Proposal for an I-Team UTRA on "Dynamics and Stochastics"

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I propose an I-Team UTRA on "Dynamics and Stochastics" that would involve 5 Brown undergraduate students as part of the Summer@ICERM program at the Institute for Computational and Experimental Research in Mathematics (ICERM) here at Brown. Summer@ICERM (see [http://icerm.brown.edu/summerug/2016/](http://icerm.brown.edu/summerug/2016/)) is a highly successful 8-week summer research experience for around 14 domestic students: the summer 2016 program will be organized by Margaret Beck (BU), Todd Kapitula (Calvin College), and myself. Given the demand for research opportunities from Brown students interested in applied mathematics, I would be very interested in adding Brown students to this program: ICERM funding comes from the NSF and is restricted to domestic students; in addition, NSF requires that students are selected with geographic diversity in mind, and we are therefore not able to accommodate a significant group of Brown undergraduates. I therefore propose this I-Team UTRA as a way to fund participation by Brown undergraduates.

For Summer@ICERM, I collected projects that are all closely connected to my research: students will work on these problems in teams of 3-4 students, supported by myself, the other faculty organizers, and a team of my graduate students and postdocs. Below, I will give one sample project on stripe formation in zebrafish and outline a few additional topics.

1 Sample project: patterning on caudal fins of zebrafish

Zebrafish is a small fish with black and yellow stripes, and, as a vertebrate with a fully sequenced genome and nearly transparent embryo, it is a widely studied organism in developmental biology.

Stripes in wild-type and as well as spots, widened stripes, and labyrinth patterns on mutants are due to the interaction of melanophore, xanthophore, and iridophore pigment cells. In a recent paper, Alexandria Volkening (one of my graduate students) and I have developed a model for stripe formation on zebrafish skin in which melanophores and xanthophores are modelled as cells that interact on a growing fish body through cell differentiation, death, and migration. Cell migration is governed by differential equations for the positions of melanophores and xanthophores, which are paired with discrete-time stochastic rules for cell differentiation and death. Informed by experimental data, we consider both short-range interactions (possibly regulated by cell dendrites or contact) and long-range interactions (possibly regulated by pseudopodia, which can be up to half a stripe width long).

Our goal is to extend this model for patterning on the body of zebrafish to stripes formed on their caudal (back) fins. Pattern formation on fins is not well understood: for example, it is unclear what role bones play in stripe formation on fins and how strong the interplay between pattern formation on the caudal fin and the body is. Furthermore, the stripes on the caudal fin do not initially match up with their counterparts on the body; it is only at
later stages that they connect. Finally, some mutations are known to disrupt patterns on the body but not the fin, while others impact patterning across the entire fish. The plan is to (1) model the radial outgrowth of fin bones which seem to align melanophores, (2) develop a model for pigment cells on fins to reproduce wild-type patterns, and (3) test the model on regeneration experiments. Finally, we will use the model to explore zebrafish mutations and the body-fin interface, including long-fin mutations that seem to feature bifurcating stripes (unlike on the body, where additional stripes always appear sequentially at the edge of the domain). We believe that this model will contribute to an improved understanding of why some mutations impact both body and fin, while others impact either only the body or the fins. The model would be informed by recent experiments: Christiane Nüsslein-Volhard (Tübingen/Germany), a Nobel prize winner who leads one of the leading labs in this area, has agreed to share data with us over the coming year that would allow us to incorporate recent experimental observations into our model.

This project would be ideal for a team of 3-4 students who would develop the underlying model, implement the code needed to explore the model numerically, and run simulations. The background required for this project would consist of a differential equations course such as APMA 0330 or APMA 0350; an introductory probability course such as APMA 1650 or programming experience in Matlab would be advantageous but will not be required.

2 Other research projects

I list three additional sample projects that we will offer to student teams:

1. Localization of mRNA in Xenopus oocytes: Establishing spatial directionality is an important part of the early development of organisms. In Xenopus, directionality is achieved by localization of mRNA, first at the nucleus and then at the vegetal cortex. The process of moving mRNA cargo from the nucleus to the vegetal cortex involves several kinds of molecular motors, but their precise role and the role of anchoring of mRNA to the cortex is not well understood. This project, a collaboration of my group with Kim Mowry’s lab in MCB, would extend recent agent-based one-dimensional models to planar models that involve transport along complex networks of microtubules.

2. Pattern formation in the Swift-Hohenberg equation: The Swift-Hohenberg equation is a paradigm model for localized stripe patterns that emerge, for instance, as vegetation patches in deserts and convection rolls in fluids. These patterns depend sensitively on parameters and exhibit intriguing bifurcation diagrams: our goal is to understand how bifurcation diagrams depend on the dimensionality of the underlying patterns, and how geometric structures associated with these patterns affect the global bifurcation structure: these topics will be approached by applying analytical methods to simpler geometric models and using detailed numerical explorations.

3. Low-frequency forcing of spiral waves: Spiral waves play an important role in sustaining certain cardiac arrhythmias such as tachycardia. Pacemaker waves can get stuck in damaged cardiac tissue and evolve into spiral waves that excite cardiac muscle cells with a much higher temporal frequency, thus potentially leading to fibrillation. Recent
experiments have shown that low-energy far-field pacing can remove spiral waves from inhomogeneous damaged tissue. However, the theoretical underpinnings for this mechanism are not known: this project will use theoretical and numerical approaches to study simpler equations that could help shed light on why far-field pacing is successful.

3 Mentoring and research environment

When working with undergraduates, I see my role as being a combination of teacher, mentor, and collaborator. One of my responsibilities is to help students make the transition from learning about mathematics to using mathematics in an applied context: this means asking questions, identifying ways in which these questions can be addressed, selecting (and learning) the right techniques, applying them to the project, questioning and testing the results, and, finally, interpreting them.

My goal for the proposed I-Team UTRA is to form a diverse group of students, including women and members from underrepresented minorities, and students from different types of colleges and universities. I am very interested in including Brown first-year students and sophomores: there is a number of excellent students in my APMA 0350 class who expressed interest in applied mathematics research, for instance.

ICERM provides an excellent research environment for summer research: except for one or two weeks where some of the space will be shared with workshops held at ICERM, participants will have office and collaborative spaces on both floors available for their research. Margaret, Todd, I, and graduate and postdoc TAs will reside at ICERM and work closely with students throughout the program. I plan to meet with the groups I mentor at least once per day: we will have weekly meetings of the entire cohort to update each other on our progress. Throughout the program, we will run several minicourses to provide background on the various techniques needed to approach the proposed projects. In addition, we envision to offer the following:

- Tutorials on Matlab and LaTeX;
- Introduction on how to give presentations: we will ask each team to give a 10-15 minute presentation each Friday afternoon followed by feedback from the entire group to strengthen presentation skills (this worked very well in previous REUs I organized);
- Introduction on how to write papers;
- Panel on graduate school with Brown graduate students, focusing on applications and how to decide which schools to apply to;
- Panel on graduate fellowships (NSF GRFP, DoD NDSEG, DOE CSGF) with Brown graduate students.

Student teams are expected to finish a paper by the end of the program: some of these papers may be submitted to research journals, while others would go to dedicated mathematics undergraduate journals that are hosted by professional societies such as SIAM.