Abstract

This thesis attempts to propose a possible explanation for non-equilibrium cooperation observed in public goods game experiments by bounded-rationality and a latent realistic background. This paper proposes a public goods game modeling real situations people face and researches on how rule of thumb plays an important role in equilibrium selection.

1. Introduction

This attempt of thesis is motivated by the phenomena of non-equilibrium cooperation observed in some Public Goods experiments. The classical Public Goods
game, where each player decides his/her own contribution to the Public Goods from the initial endowment, has a unique equilibrium where each player makes zero contribution. In various experiments, however, substantial contributions are observed. Those contributions mostly divide into two categories.

First, substantial contribution in early periods followed by a decline was widely observed. Some may argue that this in a merely a result of learning the unfamiliar rules and outcome correspondence. However findings by Andreoni (1988) and Isaac and Walker (1988) showed that there is a “restart” effect when a second finitely repeated game takes place, therefore it cannot be explained as such learning behavior. The second type of non-equilibrium contribution is observed in some experiments with additional mechanisms, such as redistribution proportional to individual contribution, sanction and communication. In some groups, subjects maintained a certain level of cooperation, all contributing a positive amount, throughout the entire session.

Although these behaviors could be explained by reciprocity, altruism or social norms, the formation of such social preferences is not well explained. Purely claiming these social preferences as a given fact is somewhat unsatisfactory. Evolutionary explanation of the formation of social norms has been a popular attempt (Ostrom 2000), however, this thesis is going to take another approach. This approach is based on bounded rationality and attempts to posit the classical Public Goods Game in a larger frame. Instead of assuming that people have a fixed type as in the evolutionary explanation, this thesis claims that subjects, due to bounded
rationality, have a rule of thumb, which is optimal in reality. That is the reason for the existence non-equilibrium in the experiments. This thesis is going to construct a variant of Public Goods game model with group contribution and redistribution, in an attempt to capture the features in reality that makes cooperation profitable. Then a computational method called agent-based modeling is adopted to simulate the process of formation of such rule of thumb and how the characteristics would affect the distribution of equilibrium outcomes.

2. Theoretical Model

2a. Model Specification

Here we introduce a specific type of equilibrium of group competition robust to randomness.

**Definition:** An error-proof Nash equilibrium is a Nash equilibrium where for any group, when the contribution of all group members rises or falls by a small amount simultaneously, the optimal action for each member is to retain the previous contribution level.

Mathematically, suppose in an equilibrium, the contribution level of all members are $c_1, c_2, \ldots, c_n$ respectively, then there exists a positive number $\varepsilon_0$, such that for any member $i$, any $\varepsilon<\varepsilon_0$, i's optimal contribution level facing $c_i=(c_1-\varepsilon, c_2-\varepsilon, \ldots, c_{i-1}-\varepsilon, c_i+1-\varepsilon, \ldots, c_n-\varepsilon)$ or $(c_1+\varepsilon, c_2+\varepsilon, \ldots, c_{i-1}+\varepsilon, c_{i+1}+\varepsilon, \ldots, c_n+\varepsilon)$ is $c_i$. 
2b. Public Goods game

**Definition:** A Public Goods game consists of a group of N members. Each member is endowed with an initial endowment and each independently decides the amount to contribute to the group account and rest is invested to the member’s private account. The resources in the group account gain a return of $R \in (1,N)$ and the return is evenly redistributed to each member and the return to the resources in the private account is 1, solely belonging to the respective subject.

**Nash Equilibrium:** The MPCR (marginal per capita return) for contribution is $R/N$, strictly smaller than 1, which is the MPCR for investment in private account. Thus in equilibrium, each subject makes zero contribution.

**Error-proof equilibrium:** as the MPCR’s are constant, the Nash equilibrium is error-proof.

2c. Public Goods Game with efficiency redistribution

One feature popularly studied, not only in game theory, is redistribution (Balafoutas et al, 2013). In the classic Public Goods Game, the mechanism of redistribution is just equal division. If part of the return from collective production is redistributed according to contribution, the MPCR of contribution for each member will increase. This then is no longer a pure public goods game but still it is of interest here as it
captures some characteristics of realistic situations people face, when the
distribution of public goods can be asymmetric, such as education and highways.

**Definition:** A Public Goods game with efficiency redistribution consists of a group of
N members. Each member is endowed with an initial endowment and each
independently decides the amount to contribute to the group account and rest is
invested to the private account. The resources in the group account gain a return of
R ∈ (1,N) and a proportion α of the return is redistributed according to contribution
and the rest is evenly redistributed to each member and the return to the resources
in the private account is 1, solely belonging to the respective subject.

**Nash Equilibrium:** The MPCR (marginal per capita return) for contribution is
Rα+R(1 – α)/N. If it’s strictly smaller than 1, which is the MPCR for investment in
private account, then in equilibrium, each subject makes zero contribution. If it’s
strictly larger than 1, then in equilibrium, full contribution is achieved.

**Error-proof equilibrium:** as the MPCR’s are constant, the Nash equilibrium is
error-proof.

**2d. Group Competition**

In reality, the return of the collective production is not fixed. In fact, the contribution
to the group, in many circumstances, could make a difference in the return, for
example, NBA teams may be competing for the ranking, where the higher ranked
teams generate more income, and the more effort put in, the higher the team is
ranked. The classical Public Goods game does not try to model that. The following is an attempt to capture that incentive.

**Definition:** A Public Goods game with group competition consists of G groups, each with N members. Each member, with the same initial endowment, decides independently the amount of contribution to the respective group account and the rest is invested on the member’s private account. The groups are ranked in descending order by their aggregate contribution within each group. Each group gains a MPCR $r_i$, when the group is ranked $i$th, where $r_1 > r_2 > ... > r_G$. Ties are broken by the tied groups sharing the average MPCR among them. The return from each group account is evenly redistributed to each member within the respective group.

**Nash Equilibrium:** The situation where the MPCR from group account is exactly 1 is not of our interest here and thus ignored. Since MPCR is not 1, in equilibrium, each member is either contributing nothing or everything, and within each group, members should make the same contribution. Suppose that in equilibrium, there are $S$ groups achieving full contribution and the rest achieve zero contribution. The equilibrium induces two inequalities.

First, the MPCR for the contributing members is $(r_1 + r_2 + ... + r_S)/(S*N)$ and it should be larger than 1.
Second, the MPCR from contributing for a deviating member in a non-contributing group is $r_{S+1}/N$ and it should be smaller than 1.

Therefore we have that $L \leq S \leq U$, where $U = \max\{l : (r_1 + r_2 + \ldots + r_l)/l > N\}$ and $L = \min\{u : r_{u+1} < N\}$. L and U are the lower and upper bound of the number of groups cooperating in an equilibrium. Thus for any $S$, where $L \leq S \leq U$, there exists a class of equilibria where exactly $S$ groups achieve full contribution and the rest of groups achieve zero contribution.

**Error-proof Equilibrium:** For any $S > L$, $r_S < N$, there exists no error-proof equilibrium as if all group members in a contributing group reduce contribution by a small amount at the same time, then the optimal level of contribution for the members in this group will become zero.

Thus the only error-proof equilibria are the class of equilibria where exactly $L$ groups achieve full contribution and the rest achieve zero contribution.

**2e. Group Competition with efficiency redistribution**

The proportion of collective return redistributed according to contribution is an interesting character to differentiate the groups. Although alpha's close enough to 1 will allow all groups to benefit from contribution regardless the outcome of the
competition, while constrained, assigning different redistribution parameter $\alpha$'s creates a spectrum of incentives to contribute among groups.

**Definition:** A Public Goods game with group competition and efficiency redistribution consists of $G$ groups, each with $N$ members. The $i$th group is assigned a redistribution parameter $\alpha_i$, where $\alpha_1 > \alpha_2 > ... > \alpha_G$. Each member, with the same initial endowment, decides independently the amount of contribution to the respective group account and the rest is invested on the private account. The groups are ranked in descending order by their aggregate contribution within each group. Each group gains a MPCR $r_i$, when the group is ranked $i$th, where $r_1 > r_2 > ... > r_G$. Ties are broken by tied groups sharing the average MPCR among them. A proportion $\alpha_i$ of the return is redistributed according to contribution and the rest is evenly redistributed to each member within the $j$th group.

**Nash Equilibrium:** Let $f(\alpha)=N/(1-\alpha+N\alpha)$. From similar inequalities from 2d, we can get the similar equilibria. For only groups $\{k_1, k_2, ..., k_S\}$ with $k_1 < k_2 < ... < k_S$, achieve full contribution and rest achieve zero contribution.

First the MPCR of contribution is $(r_1+r_2+...+r_S)\times(\alpha_{k_S}+(1-\alpha_{k_S})/N)/S$ should be larger than 1.

Second, the MPCR from contributing for a deviating member in a non-contributing group is $r_{S+1}\times(\alpha_{k_S}+(1-\alpha_{k_S})/N)$ and it should be smaller than 1.

Therefore the equilibrium requires that
\( \frac{(r_1+r_2+...+r_S)}{S} > \max \{ f(\alpha_{k,s}): s \leq S \} = f(\alpha_{k,s}) \), and \( r_{S+1} \leq \min \{ f(\alpha_{k,s}): s \geq S+1 \} = f(\alpha^*) \),
where \( k^* = \min \{ 1,2,...,N \} \setminus \{ k_1, k_2, ..., k_s \} \).

Therefore for each set of groups \( \{ k_1, k_2, ..., k_s \} \) such that \( k_1 < k_2 < ... < k_s \),
\( \frac{(r_1+r_2+...+r_S)}{S} > f(\alpha_{k,s}) \), and \( r_{S+1} < f(\alpha^*) \), where \( k^* = \min \{ 1,2,...,N \} \setminus \{ k_1, k_2, ..., k_s \} \), there exists an equilibrium where the groups \( \{ k_1, k_2, ..., k_s \} \) achieve full contribution and the rest achieve zero contribution.

**Error-proof Equilibrium:** For each set of groups \( \{ k_1, k_2, ..., k_s \} \) such that \( k_1 < k_2 < ... < k_s \), such that \( \frac{(r_1+r_2+...+r_S)}{S} > f(\alpha_{k,s}) \), \( r_{S+1} < f(\alpha^*) \), and \( r_s > f(\alpha_{k,s}) \), there exists an error-proof equilibrium where the groups \( \{ k_1, k_2, ..., k_s \} \) achieve full contribution and the rest achieve zero contribution. Such equilibria is obtained by the addition of the constraint \( r_s > f(\alpha_{k,s}) \) (noting that \( f(\alpha) \) is the minimum MPCR required for a group with redistribution factor \( \alpha \)), which makes the members of the group \( k_s \) able to sustain cooperating when all the members deviate by a small amount and thus only getting \( r_s \) instead of \( \frac{(r_1+r_2+...+r_S)}{S} \).

This result is still not free of multiplicity as the constraints can still be satisfied by multiple equilibria.

3. Agent-based Modeling

3a. Overview

The multiplicity we observed from the result in 2e almost makes it meaningless.

While having derived the possible equilibria, yet it’s still not clear what the
probabilistic distribution among those from different initial conditions is. In order to tackle that, in this section, we are going to adopt the method of agent-based modeling, which is a variant of Monte Carlo method. We will adopt autonomous agents who start from a random initial contribution level and in each period, adjust their levels of contribution according to their rules of thumb. The initial contribution level of each agent is randomly assigned and randomness is also introduced in the adjustment in each period. We repeat this process until the outcomes converge to an equilibrium (we will guarantee that it happens almost surely by repeating it for 300 to 500 rounds) and once a game reaches an equilibrium, then the adjustments will become zero and the situation will be simply repetitive thereafter. With a large number of such samples (we set the number of samples to be 50000 here) we can estimate the probabilistic distribution of equilibria in relation to the characteristics of the rules of thumb. In this section, we study two features of the rules of thumb: adjustment rate and learning mechanism. Because in fact, this simulation is essentially how people adapt to fit into the game. These are actually two of the most important characteristics of such adaption: how quickly people adapt according to the feedback they get and what they adapt according to. People may be cautious or bold in adapting. Also people may not be able to find out the real factors or real marginal rate of return at the beginning. They then have to make their decisions according to what they can observe, which is the average return from their investments in the group account. We attempt to capture the first by adjustment rate, which is the magnitude of adjustment made by an agent in one period. The second characteristic is of importance as information is a very
crucial component of a game and in reality not all information is always available. If players can’t receive information of how much others have contributed, they might need a while to learn how the mechanism works and what their marginal returns are, which might even not happen at all. Therefore it’s reasonable to assume, under bounded rationality or imperfect information, that agents would consider the average return (the ratio of amount returned over the amount invested) before they gain more information and learn about the marginal return. This important trait is presented by a learning period, during which the average return is assigned a diminishing weight.

3b. Treatments

We study six treatments in this section – Basic, Adjustment Rate, Learning, Redistribution, Fixed Learning/Adjustment Rate, and Fixed Learning/Redistribution. In the Basic Treatment (Treatment B), we follow the basic group competition setting described in 2d. The agents start out each round with initial endowment 10. The initial contribution level for each agent is independently drawn from a uniform random distribution on [0, 10] and the agents adjust their contribution in by $s^\ast h^\ast (\text{MPCR}-1)$, where $s$ is the adjustment rate, which is the same for all agents in this treatment, and $h$ is a random variable distributed uniformly on [0,1] and MPCR refers to that specific round. As we can see, if MPCR for one agent is larger than 1, that agent will increase his contribution level in the next round, and if
MPCR is smaller than 1, that agent will decrease his contribution level, which seems intuitive.

In the Adjustment Rate Treatment (Treatment AR), now each group is assigned a different adjustment rate. In the Learning Treatment, agents consider both their marginal return and average return from the group accounts. In this treatment, each group is assigned a learning period $L_i$, and the weight of the marginal return increases from $1/L_i$ in the first period to 1 in the $L_i$’s period and thereafter. In the Redistribution Treatment (Treatment R), each group is assigned a redistribution factor $\alpha_i$, which is the proportion of the return from the group account distributed according to contribution and the rest is still evenly distributed. In the Fixed Learning/Adjustment Rate Treatment (Treatment FLAR), all groups have the same learning period but different adjustment rates. In the Fixed Learning/Redistribution (Treatment FLR), all groups have the same learning period but different redistribution factors. The treatments are summarized in the following table:

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Adjustment Rate (s)</th>
<th>Learning Period (L)</th>
<th>Redistribution Factor ($\alpha$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>s for all groups</td>
<td>$L=0$</td>
<td>$\alpha=0$</td>
</tr>
<tr>
<td>AR</td>
<td>$s_i$ respectively</td>
<td>$L=0$</td>
<td>$\alpha=0$</td>
</tr>
<tr>
<td>L</td>
<td>s for all groups</td>
<td>$L_i&gt;0$ respectively</td>
<td>$\alpha=0$</td>
</tr>
<tr>
<td>R</td>
<td>s for all groups</td>
<td>L=0</td>
<td>$\alpha_i$ respectively</td>
</tr>
<tr>
<td>---------</td>
<td>------------------</td>
<td>--------------</td>
<td>-------------------------</td>
</tr>
<tr>
<td>FLAR</td>
<td>$s_i$ respectively</td>
<td>L&gt;0 for each group</td>
<td>$\alpha=0$</td>
</tr>
<tr>
<td>FLR</td>
<td>s for all groups</td>
<td>L&gt;0 for each group</td>
<td>$\alpha_i$ respectively</td>
</tr>
</tbody>
</table>

4. Results

The results are discussed treatment by treatment. We simulate the game process for $n=50000$ times for set of parameters and use the average of the sample as an estimator of the probabilistic distribution. Illustrative graphs can be found in the appendix demonstrating the dynamics within a single simulation and across all the simulations. Illustrative graphs are available in the appendix.

4a. Treatment B

In this treatment, we verify that the simulation does induce equilibria, and in particular, error-proof equilibria with equal chances. These two hypotheses come from the fact that the adjustment in each period is stochastic and the fact that initial contribution level of each agent is randomly assigned individually, respectively.
We set the number of groups $ng = 3$, number of members in each group $nm = 10$, initial endowment of resources in each period for each individual to be 10 and the adjustment rate to be $s=1$. We then set the MPCR to be 1.2, 0.9, 0.7 for groups ranked at 1st, 2nd, and 3rd. Note that there are six Nash equilibria with this setting and three of them are error-proof. Let’s denote an equilibrium by the groups fully contributing. Then the six equilibria are \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\} and by checking the conditions in 2d, we have that only the first three are error-proof, as $r_2<1$, $U=1$, which is the upper bound of the number of cooperating groups in an equilibrium. The result given by simulation is $A=[0.33244, 0.33082, 0.33674]$, where $A$ is the frequency distribution (which is just an estimator of the probabilistic distribution) of the first three equilibria. Note that $A$ is the distribution of equilibria and in each equilibria, for each group, either all members end up contributing everything or all members end up contributing nothing. For example, in equilibrium \{1\}, all the members from group number 1 contribute everything and the rest of agents contribute zero. The fact that the sum of the entries of $A$ is exactly 1 (meaning the probability of equilibria with two groups fully contributing is zero, which indeed didn’t appear in the simulations) shows that outcomes of such autonomous agent simulation indeed converge to error-proof equilibria. In addition, a chi-square test shows that the outcomes converge to the equilibria with equal chances:

$$\Sigma (n_i-n_i^0)^2/n_i^0=50000\times[(3.3244-1/3)^2+(3.3082-1/3)^2+(3.3674-1/3)^2]/(1/3)$$

$$=2.8080<3.84 \text{ (insignificant at 5\% level).}$$
This treatment serves as a benchmark in comparison to the following treatments.

4b. Treatment AR

In this treatment, the adjustment rate is no longer the same across groups. We set them to be 2, 1, 0.5 respectively. The other settings are the same. The simulation gives \( A = [0.33694, 0.32982, 0.34467] \), where \( A \) is the estimated probabilistic distribution of equilibria \{1\}, \{2\} and \{3\}.

Our null hypothesis is that the probabilistic distribution is \([1/3, 1/3, 1/3]\). We use a multinomial test (chi-square test). The chi-square statistic is

\[
\sum (n_i - n_i^0)^2 / n_i^0 = 50000 \times \left[ (0.33694-1/3)^2 + (0.32982 -1/3)^2 + (0.34467 -1/3)^2 \right] / (1/3)
\]

\[= 517.627 >> 13.82.\]

This indicates that the difference is significant at 0.1% level.

To determine whether the relative order of outcomes is related with the order of adjustment rates or with the level of adjustment rates, we ran another simulation with \( s = [3, 1.5, 0.75] \) and we get \( A = [0.32900, 0.33482, 0.33618] \).

Therefore the relative order is indeed related with the level of adjustment however the functional relation between the adjustment rate and the outcomes is beyond the scope of this thesis.
4c. Treatment L

In this treatment, adjustment rate is the same across the groups but each group now has a learning period. During the learning period, the members put a linearly diminishing weight on the average return from investing in the group account, which is the ratio of the resources gained from the group account over the amount of resources invested. We set the length of the learning periods to be 20, 40, and 60 for each group respectively. The simulation gives $A = [0.31950, 0.33450, 0.34600]$, where $A$ is the estimated probabilistic distribution of equilibria \{1\}, \{2\}, and \{3\}. Note that while considering the average return, the members are encouraged to invest more in the group account by the contribution by other members in the same group. The longer the learning period is, the more likely a group would gain advance in the group competition. This is verified by a chi-square test:

$$\sum (n_i - n_i^0)^2/n_i^0 = 50000 \times \left[ (0.31950 - 1/3)^2 + (0.33450 - 1/3)^2 + (0.34600 - 1/3)^2 \right] / (1/3)$$

$$= 52.9750 >> 13.82 \text{ (significant at 0.1\% level).}$$

4d. Treatment FLAR

In this treatment, all is the same as in treatment AR except for that a same learning period is added to each of the groups. We compare the results with those in treatment AR. The simulation gives $A = [0.50830, 0.29154, 0.20761]$ with adjustment rates [2, 1, 0.5] and $A = [0.51781, 0.30624, 0.17595]$ with adjustment rates [3, 1.5, 0.75], where $A$ is the estimated probabilistic distribution of equilibria \{1\}, \{2\}, and
Comparing these results with the results from treatment AR, we can conclude that with the learning period, groups with higher adjustment rates are more likely to thrive in competition. With a learning period, the effect of different adjustment rates on the distribution of equilibrium outcomes is larger and more oriented.

4e. Treatment R

In this treatment, instead of distributing all the resources in the group account evenly to each member, part of the resources is distributed to group members proportional to their individual contribution. We set the redistribution factors (the proportion of the return from the group account redistributed according to individual contribution) to be 1/51, 1/171, 0 respectively for the three groups. A second simulation is run with redistribution factors 1/51, 1/111, 0. We run a third and a fourth simulation with redistribution factors 1/51, 1/171, 0 and 1/51, 1/111, 0 respectively, with adjustment rate s=2. Checking the conditions stated in 2e, we can get the error-proof equilibria: {1}, {2}, {3}, {1, 2}, and {1, 3} (checking one-by-one all the possible combinations: {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, and {1, 2, 3}). The estimated probabilistic distributions of equilibria {1}, {2}, {3}, {1, 2}, and {1, 3} are as follows (they are estimated by their frequencies in the sample).

<table>
<thead>
<tr>
<th>Equilibria (Groups fully contributing)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1, 2</th>
<th>1, 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>1/3</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>Probability 1</td>
<td>0.3330</td>
<td>0.1683</td>
<td>0.1665</td>
<td>0.1667</td>
<td>0.1655</td>
</tr>
<tr>
<td>--------------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>Probability 2</td>
<td>0.2529</td>
<td>0.2508</td>
<td>0.0013</td>
<td>0.2466</td>
<td>0.2484</td>
</tr>
<tr>
<td>Probability 3</td>
<td>0.3354</td>
<td>0.1645</td>
<td>0.1660</td>
<td>0.1659</td>
<td>0.1682</td>
</tr>
<tr>
<td>Probability 4</td>
<td>0.3291</td>
<td>0.1666</td>
<td>0.1671</td>
<td>0.1671</td>
<td>0.1686</td>
</tr>
</tbody>
</table>

Before looking into the data, we first calculate the MPCR required for contribution to be profitable for each group. These benchmark rates are [0.85, 0.95, 1] and [0.85, 0.925, 1] respectively. The two the possible outcomes of MPCR’s in equilibrium are [1.2, 0.8, 0.8], [1.05, 1.05, 0.7]. Ignoring the case of ties at the beginning, the initial outcome of MPCR is [1.2, 0.9, 0.7] almost for sure. If the adjustments to optimum are made right in the first period without randomness, we can get the benchmark probabilities (both setting’s gives the benchmarks since 0.95 and 0.925 are both within the interval (0.9, 1.05)). Multinomial chi-square tests gives

\[
\sum (n_i - n_i^0)^2 / n_i^0
\]

\[
= 50000 \times [(0.3330 - 1/3)^2/(1/3) + (0.1683 - 1/6)^2/(1/6) + (0.1665 - 1/6)^2/(1/6) + (0.1667 - 1/6)^2/(1/6) + (0.1655 - 1/6)^2/(1/6)]
\]

\[
= 1.2340 < 9.49 \text{ (insignificant at 5\% level). (For redistribution factors 1/51, 1/171, 0)}
\]

\[
\sum (n_i - n_i^0)^2 / n_i^0
\]

\[
= 50000 \times [(0.2529 - 1/3)^2*3 + (0.2508 - 1/6)^2*6 + (0.0013 - 1/6)^2*6 + (0.2466 - 1/6)^2*6 + (0.2484 - 1/6)^2*6]
\]
Therefore the difference between the simulations with different redistribution factors is induced by the effect on how the redistribution factors affect the speed of adjustment each round.

**4f. Treatment FLR**

This treatment is the same as treatment R except for that all agents adopt a learning period of the same length 30. We run two simulations with redistribution factors \([1/51, 1/171, 0]\) and \([1/51, 1/111, 0]\) respectively. The distributions of equilibrium outcomes are listed as follows.

<table>
<thead>
<tr>
<th>Equilibria (Groups fully contributing)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1,2</th>
<th>1,3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.3325</td>
<td>0.1657</td>
<td>0.1680</td>
<td>0.1665</td>
<td>0.1672</td>
</tr>
<tr>
<td>Probability</td>
<td>0.2633</td>
<td>0.2359</td>
<td>0.0291</td>
<td>0.2384</td>
<td>0.2334</td>
</tr>
</tbody>
</table>

We can observe that the results are quite similar to the results in 4e, which means that the effect of redistribution does not depend on learning behavior. This is verified by chi-square tests:

\[ \Sigma (n_i - n_i^0)^2 / n_i^0 \]
50000*[(0.3330-0.3325)^2/0.3330+(0.1683-0.1657)^2/0.1683+(0.1665-0.1680)^2/0.1665 +(0.1667-0.1665)^2/0.1667+(0.1655-0.1672)^2/0.1655]

=3.6066<9.49 (insignificant at 5% level).

Σ[(ni-ni^0)^2/n_i^0

=50000*[(0.2529-0.2633)^2/0.2529+(0.2508 -0.2359)^2/0.2508 +(0.0013-0.0291)^2/0.0013+(0.2466-0.2384)^2/0.2466+(0.2484-0.2334)^2/0.2484]

=0.0003<9.49 (insignificant at 5% level).

5. Discussion

This thesis attempts to explain cooperation in lab experiments of Public Goods games by bounded rationality. People enter lab experiments following their rule of thumb they formed from their everyday life. However the situation they face in reality may seem similar to lab conditions but may actually be different. Here we discuss how the role of empirical rule explains cooperation. Firstly, the Group Competition model proposed by this thesis is meant to capture some features of realistic situation people face similar to the public goods game carried out in labs, except for that contribution may actually be profitable. For example, in a sports league, such as NBA, teams are rewarded, in cash or in reputation, for their rankings in the league. Players gain larger contracts if their teams play well but the contribution may not be perfectly recognized. Also in working places, such as offices, when people work on a project together, their
individual effort may not be perfectly recognized but the relative competitiveness of their project determines the cash bonuses they will gain from it. In these situations, cooperating may indeed be rewarding if enough effort is being made towards prizes large enough. We can also see that in all the equilibria, all the group members within a specific group have the same amount of contribution, either full contribution or zero contribution. This may also explain the fact that people may positively respond to the cooperation of other players in the sense that from their realistic experiences that implies they are in a winning group. This model proposes that the seemingly non-equilibrium behaviors may be equilibrium behaviors in a similar game people face in reality.

Secondly, from the simulation, we can see that bounded rationality and rules of thumb play an important role in selecting equilibria in the Group Competition model. The empirical features such as learning (due to bounded rationality or lack of information) and caution (captured by adjustment rate) are what differentiate the groups.

Caution can be measured by adjustment rates of agent. Compare the results from treatment AR and treatment FLAR. With enough information of the marginal return, a proper level of caution is most beneficial. There is a tradeoff between increasing contribution quickly to outplay other groups (high adjustment rate) and decreasing contribution slowly to avoid loss of competitiveness. However, when the information about marginal return is no longer available from the beginning, a higher adjustment rate leads to a higher chance of winning.
Compare treatment R and treatment FLR. From the chi-square tests, the effect of redistribution factors is not affected by the learning mechanism, thus does not depend on information. However it depends on adjustment rate as shown in treatment R.

From treatment L, we can observe that the longer the learning period, the more advantageous it is to the group. If no information about the marginal return is available throughout the game, people may even never realize what the marginal return actually is. This is in fact not rare. In cases such as tax, people have little information of how much each individual contributes. And in bureaucratic systems, typically in China, little effort is paid to keep track of the contributions of the bureaucratic members, neither individual nor aggregate data.

Thirdly, people in reality may also be facing different games. Some may face situations where more groups are fighting for limited award, which is more competitive. This might affect the willingness to contribute. Also the efficiency of redistribution could vary, which accounts for to what extent individual effort is recognized and rewarded. We can use family as an example as it has a great impact on individuals. Poor families tend to either hold extremely tight or suffer a total breakdown, because the competition between groups is severe, and if members see no chance of getting rewarded, they simply choose to contribute less. Also the recognition of effort and contribution within a family is a good example of the redistribution described in this thesis if we consider nonmaterial utilities. People that grew up in families that respond to individual contributions may tend to be
more cooperative as they have been enjoying a higher MPCR. This real life
observation lines up with what we observe in treatment R and FLR, that the higher
the redistribution factor is for a group, the greater are the chances of the group
achieving cooperation
Reference


Appendix. A Graphs

In this section we show two sets of graphs revealing the dynamic process within each simulation and across simulations. The graphs are from the simulation of the treatments described above, with three groups. In the first subsection, the graphs are from a typical simulation. The horizontal axis represents the number of rounds and the vertical axis represents the total contribution of a group. Each group is represented by a line of different colors. Note that in some graphs we only showed the first proportion of the whole simulation in order to get a clearer look at the volatility, which mainly concentrated in the early rounds. In the second subsection, we demonstrate the convergence of frequency distributions as the results of ever larger numbers of simulations are averaged together. Each graph in the second subsection represents a total of 50000 simulations, with the number of simulations accounted for rising from left to right. The horizontal axis represents the number of simulations and the vertical represents the frequency of an equilibrium so far. Each equilibrium, that is, \{1\} (i.e., the equilibrium in which only members of group 1 cooperate), \{2\} and \{3\}, is represented by a line of different colors.

A1. Dynamic Within Simulation

Treatment B

As we can see, only one group prevailed and the other two ended with zero contribution. In this treatment, the final outcome in largely related to the initial positions of the group contribution level as groups are identical.
Treatment AR

In this treatment, the adjustment rate does make a difference as we expected. We can see a crossing at the beginning, which is caused by the different rate of adjustment. We only showed the first 100 rounds to make this crossing clear enough.
Treatment L

In this graph we can see an abrupt change in at the end of a group’s learning period. This means that the difference between average return and marginal return is quite significant so that the last weight change in considering average return make such a difference. We can observe in this graph that there is tradeoff considering average return. When a group’s total contribution level is high, the average return may be much higher than the marginal return but when the total level is low, the average return may be much lower than the marginal return, which is the case here in this graph.
Treatment FLAR

The effect at the end of learning period is similar to the graph right above.

![Graph showing FLAR treatment effectiveness](image)

Treatment R

As shown in this graph, a group with higher redistribution factor can beat the other groups even if the initial position is disadvantageous.

![Graph showing R treatment effectiveness](image)
A2. Convergence across simulations

We can see that the lines approach to be level as the number of simulations increase.
Treatment AR

We only showed the first 500 simulations to illustrate the volatility.

Treatment L
Treatment FLAR
Treatment R

Note that in this treatment and the next treatment, there are five lines instead of three lines in the graphs above since there are five equilibria possible now.
Treatment FLR
Appendix. B Simulation Code

1. Treatment B

```matlab
%Basic_MC
r=[1.2,0.9,0.7];
ng=3;
nm=10;
n=300;
s=1;
N=50000;
A=zeros(3,1);
for t=1:N
    C=10*rand(ng,nm);
    for i=1:n
        TC=sum(C,2);
        R=sortedreturn(TC,r,ng);
        for g=1:ng
            for m=1:nm
                c=rand*s*(R(g)-1)+C(g,m);
                C(g,m)=min(max(c,0),10);
            end
        end
    end
    A=A+sum(C,2);
end
A=A/N/nm;
```

2. Treatment AR

```matlab
%Ajustment Rate_MC
r=[1.2,0.9,0.7];
ng=3;
nm=10;
n=300;
s=[2,1,0.5];
N=50000;
A=zeros(3,1);
for t=1:N
    C=10*rand(ng,nm);
    for i=1:n
        TC=sum(C,2);
        R=sortedreturn(TC,r,ng);
        for g=1:ng
            for m=1:nm
                c=rand*s*(R(g)-1)+C(g,m);
                C(g,m)=min(max(c,0),10);
            end
        end
    end
    A=A+sum(C,2);
end
A=A/N/nm;
```
TC = sum(C, 2);
R = sortedreturn(TC, r, ng);
for g = 1:ng
    for m = 1:nm
        c = rand * s(g) * (R(g) - 1) + C(g, m);
        C(g, m) = min(max(c, 0), 10);
    end
end
A = A + sum(C, 2);
end
A = A / N / nm;

3. Treatment L

% Learning MC
r = [1.2, 0.9, 0.7];
ng = 3;
nm = 10;
n = 300;
s = 1;
N = 50000;
nl = [20, 40, 60];
A = zeros(3, 1);
for t = 1:N
    C = 10 * rand(ng, nm);
    for i = 1:n
        TC = sum(C, 2);
        R = sortedreturn(TC, r, ng);
        for g = 1:ng
            for m = 1:nm
                k = nl(g);
                c = rand * s * ((min(i, k) / k + max(k - i, 0) / k) * TC(g) / nm / C(g, m)) * R(g) - 1 + C(g, m);
                C(g, m) = min(max(c, 0), 10);
            end
        end
    end
    A = A + sum(C, 2);
end
A = A / N / nm;

4. Treatment FLAR
% Fixed Learning+Adjustment Rate
r=[1.2,0.9,0.7];
ng=3;
nm=10;
n=300;
s=[2,1,0.5];
N=50000;
nl=30;
A=zeros(3,1);
for t=1:N
    C=10*rand(ng,nm);
    for i=1:n
        TC=sum(C,2);
        R=sortedreturn(TC,r,ng);
        for g=1:ng
            for m=1:nm
                c=rand*s(g)*((min(i,nl)/nl+max(nl-i,0)/nl*TC(g)/nm/C(g,m))*R(g)-1)+C(g,m);
                C(g,m)=min(max(c,0),10);
            end
        end
    end
    A=A+sum(C,2);
end
A=A/N/nm;

% Fixed Learning+Adjustment Rate
r=[1.2,0.9,0.7];
ng=3;
nm=10;
n=300;
s=[3,1.5,0.75];
N=50000;
nl=30;
A=zeros(3,1);
for t=1:N
    C=10*rand(ng,nm);
    for i=1:n
        TC=sum(C,2);
        R=sortedreturn(TC,r,ng);
        for g=1:ng
            for m=1:nm
                c=rand*s(g)*((min(i,nl)/nl+max(nl-i,0)/nl*TC(g)/nm/C(g,m))*R(g)-1)+C(g,m);
                C(g,m)=min(max(c,0),10);
            end
        end
    end
    A=A+sum(C,2);
end
A=A/N/nm;
A=A+sum(C,2);
end
A=A/N/nm;

5. Treatment R

r=[1.2,0.9,0.7];
a=[1/51,1/171,0];
ng=3;
nm=10;
n=500;
s=1;
N=50000;
AA=zeros(3,7);
AA(:,1)=[10,0,0]';
AA(:,2)=[0,10,0]';
AA(:,3)=[0,0,10]';
AA(:,4)=[10,10,0]';
AA(:,5)=[10,10,0]';
AA(:,6)=[0,10,10]';
AA(:,7)=[10,10,10]';
A=zeros(3,N);
B=zeros(1,7);
for t=1:N
    C=10*rand(ng,nm);
    for i=1:n
        TC=sum(C,2);
        R=sortedreturn(TC,r,ng);
        for g=1:ng
            for m=1:nm
                c=rand*s*((a(g)*nm+1-a(g))*R(g)-1)+C(g,m);
                C(g,m)=min(max(c,0),10);
            end
        end
        TC=sum(C,2);
        A(:,t)=TC;
    end
    A=A/nm;
end
for t=1:N
    TC=A(:,t);
    for k=1:7
        if sum(AA(:,k)==TC)==3
            end
        end
    end
end
A=A+sum(C,2);
end
A=A/N/nm;
B(k)=B(k)+1;
end
end
end
B=B/sum(B);

%Redistribution_MC
r=[1.2,0.9,0.7];
a=[1/51,1/111,0];
ng=3;
nm=10;
n=500;
s=1;
N=50000;
AA=zeros(3,7);
AA(:,1)=[10,0,0]’;
AA(:,2)=[0,10,0]’;
AA(:,3)=[0,0,10]’;
AA(:,4)=[10,10,0]’;
AA(:,5)=[10,0,10]’;
AA(:,6)=[0,10,10]’;
AA(:,7)=[10,10,10]’;
A=zeros(3,N);
B=zeros(1,7);
for t=1:N
    C=10*rand(ng,nm);
    for i=1:n
        TC=sum(C,2);
        R=sortedreturn(TC,r,ng);
        for g=1:ng
            for m=1:nm
                c=rand*s*((a(g)*nm+1-a(g))*R(g)-1)+C(g,m);
                C(g,m)=min(max(c,0),10);
            end
        end
        TC=sum(C,2);
        A(:,t)=TC;
    end
    A=A/nm;
    for t=1:N
        TC=A(:,t);
        for k=1:7
            if sum(AA(:,k)==TC)==3
                B(k)=B(k)+1;
            end
        end
    end
end
\begin{verbatim}
end
end
B=B/sum(B);

6. Treatment FLR

%Fixed Learning+Redistribution_MC
r=[1.2,0.9,0.7];
a=[1/51,1/171,0];
ng=3;
nm=10;
nl=20;
n=500;
s=1;
N=50000;
AA=zeros(3,7);
AA(:,1)=[10,0,0]';
AA(:,2)=[0,10,0]';
AA(:,3)=[0,0,10]';
AA(:,4)=[10,10,0]';
AA(:,5)=[10,0,10]';
AA(:,6)=[0,10,10]';
AA(:,7)=[10,10,10]';
A=zeros(3,N);
B=zeros(1,7);
for t=1:N
    C=10*rand(ng,nm);
    for i=1:n
        TC=sum(C,2);
        R=sortedreturn(TC,r,ng);
        for g=1:ng
            for m=1:nm
                c=rand*s*((min(nl,i)/nl*(a(g)*nm+1-a(g))+max(nl-i,0)/nl*(TC(g)*(1-a(g))/nm/C(g,m)+a(g)))*R(g)-1)+C(g,m);
                C(g,m)=min(max(c,0),10);
            end
        end
        TC=sum(C,2);
        A(:,t)=TC;
    end
    A=A/nm;
    for t=1:N
        TC=A(:,t);
\end{verbatim}
for k=1:7
    if sum(AA(:,k)==TC)==3
        B(k)=B(k)+1;
    end
end
end
B=B/sum(B);

%Fixed Learning+Redistribution_MC
r=[1.2,0.9,0.7];
a=[1/51,1/111,0];
ng=3;
nm=10;
nl=20;
n=500;
s=1;
N=50000;
AA=zeros(3,7);
AA(:,1)=[10,0,0]';
AA(:,2)=[0,10,0]';
AA(:,3)=[0,0,10]';
AA(:,4)=[10,10,0]';
AA(:,5)=[10,0,10]';
AA(:,6)=[0,10,10]';
AA(:,7)=[10,10,10]';
A=zeros(3,N);
B=zeros(1,7);
for t=1:N
    C=10*rand(ng,nm);
    for i=1:n
        TC=sum(C,2);
        R=sortedreturn(TC,r,ng);
        for g=1:ng
            for m=1:nl
                c=rand*s*((min(nl,i)/nl*(a(g)*nm+1-a(g))+max(nl-i,0)/nl*(TC(g)*(1-a(g))/nm/C(g,m)+a(g)))*R(g)-1)+C(g,m);
                C(g,m)=min(max(c,0),10);
            end
        end
    end
    TC=sum(C,2);
    A(:,t)=TC;
end
A=A/nm;
for t=1:N
    TC=A(:,t);
for k=1:7
    if sum(AA(:,k)==TC)==3
        B(k)=B(k)+1;
    end
end
B=B/sum(B);