Solution to Set # 1: Concentration Measures

(a)

(i) $I_4 = 60 + 10 + 5 + 5 = 80$. $I_{\mathrm{HH}} = 60^2 + 10^2 + 6 \cdot 5^2 = 3850$.

(ii) Firm $\overline{23}$ has a 15% market share. Hence, $\hat{I}_4 = 60 + 15 + 5 + 5 = 85$ and $\hat{I}_{\mathrm{HH}} = 60^2 + 15^2 + 5 \cdot 5^2 = 3950$.

(iii) $\Delta I_4 = 85 - 80 = 5$. $\Delta I_{\mathrm{HH}} = 3950 - 3850 = 100$.

(iv) $\bar{I}_4 = I_4 = 60 + 15 + 10 + 5 = 90$. $\bar{I}_{\mathrm{HH}} = 60^2 + 10^2 + 3 \cdot 5^2 + 15^2 = 4000$.

(v) $\Delta I_4 = 90 - 80 = 10$. $\Delta I_{\mathrm{HH}} = 4000 - 3850 = 150$.

(vi) For the merger between 2 and 3, $\hat{I}_{\mathrm{HH}} = 3950 > 1800$ and $\Delta I_{\mathrm{HH}} = 100 > 50$. Hence, this merger may be challenged.

For the merger between 6, 7, and 3, $\hat{I}_{\mathrm{HH}} = 4000 > 1800$ and $\Delta I_{\mathrm{HH}} = 150 > 50$. Hence, this merger may also be challenged.

(b) (i) According to the four-largest firm concentration index, industry A is more concentrated than industry B since

$$I_4^A = 40 + 15 + 15 + 15 = 85 > 78 = 45 + 11 + 11 + 11 = I_4^B.$$

According to the Hirschman-Herfindahl concentration index, industry B is more concentrated than industry A since

$$I_{HH}^A = 40^2 + 4 \cdot 15^2 = 2500 < 2630 = 45^2 + 5 \cdot 11^2 = I_{HH}^B.$$

Country	Firms						Concentration Index	
	1	2	3	4	5	6	I_4	I_{HH}
Albania	40%	15%	15%	15%	15%	0%	85	2500
Bolivia	45%	11%	11%	11%	11%	11%	78	2630

(ii) The post-merger $I_{\mathrm{HH}} = 45^2 + (11 + 11)^2 + 3 \cdot 11^2 = 2872 > 1800$. The change in this index as a result of this merger is $2872 - 2630 = 242 > 50$. Therefore, the merger is likely to be challenged according to the merger guidelines.

Solution to Set # 2: Normal-form Games

(a) (i) The firms' best-response functions are given by

$$p_G = R_G(p_F) = \begin{cases} p^L & \text{if } p_F = p^H \\ p^L & \text{if } p_F = p^L \end{cases} \quad \text{and} \quad p_F = R_F(p_G) = \begin{cases} p^L & \text{if } p_G = p^H \\ p^L & \text{if } p_G = p^L. \end{cases}$$

Remark: For students who know a little bit about Game Theory, firm G's best-response function implies that p^L is a *dominant* action. Similarly, p^L is a *dominant* action for firm F.

(ii) The two best-response function "intersect" at $\langle p_G, p_F\rangle = \langle p^L, p^L\rangle$. Hence, the outcome $\langle p^L, p^L\rangle$ constitutes a unique Nash equilibrium.

To formally prove that $\langle p^L, p^L\rangle$ that constitutes a Nash is equilibrium we must show that no firm can enhance its profit by deviating from p^L given that the other firm maintains p^L. Formally,

$$\pi^G(p^L, p^L) = 100 > 0 = \pi^G(p^H, p^L)$$
$$\pi^F(p^L, p^L) = 100 > 0 = \pi^F(p^L, p^H).$$

(iii) Yes, the outcome $\langle p^H, p^H\rangle$ Pareto dominates $\langle p^L, p^L\rangle$ since $\pi^G(p^H, p^H) = 200 > 100 = \pi^G(p^L, p^L)$, and $\pi^F(p^H, p^H) = 250 > 100 = \pi^F(p^L, p^L)$.

(iv) Industry profit is maximized when GM sets $p_G = p^L$ and FORD sets $p_F = p^H$, in which case $\pi_G + \pi_F = 500 + 0 = 500$.

(b)

$$t_A = BR_A(t_B) = \begin{cases} N & \text{if } t_B = N \\ O & \text{if } t_B = O \end{cases} \quad \text{and} \quad t_B = BR_B(t_A) = \begin{cases} O & \text{if } t_B = N \\ N & \text{if } t_B = O. \end{cases}$$

There is no Nash equilibrium in this game (no outcome lies on both best-response functions). To see this, note that

$$t_A = N \Longrightarrow t_B = O \Longrightarrow t_A = O \Longrightarrow t_B + N \Longrightarrow t_A = N \ldots.$$

Solution to Set # 3: Extensive-form Games

(a) (i) The firms' best-response functions are given by

$$t_A = R_A(t_B) = \begin{cases} N & \text{if } t_B = N \\ O & \text{if } t_B = O \end{cases} \quad \text{and} \quad t_B = R_B(t_A) = \begin{cases} N & \text{if } t_A = N \\ O & \text{if } t_A = O. \end{cases}$$

Remark: This game is known as the "Battle-of-the-Sexes."

(ii) The two best-response functions "intersect" twice: At $\langle t_A, t_B\rangle = \langle N, N\rangle$ and at $\langle t_A, t_B\rangle = \langle O, O\rangle$. That is, $N = R_A(N)$ and $N = R_B(N)$. Also, $O = R_A(O)$ and $O = R_B(O)$. Hence, there are two Nash equilibria: $\langle O, O\rangle$ and $\langle N, N\rangle$.

(iii) The second mover, firm B, sets its strategy to equal its best-response function $t_B(t_A)$ already computed in problem aii.

The first mover, firm A's SPE strategy is $t_A = N$ thereby earning $\pi_A(N, N) = 6$ since firm B responds with $t_B(N) = N$.

To demonstrate why this is the case suppose instead that firm A sets $t_A = O$. Then, firm B responds with $t_B(O) = O$, in which case firm A earns $\pi_A(O, O) = 4 < 6$. Hence, $t_A = O$ cannot be a SPE strategy.

(iv) The second mover, firm A, sets its strategy to equal its best-response function $t_A(t_B)$ already computed in problem aii.

Therefore, the first mover's (firm B) SPE strategy is $t_B = O$ thereby earning $\pi_A(O, O) = 5$, and this is because firm A responds with $t_B(O) = O$.

To demonstrate why this is the case suppose firm B sets $t_B = N$ instead of $t_B = O$. In this case, firm A responds with $t_B(N) = N$, in which case firm B earns $\pi_A(N, N) = 4 < 5$. Hence, $t_B = N$ cannot be a SPE strategy.

(b) (i)

$$t_A = BR_A(t_B) = \begin{cases} N & \text{if } t_B = N \\ O & \text{if } t_B = O \end{cases} \quad \text{and} \quad t_B = BR_B(t_A) = \begin{cases} O & \text{if } t_B = N \\ N & \text{if } t_B = O. \end{cases}$$

(ii) The game tree is not drawn here. From firm B's best response function given above, if $t_A = N$, $t_B(N) = O$, in which case firm A earns $\pi_A(N, O) = 0$. Instead, if $t_A = O$, $t_B(O) = N$, in which case firm A earns $\pi_A(O, N) = 50 > 0$. Therefore, the subgame-perfect equilibrium for this game is:

$$t_A = O \quad \text{and} \quad t_B = BR_B(t_A) = \begin{cases} O & \text{if } t_A = N \\ N & \text{if } t_A = O. \end{cases}$$

Note: Although there is no Nash equilibrium of the normal-form game a SPE of extensive-form game does exist, because the extensive-form game is somewhat different than the normal-form game.

(iii) The tree drawing is not provided here. From firm A's best response function given above, if $t_B = N$, $t_A(N) = N$, in which case firm B earns $\pi_B(N, N) = 0$. Instead, if $t_B = O$, $t_A(O) = O$, in which case firm B earns $\pi_B(O, O) = 50 > 0$. Therefore, the subgame-perfect equilibrium for this game is:

$$t_B = O \quad \text{and} \quad t_A = BR_A(t_B) = \begin{cases} N & \text{if } t_B = N \\ O & \text{if } t_B = O. \end{cases}$$

(iv) In the game (ii) $\pi_A(O, N) = 50$ and $\pi_B(O, N) = 100$.
In the game (iii) $\pi_A(O, O) = 100 > 50$ and $\pi_B(O, O) = 50$.

This game is interesting because A has a first-mover *disadvantage*. Not every game yields this result. This is because A would like to "match" B's technology, whereas B

gains from introducing a different technology. By letting B making the first choice, A is able to match its technology choice with B's choice.

(c) (i)

$$p_G = BR_G(p_F) = \begin{cases} p^L & \text{if } p_F = p^L \\ p^H & \text{if } p_F = p^M \\ p^M & \text{if } p_F = p^H \end{cases} \quad \text{and} \quad p_F = BR_F(p_G) = \begin{cases} p^L & \text{if } p_G = p^L \\ p^H & \text{if } p_F = p^M \\ p^M & \text{if } p_F = p^H \end{cases}$$

Therefore, there are three Nash equilibria:
$\langle p_G, p_F\rangle = \langle p^H, p^M\rangle$, $\langle p_G, p_F\rangle = \langle p^M, p^H\rangle$, and $\langle p_G, p_F\rangle = \langle p^L, p^L\rangle$.

(ii) No, because $\pi_G(p^H, p^M) = 250 < 300 = \pi_G(p^H, p^H)$,
but $\pi_F(p^H, p^M) = 350 > 300 = \pi_F(p^H, p^H)$.

(iii) The equilibrium strategies are:

$$p_F = p^M \quad \text{and} \quad p_G = BR_G(p_F) = \begin{cases} p^L & \text{if } p_F = p^L \\ p^H & \text{if } p_F = p^M \\ p^M & \text{if } p_F = p^H. \end{cases}$$

In this equilibrium $p_G = p^H$ and hence $\pi_F(p^M, p^H) = 350$ and $\pi_G(p^M, p^H) = 250$.
To prove that the above is a SPE, note that GM's strategy is its best-response function. Next, if Ford sets different prices then if

$$p_F = p^L \Longrightarrow p_G = p^L \Longrightarrow \pi_F(p^L, p^L) = 100 < 350$$

and if

$$p_F = p^H \Longrightarrow p_G = p^M \Longrightarrow \pi_F(p^H, p^M) = 250 < 350.$$

(d) (i)

$$p_G = BR_G(p_F) = \begin{cases} p^L & \text{if } p_F = p^L \\ p^H & \text{if } p_F = p^M \\ p^M & \text{if } p_F = p^H \end{cases} \quad \text{and} \quad p_F = BR_F(p_G) = \begin{cases} p^L & \text{if } p_G = p^L \\ p^H & \text{if } p_F = p^M \\ p^M & \text{if } p_F = p^H \end{cases}$$

Note first that firm G does not have a dominant action. This follows from the above best-response function by observing that firm G sets a low price, p^L, if firm F sets p^L. However, firm G sets a high price, p^H, if firm F sets p^M.
Now, a pair of prices $\langle p_G, p_F\rangle$ constitutes an equilibrium in dominant actions if each firm plays its dominant action. However, since firm G does not have a dominant action, such as equilibrium does not exist.

(ii) No, because $\pi_F(p^H, p^M) = 350 > 300 = \pi_F(p^H, p^H)$. Therefore, given that firm G sets $p_G = p^H$, firm F can increase its profit by deviating from $p_F = p^H$ to $p_F = p^M$.

(iii) There are two NE outcomes in the restricted game: $\langle p_G, p_F\rangle = \langle p^L, p^L\rangle$ and $\langle p_G, p_F\rangle = \langle p^M, p^M\rangle$. This follows from

$$\pi_G(p^L, p^L) = 100 \geq 50 = \pi_G(p^M, p^L) \quad \text{and} \quad \pi_F(p^L, p^L) = 100 \geq 50 = \pi_F(p^L, p^M)$$

and

$$\pi_G(p^M, p^M) = 200 \geq 150 = \pi_G(p^L, p^M) \quad \text{and} \quad \pi_F(p^M, p^M) = 200 \geq 150 = \pi_F(p^M, p^L).$$

Another way of proving this would be to construct the following two best-response functions

$$p_G = BR_G(p_F) = \begin{cases} p^L & \text{if } p_F = p^L \\ p^M & \text{if } p_F = p^M \end{cases} \quad \text{and} \quad p_F = BR_F(p_G) = \begin{cases} p^L & \text{if } p_G = p^L \\ p^M & \text{if } p_F = p^M \end{cases}$$

The two equilibria are on the firms' best-response functions.

(iv) The equilibrium strategies are:

$$p_F = p^M \quad \text{and} \quad p_G = BR_G(p_F) = \begin{cases} p^L & \text{if } p_F = p^L \\ p^M & \text{if } p_F = p^M \end{cases}$$

In this equilibrium $p_G = p^M$ and hence $\pi_F(p^M, p^M) = 200$ and $\pi_G(p^M, p^M) = 200$. To prove that the above is a SPE, note that GM's strategy is its best-response function. Next, if Ford sets different prices then if

$$p_F = p^L \Longrightarrow p_G = p^L \Longrightarrow \pi_F(p^L, p^L) = 100 < 200.$$

So, $p_F = p^M$ yields a higher profit to Ford.

Solution to Set # 4: Simple Monopoly

(a) (i) The marginal and average cost functions are given by

$$MC(Q) = \frac{\mathrm{d}C(Q)}{\mathrm{d}Q} = 2 \quad \text{and} \quad AC(Q) = \frac{C(Q)}{Q} = \frac{4}{Q} + 2.$$

The marginal revenue function is $MR(Q) = 12 - Q$.

(ii) The monopoly equates $MR(Q) = 12 - Q = 2 = MC(Q)$ to obtain the profit-maximizing output level $Q = 10$. The monopoly price is then $p = 12 - Q/2 = \$7$. Finally, the profit is $\pi = 7 \cdot 10 - 2 \cdot 10 - 4 = \$46$.

(iii) The direct demand function is $Q(p) = 24 - 2p$. Then, the price elasticity is

$$\epsilon_p = \frac{\mathrm{d}Q}{\mathrm{d}p}\frac{p}{Q} = -2\frac{7}{10} = -1.4.$$

Thus, the demand is elastic since $|e_p| > 1$.

(iv) The marginal and average cost functions are given by

$$MC(Q) = \frac{\mathrm{d}C(Q)}{\mathrm{d}Q} = 2Q \quad \text{and} \quad AC(Q) = \frac{C(Q)}{Q} = \frac{4}{Q} + Q.$$

The marginal revenue function is the same, $MR(Q) = 12 - Q$.

(v) The monopoly equates $MR(Q) = 12 - Q = 2Q = MC(Q)$ to obtain the profit-maximizing output level $Q = 4$. The monopoly price is then $p = 12 - Q/2 = \$10$. Finally, the profit is $\pi = 10 \cdot 4 - 4 - 4^2 = \$20$.

(vi) In class we have proved that at the profit-maximizing output, the monopoly sets the price to satisfy

$$p\left(1 + \frac{1}{\epsilon_p(Q)}\right) = MC(Q).$$

In the present case, the price elasticity is constant and is given by $e_p(Q) = -2$. Hence, the above "formula" becomes

$$p\left(1 + \frac{1}{-2}\right) = \frac{1}{2} \Longrightarrow p = \$1.$$

Hence, output level is $q = 120$, and the profit is $\pi = (1 - 0.5)120 - 4 = \$56$.

(b) The monopoly equates marginal revenue to marginal cost, c. Therefore,

$$p^m\left(1 + \frac{1}{\text{elas}}\right) = p^m\left(1 + \frac{1}{-3}\right) = \frac{2p^m}{3} = c = 8.$$

Therefore, $p^m = \$12$. Next, $Q = 34560\, p^{-3} = 20$ units. Hence, $\pi = (p - c)Q = (12 - 8)20 = \$80$.

(c) The direct demand function facing this monopoly is:

$$Q(p) = \begin{cases} 0 & \text{if } p > 500 \\ 1000 & \text{if } 300 < p \le 500 \\ 4000 & \text{if } 200 < p \le 300 \\ 9000 & \text{if } p \le 200 \end{cases} \quad \text{hence} \quad \pi(p) = \begin{cases} 0 & \text{if } p > 500 \\ (500 - 100)1000 & \text{if } p = 500 \\ (300 - 100)4000 & \text{if } p = 300 \\ (200 - 100)9000 & \text{if } p = 200. \end{cases}$$

Therefore, the monopoly's profit-maximizing price is $p = 200$ yielding a profit of $\pi = (200 - 100)9000 = \$900,000$.

Solution to Set # 5: Discriminating Monopoly

(a) (i) Equating the marginal revenue to marginal cost in the market for nonstudents yields $MR_N = 12 - 2q_N = 2$, hence, $q_N = 5$. Therefore, $p_N = 12 - 5 = \$7$. Similarly, in the market for students $MR_S = 6 - 2q_S = 2$, hence, $q_S = 2$. Therefore, $p_S = 6 - 2 = \$4 < p_N$. Thus, students indeed receive a discount of $p_N - p_S = 7 - 4 = \$3$. Next, combined total profit from selling in both markets is

$$\pi^D = (7-2)5 + (4-2)2 - 10 = \$19.$$

(ii) We first check how much profit can be earned if the seller sets a sufficiently low price so the entire market is served. To compute this, we first must find the aggregate market demand curve. Inverting the two demand curves yield $q_N = 12 - p$ and $q_S = 6 - p$. The aggregate demand curve is then $Q = q_N + q_S = 18 - 2p$. The resulting inverse aggregate demand function and the marginal revenue function are

$$p = \frac{18 - Q}{2} \quad \text{and} \quad MR = 9 - Q.$$

Solving $MR = 9 - Q = 2 = MC$ yields $Q = 7$, therefore, $p = (18-7)/2 = 11/2 < 6$. Altogether, the profit when both markets are served is

$$\pi^{\text{both}} = \left(\frac{11}{2} - 2\right)7 - 10 = \frac{29}{2} = 14.5.$$

We are now able to compute the profit assuming that the monopoly sets a sufficiently high price so only nonstudents can "afford" to purchase concert tickets. Thus, under a sufficiently high price, $q_S = 0$, and the monopoly solves $MR_N(q_N) = 12 - 2q_N = 2$ yielding $q_N = 5$. The price should set to $p = 12 - 5 = \$7$. Actually, we have already calculated these figures in the analysis of price discrimination above. What is important to check is that $p = 7 > 6$ which is the intercept of the students' inverse demand function. Hence, students don't buy at this price. Total profit is then given by

$$\pi^{\text{ND}} = \pi^{\text{1 only}} = (7-2)\,5 - 10 = 15 > 14.5.$$

Since $\pi^{\text{1 only}} > \pi^{\text{both}}$, the monopoly earns a higher profit when setting a sufficiently high price so only nonstudents purchase concert tickets.

(iii) The gain from price discrimination is therefore $\pi^{\text{1 only}} - \pi^{\text{both}} = 19 - 15 = \$4$.

(b) (i) The aggregate demand curve should be drawn according to the following formula:

$$Q(p) = \begin{cases} 0 & \text{if } p > \$20 \\ 200 & \text{if } \$10 < p \le \$20 \\ 200 + 300 & \text{if } p \le \$10. \end{cases}$$

(ii) Setting a high price, $p = \$20$ generates $Q = 200$ consumers and a profit of $\pi_H = (20 - 5)200 = \$3000$.

Setting a low price, $p = \$10$ generates $Q = 200 + 300$ consumers and a profit of $\pi_H = (10-5)500 = \$2500 < \$3000$. Hence, $p = \$20$ is the profit-maximizing price. Type L consumers will not buy under this prices.

(iii) The monopoly will charge $p = \$20$ in market H and $p = \$10$ in market L. Hence, total profit is given by

$$\Pi = \pi_H + \pi_L = (20 - 5)200 + (10 - 5)300 = 3000 + 1500 = \$4500 > \$3000.$$

Clearly, the ability to price discriminate cannot reduce the monopoly profit since even with this ability, the monopoly can always set equal prices in both markets. The fact that the monopoly chooses different prices implies that profit can only increase beyond the profit earned when the monopoly is unable to price discriminate.

(c) The demand price elasticity is -2 in the nonstudents' market, and -3 in the students' market. In the nonstudents' market, the monopoly sets p_N to solve

$$p_N \left[1 + \frac{1}{-2}\right] = \$2 \quad \text{yielding } p_N = \$4 \text{ and hence } q_N = \frac{240}{4^2} = 15.$$

In the students' market, the monopoly sets p_S to solve

$$p_S \left[1 + \frac{1}{-3}\right] = \$2 \quad \text{yielding } p_S = \$3 \text{ and hence } q_S = \frac{540}{3^3} = 20.$$

(d) (i) We must analyze two cases. $p > 50¢$, in which case only market 1 is served; and $p \le \$50$ in which markets 1 and 2 are both served.

If only market 1 is served, the monopoly sets $MR_1 = 60 - 2q_1 = 30$ yielding $q_1 = 15$ and hence $p_1 = 60 - 15 = 45¢$. Since $p_1 < 50¢$, consumers in market 2 also buy, hence both markets are served and this computation becomes irrelevant.

If both market 1 and 2 are served, the aggregate direct demand function is $q_{12} = 60 - p + 50 - p = 110 - 2p$. The inverse aggregate demand is $p_{12} = 55 - q_{12}/2$. Therefore, the monopoly solves

$$MR_{12} = 55 - q_{12} = 30 \quad \text{yielding } q_{12} = 25 \text{ and hence } p_{12} = 55 - \frac{25}{2} = \frac{85}{2} = 42.5¢.$$

Observe that since $p_{12} < 50¢$, the monopoly indeed sells in both markets.

To compute the profit,

$$\pi_{12} = \left(\frac{85}{2} - 30\right) 25 = \frac{625}{2} = 312.5¢.$$

(ii) The previous section demonstrated that the profit-maximizing price is $p_{12} = 42.5$ when selling to markets 1 and 2 only. Since $p_{12} = 42.5 > 40$, consumers in market 3 will not buy at this price. Therefore, it remains to investigate whether reducing the price below 40¢ (thereby serving consumers in all 3 markets) would enhance the monopoly profit beyond the profit made when only markets 1 and 2 are served.

So, suppose that $p_{123} \leq 40$ so all 3 market are served. The aggregate direct demand function is $q_{123} = 60-p+50-p+40-p = 150-3p$. The inverse demand function is $p_{123} = 50 - q_{123}/3$. Therefore, the monopoly solves

$$MR_{123} = 50 - \frac{2}{3}q_{123} = 30, \quad \text{yielding } q_{123} = 30 \text{ and hence } p_{123} = 50 - \frac{30}{3} = 40¢.$$

The resulting profit is

$$\pi_{123} = (40-30)30 = 300 < \frac{625}{2} = \pi_{12}.$$

Therefore, $p = 85/2 = 42.5$ is the profit-maximizing price. Under this price, the quantity sold in each market is

$$q_1 = 60 - \frac{85}{2} = 17.5, \quad q_2 = 50 - \frac{85}{2} = 7.5, \quad \text{and} \quad q_3 = 0.$$

(e) Section 2 of the 1914 Clayton Act states that price discrimination is unlawful if its effect is "to lessen competition or tend to create a monopoly...or to injure destroy or prevent competition." In addition, price differentials are also allowed to account for "differences in the cost of manufactures, sale or delivery."

This, in part, implies that price discrimination that does not reduce competition should not be viewed as illegal.

(f) In the market for nonstudents,

$$MR_N = p_N = \left[1 + \frac{1}{-3}\right] = c = 6 \Longrightarrow p_N = \$9.$$

$$MR_S = p_S = \left[1 + \frac{1}{-4}\right] = c = 6 \Longrightarrow p_N = \$8.$$

To find the amount of tickets sold to each group, solve

$$q_N = 7290 \cdot 9^{-3} = 10 \quad \text{and} \quad q_S = 40960 \cdot 8^{-4} = 10 \quad \text{hence} \quad Q = q_N + q_S = 20.$$

(g) (i) In the absence of capacity constraint, the price discriminating monopoly solves $MR_1 = 120 - 2q_1 = c = 30$ and $MR_2 = 120 - 2q_2/3 = c = 30$ yielding $q_1 = 45$ and $q_2 = 135$. Hence, $p_1 = 120 - 45 = 75$ and $p_2 = 120 - 135/3 = 75$. Hence, total profit is

$$\Pi = \pi_1 + \pi_2 = (75 - 30)45 + (75 - 30)135 = 8100.$$

(ii) The above computation showed that with no capacity limit $q_1 + q_2 = 180 > 160$. Hence, the capacity constraint is binding and the monopoly will produce at the maximum possible level, $Q = 160$.

Under capacity constraint, the monopoly solves for sales levels q_1 and q_2 that solve

$$MR_1 = 120 - 2q_1 = 120 - \frac{2q_2}{3} = MR_2 \quad \text{and} \quad q_1 + q_2 = 160$$

yielding $q_1 = 40$ and $q_2 = 120$. Hence, $p_1 = 120 - 40 = 80$ and $p_2 = 120 - 120/3 = 80$. The resulting profit is

$$\Pi = (p_1 - c)q_1 + (p_2 - c)q_2 = (80 - 30)40 + (80 - 30)120 = 8000 < 8100.$$

Clearly, the monopoly earns a lower profit if it is forced to reduce production below its profit-maximizing levels.

Remark: An alternative solution would be to solve

$$\max_{q_1} \Pi = (120 - q_1)q_1 + \left(120 - \frac{160 - q_1}{3}\right) - 30 \cdot 160$$

and then to set $q_2 = 160 - q_1$.

(h) First, we should solve for the direct demand functions: $q_1 = 36 - p_1$, and $q_2 = 48 - 2p_2$, and $q_3 = 24 - 2p_3$. Next, we should examine three possible price ranges, and compare the resulting profit levels.

Let $p > 24$, which means that $q_2 = q_3 = 0$. Solving $MR_1 = 36 - 2q_1$ yields $q_1 = 18$ and $p = 36 - 18 = 18 < 24$. A contradiction to our assumption that $p > 24$.

Let $12 < p \leq 24$, in which case $q_3 = 0$. Aggregate demand facing this monopoly is therefore $q_{12} = q_1 + q_2 = 84 - 3p$. Thus, $p = (84 - q_{12})/3$ and hence $MR_{12} = (84 - 2q_{12})/3 = c = 0$ yields $q_{12} = 42$. Hence, $p = (84 - 42)/3 = 14$. The resulting profit (revenue, since production is costless) is $\pi_{12} = 14 \cdot 42 = 588$.

Lastly, let $p < 12$. Aggregate demand is $q_{123} = q_1 + q_2 + q_3 = 108 - 5p$. Inverse demand is therefore $p = (108 - q_{123})/5$. Solving $MR_{123} = (108 - 2q_{123})/5 = c = 0$ yields $q_{123} = 54$ and hence $p = 54/5$. Profit (revenue) is therefore $\pi_{123} = 583.2 < 588$.

To summarize, the profit-maximizing price of this non-discriminating monopoly is $p = 14$. The monopoly sells in markets 1 and 2 only and earns a profit of $\pi = 588$.

Solution to Set # 6: Cournot Competition (Static)

(a) (i) Firm A takes q_B as given and solves

$$\max_{q_A} \pi_A = (p - c_a)q_A = \left(120 - \frac{q_A + q_B}{2} - 1\right) q_A.$$

(ii) The first-order condition for a maximum is $0 = \partial \pi_A / \partial q_A = (238 - 2q_A + q_B)/2$. Therefore, firm A's output best-response function is given by

$$q_A = R_A(q_B) = 119 - \frac{1}{2}\, q_B.$$

Observe that the second-order condition for a maximum is fulfilled since $\partial^2 \pi_A / \partial (q_A)^2 = -1 < 0$.

(iii) Firm B takes q_A as given and solves

$$\max_{q_B} \pi_B = (p - c_B)q_B = \left(120 - \frac{q_A + q_B}{2} - 2\right) q_B.$$

(iv) The first-order condition for a maximum is $0 = \partial \pi_B / \partial q_B = (236 - 2q_B + q_A)/2$. Therefore, firm B's output best-response function is given by

$$q_B = R_B(q_A) = 118 - \frac{1}{2}\, q_A.$$

Observe that the second-order condition for a maximum is also fulfilled since $\partial^2 \pi_B / \partial (q_B)^2 = -1 < 0$.

(v) Not answered.

(vi) The above two best-response functions constitute a system of two linear equations with two variables, q_A and q_B. The unique solution for this system is given by $q_A = 80$ and $q_B = 78$. Clearly, $q_A > q_B$ since firm A is more efficient in the sense that it produces each gallon at half the cost of firm B.

(vii) Aggregate industry output is $Q = q_A + q_B = 158$ gallons. The corresponding price is $p = 120 - 0.5 \cdot 158 = 41$.

(viii) $\pi_A = (p - 1)q_A = 3200$ and $\pi_B = (p - 2)q_B = 3042$. Clearly, because firm A is more efficient (lower unit cost), it earns a higher profit than firm B. Aggregate industry profit is given by $\pi_A + \pi_B = 6242$.

(b) (i) The best-response functions are given by

$$q_A(q_B) = \frac{9 - q_B}{2} \quad \text{and} \quad q_B(q_A) = \frac{9 - q_A}{2}.$$

(ii) The above best-response function constitute two linear equations with two variables, q_A and q_B. The unique solution is $q_A^c = q_B^c = 3$ gallons. Both firms produce the same amount since there equally efficient in the sense that the bear identical production costs.

(iii) $Q = q_A + q_B = 3 + 3 = 6$ gallons. The equilibrium price is $p = 12 - Q = \$6$.

(iv) Since there are no fixed costs, $\pi_A = (p - c_A)q_A = (6 - 3)3 = \$9$. Similarly, $\pi_B = (p - c_B)q_B = (6 - 3)3 = \$9$. Industry profit is then $\Pi = \pi_A + \pi_B + 9 + 9 = \$18$. Both firms earn the same profit since the bear identical production costs.

(c) The solution is: $q_A = 40$, $q_B = 30$, $p = \$50$, $\pi_A = \$1600$, $\pi_B = \$900$. Industry output and profit levels are: $Q = q_A + q_B = 70$ and $\pi_A + \pi_B = \$2500$.

(d) The solution is: $q_A = 160$, $q_B = 140$, $p = \$90$, $\pi_A = \$12,800$, $\pi_B = \$9800$. Industry output and profit levels are: $Q = q_A + q_B = 300$ and $\pi_A + \pi_B = \$22,600$.

Solution to Set # 7: Sequential Moves (Quantity Game)

(a) (i) See the solution to problem (a)(iv) in the Solution to Set # 6:.

(ii) Firm A takes into consideration the above firm B's best-response functions and solves

$$\max_{q_A} \pi_A = \left(120 - \frac{q_A + R_B(q_A)}{2} - 1\right) q_A = \left(120 - \frac{q_A + 118 - \frac{1}{2}\, q_A}{2} - 1\right) q_A.$$

The first-order condition for a maximum is $0 = \partial\pi_A/\partial q_A = 60 - q_A/2$. Therefore, $q_A = 120$.

(iii) Using B's best-response function, $q_B = 118 - q_A/2 = 58$.

(iv) Aggregate industry output is $Q = q_A + q_B = 178$. Hence, $p = 120 - Q/2 = 31$.

(v) $\pi_A = (p - 1)q_A = 3600$ and $\pi_B = (p - 2)q_B = 1682$.

(vi) $\Delta q_A = q_A^s - q_A^c = 120 - 80 = 40 > 0$. Thus, firm A uses its first-mover advantage to expand production. $\Delta\pi_A = \pi_A^s - \pi_A^c = 3600 - 3200 = 400 > 0$.

(vii) $\Delta q_B = q_B^s - q_B^c = 58 - 80 = -20 < 0$. Thus, firm B responds with a lower output than in a Cournot equilibrium to avoid letting the price fall too much. $\Delta\pi_B = \pi_B^s - \pi_B^c = 1682 - 3042 = -1360 < 0$.

(viii) $\Delta Q = Q^s - Q^c = 178 - 158 = 20 > 0$. Thus, aggregate industry output in a sequential-move equilibrium is higher than that of under Cournot. This means that the price must be lower, in fact, $\Delta p = p^s - p^c = 31 - 41 = -10 < 0$. Finally, $(\pi_A^s + \pi_B^s) - (\pi_A^c + \pi_B^c) = 5282 - 6242 = -960 < 0$, which means that aggregate industry profit is lower under the sequential-move equilibrium.

(b) (i) The solution is: $q_A = 12$, $q_B = 4$, $p = \$4$, $\pi_A = \$36$, $\pi_B = \$8$. Industry output and profit levels are: $Q = q_A + q_B = 16$ and $\pi_A + \pi_B = \$44$.

(ii) The solution is: $q_A = 6.5$, $q_B = 9$, $p = \$4.25$, $\pi_A = \$21.125$, $\pi_B = \$20.25$. Industry output and profit levels are: $Q = q_A + q_B = 15.5$ and $\pi_A + \pi_B = \$41.375$.

(c) (i) The solution is: $q_A = 6$, $q_B = 2$, $p = \$4$, $\pi_A = \$18$, $\pi_B = \$4$. Industry output and profit levels are: $Q = q_A + q_B = 8$ and $\pi_A + \pi_B = \$22$.

(ii) The solution is: $q_A = 3.25$, $q_B = 4.5$, $p = \$4.25$, $\pi_A = \$10.5625$, $\pi_B = \$10.125$. Industry output and profit levels are: $Q = q_A + q_B = 7.75$ and $\pi_A + \pi_B = \$20.6875$.

(d) (i) Firm A sets q_A to solve

$$\max_{p_A} \pi_A = \left(12 - \frac{q_A + q_B}{3}\right) q_A - 2q_A$$

The first order condition $0 = 12 - 2q_A/3 - q_B/3 - 2$ yields

$$q_A = BR_A(q_B) = 15 - \frac{1}{2} q_B.$$

Similarly, firm B chooses q_B to solve

$$\max_{p_B} \pi_B = \left(12 - \frac{q_A + q_B}{3}\right) q_B - 2q_B$$

The first order condition $0 = 12 - q_A/3 - 2q_B/3 - 2$ yields

$$q_B = BR_B(q_A) = 15 - \frac{1}{2} q_A.$$

Solving the above two best-response function yields $q_A^c = q_B^c = 10$.

(ii) Aggregate industry output is $Q^c = q_A^c + q_B^c = 20$. Therefore, the equilibrium price is $p^c = 12 - 20/3 = 16/3$. The profit of firm i ($i = A, B$) is:

$$\pi_A = p^c q_i^c - 2q_i = \left(\frac{16}{3} - 2\right) 10 = \frac{100}{3}.$$

(iii) Firm A takes B's best-best response function into account (instead of taking q_B as given) and chooses q_A to solve

$$\max_{p_A} \pi_A = \left[12 - \frac{q_A}{3} - \frac{1}{3}\left(15 - \frac{1}{2} q_A\right)\right] q_A - 2q_A = \left(7 - \frac{q_A}{6}\right) q_A - 2q_A.$$

The first-order condition yields $0 = 7 - q_A/3 - 2$, and hence $q_A^{sl} = 15$. Substituting into B's best-response function yields $q_B^{sl} = 15 - q_A/2 = 15/2$.

(iv) Total output is $Q^s = q_A^{sl} + q_B^{sf} = 45/2$. Substituting into the demand function obtains $p^s = 12 - 45/6 = 9/2 = \$4.5$. Substituting into the firms' profit functions yield

$$\pi_A^{sl} = \left(\frac{9}{2} - 2\right) 15 = \frac{75}{2} = \$37.5$$

$$\pi_B^{sf} = \left(\frac{9}{2} - 2\right) \frac{15}{2} = \frac{75}{4} = \$18.75$$

(e) In stage $t = 3$, firm 3 takes q_1 and q_2 as given and solves

$$\max_{q_3} \pi_3 = (120 - q_1 - q_2 - q_3)q_3 \quad \text{yielding} \quad q_3 = BR_3(q_1, q_2) = \frac{120 - q_1 - q_2}{2}.$$

In stage $t = 2$, firm 2 takes q_1 and $BR_3(q_1, q_2)$ as given and solves

$$\max_{q_2} \pi_2 = \left[120 - q_1 - q_2 - \frac{120 - q_1 - q_2}{2}\right] q_2 \quad \text{yielding} \quad q_2 = BR_2(q_1) = \frac{120 - q_1}{2}.$$

Substituting $q_2 = BR_2(q_1)$ into $BR_3(q_1, q_2$ yields

$$q_3 = BR_3(q_1) = \frac{120 - q_1}{4}.$$

In stage $t = 1$, firm 1 chooses q_1 to solve

$$\max_{q_1} \pi_1 = (120 - q_1 - q_2 - q_3)q_1 = \left(120 - q_1 - \frac{120 - q_1}{2} - \frac{120 - q_1}{4}\right) q_1.$$

The solution to firm 1's profit maximization problem is

$$q_1 = 60, \quad q_2 = \frac{120 - q_1}{2} = 30, \quad \text{and} \quad q_3 = \frac{120 - q_1}{4} = 15.$$

Aggregate industry production and the market price are therefore

$$Q = q_1 + q_2 + q_3 = 105 \quad \text{hence} \quad p = 120 - 105 = 15.$$

Profits (same as revenue because production is costless) are therefore

$$\pi_1 = 15 \cdot 60 = 900, \quad \pi_2 = 15 \cdot 30 = 450, \quad \text{and} \quad \pi_3 = 15 \cdot 15 = 225.$$

Solution to Set # 8: Bertrand Price Competition (Static and Sequential)

(a) (i) The first case to be checked is where the efficient firm A undercuts B by setting $p_A = c_B - \epsilon = 8 - \epsilon$, where ϵ is a small number. Firm B sets $p_B = c_B = \$8$. In this case, all consumers buy brand A only, hence, solving $8 = 12 - q_A/2$ yields $q_A = 8$ and $q_B = 0$. The profits are then $\pi_A = (8-6)8 = \$16$ and $\pi_B = 0$.

The second case to be checked is where A sets a monopoly price. Solving $MR = 12 - q_A = c_A = 6$ yields $q_A = 6$. Hence, $p = 12 - 6/2 = \$9 > \$8 = p_B$. Therefore, in this case, firm A cannot charge its monopoly price.

Altogether, the firms' best-response functions are given by

$$p_A = BR_A(p_B) = \begin{cases} 9 & \text{if } p_B > 9 = p_A^m \\ p_B - \epsilon & \text{if } 6 < p_B \le 9 \\ 6 & \text{if } p_B \le 6 \end{cases}$$

and

$$p_B = BR_B(p_A) = \begin{cases} 10 & \text{if } p_A > 10 = p_B^m \\ p_A - \epsilon & \text{if } 8 < p_A \le 10 \\ 8 & \text{if } p_B \le 8. \end{cases}$$

Therefore, a Nash-Bertrand equilibrium is $p_A^b = \$8 - \epsilon$ and $p_B^b = \$8$. The equilibrium profits are therefore $\pi_A^b = (8-6)8 = \$16$ and $\pi_B^b = 0$.

(ii) The first case to be checked is whether the efficient firm A undercuts B by setting $p_A = c_B - \epsilon = 8 - \epsilon$, where ϵ is a small number. Firm B sets $p_B = c_B = \$8$. In this case, all consumers buy brand A only, hence, solving $8 = 12 - q_A/2$ yields $q_A = 8$ and $q_B = 0$. The profits are then $\pi_A = (8-2)8 = \$48$ and $\pi_B = 0$.

The second case to be checked is where A sets a monopoly price. Solving $MR = 12 - q_A = c_A = 2$ yields $q_A = 10$. Hence, $p = 12 - 10/2 = \$7 < \$8 = p_B$. Therefore, $\pi_A = (7-2)10 = \$50 > \$48$.

Altogether, a Nash-Bertrand equilibrium is $p_A^b = \$7$ and $p_B^b = \$8$. The equilibrium profits are therefore $\pi_A^b = (7-2)10 = \$50$ and $\pi_B^b = 0$.

(b) (i) Let ϵ (epsilon) denote a small number, or simply the smallest currency denomination. The firms' best-response functions are:

$$p_A = BR_A(p_B) = \begin{cases} 7 & \text{if } p_B > 7 \\ p_B - \epsilon & \text{if } 2 < p_B \le 7 \\ 2 & \text{if } p_B \le 2, \end{cases}$$

and

$$p_B = BR_B(p_A) = \begin{cases} 7 & \text{if } p_A > 7 \\ p_A - \epsilon & \text{if } 2 < p_A \le 7 \\ 2 & \text{if } p_A \le 2. \end{cases}$$

Note that $p^m = \$7$ is the monopoly price, which is computed by $MR^m = 12 - 2q^m/3 = 2$ implying that $q^m = 15$ units and hence $p^m = 12 - 15/3 = \$7$.

(ii) The unique Bertrand-Nash equilibrium is $p_A^b = p_B^b = \$2$ (since the firms have identical unit costs, price competition leads to unit cost pricing). Clearly, each firm earns zero profit, so that $\pi_A^b = \pi_B^b = 0$. Total output is solved from $2 = 12 - Q/3$ implying that $Q^b = 30$. Hence, $q_A^b = q_B^b = 15$ units.

(iii) There are "many" equilibria in the form of $p_A \geq 2$ (equilibrium strategy of firm A) and

$$p_B = BR_B(p_A) = \begin{cases} 7 & \text{if } p_A > 7 \\ p_A - \epsilon & \text{if } 2 < p_A \leq 7 \\ 2 & \text{if } p_A \leq 2, \end{cases}$$

which is the equilibrium strategy of firm B. Notice that firm A (first mover) is indifferent between setting $p_A = \$2$ and $p_A > \$2$ since it is being undercut by firm B in either case, and therefore makes zero profit.

(c) (i) In the second stage of this game, firm B solves

$$\max_{q_B} \pi_B = \left(12 - \frac{q_A}{3} - \frac{q_B}{3}\right) q_B - 0 \cdot q_B,$$

yielding B's best-response function

$$q_B(q_A) = 18 - \frac{q_A}{2}.$$

In the first stage, firm A solves

$$\max_{q_A} \pi_A = \left[12 - \frac{1}{3}\, q_A - \frac{1}{3}\left(18 - \frac{q_A}{2}\right)\right]$$

yielding $q_A = 18$. Therefore $q_B = 18 - 18/2 = 9$, $Q = 18 + 9 = 27$, $p = 12 - 27/3 = \$3$. Hence, $\pi_A = (3-0)18 = \$54$ and $\pi_B = (3-0)9 = \$27$.

(ii) We first compute the monopoly price from $MR = 12 - 2Q/3 = c = 0$ yielding $Q^m = 18$ hence $p^m = 12 - 18/3 = \$6$. Therefore, B's best response function (second stage) is

$$p_B(p_A) = BR_B(p_A) = \begin{cases} 6 & \text{if } p_A > 6 \\ p_A - \epsilon & \text{if } 0 < p^A \leq 6 \\ 0 & \text{if } p_A = 0. \end{cases}$$

Notice that firm A is indifferent among all prices $p_A \geq 0$ since it makes zero profit regardless of which price it sets. Therefore, there are many equilibria consisting of the above B's best-response function and $p_A \geq 0$ (including $p_A = 0$, in which case $p_B = 0$.

To summarize the above analysis, any SPE takes the form of $p_A \geq 0$ and $p_B = BR_B(p_A)$ where the best-response function $BR_B(p_A)$ is defined above.

(iii) We have already shown that the monopoly price (for firm A) is $p_A^m = \$6$. The monopoly price for firm B is computed from $MR = 12 - 2Q/3 = c_B = 4$ yielding $q_B^m = 12$ and hence $p_B^m = 12 - 12/3 = \$8$. Hence, B's best-response function is now given by

$$p_B(p_A) = \begin{cases} 8 & \text{if } p_A > 8 \\ p_A - \epsilon & \text{if } 4 < p^A \le 8 \\ 4 & \text{if } p_A \le 4. \end{cases}$$

Firm A sets its monopoly price $p_A = 4 - \epsilon$ and grabs the entire market.

To summarize the above analysis, the SPE strategies are: $p_A = 4 - \epsilon$ and $p_B = BR_B(p_A)$ where the best-response function $BR_B(p_A)$ is defined above. In this equilibrium, $q_A^b = 24$, $q_B^b = 0$, and $\pi_A^b = (4 - 0)24 = 96$.

Solution to Set # 9: Self-enforcing Collusion

(a) (i) The trigger strategy of GM is to keep charging p^H in each period τ as long as both firms charge p^H in all earlier periods $t = 0, 1, 2, \ldots, \tau - 1$. If one of the firms deviates in one of these earlier periods, GM sets p^L forever. Formally,

$$p_G(\tau) = \begin{cases} p^H & \text{if } p_G(t) = p_F(t) = p^H \text{ in each period } t = 0, 1, 2, \ldots, \tau - 1 \\ p^L & \text{otherwise.} \end{cases}$$

Similarly for FORD,

$$p_F(\tau) = \begin{cases} p^H & \text{if } p_G(t) = p_F(t) = p^H \text{ in each period } t = 0, 1, 2, \ldots, \tau - 1 \\ p^L & \text{otherwise.} \end{cases}$$

(ii) The infinite discounted sum of the stream of profits for each firm are

$$\Pi_G = \sum_{t=0}^{\infty} \rho^t \cdot 200 = \frac{200}{1-\rho} \quad \text{and} \quad \Pi_F = \sum_{t=0}^{\infty} \rho^t \cdot 250 = \frac{250}{1-\rho}.$$

(iii) Given $p_F(0) = p^H$, if GM reduces the price to $p_G = p^L$ it earns $\pi_G(p^L, p^H) = 500$.

(iv) Given $p_G(0) = p^H$, if FORD reduces the price to $p_F = p^L$ it earns $\pi_F(p^H, p^L) = 300$.

(v) If GM deviates from the collusive price in $t = 0$, FORD will lower its price from period $t = 1$ and on. Hence,

$$\Pi_G = 500 + \rho \cdot \frac{100}{1-\rho}.$$

(vi) If FORD deviates from the collusive price in $t = 0$, GM will lower its price from period $t = 1$ and on. Hence,

$$\Pi_F = 300 + \rho \cdot \frac{100}{1-\rho}.$$

(vii) GM will not unilaterally deviate from the collusive price if

$$\frac{200}{1-\rho} \geq 500 + \rho \cdot \frac{100}{1-\rho} \quad \text{hence if } \rho \geq \frac{3}{4}.$$

Similarly, FORD will not unilaterally deviate from the collusive price if

$$\frac{250}{1-\rho} \geq 300 + \rho \cdot \frac{100}{1-\rho} \quad \text{hence if } \rho \geq \frac{1}{4}.$$

Therefore, self-enforcing collusion can be sustained for all discount factors satisfying $\rho \geq 3/4$.

(b) (i) GM's discounted sum of profits when it does not deviate from the collusive high price, and when it deviates from the collusive price are given by

$$\pi_G = \frac{5}{1-\rho} \quad \text{and} \quad \pi_G' = 5 + \rho\,\frac{4}{1-\rho}.$$

Hence, $\pi_G \geq \pi_G'$ for every ρ satisfying $0 < \rho < 1$. Intuitively, it follows directly from the profit levels in the above table that GM cannot benefit even from one-period deviation since $\pi_G(p^L, p^H) = 5 = \pi_G(p^H, p^H)$.

(ii) Ford's discounted sum of profits when it does not deviate from the collusive high price, and when it deviates from the collusive price are given by

$$\pi_F = \frac{4}{1-\rho} \quad \text{and} \quad \pi_F' = 6 + \rho\,\frac{3}{1-\rho}.$$

Hence, $\pi_F \geq \pi_F'$ if $\rho > 2/3$.

(c) (i) The monopoly price is $p^m = 10$. Hence, the best response function of the firms are

$$p_A(p_B) = \begin{cases} 10 & \text{if } p_B > 10 \\ p_B - \epsilon & \text{if } 2 < p_B \leq 10 \\ 2 & \text{if } p_B \leq 2 \end{cases} \quad \text{and} \quad p_B(p_A) = \begin{cases} 10 & \text{if } p_A > 10 \\ p_A - \epsilon & \text{if } 2 < p_A \leq 10 \\ 2 & \text{if } p_A \leq 2. \end{cases}$$

Hence, the unique Nash equilibrium is $p_A = p_B = \$2$.

(ii) Trigger price strategy of A is at each period τ

$$p_A^\tau = \begin{cases} 10 & \text{if } p_A^t = p_B^t = 10 \text{ for all } t = 1, 2, \ldots, \tau - 1 \\ 2 & \text{otherwise.} \end{cases}$$

Trigger price strategy of B is at each period τ

$$p_B^\tau = \begin{cases} 10 & \text{if } p_A^t = p_B^t = 10 \text{ for all } t = 1, 2, \ldots, \tau - 1 \\ 2 & \text{otherwise.} \end{cases}$$

If firm A does not deviate, its discounted stream of profit is

$$\pi_A = \sum_{t=0}^{\infty} \rho^t (10 - 2)\frac{N}{2} = \frac{4N}{1-\rho}.$$

If firm A deviates in period $t = 0$, its discounted stream of profit is

$$\pi_A' = (10 - 2 - \epsilon)N + \rho\frac{0}{1-\rho} \approx 8N.$$

Deviation is not profitable for firm A if $\pi_A \geq \pi_A'$ or

$$\frac{4N}{1-\rho} \geq 8N \quad \text{hence} \quad \rho \geq \frac{1}{2}.$$

Because firm B is identical to firm A, $\rho \geq 1/2$ is also sufficient for having firm B not deviating from the collusive price.

(d) (i) First, compute the monopoly's price. $MR = 140 - 4Q = 20$ yields $Q^m = 30$, and hence $p^m = 120 - 2Q = 80$. Next, the price best-response function of each firm i to the price set by firm j is

$$p_i = BR_i(p_j) = \begin{cases} 80 & \text{if } p_j > 80 \\ p_j - \epsilon & \text{if } 20 < p_j \leq 80 \\ 20 & \text{if } p_j < 20. \end{cases} \quad i, j = A, B; \quad i \neq j.$$

The unique Nash-Bertrand equilibrium is therefore $p_A^b = p_B^b = 20$ (each firm replies to a price of 20 by setting also a price of 20).

(ii) When both firm cooperate by setting the monopoly price $p = 80$, they jointly produce $Q = (140 - 80)/2 = 30$ units. Assuming equal production, each firm produces $q_A = q_B = 15$. Hence, each firm earns a profit of $\pi_i(t) = (80 - 30)15 = 900$ in each period of cooperation t. Thus, if both firms cooperate, they earn a discounted profit of

$$\sum_{t=0}^{\infty} = \rho^t \cdot 900 = \frac{900}{1-\rho}.$$

Now, suppose that firm A deviates and undercut firm B by setting $p_A' = 80 - \epsilon$. Then, firm A sells to the entire market, so $q_A' = 30$. In the period of deviation, the firm earns $\pi_A' \approx (80 - 20)30 = 1800$. But, according to the trigger strategy, both firms set $p_A = p_B = 20$ in all subsequent period.

Deviation is not profitable for firm A (by symmetry, also for firm B) if

$$\frac{900}{1-\rho} \geq 1800 + \rho\frac{0}{1-\rho} = 1800$$

hence if $\rho > 0.5$.

Solution to Set # 10: Differentiated Brands

(a) (i) Firm A takes q_B as given and chooses q_A to solve

$$\max_{q_A} \pi_A(q_A, q_B) = \left(60 - \frac{3}{2}\, q_A - q_B\right) q_A.$$

Similarly, Firm B takes q_A as given and chooses q_B to solve

$$\max_{q_B} \pi_B(q_A, q_B) = \left(60 - \frac{3}{2}\, q_B - q_A\right) q_B.$$

(ii) A's first-order condition for a maximum is $0 = \partial\pi_A/\partial q_A = 60 - 3q_A - q_B$. Similarly, $0 = \partial\pi_B/\partial q_B = 60 - 3q_B - q_A$. Hence, the best response functions are downward sloping and are given by

$$q_A = R_A(q_B) = 20 - \frac{q_B}{3} \quad \text{and} \quad q_B = R_B(q_A) = 20 - \frac{q_A}{3}.$$

(iii) Solving the two best-response functions yields $q_A = q_B = 15$. Substituting into the demand functions yield $p_A = p_B = 45/2 = 22.5$. Substituting into the profit functions yields $\pi_A = \pi_B = 675/2 = 337.5$.

(iv) Solving for q_A and q_B from the above inverse demand functions obtains

$$q_A = 24 - \frac{6}{5}\, p_A + \frac{4}{5}\, p_B \quad \text{and} \quad q_B = 24 - \frac{6}{5}\, p_B + \frac{4}{5}\, p_A.$$

(v) Firm A takes p_B as given and chooses p_A to solve

$$\max_{p_A} \pi_A(p_A, p_B) = p_A \left(24 - \frac{6}{5}\, p_A + \frac{4}{5}\, p_B\right).$$

Firm B takes p_A as given and chooses p_B to solve

$$\max_{p_B} \pi_B(p_A, p_B) = p_B \left(24 - \frac{6}{5}\, p_B + \frac{4}{5}\, p_A\right).$$

(vi) A's first-order condition is: $0 = \partial\pi_A/\partial p_A = 4(30 - 3p_A + p_B)/5$. B's first-order condition is $0 = \partial\pi_B/\partial p_B = 4(30 - 3p_B + p_A)/5$. Hence, the upward sloping best-response functions are

$$p_A = R_A(p_B) = 10 + \frac{1}{3}\,p_B \quad \text{and} \quad p_B = R_B(p_A) = 10 + \frac{1}{3}\,p_A.$$

(vii) Solving the two price best-response functions yield $p_A = p_B = 15$. Substituting into the above direct demand functions obtains $q_A = q_B = 18$. The resulting profits are $\pi_A = \pi_B = 270$.

(viii) The price game yields a lower price than the quantity game (15 versus 22.5). Therefore, consumers buy 6 more units of each brand (36 versus 30). The price game yields lower profit to each brand-producing firm (270 as opposed to 337.5). This means that a price game results in a more intense competition relative to a quantity-setting competition game.

(b) (i) For a given p_B, firm A chooses p_A to solve

$$\max_{p_A} \pi_A = p_A q_A = (180 - 2p_A + p_B) \Longrightarrow 0 = \frac{\mathrm{d}\pi_A}{\mathrm{d}p_A} = 180 - 4p_A + p_B$$
$$\Longrightarrow p_A = BR_A(p_B) = 45 + \frac{1}{4}p_B.$$

(ii) For a given p_A, firm B chooses p_B to solve

$$\max_{p_B} \pi_B = p_B q_B = (120 - 2p_B + p_A) \Longrightarrow 0 = \frac{\mathrm{d}\pi_B}{\mathrm{d}p_B} = 120 - 4p_B + p_A$$
$$\Longrightarrow p_B = BR_B(p_A) = 30 + \frac{1}{4}p_A.$$

(iii) Solving the above two best-response functions yields $p_A^b = \$56$ and $p_B^b = \$44$. Substituting prices into the direct demand functions yields

$$q_A^b = 180 - 2 \cdot 56 + 44 = 112 \quad \text{and} \quad q_B^b = 120 - 2 \cdot 44 + 56 = 88.$$

Hence, $\pi_A^b = 56 \cdot 112 = \$6272$ and $\pi_B^b = 44 \cdot 88 = \$3872$. Finally, aggregate industry profit is: $\Pi^b = \pi_A^b + \pi_B^b = \$10,144$.

(iv) Setting $p = p_A = p_B$, this cartel's joint profit is

$$\pi_A + \pi_B = (180 - 2p + p)p + (120 - 2p + p)p = 300p - 2p^2.$$

Maximizing $\pi_A + \pi_B$ with respect to p yields

$$0 = \frac{\mathrm{d}\pi_A + \pi_B}{\mathrm{d}p} = 300 - 4p \Longrightarrow p = \$75 \Longrightarrow \pi_A + \pi_B = \$11,250 > \$10,144$$

which is the aggregate industry profit earned under Bertrand competition. Note that $\pi_A = \$7875 > \$6272$ and $\pi_B = \$3375 < \$3872$. Hence, if the two firms become a cartel, firm A will have to compensate firm B for its loss, or simply, the merged firms will have to decide about how joint profits are shared. Finally, in this equilibrium, $q_A = 105$ and $q_B = 45$.

(v) The merged firm sets p_A and p_B to solve

$$\max_{p_A,p_B}(\pi_A + \pi_B) = p_A(180 - 2p_A + p_B) + p_B(120 - 2p_B + p_A)$$

yielding two first-order conditions given by

$$0 = \frac{\partial(\pi_A + \pi_B)}{\partial p_A} = 2(2p_A - p_B - 90) \quad \text{and} \quad 0 = \frac{\partial(\pi_A + \pi_B)}{\partial p_B} = 2[p_A - 2(p_B - 30)].$$

Solving the two first order conditions yields $p_A^j = \$80 > \$56$ and $p_B^j = \$70 > \$44$. Hence, $q_A = 95$ and $q_B = 50$. Substituting equilibrium prices into the profit of the merged firm yields $\pi_A^j + \pi_B^j = \$11,400 > \$10,144$. Clearly the merger enhances joint profit compared with Bertrand competition.

(c) (i) Under Cournot (quantity) game: $q_A^c = q_B^c = 24$, $p_A^c = p_B^c = 48$, and $\pi_A^c = \pi_B^c = 1152$.

(ii) The corresponding system of direct demand functions is:

$$q_A = 40 - \frac{2}{3}p_A + \frac{1}{3}p_B \quad \text{and} \quad q_B = 20 - \frac{2}{3}p_B + \frac{1}{3}p_A.$$

(iii) Under Bertrand (price) game: $q_A^b = q_B^b = 26\frac{2}{3}$, $p_A^b = p_B^b = 40$, and $\pi_A^b = \pi_B^b = 1066\frac{2}{3}$.

(iv) $q_i^c - q_i^b = -2\frac{2}{3} < 0$, $p_i^c - p_i^b = 8 > 0$, $\pi_i^c - \pi_i^b = 85\frac{1}{3} > 0$. Therefore, competition in prices is more intense than competition in quantity sold as reflected by lower prices, lower profits, and more output sold.

(d) (i) Under Cournot (quantity) game: $q_A^c = q_B^c = 12$, $p_A^c = p_B^c = 48$, and $\pi_A^c = \pi_B^c = 576$.

(ii) The corresponding system of direct demand functions is:

$$q_A = 20 - \frac{1}{3}p_A + \frac{1}{6}p_B \quad \text{and} \quad q_B = 40 - \frac{1}{3}p_B + \frac{1}{6}p_A.$$

(iii) Under Bertrand (price) game: $q_A^b = q_B^b = 13\frac{1}{3}$, $p_A^b = p_B^b = 40$, and $\pi_A^b = \pi_B^b = 533\frac{1}{3}$.

(iv) $q_i^c - q_i^b = -1\frac{1}{3} < 0$, $p_i^c - p_i^b = 8 > 0$, $\pi_i^c - \pi_i^b = 42\frac{2}{3} > 0$. Therefore, competition in prices is more intense than competition in quantity sold as reflected by lower prices, lower profits, and more output sold.

(e) (i) Firm A solves

$$\max_{q_A} \pi_A = 80q_A - \frac{3}{2}(q_A)^2 - q_B q_A \quad \Longrightarrow \quad 0 = \frac{\partial \pi_A}{\partial q_A} = 80 - 3q_A - q_B.$$

Firm B solves

$$\max_{q_B} \pi_B = 80q_B - \frac{3}{2}(q_B)^2 - q_A q_B \quad \Longrightarrow \quad 0 = \frac{\partial \pi_B}{\partial q_B} = 80 - 3q_B - q_A.$$

Second-order conditions are clearly satisfied. Solving two equations with two variables yields $q_A^c = q_B^c = 20$ units. Hence, $p_A^c = p_B^c = \$30$, and $\pi_A^c = \pi_B^c = 30 \cdot 20 = \$600$.

(ii)

$$p_B = 80 - \frac{3}{2}q_B - q_A \quad \Longrightarrow \quad q_A = 80 - \frac{3}{2}q_B - p_B \quad \Longrightarrow \quad p_A = 80 - \frac{3}{2}\left(80 - \frac{3}{2}q_B - p_B\right) - q_B$$

Hence,

$$p_A = -40 + \frac{5}{4}q_B + \frac{3}{2}p_B \quad \Longrightarrow \quad \frac{5}{4}q_B = p_A + 40 - \frac{3}{2}p_B \quad \Longrightarrow \quad q_B = 32 - \frac{6}{5}p_B + \frac{4}{5}p_A.,$$

which is the direct demand function for brand B. Next, substituting the direct demand function for q_B into the second equation above.

$$q_A = 80 - \frac{3}{2}\left(32 + \frac{4}{5}p_A - \frac{6}{5}p_B\right) - p_B = 32 - \frac{6}{5}p_A + \frac{4}{5}p_B.$$

(iii) Firm A chooses p_A to solve

$$\max_{p_A} \pi_A = 32p_A - \frac{6}{5}(p_A)^2 + \frac{4}{5}p_B p_A \quad \Longrightarrow \quad = \frac{\partial \pi_A}{\partial p_A} = 32 - \frac{12}{5}p_A + \frac{4}{5}p_B.$$

Firm B chooses p_B to solve

$$\max_{p_B} \pi_B = 32p_B - \frac{6}{5}(p_B)^2 + \frac{4}{5}p_B p_A \quad \Longrightarrow \quad = \frac{\partial \pi_B}{\partial p_B} = 32 - \frac{12}{5}p_B + \frac{4}{5}p_A.$$

Solving 2 equations with 2 variables yields $p_A^b = p_B^b = \$20$. Substituting into the direct demand functions yields $q_A^b = q_B^b = 24$ units. Hence, $\pi_A^b = \pi_B^b = 20 \cdot 24 = \$480$.

A comparison of the quantity game with the price game implies

$$p_i^b = \$20 < \$30 = p_i^c, \quad q_i^b = 24 > 20 = q_i^c, \quad \text{and} \quad \pi_i^b = \$480 < \$600 = \pi_i^c, \quad \text{for firm } i = A, B.$$

Thus, competition is more intense when firm play a price game compared with a quantity game.

(f) (i) The firm chooses a uniform price p to solve

$$\max_p \Pi = \pi_O + \pi_G = pq_O + pq_G = p(24 - 2p + p) + p(12 - 2p + p).$$

The first- and second order conditions for maximum profit are

$$0 = \frac{d\Pi}{dp} = 36 - 4p \quad \text{and} \quad \frac{d^2\Pi}{dp^2} = -4 < 0.$$

Hence, $p = 36/4 = 9$. Hence, the quantities sold are $q_O = 15$ and $q_G = 3$. Hence, $\Pi = 9(15 + 3) = 162$.

(ii) The firm chooses the prices p_O and p_G to solve

$$\max_{p_O, p_G} \pi_O + \pi_G = p_O q_O + p_G q_G = p_O(24 - 2p_O + p_G) + p_G(12 - p_G + p_O).$$

The first-order conditions for a maximum are

$$0 = \frac{\partial\Pi}{\partial p_O} = 24 - 4p_O + 2p_G \quad \text{and} \quad 0 = \frac{\partial\Pi}{\partial p_G} = 12 - 4p_G + 2p_O,$$

yielding $p_O = 10$, $p_G = 8$. Hence, the quantities sold are $q_O = 12$ and $q_G = 6$. Hence, $\Pi = 168 > 162$. Clearly JUICIANA earns no less profit when it sets different prices for orange and grape juice compared with uniform pricing. Note that JUICIANA can still set $p_O = p_G$, so the fact that it chooses $p_O \neq p_G$ implies that nonuniform pricing yields a higher profit.

Solution to Set # 11: Location Models

(a) (i)

$$n_A = \begin{cases} 0 & \text{if } p_A > p_Y + 1 \\ 120 & \text{if } p_Y - 2 \le p_A \le p_Y + 1 \\ 240 & \text{if } p_A < p_Y - 2. \end{cases}$$

(ii)

$$n_Y = \begin{cases} 0 & \text{if } p_Y > p_A + 2 \\ 120 & \text{if } p_A - 1 \le p_Y \le p_A + 2 \\ 240 & \text{if } p_Y < p_A - 1. \end{cases}$$

(iii) The Ann Arbor store can undercut the Ypsilanti store by setting $p_A < p_Y - 2$. Under this price, $\pi_A = (120 + 120)p_A \approx 240(p_Y - 2)$.

(iv) The Ypsilanti store can undercut the Ann Arobr store by setting $p_Y < p_A - 1$. Under this price, $\pi_Y = (120 + 120)p_Y \approx 240(p_A - 1)$.

(v) In an UPE, store A sets the highest p_A subject to

$$\pi_Y = 120p_Y \geq (120 + 120)(p_A - 1).$$

Similarly, store Y sets the highest p_Y subject to

$$\pi_A = 120p_A \geq (120 + 120)(p_Y - 2).$$

Solving two equations with two variables for the case of equality, yields $p_A = 8/3$ and $p_Y = 10/3$. Store A charges a lower price than store Y since the switching costs of Ann Arbor's residents are lower than that of Ypsilanti's residence. Loosely speaking, Ypsilanti's residents are less likely to travel to Ann Arbor than Ann Arbor's residents traveling to Ypsilanti due to higher transportation costs.

(vi) In equilibrium, $\pi_A = 120 \cdot 8/3 = \$320$, and $\pi_Y = 120 \cdot 10/3 = \$400$.

(b) (i) In an UPE, store A sets the highest p_A subject to

$$\pi_Y = 200p_Y \geq (200 + 200)(p_A - 3).$$

Similarly, store Y sets the highest p_Y subject to

$$\pi_A = 200p_A \geq (200 + 200)(p_Y - 3).$$

Solving two equations with two variables for the case of equality, yields $p_A^U = \$6$ and $p_Y^U = \$6$. Hence, $\pi_A^U = 200 \cdot 6 = \$1200$ and $\pi_Y^U = 200 \cdot 6 = \$1200$.

(ii) In an UPE, store A sets the highest p_A subject to

$$\pi_Y = 200(p_Y - 4) \geq (200 + 200)(p_A - 4 - 3).$$

Similarly, store Y sets the highest p_Y subject to

$$\pi_A = 200(p_A - 1) \geq (200 + 200)(p_Y - 1 - 3).$$

Solving two equations with two variables for the case of equality, yields $p_A^U = \$9$ and $p_Y^U = \$8$. Hence, $\pi_A^U = 200(9 - 1) = \$1600$ and $\pi_Y^U = 200(8 - 4) = \$800$.

Solution to Set # 12: Choice of Location

(a) Solving backwards, in stage 2, firm 2's best-response function is given by

$$x_2 = BR_2(x_1) = \begin{cases} x_1 + \epsilon & \text{if } x_1 \leq 1/2 \\ x_1 - \epsilon & \text{if } x_1 > 1/2, \end{cases} \quad \text{hence, in } t = 1, \quad x_1 = \frac{1}{2}.$$

The equilibrium market shares are $\pi_1 = \pi_2 = 1/2$.

(b) Solving backwards, in stage 3, firm 3's best-response function is given by

$$x_3 = BR_3(x_1, x_2) = \begin{cases} x_2 + \epsilon & \text{if } x_2 \le 2/3 \\ 1/3 - \epsilon & \text{if } x_2 > 2/3, \end{cases} \quad \text{hence, in } t = 2, \quad x_2 = \frac{2}{3} + \epsilon.$$

The resulting market shares are

$$\pi_3 = \frac{1}{3}, \quad \pi_1 = \left(\frac{2}{3} - \frac{1}{3}\right) \Big/ 2 = \frac{1}{6}, \quad \text{and} \quad \pi_2 = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}.$$

Here, firm 2 manages to capture the highest market share.

(c)

$$x_3 = BR_3(x_1, x_2) = \begin{cases} 1/2 + \epsilon & \text{if } x_2 \le 1/2 \\ 1/2 - \epsilon & \text{if } x_2 > 1/2, \end{cases} \quad \text{hence, in } t = 2, \quad x_2 \in \left\{ \frac{1}{2} - \epsilon \,;\, \frac{1}{2} + \epsilon \right\},$$

that is, firm 2 is indifferent between these 2 locations. The equilibrium market shares are therefore

$$\pi_3 = \pi_2 = \frac{1}{2} \quad \text{and} \quad \pi_1 = 0.$$

(d)

$$x_3 = BR_3(x_1, x_2) = \begin{cases} x_2 - \epsilon & \text{if } \frac{x_2}{2} \ge 1 - x_2 \\ x_2 + \epsilon & \text{if } \frac{x_2}{2} < 1 - x_2 \end{cases} = \begin{cases} x_2 - \epsilon & \text{if } x_2 \ge \frac{2}{3} \\ x_2 + \epsilon & \text{if } x_2 < \frac{2}{3} \end{cases}$$

Hence, in $t = 2$

$$x_2 = \frac{2}{3} + \epsilon$$

The equilibrium market shares are therefore

$$\pi_3 = \pi_2 = \pi_1 = \frac{1}{3}.$$

Note that the above equilibrium location of firm 2 is not unique because firm 2 makes the same profit on any location between 0 and $2/3$.

(e) To construct firm 3's best response function, we search for a threshold $\tilde{x}_2$ under which firm 3 is indifferent between locating at $\tilde{x}_2 + \epsilon$ and $\tilde{x}_2 - \epsilon$. If firm 3 locates to the "left" of firm 2 it earns $\pi_3^L = \tilde{x}_2 - 0 = \tilde{x}_2$. If firm 3 locates to the "right" of firm 2, it equally splits the profit with firm 1, hence earns $\pi_3^R = (1 - \tilde{x}_2)/2$. Hence, $\tilde{x}_2$ must satisfy

$$\tilde{x}_2 = \frac{1 - \tilde{x}_2}{2} \quad \Longrightarrow \tilde{x}_2 = \frac{1}{3}.$$

Hence, the best-response function of restaurant 3 is

$$x_3 = BR_3(x_2) = \begin{cases} x_2 - \epsilon & \text{if } x_2 > 1/3 \\ x_2 + \epsilon & \text{if } x_2 \le 1/3 \end{cases}$$

Therefore, $x_2 = 1/3 - \epsilon$, $x_3 = 1/3 + \epsilon$ so $\pi_1 = \pi_2 = \pi_3 = 1/3$.

Therefore, $x_2 = 1/3 - \epsilon$, $x_3 = 1/3 + \epsilon$ so $\pi_1 = \pi_2 = \pi_3 = 1/3$.

Remark: Note that the equilibrium location of firm 3 is not unique in the sense that it is indifferent between locating at any point $x_2 < x_3 < 1$. Therefore,

$$x_3 = BR_3(x_2) = \begin{cases} x_2 - \epsilon & \text{if } x_2 > 1/3 \\ (x_2, 1) & \text{if } x_2 \leq 1/3 \end{cases}$$

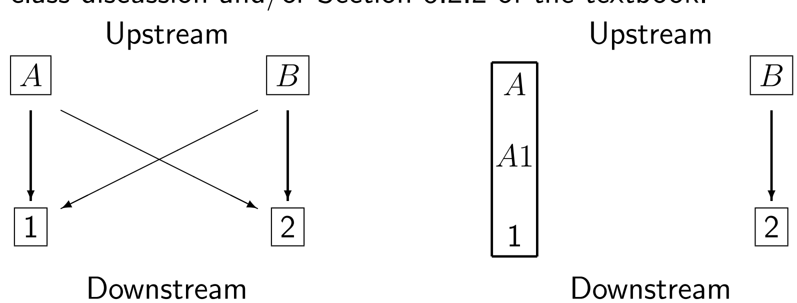
Hence, the following locations also constitute an equilibrium: $x_2 = 1/3$, $x_3 = 2/3$ yielding the equilibrium payoffs

$$\pi_1 = \frac{1}{6}, \quad \pi_2 = \frac{1}{3} + \frac{1}{6}, \quad \text{and} \quad \pi_3 = \frac{1}{3}.$$

Solution to Set # 13: Mergers and Entry Barriers

(a) Please refer to class discussion and/or Section 8.2 of the textbook. In your answer, you should briefly describe (a) horizontal merger, (b) vertical merger, and (c) conglomerate merger.

(b) Please refer to class discussion and/or Section 8.2.2 of the textbook.



The antitrust authority may want to check whether the merger of component supplier A with final good producer 1 may result in a foreclosure on firm 2 in the market for the final good. This may happen if, for some reason, component producer B goes out of business.

(c) (i) Since $60 - \epsilon > \phi - \epsilon$. firm A's SPE strategy is

$$s_A = \begin{cases} \text{stay} & \text{if } s_B = \text{enter (because } 60 - \epsilon > \phi - \epsilon) \\ \text{stay} & \text{if } s_B = \text{out (because } 100 - \epsilon > \phi - \epsilon) \end{cases}$$

The SPE strategy of firm B (first mover) is $s_B = $ enter (because $60 - \epsilon > 0$).

(ii) Now, $60 - \epsilon < \phi - \epsilon$. Therefore, Firm A's SPE strategy is

$$s_A = \begin{cases} \text{exit} & \text{if } s_B = \text{enter (because } 60 - \epsilon < \phi - \epsilon) \\ \text{stay} & \text{if } s_B = \text{out (because } 100 - \epsilon > \phi - \epsilon) \end{cases}$$

The SPE strategy of firm B (first mover) is $s_B =$ enter (because $100 - \epsilon > 0$).

(d) (i) The profit earned by firm A when $p_A = p_B = \$60$ and $c_A = c_B = \$40$ is

$$\pi_A(60, 60) = (p_A - c_A)(120 - 2p_A + p_B) = (60 - 40)(120 - 2 \cdot 60 + 60) = \$1200.$$

Now, suppose that firm A raises its price by 5% so that $p_A = \$63 > \$60 = p_B$. Then,

$$\pi_A(63, 60) = (p_A - c_A)(120 - 2p_A + p_B) = (63 - 40)(120 - 2 \cdot 63 + 60) = \$1242 > \$1200.$$

Hence, the market in which firm A is selling should be considered as the "relevant" market because firm A can benefit from raising its price by 5% (thereby exercising some monopoly power).

(ii) The profit earned by firm A and B when $p_A = p_B = \$60$ and $c_A = c_B = \$30$ is

$$\pi_i(60, 60) = (p_i - c_i)(120 - 2p_i + p_j) = (60 - 30)(120 - 2 \cdot 60 + 60) = \$1800 \quad i = A, B.$$

Now, suppose that firm A raises its price by 5% so that $p_A = \$63 > \$60 = p_B$. Then,

$$\pi_A(63, 60) = (p_A - c_A)(120 - 2p_A + p_B) = (60 - 30)(120 - 2 \cdot 63 + 60) = \$1782 < \$1800.$$

Hence, the market in which firm A is selling should not be considered as the "relevant" market because firm A cannot benefit from raising its price by 5%.

Next, suppose that firm A acquires firm B. Should the combined A and B markets be considered as the "relevant" market? First, note that the firms' joint profits is

$$\pi_A(60, 60) = \pi_B(60, 60) = \$1800 + \$1800 = \$3600.$$

To check that whether the combined market should be considered as the relevant market, let us raise p_A by 5% so that $p_A = \$63 > \$60 = p_B$. We have already shown that $\pi_A(63, 60) = \$1782$. So we now compute

$$\pi_B(63, 60) = (p_B - c_B)(120 - 2p_B + p_A) = (60 - 30)(120 - 2 \cdot 60 + 63) = \$1890.$$

The joint profit is

$$\pi_A(63, 60) + \pi_B(63, 60) = \$1782 + \$1890 = \$3672 > \$3600.$$

Hence, the combined A and B market share should be considered as the relevant market because an increase in p_A causes lots of consumers to switch from A to B thereby increasing the profit of firm B more than the loss of profit of firm A.

(e) Under the observed prices, the ORANGADA company's profit is

$$\pi_O(60,40,40) = (60-20)q_O = (60-20)(90-2\cdot 60+40+40) = 2000.$$

Similarly,

$$\pi_G(60,40,40) = (40-20)q_G = (40-20)(100-2\cdot 40+60+40) = 2400,$$

and

$$\pi_T(60,40,40) = (40-20)q_T = (40-20)(120-2\cdot 40+60+40) = 2800.$$

We first check whether the market for orange juice alone is the relevant market for the ORANGADA company by raising p_O by 5% from $p_O = 60$ to $p'_O = 63$. Then,

$$\pi_O(63,40,40) = (63-20)q_O = (63-20)(90-2\cdot 63+40+40) = 1892 < 2000.$$

Thus, the market for orange juice alone should *not* be considered as the relevant market for ORANGADA.

Next, we ask whether the combined market for orange and grape juice should be considered as the relevant market? Setting again $p'_O = 63$ makes the combined profit in both markets is

$$\pi_O(63,40,40)+\pi_G(63,40,40) = 1892+(40-20)(100-2\cdot 40+63+40) = 4352 < 2000+2400.$$

Thus, the combined market for orange and grape juice should *not* be considered as the relevant market for ORANGADA.

Next, should the markets for the three juices combined be considered as the relevant market?

$$\pi_O(63,40,40)+\pi_G(63,40,40)+\pi_T(63,40,40) = 4352+(40-20)(120-2\cdot 40+63+40) = 7212$$
$$> 7200 = \pi_O(60,40,40)+\pi_G(60,40,40)+\pi_T(60,40,40).$$

Yes, the relevant market for ORANGADA is the market for the three juices combined.

(f) See Figure 8.11 on p.208 in the textbook. In a contestable market equilibrium, the incumbent firm sets the highest price subject to the constraint that no other firm can undercut its price while making positive profits.

The above definition means that the incumbent's price cannot exceed average cost $p^I \leq AC(Q)$, as otherwise, a potential entrant would be able to undercut the incumbent by setting $AC(Q) < p^E < p^I$ while still making a strictly positive profit. Therefore,

$$p = 22 - \frac{Q}{2} = AC(Q) = \frac{40+10Q}{Q} \quad \Longrightarrow Q = 20 \quad \Longrightarrow p = 12.$$

Solution to Set # 14: Innovation & Patent Races

(a) Innovation is drastic (major) if firm A can set a monopoly price, p_A^m and undercut firm B given the new cost structure. That is, innovation is major if $p_A^m(c_1) < c_0 = \$80$.

Let's check it by computing A's monopoly price. If A is a monopoly, it solves $MR = 120 - Q = c_1$. Hence, $Q_A^m = 120 - c_1$. Substituting into the inverse demand function yields

$$p_A^m(c_1) = \frac{120 + c_1}{2} < 80 \quad \text{if} \quad c_1 < 40.$$

To conclude, A's innovation is called drastic (major) if $c_1 < 40$ and minor if $c_1 \geq 40$.

(b) We show that this innovation should be classified as major/drastic if $c_0 > 140$, and minor if $c_0 < 140$. If the innovator (firm A) can exercise full monopoly power, it solves

$$MR = 240 - 4Q = c_1 = 40 \quad \Longrightarrow Q = 50 \quad \Longrightarrow p^m(c_1 = 40) = 140.$$

However, if $c_0 < 140$, firm A won't be able to charge the monopoly price $p^m = 140$ as it be will undercut by firm B that can still make a profit by setting $c_0 < p_B < 140$, which is the case of minor (nondrastic) innovation.

(c) (i) The expected profit of firm 1 when only firm 1 invests in R&D is

$$\pi_1(1) = \frac{1}{3}150 = 40 = \$10 > 0.$$

(ii) The expected profit of firm 1 when, both, firm 1 and firm 2 engage in R&D is

$$\pi_1(2) = \underbrace{\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{150}{2}\right)}_{\text{both firms discover}} + \underbrace{\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)150}_{\text{only firm 1 discovers}} - 40 = \frac{5}{3} > 0.$$

(iii) The expected profit of firm 1 when all three firms engage in R&D is

$$\pi_1(3) = \underbrace{\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{150}{3}\right)}_{\text{all 3 firms discover}} + \underbrace{\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{150}{2}\right)}_{\text{firms 1 and 2 firms discover}} + \underbrace{\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{150}{2}\right)}_{\text{firms 1 and 3 firms discover}}$$
$$+ \underbrace{\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)150}_{\text{only 1 discovers}} - 40 = -\frac{130}{27} < 0.$$

(iv) The above computation reveal that in equilibrium exactly two firms will engage in R&D.

(v) Social welfare when only one firm engages in R&D is

$$W(1) = \frac{1}{3}150 - 40 = \$10.$$

(vi) Social welfare when exactly two firms engage in R&D is

$$W(2) = \left[1 - \underbrace{\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)}_{\text{prob. both don't discover}}\right] 150 - 40 - 40 = \frac{\$10}{3} < \$10.$$

(vii) Social welfare when exactly three firms engage in R&D is

$$W(3) = \left[1 - \underbrace{\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)}_{\text{prob. all 3 don't discover}}\right] 150 - 40 - 40 - 40 = -\frac{\$130}{9} < \$10.$$

(viii) The above computations reveal that from a social welfare point of view, only one firm should be engaged in R&D. Yes, there is a market failure since in equilibrium two firms will engage in R&D. Thus, from a social welfare view point, this patent race leads to excessive R&D.

(d) **Option A:** The probability that both labs do not discover is: $(1 - 0.75)^2 = 1/16$. Therefore, expected profit is given by

$$\pi = \left(1 - \frac{1}{16}\right) 16 - 2 \cdot 2 = \$11.$$

Option B: The probability that all three labs do not discover is: $(1 - 0.5)^3 = 1/8$. Therefore, expected profit is given by

$$\pi = \left(1 - \frac{1}{8}\right) 16 - 3 \cdot 1 = \$11.$$

Therefore, both options yield the same expected profit.

(e) (i) Suppose that all 3 labs enter the race. The expected profit to lab C is then

$$\pi_C(A,B,C) = 240 \left[\underbrace{\frac{1}{3}\frac{2}{3}\frac{2}{3}}_{\text{only } C} + \underbrace{\frac{1}{3}\frac{1}{3}\frac{1}{3}\frac{1}{3}}_{A,B,C \text{ discover}} + \underbrace{\frac{1}{2}\frac{1}{3}\frac{1}{3}\frac{2}{3}}_{B,C} + \underbrace{\frac{1}{2}\frac{1}{3}\frac{2}{3}\frac{1}{3}}_{A,C}\right] - 70 = \frac{1520}{27} - 70 < 0.$$

Hence, lab C makes a loss if all firms race for this patent. This proves that an equilibrium in which 3 labs innovate does not exist.

(ii) Suppose that A and B enter the race. Then,

$$\pi_A(A,B) = 240\left[\underbrace{\frac{1}{3}\frac{2}{3}}_{\text{only } A \text{ discovers}} + \underbrace{\frac{1}{2}\frac{1}{3}\frac{1}{3}}_{A,B \text{ discover}}\right] - 40 = \frac{200}{3} - 40 > 0$$

$$\pi_B(A,B) = 240\left[\underbrace{\frac{1}{3}\frac{2}{3}}_{\text{only } B \text{ discovers}} + \underbrace{\frac{1}{2}\frac{1}{3}\frac{1}{3}}_{A,B \text{ discover}}\right] - 60 = \frac{200}{3} - 60 > 0$$

This proves that there exists an equilibrium where labs A and B race for the patent while lab C stays out. It remains to check whether other equilibria exist, or whether this is the only equilibrium.

Suppose now lab A and C enter while lab B stays out. The, the profit of lab C is given by

$$\pi_C(A,C) = 240\left[\underbrace{\frac{1}{3}\frac{2}{3}}_{\text{only } C \text{ discovers}} + \underbrace{\frac{1}{2}\frac{1}{3}\frac{1}{3}}_{A,C \text{ discover}}\right] - 70 = \frac{200}{3} - 70 < 0.$$

Hence, there is no equilibrium in which lab C and lab A or lab B also enter.

(iii) Since an equilibrium in which labs A and B enter exist, there does *not* exist an equilibrium in which there is only one firm racing for the patent. That is, if A enters B will enter, and if B enters A will also enter

(iv) If only A operates

$$\pi_A^s = \frac{1}{3}240 - 40 = \$40.$$

If A and B operate, the joint expected profit is

$$\pi_{A,B}^s = 240\left[\underbrace{\frac{1}{3}\frac{1}{3}}_{\text{both discover}} + \underbrace{\frac{1}{3}\frac{2}{3}}_{A \text{ discovers}} + \underbrace{\frac{2}{3}\frac{1}{3}}_{B \text{ discovers}}\right] - 40 - 60 = \frac{100}{3} < \$40.$$

If A, B, and C operate, the joint expected profit is

$$\pi_{A,B,C}^s = 240\left[\underbrace{\frac{1}{3}\frac{1}{3}\frac{1}{3}}_{A,B,C} + \underbrace{\frac{1}{3}\frac{1}{3}\frac{1}{3}}_{\text{3 labs discover}} + \underbrace{\frac{1}{3}\frac{1}{3}\frac{2}{3}\cdot 3}_{\text{2 labs}} + \underbrace{\frac{1}{3}\frac{2}{3}\frac{2}{3}\cdot 3}_{\text{1 lab}}\right] - 170 \approx -1.1 < \$40.$$

Hence, Google will operate only lab A and will close labs B and C.

Remark: This problem demonstrates a market failure in the sense that equilibrium generates excessive R&D. More precisely, in equilibrium labs A and B will race, whereas operating A alone is socially efficient.

(f) (i) In equilibrium, both firms will enter the R&D race. To prove this we merely have to show that both make nonnegative profits when both labs enter this race.

$$\pi_A(A,B) = 240\left[\underbrace{\frac{1}{4}\frac{2}{3}}_{\text{only } A \text{ discovers}} + \underbrace{\frac{1}{2}\frac{1}{4}\frac{1}{3}}_{A,B \text{ discover}}\right] - 40 = 50 - 40 > 0.$$

$$\pi_B(A,B) = 240\left[\underbrace{\frac{3}{4}\frac{1}{3}}_{\text{only } B \text{ discovers}} + \underbrace{\frac{1}{2}\frac{1}{4}\frac{1}{3}}_{A,B \text{ discover}}\right] - 60 = 70 - 60 > 0.$$

Clearly, there is no equilibrium in which only one lab innovates since we have just shown that it is profitable to the second lab to enter this race.

(ii) If the owner (or the social planner) operates only lab A, she earns $\pi^s_A = \frac{1}{4}240 - 40 = \$20$. If the owner operates only lab B, she earns $\pi^s_B = \frac{1}{3}240 - 60 = \$20$.

If the owner operates both labs, the expected profit becomes

$$\pi^s_{A,B} = 240\left[\underbrace{\frac{1}{4}\frac{1}{3}}_{\text{both discover}} + \underbrace{\frac{1}{4}\frac{2}{3}}_{A \text{ discovers}} + \underbrace{\frac{3}{4}\frac{1}{3}}_{B \text{ discovers}}\right] - 40 - 60 = \$20.$$

Therefore, all three options (operating A only, B only, or both) yield the same profit, so the owner (social planner) should be indifferent among them.

(g) (i) All the firms have identical R&D technologies. Hence, it is sufficient to compute the profit of one representative firm, say firm 1. The profit of firm 1 when all 3 firms engage in R&D is

$$\pi_1(3) = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\frac{640}{3} + 2\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\frac{640}{2} + \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)640 - 120 = \frac{10}{3} > 0.$$

Hence, in equilibrium 3 firms will enter the R&D patent race.

(ii) Expected profit when the investor operates all 3 labs is:

$$\pi^s(3) = 640\left[1 - \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\right] - 3\cdot 120 = \$10.$$

Expected profit when the investor operates 2 labs is:

$$\pi^s(2) = 640\left[1 - \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\right] - 2\cdot 120 = \$40.$$

Expected profit when the investor operates 1 lab is:

$$\pi^s(1) = 640\left(\frac{1}{4}\right) - 120 = \$40.$$

Hence, under single ownership, the owner will choose to operate either 2 labs or 1 lab only.

(h) The solution calls for computing expected profits using increasing number of labs. Thus,
$\pi(1) = \alpha V - I = \$496.$
$\pi(2) = [1-(1-\alpha)^2]V - 2I = \$736.$
$\pi(3) = [1-(1-\alpha)^3]V - 3I = \$848.$
$\pi(4) = [1-(1-\alpha)^4]V - 4I = \$896.$
$\pi(5) = [1-(1-\alpha)^5]V - 5I = \$912.$
$\pi(6) = [1-(1-\alpha)^6]V - 6I = \$912.$
$\pi(7) = [1-(1-\alpha)^7]V - 7I = \$904 < \$912.$
$\pi(8) = [1-(1-\alpha)^8]V - 8I = \$892 < \$912.$
Thus, the Lazy maximizes expected profit from R&D when it invests in 5 or 6 labs.

If you like to use Calculus, you can solve the following problem: Let n be the number of labs. The CEO sets n to solve

$$\max_n \mathsf{E}\pi = \left[1 - \left(\frac{1}{2}\right)^n\right] 1024 - 16n = 1024 - 1024 \cdot 2^{-n} - 16n.$$

The first-order condition for a maximum is

$$0 = \frac{\mathsf{d}\mathsf{E}\pi}{\mathsf{d}n} = 1024 \cdot 2^{-n}\ln(2) - 16 \Longrightarrow 2^n = 64\ln(2) \Longrightarrow n\cdot\ln(2) = \ln\left(64\cdot\ln(2)\right).$$

Hence,

$$n = \frac{\ln\left(64\cdot\ln(2)\right)}{\ln(2)} \approx 5.4712 \text{ labs.}$$

Since n is an integer, you only need to evaluate the expected profit for $n = 5$ and $n = 6$ labs.

(i) From class discussion and/or your reading of Section 9.7, your answer should be that patents are granted for (i) products, (ii) processes, (iii) plants, and (iv) design.

The three requirements are: (i) Novelty, (ii) nonobviousness, and (iii) usefulness.

Solution to Set # 15: Subsidies to R&D

(a) (i) A subsidy of 10 would turn "Produce" into a dominant strategy for Airbus. That is, the profit from producing is no less than not producing regardless of the actions taken by the rival firm, Boeing.

(ii) A subsidy of 10 would turn "Produce" into a dominant strategy for Boeing. That is, the profit from producing is no less than not producing regardless of the actions taken by the rival firm, Airbus.

(iii) No, since "Produce" is a dominant strategy for Airbus.

(iv) The world loses for this subsidy competition among government since it leads to socially excessive investment in R&D.

(b) No, for the same reason as above.

(c) Deterrence is not possible for the same reason as the above.

(d) Boeing earns a higher profit when it chooses to develop regardless of the choice made by Airbus. Formally, $\pi_B(dev, dev) = 2 > 0 = \pi_B(not, dev)$ and $\pi_B(dev, not) = 50 > 0 = \pi_B(not, not)$ which means that Develop is a dominant action for Boeing. Hence, the EU cannot prevent Boeing from developing the aircraft.

Solution to Set # 16: Advertising

(a) Because

$$e_a = \frac{\%\Delta q}{\%\Delta a} = 0.04 \quad \text{and} \quad e_p = \frac{\%\Delta q}{\%\Delta p} = -0.2,$$

by the Dorfman-Steiner formula the profit-maximizing ratio of advertising expenditure to sales revenue is given by

$$\frac{A}{p\,q} = \frac{A}{\$50 \text{ million}} = \frac{1}{5} = \frac{0.04}{-(-0.2)} = \frac{e_a}{-e_p}.$$

Hence, $A = \$10$ million.

(b) The Dorfman-Steiner condition implies

$$\frac{A}{pQ} = \frac{20}{100} = \frac{1}{5} = \frac{\epsilon_A}{-\epsilon_p} = \frac{\epsilon_A}{2}.$$

Hence, the advertising elasticity if $\epsilon_A = 0.4$.

Solution to Set # 17: Monopoly & Durability

(a) (i) The maximum price a consumer is willing to pay *per month of use* of a long-durability battery is

$$\frac{p_L}{40} = v - \frac{t}{40} = 20 - \frac{120}{40} = \$17.$$

The maximum price a consumer is willing to pay *per month of use* of a short-durability battery is

$$\frac{p_S}{30} = v - \frac{t}{30} = 20 - \frac{120}{30} = \$16.$$

Therefore, the *per month of use* profit made from long-lasting battery is

$$\pi_L = \frac{p_L}{40} - \frac{c_L}{40} = 17 - \frac{240}{40} = \$11.$$

Similarly, the *per month of use* profit made from short-lasting battery is

$$\pi_S = \frac{p_S}{30} - \frac{c_S}{30} = 16 - \frac{180}{30} = \$10.$$

Hence, selling long-lasting battery is more profitable for this monopoly seller.

(ii) Perfect competition reduces battery prices to unit cost. Therefore, $p_L = c_L = \$240$ and $p_S = c_S = \$180$. The *per month of use* utility from buying a long-lasting battery is

$$U_L = v - \frac{p_L}{40} - \frac{t}{40} = 20 - \frac{240}{40} - \frac{120}{40} = 11.$$

Similarly, the *per month of use* utility from buying a short-lasting battery is

$$U_S = v - \frac{p_S}{30} - \frac{t}{30} = 20 - \frac{180}{30} - \frac{120}{30} = 10.$$

Since $U_L > U_S$ consumers will buy only long-lasting batteries. Hence, short-durability batteries will not be sold.

To summarize, this exercise demonstrates Swan's durability theorem which states that manufacturer's choice of durability is not affected by market structure.

(b) (i) A monopoly, extracting entire consumer surplus, will set the price on the basis of \$20 per month of service. Hence, $p_L = 30 \cdot 60 = \$1800$ and $p_S = 30 \cdot 40 = \$1200$. The profits per month of use are:

$$\frac{\pi_L}{60} = 30 - \frac{c_L}{60} = 30 - \frac{120}{60} = \$28 \quad \text{and} \quad \frac{\pi_S}{40} = 30 - \frac{c_S}{40} = 30 - \frac{80}{40} = \$28.$$

Hence, a monopoly seller would be indifferent between selling long and short lasting batteries (or both types).

(ii) In a competitive industry, prices drop to marginal costs. Hence, $p_L = \$120$ and $p_S = \$80$. Consumers' utility (per month of service) from each battery type are given by

$$\frac{U_L}{60} = 30 - \frac{p_L}{60} = 30 - \frac{120}{60} = 28 \quad \text{and} \quad \frac{U_S}{40} = 30 - \frac{p_S}{40} = 30 - \frac{80}{40} = 28.$$

Hence, both types of batteries will be demanded when produced by an competitive industry.

Solution to Set # 18: Warranties

(a) (i) The maximum price a consumer will be willing to pay for a product with no warranty is $p_N = \rho V = 0.8 \times \$120 = \$96$.

(ii) The profit made from selling without a warranty is $\pi_N = p_N - c = 96 - 60 = \$36$.

(iii) Expected production cost under a full replacement warranty is

$$c_F = c + (1-\rho)c + (1-\rho)^2 c + \cdots = \frac{c}{1-(1-\rho)} = \frac{c}{\rho} = \frac{60}{0.8} = \$75.$$

(iv) With a full-replacement warranty, the monopoly can raise the price to $p_F = \$120$ because the buyer does not face any risk of using a defective product. Expected profit is therefore $\pi_F = p_F - c_F = 120 - 75 = \$45 > \$36$. Thus, providing a full-replacement warranty is profitable.

(v) A consumer's expected utility, which is the maximum willingness to pay, with a one-time replacement warranty is

$$\underbrace{\rho V}_{\text{nondefective 1}^{\text{st}}\text{ time}} + \underbrace{(1-\rho)\rho V}_{\text{defective 1}^{\text{st}}\text{, nondefective replacement}} = [0.8+(1-0.8)0.8]120 = \frac{576}{5} = 115.2.$$

Thus, the monopoly will charge $p_1 = \$115.20$ for the product bundled with a one-time replacement warranty.

Next, the monopoly's expected total production cost is

$$c + \underbrace{(1-\rho)c}_{\text{replacement cost}} = 60 + (1-0.8)60 = \$72.$$

Altogether, expected profit under this warranty type is $\pi_1 = 115.2 - 72 = \$43.20$.

(vi) If we rank the profit levels among the three warranty types, we obtain

$$\pi_N = \$36 < \pi_1 = \$43.20 < \pi_F = \$45.$$

Therefore, the most profitable warranty type is the full-replacement warranty.

(b) (i) $p^{NW} = \rho V = 0.8 \cdot 40 = \$32$. $\pi^{NW} = \rho V - c = 0.8 \cdot 40 - 10 = \$22$

(ii) $p^{FW} = V = \$40$.

$$\pi^{FW} = p^{FW} - \frac{c}{\rho} = 40 - \frac{10}{0.8} = \frac{320 - 100}{8} = \frac{55}{2} = \$27.5.$$

(iii)

$$p^{PW} = \rho V + (1-\rho)20 = 0.8 \cdot 40 + 0.2 \cdot 20 = \$36.$$

Therefore, the profit is

$$\pi^{PW} = p^{PW} - c - (1-\rho)20 = 36 - 10 - 0.2 \cdot 20 = \$22 = \pi^{NW}.$$

Hence, this type of warranty yields the same profit as with no warranty.

(iv) We first must formulate a consumer's expected utility (expected benefit) function. Thus,

$$U = \begin{cases} \rho V - p^{NW} = 32 - p^{NW} & \text{no warranty} \\ \rho V - p^{W} + (1-\rho)p^{W} = \rho(V - p^{W}) = 0.8(40 - p^{W}) & \text{with warranty} \\ 0 & \text{does not buy.} \end{cases}$$

If the monopoly does not provide any warranty, the monopoly price is $p^W = \rho V = \$32$. The profit per customer is therefore $\pi^{NW} = p^{NW} - c = \rho V - c = 32 - 10 = \$22$.

Under the money-back guarantee, $p^W = V = \$40$. The resulting profit per customer is

$$\pi^{W} = -c + \rho p^{W} + (1-\rho)0 = \rho V - c = 32 - 10 = \$22 = \pi^{NW}.$$

Therefore, providing money-back warranty does not enhance monopoly profit relative to providing no warranty.

(c) (i)

$$p^{NW} = \rho V = \frac{3}{4}\,120 = \$90. \quad \text{hence} \quad \pi^{NW} = 90 - 60 = \$30.$$

(ii) Since consumers are fully protected against defects, they are willing to pay $p^W = \$120$, which is the price charged by a monopoly seller. The monopoly's expected cost is $c + (1-\rho)R$. Hence, the monopoly profit is

$$\pi^{W} = p^{W} - c - (1-\rho)R = 120 - 60 - \frac{1}{4}\,40 = \$50 > \$30 = \pi^{NW}.$$

(d) (i) First, we check a consumer's willingness to pay for this type of warranty by evaluating the net utility

$$\rho V + (1-\rho)\rho V + (1-\rho)^2 \rho V - p \geq 0 \Longrightarrow p \leq \$210.$$

An alternative method of computing the maximum price that the monopoly can charge is to use the fact that the product fails 3 times with probability $(1-\rho)^3$. Hence, the product fails at most twice with probability $[1-(1-\rho)^3]$. Hence,

$$p \le \left[1-(1-\rho)^3\right] V = 240\left(1-0.5^3\right) = \$210.$$

Next, we compute the monopoly's expected cost to be

$$c + (1-\rho)c + (1-\rho)^2 c = \$105.$$

Therefore, expected profit is $\pi = 210 - 105 = \$105$.

(ii) First check a consumer's willingness to pay for this warranty type.

$$\rho V - p + (1-\rho)p \ge 0 \Longrightarrow p = V = \$240.$$

That is, since the consumer is fully insured, the consumer is willing to pay her entire valuation under the money-back guaranty.

Expected cost is

$$c + (1-\rho)p = 60 + (1-0.5)240 = 180.$$

Hence, expected profit is $\pi = 240 - 180 = \$60$.

Notice that this is the same profit that the monopoly can obtain by selling without any warranty. As we learned in class, money back guaranty may not be profitable to the monopoly (as opposed to replacement warranty).

Solution to Set # 19: Peak-load pricing

(a) (i) If summer turns out to be the peak season, the airline should solve

$$\begin{aligned} \mathit{MR}_S(q_S) = 12 - q_S &= \$2 + \$2 = \mu_k + \mu_o, \Longrightarrow q_S^{\mathit{pl}} = k^{\mathit{pl}} = 8 \\ \mathit{MR}_W(q_W) = 24 - 4q_W &= \$2 = \mu_o, \Longrightarrow q_W^{\mathit{pl}} = \frac{11}{2} < k^{\mathit{pl}}. \end{aligned}$$

Therefore, $p_S^{\mathit{pl}} = \$8$ and $p_W^{\mathit{pl}} = \$13$. The profit is then given by

$$\pi^{\mathit{pl}} = (p_W^{\mathit{pl}} - \mu_o)q_W^{\mathit{pl}} + (p_S^{\mathit{pl}} - \mu_k - \mu_o)q_S^{\mathit{pl}} = (13-2)\frac{11}{2} + (8-2-2)8 - 0 = \$92.5.$$

(ii) Let $p = p_W = p_S$. In this case, the direct demand functions are:

$$q_S = 2(12-p) \quad \text{and} \quad q_W = 12 - \frac{p}{2}.$$

We first would like to "estimate" which would be the "high" season. From the above $q_S \le q_W$ if $p \ge 8$. So, let us assume (and later verify), that Winter is the "high" season.

If p_W is indeed the high season (which means that the equilibrium price should satisfy $p > \$8$, then the seller solves

$$\max_p \pi = p(q_W + q_S) - (2+2)q_W - 2q_S = 42p - \frac{5}{2}p^2 - 96$$

The first-order condition yields $0 = d\pi/dp = 42 - 5p$. The second-order condition for a maximum is satisfied since $d^2\pi/dp^2 = -5 < 0$. Therefore,

$$p = \frac{42}{5} = \$8.4 \quad \text{and} \quad \pi = \$80.4 < \$92.6$$

which is the price obtained under peak-load price discrimination.

Notice that $p = 8.4 > 8$ which confirms that Winter is indeed the peak season. Alternatively, we can also confirm that Winter is the peak season by looking at the equilibrium quantities: $q_W = 7.8 > 7.2 = q_S$.

Finally, we are not done until we check one more possibility which is raising the price above $p = \$12$ thereby serving only Winter consumers. In this case, solving $MR_W = 24 - 4q_W = 2 + 2 = r + c$ yields $q_W = 5$, hence $p = 24 - 2q_W = \$14 > \$12$. Under $p = \$14$, the profit is

$$\pi = pq_W - (c+r)q_W = 14 \cdot 5 - 4 \cdot 5 = \$50 < \$64.9.$$

Therefore, $p = \$8.4$ is the profit-maximizing price when the monopoly is forced to set uniform prices across seasons.

(b) (i) If summer turns out to be the peak season, the airline should solve

$$MR_S(q_S) = 36 - q_S = \$2 + \$4 = c + r, \quad \Longrightarrow q_S^{pl} = k^{pl} = 30$$
$$MR_W(q_W) = 36 - 2q_W = \$2 = c, \quad \Longrightarrow q_W^{pl} = 17 < k^{pl}.$$

Therefore, $p_S^{pl} = \$21$ and $p_W^{pl} = \$19$.

(ii) Let $p = p_W = p_S$. In this case, the direct demand functions are:

$$q_S = 2(36 - p) > q_W = 36 - p, \quad \text{for all } p \ge 0.$$

The seller solves

$$\max_p \pi(p) = p(q_W + q_S) - (2+4)q_S - 2q_W$$
$$= p[2(36-p)] + p(36-p) - (2+4)[2(36-p)] - 2(36-p) = -3p^2 + 122p - 504.$$

The first-order condition yields $0 = d\pi/dp = 122 - 6p$. The second-order condition for a maximum is satisfied since $d^2\pi/dp^2 = -6 < 0$. Therefore, $p_{S,W} = 61/3 \approx \$20.33$.

(c) Try first assuming that Day is the "high season." Therefore,

$$MR_D = 12 - q_D = r = 4 \quad \Longrightarrow \quad q_D = K = 8 \quad \Longrightarrow \quad p_D = 12 - \frac{8}{2} = 8.$$

For the Night "season"

$$MR_N = 24 - 4q_N = c = 0 \quad \Longrightarrow \quad q_N = 6 \quad \Longrightarrow \quad p_N = 24 - 2 \cdot 6 = 12.$$

It is important to confirm that Day is indeed the high season by verifying that $q_N = 6 < 8 = K$, so the night demand can be accommodated with the capacity K.

(d) Suppose that Summer is the high season. Then,

$$MR_S = 12 - q_S = r + c = 3 + 4 \Longrightarrow K = q_S = 5 \Longrightarrow p_S = 12 - \frac{5}{2} = \frac{19}{2} = \$9.5.$$

$$MR_W = 24 - 4q_W = c = 4 \Longrightarrow q_S = 5 \le K \Longrightarrow p_W = 24 - 2 \cdot 5 = \$14.$$

Note that we have verified that Summer is indeed the high season since $q_W = 5 \le K = q_S$. The profit over a cycle of one Summer and one Winter is

$$\pi = \frac{19}{2} \cdot 5 + 14 \cdot 5 - 5(3 + 4) - 5 \cdot 4 = \frac{125}{2} = \$62.5$$

Solution to Set # 20: Tying

(a) (i) With no tying, pricing R at a high rate so that only type 1 guests book a room, $p_R = \$100$ yields a profit of $\pi_R = (100 - 40)200 = \$12,000$. Reducing the price so that both types book a room, $p_R = \$60$ yields a profit of $\pi_R = (60 - 40)1000 = \$20,000$. Therefore, $p_R = \$60$ is the profit-maximizing rate.

Setting a high breakfast price so that only type 2 consumers buy breakfast, $p_B = \$10$, yields a profit of $\pi_B = (10 - 2)800 = \$6400$. Reducing the price so that both types buy, $p_B = \$5$, yields a profit of $\pi_B = (5 - 2)1000 = \$3000$. Therefore, $p_B = \$10$ is the profit-maximizing breakfast price.

Adding the profit made from selling these two services separately yields a profit of $\pi^{NT} = 20,000 + 6400 = \$26,400$.

(ii) Selling a room and breakfast in one package for a high price of $p_{RB} = 100+5 = \$105$ results in sales to type 1 consumers only. Hence, $\pi^{PT} = (105-40-2)200 = \$12,600$. Reducing the package price to $p_{RB} = 60+10 = \$70$ yields $\pi^{PT} = (70-40-2)1000 = \$28,000 > \$26,400 = \pi^{NT}$. Therefore, tying is profitable for this hotel.

(iii) Under no tying, setting $p_R = \$100$ yields $\pi_R = (100 - 40)200 = \$12,000$. Setting $p_R = \$60$ yields $\pi_R = (60 - 40)400 = \$8000$. Setting $p_B = \$10$ yields $\pi_B = (10-2)200 = \$1600$. Setting $p_B = \$5$ yields $\pi_B = (5-2)400 = \$1200$. Altogether, the maximum profit that can be earned from selling the two services separately is $\pi^{NT} = 12,000 + 1600 = \$13,600$.

(iv) With tying, setting $p_{RB} = \$105$, thereby selling to type 1 only, yields $\pi^{PT} = (105 - 40 - 2)200 = \$12,600$. Setting $p_{RB} = \$70$, thereby selling to both types, yields $\pi^{PT} = (70 - 40 - 2)400 = \$11,200$. Therefore, tying is not profitable in this example.

(b) (i) Setting a high price for CNN, $p_C = \$11$, results in 200 subscribers, hence a profit of $\pi_C = (11-1)200 = \$2000$. Setting a low price, $p_C = \$2$, results in 400 subscribers, hence a profit of $\pi_C = (2-1)400 = \$400$. Therefore, $p_C = \$11$ is profit maximizing. Similarly, BBC subscriptions should also be sold for $p_B = \$11$.

Setting a high price for HIS, $p_H = \$6$, results in 200 subscribers, hence a profit of $\pi_H = (6 - 1)200 = \$1000$. Setting a low price, $p_H = \$3$, results in 400 subscribers, hence a profit of $\pi_H = (3 - 1)400 = \$800$. Therefore, $p_H = \$6$ is the profit-maximizing price. Altogether, the total profit under no tying is $\pi^{NT} = 2000 + 2000 + 1000 = \$5000$.

(ii) Setting a high package price, $p_{CBH} = \$19$, results in 200 subscribers, hence a profit of $\pi^{PT}(19) = (19 - 3)200 = \$3200$. Setting a low price, $p_{CBH} = \$16$, results in 400 subscribers, hence a profit of $\pi^{PT}(16) = (16 - 3)400 = \$5200$. Therefore, $p_{CBH} = \$16$ is the profit-maximizing price.

(iii) Suppose now that the cable TV operator makes the following offer: Viewers can subscribe to a "news" package containing CNN and BBC for a price of $p_{CB} = \$13$ and the HIS(tory) channel for $p_H = \$6$. Inspecting the table reveals that all 400 consumers will subscribe to the "news" package whereas only 200 will subscribe to the HIS(tory) channel. Hence, total profit under mixed tying is

$$\pi^{MT} = (13 - 2)400 + (6 - 1)200 = \$5400 > \pi^{PT} = \$5200 > \pi^{NT} = \$5000.$$

Therefore, mixed tying is more profitable than either pure tying or no tying.

(c) (i) With no tying, pricing R at a high rate so that only type 1 guests book a room, $p_R = \$100$ yields a profit of $\pi_R = (100 - 40)200 = \$12,000$. Reducing the price so that both types book a room, $p_R = \$60$ yields a profit of $\pi_R = (60 - 40)1000 = \$20,000$. Therefore, $p_R = \$60$ is the profit-maximizing rate.

Setting a high breakfast price so that only type 2 consumers buy breakfast, $p_B = \$10$, yields a profit of $\pi_B = (10 - 2)800 = \$6400$. Reducing the price so that both types buy, $p_B = \$5$, yields a profit of $\pi_B = (5 - 2)1000 = \$3000$. Therefore, $p_B = \$10$ is the profit-maximizing breakfast price.

The gym should be priced at $p_G = \$10$, yielding a profit of $\pi_G = \$10,000$. Altogether, total profit with no tying is $\pi^{NT} = 20,000 + 6400 + 10,000 = \$36,400$.

(ii) Setting a high price for the package, $p_{RBG} = \$115$, attracts only 200 customers, hence yields a profit of $\pi^{PT} = (115 - 42)200 = \$14,600$. Setting a low price, $p_{RBG} = \$80$, attracts all 1000 customers, hence yields a profit of $\pi^{PT} = (80 - 42)1000 = \$38,000 > \pi^{NT}$. Therefore, pure tying is more profitable than no tying.

Solution to Set # 21: Dealerships

(a) (i) The dealer takes d as given and solves

$$MR = 120 - 4Q = d \quad \Longrightarrow \quad Q = \frac{120 - d}{4}.$$

(ii) The manufacturer sets the price d to solve

$$\max_d \pi_m = (d - c)q = (d - 40)\,\frac{120 - d}{4} = \frac{1}{4}\left(120d - d^2 - 40 \cdot 120 + 40d\right).$$

The first-order condition implies

$$0 = \frac{\partial \pi_m}{\partial d} = \frac{1}{4}\,(120 - 2d + 40) \quad \Longrightarrow \quad d_m = \$80.$$

Notice that the manufacturer charges a markup of $d - c = 80 - 40 = \$40$ above marginal cost.

(iii)

$$Q = \frac{120 - 80}{4} = 10 \quad \Longrightarrow \quad p = 120 - 2 \cdot 10 = \$100.$$

Notice that the consumer pays $\$100$ for a good that costs $\$40$ to produce. The dealer is charged $\$80$ so the dealer sets a markup of $p - d = 100 - 80 = \$20$ on each unit sold.

(iv)

$$\pi_m = (d - c)Q = (80 - 40)10 = \$400, \quad \text{and} \quad \pi_d = (p - c)Q = (100 - 80)10 = \$200.$$

(v) Consider the following contract between the HUMMER manufacturer and the dealer: The dealer shall pay $d = \$60$ for each unit sold, and a fixed fee of $\phi = \$200$. To see why this contract (called two-part tariff contract) yields a higher profit to, both the dealer and the manufacturer, observe that at a lower per-unit price the dealer sells $Q = \frac{120-60}{4} = 15$ HUMMERs. The price paid by a buyer is therefore $p = 120 - 2 \cdot 15 = \$90$. Hence,

$$\pi_m = (d - c)Q + \phi = (60 - 40)15 + 200 = \$500 > \$400, \quad \text{and}$$

$$\pi_d = (p - c)Q - \phi = (90 - 60)15 - 200 = \$250 > \$200.$$

What happens here is that a lower per-unit dealer price increases sales, as opposed to a high dealer price $d = \$80$ resulting from a double markup. Higher sales extract more revenue from consumers and the distribution between the dealer and the manufacturer is done via the fixed fee $\phi = \$200$.

(b) (i) In the second stage, the dealer takes d as given and chooses output level Q to solve

$$\max_p \pi^d = (p-d)Q - \phi = [(36-Q)-d]Q$$
$$\Longrightarrow 0 = \frac{\mathsf{d}\pi^d}{\mathsf{d}Q} = 36 - 2Q - d \quad \Longrightarrow Q = \frac{36-d}{2}.$$

Therefore,

$$p = \frac{36+d}{2} \quad \text{and} \quad \pi^d = \frac{(36-d)^2}{4} - \phi.$$

Note that $\phi = 0$ for this part of the problem.

(ii) In the first stage, the manufacturer selects a dealer price d to solve

$$\max_d \pi^m = (d-20)Q = (d-20)\left(\frac{36-d}{2}\right) \quad \Longrightarrow 0 = \frac{\mathsf{d}\pi^m}{\mathsf{d}d} = \frac{56-2d}{2} \quad \Longrightarrow d = 28.$$

Hence,

$$Q = \frac{36-d}{2} = 4, \quad p = \frac{36+d}{2} = 32,$$
$$\pi^d = \frac{(36-d)^2}{4} - \phi = 16 - \phi, \quad \text{and} \quad \pi^m = (d-20)\left(\frac{36-d}{2}\right) + \phi = 32 + \phi.$$

(iii) Consider the following contract between the manufacturer and the dealer: $d = c = 20$ and $\phi = 40$. Then,

$$\pi^d = \frac{(36-20)^2}{4} - 40 = 24 > 16 \quad \text{and} \quad \pi^m = 40 > 32.$$

THE END

