{\delta_s}{1 - \delta_s} \cdot \frac{n_b}{n_s} \cdot (\pi - c_s) - \overline{\Pi}^s \right] < 0.$$

This completes the proof by contradiction to $\min_{s \in S}\left[(\delta_s/(1 - \delta_s)) FV_{s,b}(\Delta\mathbf{m}) - \overline{\Pi}^s \right] > 0$.

Appendix B: Additional Results

PROPOSITION 1:

(1) *Consider buyers $b, b' \in B$ such that $d_{b'} \geq d_b$. Then, $FV_{s,b'}(T^\infty(\mathbf{m}, s)) \leq FV_{s,b}(T^\infty(\mathbf{m}, s))$.*

(2) *Consider sellers $s, s' \in S$ such that $d_{s'} \geq d_s$ and $\{d_b\}_{b \in B_s} \subseteq \{d_{b'}\}_{b \in B_{s'}}$, and consider buyers $b_1 \in B_s$ and $b_1' \in B_{s'}$ such that $d_{b_1'} = d_{b_1}$. Then, $FV_{s',b_1'}(T^\infty(\mathbf{m}, s')) \leq FV_{s,b_1}(T^\infty(\mathbf{m}, s))$.*

PROOF:

By Lemma 1, as $d_{b'} > d_b$ we have that $\Pr(I^t_{T^d(\mathbf{m},s,b')}(s,b')) \leq \Pr(I^t_{T^d(\mathbf{m},s,b)}(s,b))$, which when combined with Theorem 1 and equation (3) completes the proof of Part 1. Part 2 follows directly from Theorem 1 and equation (3).

PROPOSITION 2: *Consider two sellers s and s′ such that* (i) $d_{s'} = d_s$; *and* (ii) *there exist* $b_1 \in B_s$ *and* $b'_1 \in B_{s'}$ *such that* $d_{b'_1} \geq d_{b_1}$ *and* $\{d_b\}_{b \in B_S \setminus b_1} \equiv \{d_{b'}\}_{b \in B_{s'} \setminus b'_1}$. *Then, for every* $b \in B_s$ *and* $b' \in B_{s'}$ *such that* $d_{b'} = d_b$, $FV_{s',b'}(T^\infty(\mathbf{m},s')) \geq FV_{s,b}(T^\infty(\mathbf{m},s))$.

PROOF:

By Lemma 1, if $d_{b'_1} > d_{b_1}$, $\Pr(I^t_{T^d(\mathbf{m},s',b'_1)}(s',b'_1)) < \Pr(I^t_{T^d(\mathbf{m},s,b_1)}(s,b_1))$. Plugging this inequality into equation (3) for some $b \in B_s$ and $b' \in B_{s'}$ such that $d_{b'} = d_b$ yields that $FV_{s',b'}(T^\infty(\mathbf{m},s')) \geq FV_{s,b}(T^\infty(\mathbf{m},s))$.

PROPOSITION 3: *Let* $c_s = c$ *for all* $s \in S$. *Hold fixed* d^B, d^S, *and* μ. *There exist* $\overline{d^S}(d^B,\mu)$, $\overline{d^B}(d^S,\mu)$, $\overline{\mu}(d^S,d^B)$, *and* $\underline{d^S}(d^B,\mu)$, $\underline{d^B}(d^S,\mu)$, $\underline{\mu}(d^S,d^B)$ *such that*

(1) *If* $d^S > \overline{d^S}$ *then* $FV^T(d^B+1,d^S) > FV^T(d^B,d^S) > FV^T(d^B,d^S+1)$ *and if* $d^S < \underline{d^S}$ *then* $FV^T(d^B+1,d^S) < FV^T(d^B,d^S)$.

(2) *If* $d^B < \underline{d^B}$ *then* $FV^T(d^B+1,d^S) > FV^T(d^B,d^S) > FV^T(d^B,d^S+1)$ *and if* $d^B > \overline{d^B}$ *then* $FV^T(d^B+1,d^S) < FV^T(d^B,d^S)$.

(3) *If* $\mu < \underline{\mu}$ *then* $FV^T(d^B+1,d^S) > FV^T(d^B,d^S)$ *and if* $\mu > \overline{\mu}$ *then* $FV^T(d^B+1, d^S) < FV^T(d^B,d^S)$.[37]

PROOF:

We prove first all of the inequality that involve a comparison of $FV^T(d^B,d^S)$ and $FV^T(d^B,d^S+1)$. Since

$$\text{(B1)} \qquad FV^T(d^B,d^S) = (\pi - c) \cdot \Pr(I^t_{T^\infty(\mathbf{m},s,b)}(s,b)) \cdot \left[1 - \frac{1}{\mu} \cdot \Pr(I^t_{T^\infty(\mathbf{m},s,b)}(s,b))\right]^{d^S-1},$$

Lemma 1 implies that when d^S is large and when d^B and μ are small, an increase in d^S decreases $FV^T(d^B,d^S)$ by both increasing $\Pr(I^t_{T^\infty(\mathbf{m},s,b)}(s,b))$ and the power argument.

We now prove the inequalities that involve a comparison of $FV^T(d^B,d^S)$ and $FV^T(d^B+1,d^S)$. To see that when $d^S > 1$ there exist small enough $d^B \geq 1$ for which the result for small d^B hold, it is immediate that for any $d^S > 1$, $(FV^T | d^B = 1) < (FV^T | d^B = 2)$. The result for small μ follows the same reasoning. Similarly, to see

[37] Part 3 of Proposition 3 sheds light on the role of μ. We illustrate that using our job recommendations example. Recall that a low μ implies that only a small fraction of the teachers have high ability students in every period. Hence, more teachers are required to be connected to every firm to prevent the competition from being "too weak." On the other hand, high μ implies that a large fraction of the teachers have high ability students in every period and in order to restrain the fierce competition a low degree for firms or a high degree for teachers is required.

that there exists small enough d^S for which the result for small d^S hold, it is immediate that for $d^S = 1$, $(FV^T | d^B = 1) > (FV^T | d^B = 2)$.

For the remainder of the proof, we treat d^B as a continuous variable. We show that $\partial FV_{s,b}/\partial d^B < 0$ for large μ and d^B, and that $\partial FV_{s,b}/\partial d^B < 0$ for large d^S. First, note that

$$\text{(B2)} \quad \frac{\partial FV^T}{\partial d^B} = \left[1 - \frac{1}{\mu} \cdot \Pr\left(I^t_{T^\infty(\mathbf{m},s,b)}(s,b)\right)\right]^{d^S-2} \cdot \left[\partial \Pr\left(I^t_{T^\infty(\mathbf{m},s,b)}(s,b)\right)/\partial d^B\right] \cdot \left\{1 - \frac{1}{\mu} \cdot \Pr\left(I^t_{T^\infty(\mathbf{m},s,b)}(s,b)\right) \cdot d^S\right\} \cdot (\pi - c).$$

Thus, the sign of $\partial FV^T/\partial d^B$ is determined as the opposite of the sign of $1 - (1/\mu) \cdot \Pr\left(I^t_{T^\infty(\mathbf{m},s,b)}(s,b)\right) \cdot d^S$ (recall that $\Pr\left(I^t_{T^\infty(\mathbf{m},s,b)}(s,b)\right)$ is decreasing in d^B by Lemma 1). If $\Pr\left(I^t_{T^\infty(\mathbf{m},s,b)}(s,b)\right)$ and d^S are small, $1 - (1/\mu) \cdot \Pr\left(I^t_{T^\infty(\mathbf{m},s,b)}(s,b)\right) \cdot d^S > 0$ and $\partial FV^T/\partial d^B < 0$, and vice versa. It is only left to note that by Lemma 1, $(1/\mu) \cdot \Pr\left(I^t_{T^\infty(\mathbf{m},s,b)}(s,b)\right)$ is decreasing in d^B and μ, and increasing in d^S.

PROPOSITION 4: (*FG*) *Consider a market with* $S, B, \mu, \{c_s\}_{s\in S}, \{\delta_s\}_{s\in S}$, *and* $\{\overline{\Pi}^s\}_{s\in S}$. *For a given* $\mathcal{R}$ *let* $M^{\mathcal{R}}$ *be the set of buyer-seller networks for which a STNE exists. Let* $\mathcal{R}^1$ *be the reputation network in which every two buyers are connected. Then, for any* $\mathcal{R}$, $M^{\mathcal{R}} \subseteq M^{\mathcal{R}^1}$.

PROPOSITION 5: *Consider a market with* $S, B, \mu, \{c_s\}_{s\in S}, \{\delta_s\}_{s\in S}$, *and* $\{\overline{\Pi}^s\}_{s\in S}$. *Let* $M^{\beta^L,\lambda,\beta^E}$ *be the set of buyer-seller networks for which a STNE exists given* β^L, λ, *and* β^E. *If* $\widehat{\beta^L} \geq \beta^L$, $\hat{\lambda} \geq \lambda$, *and* $\widehat{\beta^E} \geq \beta^E$ *then* $M^{\beta^L,\lambda,\beta^E} \subseteq M^{\widehat{\beta^L},\hat{\lambda},\widehat{\beta^E}}$.

PROOF:

The proof is immediate and therefore omitted.

Appendix C: Repeated Games and Incomplete Knowledge of the Network

In this section, we suggest that studying environments in which individuals have only incomplete knowledge of the network is insightful beyond the tractability it provides. Clearly, repeated interactions provide sellers and buyers with opportunities to learn about their environment. However, even excluding purely behavioral considerations, there are several reasons why market participants may not be able to learn beyond their close local network and some aggregate characteristics of the global environment.

First, much of the economic literature suggests that learning is costly. Consider market participants that learn optimally given the information that they acquire and process, but have costs of information acquisition and processing.[38] Assume that market participants learn directly about the network structure (e.g., viewing

[38] Non-network examples include models of search with memory constraints (e.g., Dow 1991), or limited attention (e.g., Schwartzstein 2010), as well as models of costly information acquisition (e.g., Verrecchia 1982).

a person's links in social networking websites, going through old call or shipment records, or gathering other information on past interactions of a seller or buyer). It is easy to write a model in which assumption 1 is a result, for example, if there are increasing costs of learning information on participants that are at a large distance. On the other hand, if participants focus on frequencies of their own trade to infer the network structure, it is not clear what sellers' beliefs are likely to converge to. In the latter case, assumption 1 is a stylized approximation of the knowledge held by market participants in the long run.

Second, real world networks are dynamic structures, links are added and removed, and buyers' demand changes over time. Nevertheless, the aggregate attributes of networks (such as the degree distribution) seem to be stable over time. Moreover, the local environments of most individuals change only infrequently. The study of agents' ability to learn the network structure in a changing environment poses many interesting open questions that are beyond the scope of this paper. For now, we suggest that there are market environments in which incomplete knowledge of the network persists over time even for Bayesian agents. We offer below an example of one such environment. While the description of the environment requires more notation, it relies on simple assumptions: (i) buyers are divided into separate groups (buyers from the same group can be connected to different subsets of sellers); (ii) only a small subset of the buyers in each group have demand in a given period; (iii) all of the buyers that belong to the same group share information about past transactions; and (iv) a seller s is connected to buyers from d_s groups. Under these assumptions, as the market becomes large, it is impossible for buyers and sellers to learn much beyond $\mathbf{K}_s$ ($\mathbf{K}_b$). At the same time, the repeated nature of the interactions remains intact.

A Market Environment in which Incomplete Knowledge of the Network Persists over Time even for Bayesian Agents.—Let buyers live in different locations, in every location $l \in L$ there is a set B^l of buyers. A buyer from location l is connected to d_l sellers. A seller s has connections to buyers from d_s locations. For each location l, let the degree distribution of all of the sellers connected to buyers from l be identical to the degree distribution of sellers in the market, and be i.i.d. across buyers from l and across connections of each buyer $b \in B^l$. Denote by $\mathbf{m}^u$ the (fixed) underlying network of locations and sellers which is defined as follows: a seller and a location are connected if the seller is connected to at least one buyer from location l.

In every period, only a subset of buyers have unit demand. We call such buyers active. Let $b^{l,active}$ buyers be active in location l in every period, chosen randomly and i.i.d. across locations and periods with the following restriction: a seller s has a connection to one active buyer from each location from a (fixed) set of d_s locations in every period. Within a period, sellers and buyers that are connected meet in a random order (as described in Section III). After transacting, a buyer learns the true quality of the good, and shares it with all of the other buyers in her location.

Note that the degree distribution of the network between sellers and active buyers, $\mathbf{g} = \langle g^S, g^B \rangle$, is constant across periods and is determined by L, $\{d_l\}_{l\in L}$, S, $\{d_s\}_{s\in S}$, and $\{b^{l,active}\}_{l\in L}$.

Holding $\{b^{l,active}\}_{l\in L}$ fixed, as $|B|, |S|, |L|, |B^l| \to \infty$, the network $\mathbf{m}$ that is generated in every period has a strong random component. Focusing on large markets with random selection of active sellers and buyers creates an environment in which the network structure changes over time without changes to agents' local environments or to the degree distribution. Consequently, complete knowledge of the network is obsolete, and our analysis holds without any changes for anything between agents who know the full network structure in every period and agents who know only basic information that includes their own degree, the degree of their direct neighbors, and the degree distribution **g**. Clearly, precise conditions are required to establish that the network **m** is chosen u.a.r. from all of the networks with $\sum_{l \in L} b^{l, active}$ buyers, $|S|$ sellers, and with degree distribution **g**. We leave the exact conditions necessary as an open question for future research. However, as our analysis throughout the paper suggest, our results are not sensitive to the small changes in the details of the randomization process behind sellers' beliefs, and much of the proofs can be replicated with alternative randomization schemes for the selection of networks.

REFERENCES

Abreu, Dilip, and Mihai Manea. 2011. "Bargaining and Efficiency in Networks." http://econ-www.mit.edu/files/5982.

Akerlof, George A. 1970. "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism." *Quarterly Journal of Economics*, 84(3): 488–500.

Ali, S. Nageeb, and David Miller. 2009. "Enforcing Cooperation in Networked Societies." Unpublished.

Araujo, Luis. 2004. "Social Norms and Money." *Journal of Monetary Economics*, 51(2): 241–56.

Bala, Venkatesh, and Sanjeev Goyal. 2000. "A Noncooperative Model of Network Formation." *Econometrica*, 68(5): 1181–1229.

Bar-Isaac, Heski. 2005. "Imperfect Competition and Reputational Commitment." *Economics Letters*, 89(2): 167–73.

Bewley, Truman F. 1999. *Why Wages Don't Fall During a Recession*. Cambridge, MA: Harvard University Press.

Campbell, Arthur. 2010. "Tell your Friends! Word of Mouth and Percolation in Social Networks." http://mba.yale.edu/faculty/pdf/cambella_word_of_mouth.pdf.

Chaudhury, Iftekhar A., and Imran Matin. 2002. "Dimensions and Dynamics of Microfinance Membership Overlap—A Micro Study from Bangladesh." *Small Enterprise Development*, 13(2): 46–55.

Corominas-Bosch, Margarida. 2004. "Bargaining in a Network of Buyers and Sellers." *Journal of Economic Theory*, 115(1): 35–77.

Elliott, Matthew. 2011a. "Inefficiencies in Networked Markets." http://www.stanford.edu/~mle1/inefficiencies_in_networked_markets.pdf.

Elliott, Matthew. 2011b. "Search with Multilateral Bargaining." http://www.stanford.edu/~mle1/search_with_multilateral_bargaining.pdf.

Ellison, Glenn. 1994. "Cooperation in the Prisoner's Dilemma with Anonymous Random Matching." *Review of Economic Studies*, 61(3): 567–88.

Fafchamps, Marcel. 1996. "The Enforcement of Commercial Contracts in Ghana." *World Development*, 24(3): 427–48.

Fainmesser, Itay P. 2011. "Social Networks and Unraveling in Labor Markets." http://www.brown.edu/Research/Itay_Fainmesser/FainmesserNetworksUnraveling.pdf.

Fainmesser, Itay P., and David A. Goldberg. 2011. "Bilateral and Community Enforcement in a Networked Market with Simple Strategies." http://www.brown.edu/Research/Itay_Fainmesser/FainmesserGoldbergReputation.pdf.

Galeotti, Andrea, Sanjeev Goyal, Matthew O. Jackson, Fernando Vega-Redondo, and Leeat Yariv. 2010. "Network Games." *Review of Economic Studies*, 77(1): 218–44.

Gamarnik, David, and David A. Goldberg. 2010. "Randomized Greedy Algorithms for Independent Sets and Matchings in Regular Graphs: Exact Results and Finite Girth Corrections." *Combinatorics, Probability and Computing*, 19(1): 61–85.

Gofman, Michael. 2011. "A Network-Based Analysis of Over-the-Counter Markets." https://mywebspace.wisc.edu/gofman/web/JMP/MichaelGofmanJMP.pdf.

Golub, Benjamin, and Matthew O. Jackson. 2010. "Naive Learning in Social Networks and the Wisdom of Crowds." *American Economic Journal: Microeconomics*, 2(1): 112–49.

Goyal, Sanjeev. 2007. *Connections: An Introduction to the Economics of Networks*. Princeton: Princeton University Press.

Granovetter, Mark S. 1974. *Getting A Job: A Study of Contacts and Careers*. Cambridge, MA: Harvard University Press.

Greenhill, Catherine, Brendan D. McKay, and Xiaoji Wang. 2006. "Asymptotic Enumeration of Sparse 0-1 Matrices with Irregular Row and Column Sums." *Journal of Combinatorial Theory, Series A*, 113(2): 291–324.

Greif, Avner. 1993. "Contract Enforceability and Economic Institutions in Early Trade: the Maghribi Traders' Coalition." *American Economic Review*, 83(3): 525–48.

Haag, Matthew, and Roger Lagunoff. 2006. "Social Norms, Local Interaction, and Neighborhood Planning." *International Economic Review*, 47(1): 265–96.

Hardle, Wolfgang, and Alan Kirman. 1995. "Nonclassical Demand: A Model-Free Examination of Price-Quantity Relations in the Marseille Fish Market." *Journal of Econometrics*, 67(1): 227–57.

Jackson, Matthew O. 2008. *Social and Economic Networks*. Princeton: Princeton University Press.

Jackson, Matthew O., Tomas Rodriguez-Barraquer, and Xu Tan. Forthcoming. "Social Capital and Social Quilts: Network Patterns of Favor Exchange." *American Economic Review*.

Jackson, Matthew O., and Leeat Yariv. 2007. "Diffusion of Behavior and Equilibrium Properties in Network Games." *American Economic Review*, 97(2): 92–98.

Johnson, Simon, John McMillan, and Christopher Woodruff. 2002. "Courts and Relational Contracts." *Journal of Law, Economics, and Organization*, 18(1): 221–77.

Kandori, Michihiro. 1992. "Social Norms and Community Enforcement." *Review of Economic Studies*, 59(1): 63–80.

Karlan, Dean, Markus Mobius, Tanya Rosenblat, and Adam Szeidl. 2009. "Trust and Social Collateral." *Quarterly Journal of Economics*, 124(3): 1307–61.

Kinateder, Markus. 2008. "Repeated Games Played in a Network." Fondazione Eni Enrico Mattei (FEEM) Working Paper 22.2008.

Kirman, Alan P., and Nicolaas J. Vriend. 2000. "Learning to Be Loyal. A Study of the Marseille Fish Market." In *Interaction and Market Structure: Essays on Heterogeneity in Economics*, ed. Domenico Delli Gatti, Mauro Gallegati, and Alan Kirman, 33–56. Berlin: Springer.

Kranton, Rachel E. 1996. "Reciprocal Exchange: A Self-Sustaining System." *American Economic Review*, 86(4): 830–51.

Kranton, Rachel E., and Deborah F. Minehart. 2001. "A Theory of Buyer-Seller Networks." *American Economic Review*, 91(3): 485–508.

Krause, Manfred. 1996. "A Simple Proof of the Gale-Ryser Theorem." *American Mathematical Monthly*, 103(4): 335–37.

Lee, Robin S., and Michael Schwarz. 2009. "Interviewing in Two-Sided Matching Markets." http://pages.stern.nyu.edu/~rslee/papers/CoordinationAndInterviewing.pdf.

Lehrer, Ehud, and Ady Pauzner. 1999. "Repeated Games with Differential Time Preferences." *Econometrica*, 67(2): 393–412.

Lippert, Steffen, and Giancarlo Spagnolo. 2011. "Networks of Relations and Word-of-Mouth Communication." *Games and Economic Behavior*, 72(1): 202–17.

Manea, Mihai. 2011. "Bargaining in Stationary Networks." *American Economic Review*, 101(5): 2042–80.

McIntosh, Craig, and Bruce Wydick. 2005. "Competition and Microfinance." *Journal of Development Economics*, 78(2): 271–98.

McMillan, John, and Christopher Woodruff. 1999. "Interfirm Relationships and Informal Credit in Vietnam." *Quarterly Journal of Economics*, 114(4): 1285–1320.

Mihm, Maximilian, Russell Toth, and Corey Lang. 2009. "What Goes Around Comes Around: A Theory of Strategic Indirect Reciprocity in Networks." Cornell Center for Analytical Economics Working Paper 09-07.

Nava, Francesco. 2010. "Quantity Competition in Networked Markets." http://personal.lse.ac.uk/nava/Cournot Trade Networks.pdf.

Nava, Francesco, and Michele Piccione. 2011. "Efficiency in Repeated Two-Action Games with Local Monitoring." http://personal.dse.qc.uk/nava/Local%20Monitoring.pdf.

Ozsoylev, Han N., and Johan Walden. Forthcoming. "Asset Pricing in Large Information Networks." *Journal of Economic Theory*, 146(6): 2252–80.

Pongou, Roland, and Roberto Serrano. 2011. "A Dynamic Theory of Fidelity Networks with an Application to the Spread of HIV/AIDS." http://www.econ.brown.edu/faculty/serrano/pdfs/wp2009-02.pdf.

Rosenblat, Tanya S., and Markus M. Mobius. 2004. "Getting Closer or Drifting Apart?" *Quarterly Journal of Economics*, 119(3): 971–1009.

Schwartzstein, Joshua. 2010. "Selective Attention and Learning." http://www.dartmouth.edu/~jschwartzstein/papers/sa.pdf.

Verrecchia, Robert E. 1982. "Information Acquisition in a Noisy Rational Expectations Economy." *Econometrica*, 50(6): 1415–30.

Watts, Duncan J. 2003. *Six Degrees: The Science of a Connected Age*. Portsmouth: Heinemann.

Weisbuch, Gérard, Alan Kirman, and Dorothea Herreiner. 1996. "Market Organisation and Trading Relationships." *Economic Journal*, 110(463): 411–36.

Woodruff, Christopher. 2002. "Establishing Confidence in Business Partners: Courts, Networks, and Relationships as Pillars of Support." Paper presented at Workshop 3: Formal and Informal Cooperation, Collegium Budapest, December.