On the Allocation of Public Funds*

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Abstract

This paper investigates how political incentives affect the allocation of public funds using data from Brazil’s Federal Legislature, a setting in which federal politicians representing multi-member districts can issue budgetary amendments for public works in their respective districts. We examine these allocations in the context of a Colonel Blotto model that features three key aspects of this decision. First, politicians choose the allocation of funds to maximize the share of votes they receive in future elections and therefore their chances of staying in power. Second, while politicians are affected by political incentives, they may also have altruistic motives in deciding where to allocate these public works. Third, politicians take into account the fact that their allocation decisions are likely to influence the choices of the other incumbent politicians, and vice versa. Based on our model estimates, we find that relative to a social planner, political incentives distort around 26 percent of the allocation of public funds. Simulations illustrate the tradeoff between altruistic motives and electoral returns and the extent to which alternative electoral rules can reduce political distortions. Finally, we use the model to evaluate policies aimed at changing the electoral system. We find that an approval voting system can reduce the level of distortion in the allocation of these public funds.

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1 Introduction

A central function of government is the provision of public goods. In 2014, government expenditures represented on average more than 34 percent of their country’s GDP.\(^1\) When allocated and spent efficiently, these expenditures can be important drivers of economic development and key determinants of quality of life. Unfortunately however, the allocation of the public funds can be prone to political pathologies. As an extensive theoretical literature on redistributive politics has documented, the objectives of politicians are not always aligned with those of a benevolent social planner, and this can create distortions in the way in which public funds are spent and public goods are provided.\(^2\) While few would argue against the existence of these distortions in the allocation of public goods, there is virtually no empirical evidence on the size of these distortions and how best to design policy as a redress, which is what we ultimately care about.

This paper is one of the first attempts to fill this gap, and in the process we address two questions: To what extent do electoral incentives distort the allocation of public expenditures, and to what extent do alternative electoral rules reduce this distortion. To answer these questions, we estimate a version of the classic Colonel Blotto game using data from Brazil.\(^3\) In our adapted version of the Blotto game, incumbent legislators are endowed with a fixed budget of public funds to allocate across municipalities within their district. In each municipality, voters care about the amount of public funds that they receive, and conditional on ideological considerations, vote for the incumbent who provides them with the most public funds. The incumbents, in turn, want to maximize both the sum of votes they receive across all of the municipalities, as well as aggregate welfare.\(^4\) While other studies have used versions of the Colonel Blotto game to examine theoretical issues related to campaign spending and electoral competition (e.g. Myerson (1993), Laslier (2002), Laslier and Picard (2002), and Sahuguet and

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\(^1\)This statistic comes from the 2014 Index of Economic Freedom.

\(^2\)Myerson (1993) and Lizzeri and Persico (2001) are important studies on the effects of electoral incentives on how politicians target voters. See Persson and Tabellini (2000) for a review of the literature. The empirical literature has relied mostly on cross-country evidence (e.g. Milesi-Ferretti, Perotti, and Rostagno (2002) and Persson and Tabellini (2005)). One notable is exception is Stromberg (2008), who investigates the impact of the U.S. electoral college system on how U.S. presidential candidates allocate their campaign resources across states.

\(^3\)The original Colonel Blotto game dates back to Borel (1921) and was later studied by among others, Gross and Wagner (1950), Bellman (1969), and Shubik and Weber (1981).

\(^4\)In the original Colonel Blotto game, individuals competed over battlefields. Each individual was endowed with a fixed amount of resources, and had to decide how to allocate these resources across the battlefields in order to increase their chances of winning the war.
Persico (2006)), our paper is the first to structurally estimate one, which is what allow us to quantify the extent to which electoral incentives affect public expenditures.

We estimate our model using data from Brazil’s federal legislature, which provides an ideal setting to analyze the effects of political incentives on public expenditures. Each year the Brazilian Constitution permits each federal legislator a budget of R$ 1.5 million (US $750,000) to fund public projects in the state where the legislator is elected. This constitutional provision allows us to investigate what factors influence the politicians’ allocation decisions without worrying about the endogeneity of who has access to these funds, which is an important concern in other contexts, such as in the U.S. Congress. Moreover, these budgetary appropriations, which are commonly earmarked for large-scale development projects, have important welfare considerations.

Set within this context, our model highlights four key aspects of the allocation decisions of politicians. First, the actions of a politician are likely to affect the actions of other politicians. While this tends to be the case in general, it is particularly relevant in our setting where federal legislators are members of a multi-member district. These strategic interactions are a fundamental part of the Colonel Blotto game that we consider and are therefore taken into account in the estimation. This is to our knowledge the first empirical study in political economy to explicitly incorporate these strategic interactions, and examine their effects on an incumbent’s allocation decisions.

Second, incumbents that choose not to compete for reelection at the end of a term face limited electoral incentives. As a consequence, their allocation decisions will generally differ from the decisions of incumbents who choose to run for reelection. To account for this possibility, we augment the Colonel Blotto game by modeling, in addition to the allocation decisions, the decision to compete in the next election.

Third, our model accounts for the common belief that some politicians care more for citizens’

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5The study of Brazil’s budgetary amendments has a long tradition in the comparative politics literature. Since Ames (1995) classic study, there have been a number of empirical studies investigating both the allocation of these budget appropriations, as well as their electoral returns (e.g. Samuels (2003); Pereira and Renno (2003)). The most recent example is Firpo, Ponczek, and Sanfelice (2012). The authors show that politicians tend to reward municipalities that supported them in the previous election, and that among the legislators who seek re-election, voters reciprocate by voting for the candidates who have brought more resources to their localities. Our study complements and extends this body of work. Many of these results, which we replicate in our reduced-form analysis, help to motivate our model and its underlying assumptions.

6As we discuss in the Section 2, each state in Brazil has a given number of representatives. These elected members of the federal legislature represent the entire state and not a specific district within the state, as is the case for instance in U.S. Congress.
welfare than others. We do this by allowing for the possibility that politicians have altruistic motives in addition to electoral motives and by introducing heterogeneity in their degree of altruism. Unobserved heterogeneity has been an important limitation of most reduced-form analysis, and as we later document, allowing for this form of unobserved heterogeneity is an important feature of the model.

Lastly, the degree of altruism of a politician is generally not known to voters and other politicians. We introduce this dimension of the political process by considering a model in which the degree of altruism is private information and voters and other politicians can only form beliefs about it.

We estimate the proposed model using Simulated Method of Moments (SMM). Based on our model, we estimate that about 26 percent of these public funds are distorted relative to a social planner’s allocation. This distortion is driven but the incentives of non-altruistic politicians to target regions with more votes at the cost of poorer or more productive places. We estimate that non-altruistic politicians, who care mostly about electoral incentives, represent approximately 47 percent of the pool of candidates. Altruistic politicians are at the other extreme of the altruistic range and place much more weight on the welfare of municipalities.

Consistent with the reduced-form evidence, we also find that voters are quite responsive to public expenditures, and that this responsiveness varies by regions. For instance, according to our estimated parameters, if an incumbent transfers all of his resources from the region with the smallest political gains to the one with the largest political gains his probability of getting re-elected increases by 57 percentage points.

Incumbency advantage also matters in our model. Conditional on allocation decisions, incumbent politicians have a 39 percentage point increase in the probability of re-election. While our estimates are consistent with those found in the U.S. literature, we are to our knowledge the first to estimate incumbent advantage for federal legislators in Brazil. It is also worth noting that while the incumbency advantage does make politicians less responsive to electoral incentives, for our altruistic politicians this comes as a benefit to citizens. Altruistic politicians can place more weight on aggregate welfare when deciding to allocate its funds.

Given the presence of these political pathologies, the second question we address is: how can we design policy to reduce this distortion? Electoral reform is an obvious starting point. The importance of electoral rules for the distribution of resources across voters has long been recognized by the political science and economics literature. For instance, Myerson (1993) shows how different electoral systems can cause candidates to target minorities of voters even
when voters are identical. Similarly, Lizzeri and Persico (2001) extend the Myerson model to include public goods (as opposed to just transfers). They show that compared to a system of proportional representation, a winner-take-all system will result in the under-provision of public goods even when they are highly desirable.\(^7\)

In this vein, we explore three main policy experiments. First, we consider the effects of adopting a class of score voting rules. Scoring voting gives each candidate a certain number of points corresponding to the position in which he or she is ranked by each voter. It is widely considered a preferred voting system for its degree of expressiveness. Thus, in the first set of policy experiments we simulate the effects of various 2-person voting rules, where the rules differ in the relative number of points that first and second place candidates receive. The second set of policy experiments considers different types of approval voting schemes. Here, each ranked candidate receives the same number of points, but we vary the number of candidates that a voter can rank. The third policy experiment considers an exogenous increase in the number of challengers.

Interestingly, we find that allowing voters to vote for two candidates and varying the relative points that each candidate receives reduces the distortion generated by political incentives but by a small amount. We also find similar small effects when we double the number of challengers, which is consistent with the relatively large incumbency advantage that we estimate. A much more effective policy is approval voting. As we allow for voters to vote for more and more candidates, the distortion declines monotonically. Based on our results, if citizens in Brazil were allowed to vote for 5 candidates there would be a 11 percent reduction in the level of distortion. If voters were allowed to vote for 8 candidates, which is the size of the legislature we consider, then the distortion would reduce by 14 percent.

Overall our study highlights the importance of political institutions for the allocation of public expenditures, and in particular the type of distortion that can arise when the incentives between the politician and a social planner are not aligned. But how much we can change institutions to align incentives is still an open question, and in this case our results suggest that perhaps more emphasis should be place on attracting better types of politicians (Dal Bó, Finan, and Rossi (2013); Ferraz and Finan (2009)). Finally, our approach, although molded

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\(^7\)There is a large literature in political science and to some extent economics that examines the effects of electoral rules on candidate positioning. For example, Cox (1987) is a classic study that examines the multicandidate electoral equilibria under various voting systems. Persson and Tabellini (2005) examine the effects of different political institutions, including electoral systems, on such outcomes as: economic policy, government spending, and budget deficits. The implications of different vote-counting schemes for candidate behavior is also reviewed in a 1995 JEP issue, see for example Levin and Nalebuff (1995).
to fit the Brazilian context, is nevertheless quite general and can be easily adapted to provide insights into important questions in other settings.

The paper proceeds as follows. The next section describes Brazil’s Federal legislature and provides some institutional background. Section 3 describes the data we use to estimate the model and presents the reduced-form evidence that motivates the study. The model is then presented in Section 4, followed by a discussion of our estimation approach in Section 5. In Section 6, we discuss the identification of our model’s parameters. Section 7 presents both our estimation results, as well as our policy simulations. Section 8 concludes the paper.

2 Background: Brazil’s Federal Legislature

Several features of Brazil’s political institutions facilitate our empirical analysis. This section provides some basic background on the Brazil’s federal legislature, and highlights some of its institutional features that we exploit in our analysis.

Brazil’s federal legislature (also referred to as Chamber of Deputies) consists of 513 seats allocated across 27 states according to population size. Each state represents a multimember voting district, where candidates can receive votes from any of its municipalities. As opposed to a single-member district, incumbents not only face competition from new potential challengers, but also from the other incumbents, which is likely to influence their allocation decisions.

Nationwide elections for the legislature are held on a four-year cycle and incumbents can be elected an unlimited number of times. Despite the lack of any term limits, legislative careerism is relatively absent in Brazil. For instance, the percentage of deputies who sought reelection during the 1994 and 1998 elections was 75 percent. The D’hondt open-list proportional representation method determines how many seats in a state each political party earns, while voters’ preference select the individual candidates within each party. Voting in Brazil is mandatory, and although the electorate can vote for the political party, this option is rarely exercised as elections tend to be highly individualized. This electoral system, which fosters both inter and intra-party competition, has been a source of Brazil’s weak party system (Mainwaring and Scully 1995). It is not unusual for several elected officials to change parties during their electoral terms. In the 49th parliamentary session 55 percent of the deputies switched parties during their term. With such a low degree of party loyalty both from the standpoint of the politician as well as the electorate, our empirical analysis does not focus on party politics but instead on the individual

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8We use the terms “Deputy” and “Legislator” interchangeably.
behavior of federal deputies.

Several aspects of Brazil’s political institutions promote pork-barrel politics. Brazil’s legislature is comparatively weak and seldom legislates on issues of national concern (Ames 1995). As a Federal Deputy from Ceará stated in the Brazilian newspaper, Folha de São Paulo, on February 21, 1988: “A political career in Brazil is closely connected to success in bringing home material benefits... Especially in the poorest regions, communities judge their deputies on what they bring home”; or Federal Deputy Joaquim Haickel expressed: “The primary function of a deputy is getting resources; legislating comes second.” (Mainwaring 2002). To facilitate these objectives, federal deputies have had the right to submit pork-barrel amendments to the budget since 1988. Before 1996, members of Congress were not limited in the number budgetary proposal and between 1992 and 1995 averaged close to 137 per year per member. In its current form, Brazil’s constitution allows each member of Congress discretion over 20 budgetary amendments per year, totaling to a fixed amount typically of BRL$1.5 million, although this amount can in principal vary by year.

3 Summary Statistics and Reduced-form Evidence

In this section, we present some summary statistics and reduced-form evidence that will help develop and estimate our structural model. For convenience, we present results based on the 50th legislature, which issued budgetary amendments from 1996-1999 and faced the possibility of re-election in 1998. Although for the reduced-form analysis, we restrict the sample to a single term the patterns we present here are similar when considering alternative or additional terms (see Firpo, Ponczek, and Sanfelice (2012)).

3.1 Data sources

To estimate our model, we assemble an extensive database of political and municipal characteristics for all of Brazil from the period 1996 to 2013. The data used for this study combines secondary data from three sources.

To investigate budgetary allocations, we collected budgetary amendment data from the Federal Chamber of Deputies. Information on the author’s name, the amount and type of public investment, and the recipient municipality is provided for each budgetary amendment issued from 1996 to 2013.
Using the authors’ names, these data are merged with the characteristics and electoral results of each politician. Election data are available from the Tribunal Superior Eleitoral (TSE) in electronic form. These data contain vote total for each candidate by municipality, along with various individual characteristics: including gender, education, occupation, and party affiliation. We use this information to construct our primary measure of political support – municipal vote share – as well as various other measures of electoral performance and competition, such the candidate’s rank and vote total.

Our final data source is from the Brazilian Institute of Geography and Statistics (Instituto Brasileiro de Geografia e Estatística (IBGE)). The 2000 and 2010 population censuses provide several socioeconomic and demographic characteristics such as poverty rates, income inequality, population density, share of the population that is urban, and share of the population that is literate.

3.2 Summary Statistics

Our estimation sample consists of the 648 politicians that issued a federal budgetary amendment during the 1996-1999 legislative term. The 135 non-elected deputies were originally voted as alternates, but later replaced elected deputies who were unable to fulfill his responsibilities. Inclusion of these deputies does not affect our estimation results.

Table 1 presents some basic information on the budgetary amendments issued by federal deputies in our estimation sample. On average, each year a deputy proposes 15 budgetary amendments that are approved, with an approximate value of $1.3 million reais\(^9\). Both the number and the amount decline slightly over the electoral cycle, although differences across years are not large.

The geographic variation in the distribution of these public works is considerable. More than 10 percent of municipalities did not receive a single public work during the 1996-1999 term, with the median municipality only receiving a BRL$280,000 in budgetary amendments. This stands in stark contrast with the BRL$10,000,000 that the top one percent of municipalities receive. The degree of this geographic variation can also be seen in Figure 1.

Table 2 displays summary statistics by state corresponding to the 1998 election.\(^{10}\) On aver-

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\(^9\)As mentioned previously, federal deputies were allowed up to 20 outlays totaling up to 1.5 million per year. The limits are in general reached. However in our estimation, we only consider outlays targeted to municipality and exclude the ones that are designed to benefit either the state or the country as a whole.

\(^{10}\)We excluded the district of Brasília.
age, 68 percent of deputies from the 1996-1999 term, who had issued a budgetary amendment, ran for re-election in 1998. Of those, 65 percent were re-elected. These averages however, mask considerable variation across states. In some states reelection rates are above 80 percent, while in others they are even below 50 percent. Elected federal deputies average anywhere between 5 to 21 times the number of votes that non-elected deputies receive. Federal deputies are predominately male (90 percent), and only 60 percent of them have a college degree.

3.3 Descriptive Evidence

There are several features of the data that guide our modeling choices. Our most basic assumption is that voters care about these public works and reward the politicians who supply them. This assumption seems reasonable if we simply plot the number of votes an incumbent receives in the upcoming election by the amount of public goods they had provided to the municipality in their previous term. As Figure 2 depicts, there is a strong association between the amount of public goods provided and the number of votes. The relationship also appears to be quite linear, which is consistent with politicians targeting larger projects to more populated municipalities. Consistent with this explanation, we do see some concavity when we instead plot on the y-axis the share of votes received from the municipality (see Figure 3). Targeting even larger projects may do little to increase one’s vote share within a municipality, but can increases one’s vote totals if targeted to a more populated municipality. In Figure 4, we do see that larger projects are in fact targeted to more populated places.

In Table 3, we explore the robustness of these correlations to unobserved deputy and municipal characteristics. Because each state is a multi-member district, we can estimate these correlations controlling for both deputy and municipality fixed-effects. As it turns out, controlling for both fixed characteristics of the deputy (e.g. valence, education levels) as well as fixed characteristics of the municipality (e.g. poverty levels, party affiliation) has little impact on the relationship between electoral performance in the municipality and public spending (see columns 1-4).

Another way to visualize the data, which will motivate how we model the voting rule, is to examine how a deputy’s rank within the municipality – in terms of the amount of public goods provided – relates to the vote share he or she receives. In Figure 5, we plot the coefficients from a regression of the share of votes within municipality on a set of indicators for the deputy’s rank in the municipality, where a deputy’s rank is based on the amount of public funds he
provided to the municipality. The regression also controls for municipal fixed-effects. Deputies ranked first receive vote shares that are 27 percent point higher than those that rank above 22. Moreover, the effects of rankings decrease sharply. The vote share of second-placed deputies is only 7 percentage points higher, and the vote shares of deputies ranked third and above are statistically indistinguishable. These results suggest that when casting their ballots, voters do prefer the candidate who provided them with the most public goods.

Thus far, the evidence seems to suggest that voters do care and reward politicians who supply them with more public goods, and that politicians in turn, target larger or more projects to places with more voters. But this does not suggest, however, that electoral returns are the only motives when deciding where to target these public works. Politicians may also wish to target their public works based more on the needs of the municipality, especially if they are less concerned with electoral incentives. An example of this is when we compare the distribution patterns of public goods, distinguishing between deputies who ran for re-election and those that do not. In Panel A of Figure 6, we plot the distribution of public goods by poverty levels for those who ran for re-election and those that did not. In Panel B, we plot a similar figure but instead of poverty levels use the municipality’s human development index, which is a commonly used composite index of a country’s (or in our case a municipality’s) life expectancy, educational attainment, and income level. As both graphs indicate, incumbents who do not run for re-election are on average much more likely to target poorer and less developed municipalities, relative to those with electoral concerns.

4 Model

In this section, we develop a model of politicians’ allocation decisions with three objectives in mind. First, the model should be sufficiently rich to enable us to perform interesting policy evaluations. Second, it should be able to generate the empirical patterns documented in the previous section: (i) deputies allocate resources to municipalities with high political returns, as well as to municipalities with limited political returns; (ii) some deputies choose not to run for reelection and, if they choose not to run, they allocate resources differently; and (iii) residents are more likely to vote for deputies that transferred a larger amount of resources to their municipality during the previous term. Third, we want to allow for strategic interaction among politicians to take into account the common intuition that politicians consider the choices of their rivals when deciding to which municipalities to allocate the available funds.
To achieve these goals, we consider an economy in which, in each term $t$, $J$ deputies make two choices. First, they decide how to allocate a fixed amount of resources $\bar{Q}$ among $M$ municipalities. Second, they choose whether to run for reelection at the end of the term. The amount of resources that deputy $j$ allocates to municipality $m$ will be denoted with $q^{j,m}$. Similarly, we will denote with $q^j = \{q^{j,1}, \ldots, q^{j,M}\}$ the collection of allocations chosen by deputy $j$, with $q = \{q^1, \ldots, q^J\}$ the allocations of all deputies, and with $q^{-j} = \{q^1, \ldots, q^{j-1}, q^{j+1}, \ldots, q^J\}$ the allocations of all deputies except $j$. Finally, we will use the notation $Q^m = \sum_{j=1}^J q^{j,m}$ to describe the total amount of funds received by municipality $m$. Each municipality is populated by $N_m$ individuals who have to choose for which politician to vote for at the end of the current term.

Before we can characterize the optimal decisions of deputies, we have to introduce the preferences of deputies, the preferences of voters, the electoral rule, and how each deputy interacts with his or her political rivals. This will be the subject of the next three subsections.

### 4.1 Preferences

The allocation of deputy $j$ to municipality $m$ has a positive and increasing effect on the welfare of its residents. We will assume that the welfare of an individual in a municipality depends on the total amount of resources allocated to the municipality by the $J$ deputies, $Q^m$, a set of municipality characteristics $X^m$, and a variable $K^m$ which captures every other factor that affects the residents’ welfare. This implies that the welfare of each resident of municipality $m$ can be written in the following form:

$$w_m(Q^m, X^m, K^m).$$

The total welfare in municipality $m$ can then be easily calculated by multiplying the individual welfare by the number of individuals living in the municipality:

$$W_m(Q^m, X^m, K^m) = N_m w_m(Q^m, X^m, K^m).$$

Deputies choose the allocations to the $M$ municipalities and whether to run for reelection so as to maximize their utility. Politicians have a different utility function depending on their decision to run for reelection. If they choose to run, their utility is comprised of four parts. First, they derive utility from the opportunity of being in power. Second, the deputies derive
utility from increasing the welfare of the individuals living in a municipality through their allocations. Finally, there is a utility cost of running for reelection and a preference shock.

Specifically, let $p_j$ be the probability of winning the next elections, $v^p_j$ the utility from being in power, $v^np_j$ the egoistic part of the utility if the deputy is not in power, $\bar{C}_R$ the cost of running for reelection, $\bar{\epsilon}^j$ the preference shock, and $\alpha^j$ the weight that deputy $j$ assigns to the altruistic motive. Then, for a particular allocation of resources by all deputies $q = \{q^1, \ldots, q^J\}$, politician $j$’s utility can be written in the following way:

$$U^j_R(q) = p^j(q) v^p_j + (1 - p^j(q)) v^np_j + \alpha^j \sum_m W_m \left( \sum_j q^j,m X^m, K^m \right) - \bar{C}_R + \bar{\epsilon}^j(q)$$

The first part of the utility, $p^j(q) v^p_j + (1 - p^j(q)) v^np_j$, captures the egoistic motive of politician $j$. Provided that $v^p_j \geq v^np_j$, which is the only case in which the deputy will choose to compete in the next election, it enables us to generate the documented fact that politicians tend to allocate more resources to municipalities with higher electoral returns. Observe that generally the probability of winning the election $p^j$, which will be derived in the next subsection, depends on the allocations chosen by all deputies $q$. The second part of the utility, $\alpha^j \sum_m W_m$, describes the altruistic motive of politicians and it allows us to produce the observed fact that politicians transfers part of their funds to municipalities with limited political returns.

To allow for sufficient heterogeneity, we will assume that the shock $\bar{\epsilon}^j$ is composed of two parts. The first part depends on the allocation chosen by deputy $j$ and will be denoted with $\bar{\epsilon}^j(q)$. The second part does not vary with the chosen allocation, but it is specific to the decision of running for reelection. It will be denoted with $\bar{\nu}_R$.

Using simply manipulations, deputy $j$’s utility can be written in the following alternative form:

$$\bar{U}^j_R(q) = p^j(q) (v^p_j - v^np_j) + \alpha^j \sum_m W_m - \bar{C}_R + \bar{\epsilon}^j(q) + \bar{\nu}_R,$$

where now $\bar{C}_R$ also includes the value of not being in power. The utility from being in power, $v^p_j$, and the utility from not in power, $v^np_j$, are assumed not to vary with the allocation chosen by deputy $j$. We can therefore divide the politician’s utility by $\alpha^j + v^p_j - v^np_j$ and obtain

$$U^j_R(q_1, \ldots, q_J) = (1 - \beta_j) p^j(q) + \beta^j \sum_m W_m - C_R + \bar{\epsilon}^j(q) + \nu_R.$$

This alternative formulation of the politician’s utility makes clear the trade off that the deputy
faces when choosing how to allocate his or her budget across municipalities. The politician can allocate resources in ways that increase the probability he or she will remain in power. Alternatively, resources can be allocated to increase the welfare of the state residents. This formulation also clarifies that the significance of the two components in the deputy’s decision depends on the parameter $\beta_j = \frac{\alpha_j}{\alpha_j + v_j^p - v_j^{np}}$ which can be interpreted as the degree of altruism of deputy $j$. We will refer to $\beta_j$ as $j$’s type.

If deputy $j$ decides not to run, her or his utility will not depend on the value of being in power and on the cost of running. It will therefore take the following form:

$$\bar{U}_{NR}^j(q) = v_{np}^j + \alpha_j \sum_m W_m \left( \sum_j q^{j,m}, X^m, K^m \right) + \varepsilon^j (q^j) + \nu_{NR}.$$

If we divide the utility by the same value used for a deputy who runs for reelection, $\alpha_j + v_j^p - v_j^{np}$, we have a utility function that depends on the type $\beta_j$:

$$\bar{U}_{NR}^j(q) = \bar{v}_{np}^j + \beta_j \sum_m W_m \left( \sum_j q^{j,m}, X^m, K^m \right) + \varepsilon^j (q^j) + \nu_{NR}.$$

Given this utility function, the decisions of a deputy who chooses not to run for reelection will only be affected by welfare considerations. In this case, political incentives will play no role in the decision process. This enables us to generate the documented fact that deputies who choose not to run for reelection are more likely to allocate resources to poorer municipalities with a small number of votes.

### 4.2 Residents’ Voting Decisions and Deputies’ Strategic Interactions

The previous discussion has emphasized that, if deputy $j$ chooses to run for reelection, her utility and therefore her decisions depend on the probability of winning the next election. Since our main policy simulations are based on changing the electoral system, it is important that we model it, and hence the probability of winning the election, in a realistic way.

The probability that politician $j$ wins the election is an increasing function of the total number of residents who decides to vote for $j$ in the district. Let $q^{j,m}$ be a random variable which denotes deputy $j$’s allocation to municipality $m$ in the next term in case he wins the
election. Then, resident $i$ of municipality $m$ votes based on two variables: ideology $\xi^{i,j,m}$ and the amount of resources he expects to receive during the next term by deputy $j$, $E \left( q^{j,m} \mid Z \right)$, where $Z$ is the set of variables used to form the expectation. Specifically, resident $i$ votes for deputy $j$ if

$$j = \arg\max_{j \in J} \left\{ E \left( q^{1,m} \mid Z \right) + \xi^{i,1,m}, \ldots, E \left( q^{J,m} \mid Z \right) + \xi^{i,J,m} \right\}.$$  

We now describe how these expectations are formed. Two kinds of politicians compete in an election: incumbents and politicians who run for the first time, challengers. Consider first incumbent $j$. Voters predict the amount politician $j$ will transfer in the next term using politician $j$’s current allocation $q^{j,m}$, the allocations of its opponents $q^{-j,m}$, and the deputy’s observable characteristics $X^j$.\textsuperscript{11} As a consequence,

$$E \left( q^{j,m} \mid q^{j,m}, q^{-j,m}, X^j \right) = f \left( q^{j,m}, q^{-j,m}, X^j \right).$$

In each election, there are also $J_c$ challengers who generally have no record on previous allocations to municipality $m$. To determine how voters predict the allocation that will be chosen by challengers in the future term we make the following assumption. Voters have no information on the challengers’ type and therefore assign equal probability to the event that a challenger chooses each one of the feasible allocations.

Therefore, individual $i$ in municipality $m$ votes for politician $j$, if the following condition is satisfied:

$$j = \arg\max_{j \in J} \left\{ f \left( q^{1,m}, q^{-1,m}, X^1 \right) + \xi^{i,1,m}, \ldots, f \left( q^{J,m}, q^{-J,m}, X^J \right) + \xi^{i,J,m} \right\}. \tag{1}$$

Let $d_i^j$ be a variable equal to 1 if equation (1) is satisfied, and hence individual $i$ votes for deputy $j$, and 0 otherwise. Moreover, denote with $N = \sum_{m=1}^{M} N^m$ the total number of voters in the district. The total number of votes deputy $j$ receives in the district can be computed as follows:

$$nv \left( j \right) = \sum_{i=1}^{N} d_i^j.$$

Finally, let $S$ be the number of seats available in the elections. The probability that $j$ wins the

\textsuperscript{11}The data support this assumption. Conditional on legislator fixed-effects, a region that received an amendment in the previous election is 45.8 (robust standard error = 0.011) percentage points more likely to receive another amendment in the next election.
election can be written in the following form:

\[ P_{\text{win}}(j) = P(nv(j) > nv(k)) \text{ for all } k \text{ except at most } S - 1. \]

The probability of winning the election \( P_{\text{win}}(j) \) is a complicated statistical object that has no closed-form solution for standard choices of the probability distribution for the voter’s shock \( \xi \). In the estimation of the model, we use numerical methods to compute it.

The previous discussion emphasizes that in our model the utility and therefore the choices of deputy \( j \) depend on the decisions of all other deputies. To deal with these strategic interactions, we make two assumptions. First, deputies make simultaneous decisions. Second, deputies do not know the type of other deputies’ \( \beta \)’s. They only know the distribution function \( \pi(\beta) \) from which the types are independently drawn.

Given these assumptions, when deputy \( j \) chooses the optimal allocation, he does not know and cannot calculate the optimal allocations of the other deputies. Deputy \( j \) can only compute, given the opponents’ characteristics, the probability that his political rivals will choose whether to run and one of the possible allocations. Let \( d_R^j \) be deputy \( j \)’s decision to compete in the next election. We will denote with \( \sigma(q^h, d_R^j | X^h) \) the probability that deputy \( j \) assigns to deputy \( h \) choosing whether to run and allocation \( q^h \) given characteristics \( X^h \). Under the assumptions that types are drawn independently from the same distribution, the probability that \( j \)’s rivals choose the sequence of choices \( q^{-j} = \{q^1, \ldots, q^{j-1}, q^{j+1}, \ldots, q^J\} \) and the sequence of decisions to run \( d_R^{-j} = \{d_R^1, \ldots, d_R^{j-1}, d_R^{j+1}, \ldots, d_R^J\} \) can therefore be written in the following form

\[ \sigma_{-j}(q^{-j}, d_R^{-j} | X_m) = \Pi_{h \neq j} \sigma(q^h, d_R^h | X^h). \]

### 4.3 Deputies’s Optimal Decisions

Now that preferences, the voting decision, and the strategic interactions have been outlined, we can write down the maximization problem of deputy \( j \). We will do this in two steps. Conditional on the decision to compete in the election, we first describe how the deputy chooses the optimal allocation of resources. We then determine whether it is optimal for the deputy to run for reelection.

Consider first the case in which it is optimal for politician \( j \) to run for reelection. Conditional on running, he chooses the allocation that maximizes the expected value of his utility, where the expectation is taken over the allocations of his political rivals. Specifically, he selects the
that solves the following problem:

\[
V^j_R (X_m, X, \beta^j) = \max_q \int \left[ (1 - \beta^j) p^j(q) + \beta^j \sum_{m=1}^{M} W_m \right] d\sigma_{-j}(q^{-j}, d_R^{-j}|X) - C_R + \varepsilon^j (q^j) + \nu_R \\
\text{s.t.} \sum_{m} q^{j,m} \leq \bar{Q},
\]

where \( V^j_R (X_m, X, \beta^j) \) is the value of running for election.

Consider now the case in which it is optimal for deputy \( j \) not to run for reelection. Similarly to the previous case, the optimal allocation maximizes the deputy’s expected utility. The only difference is that, conditional on not running, his utility does not depend on the political incentives. This implies that the optimal allocation solves the following problem:

\[
V^i_{NR} (X_m, X, \beta^j) = \max_q \int \left[ \bar{v}_{np}^i + \beta^j \sum_{m=1}^{M} W_m \right] d\sigma_{-j}(q^{-j}, d_R^{-j}|X) + \varepsilon^i (q^j) + \nu_{NR} \\
\text{s.t.} \sum_{m} q^{i,m} \leq \bar{Q},
\]

where \( V^j_{NR} (X_m, X, \beta^j) \) denotes the value of not running for election.

It is now straightforward to determine whether deputy \( j \) will compete in the election. He will, if the value of running is greater than the value of not running, i.e. if

\[
V^j_R (X_m, X, \beta^j) \geq V^j_{NR} (X_m, X, \beta^j).
\]

We conclude the theoretical section by outlining the timing of the game played by the politicians and by providing a definition of its equilibrium. It is straightforward to describe the timing. First, nature reveals \( \beta^j \) to the politicians, which as discussed above is private information. Then, the politicians simultaneously decide how much to invest in each municipality subject to their resource constraint and whether to run for re-election. Finally, voters cast their ballots at the polls. The equilibrium that characterizes our model is a Bayesian Nash equilibrium, which we can now define.

**Definition 1** Allocations \( q^{1*}, \ldots, q^{J*} \) and the deputies’ decisions to run for reelection \( d_R^{1*}, \ldots, d_R^{J*} \) are a Bayesian Nash equilibrium if, for each deputy \( j \), conditional on \( q^{-j*} \) and \( d_R^{-j*} \), the deci-
sions $q^i$ and $d^i_R$ maximize deputy $j$’s utility.

We will show that an equilibrium exists in the next section, after we introduce the assumptions required for the estimation of the model.

5 Econometric Implementation

In this section, we discuss the additional assumptions imposed in the estimation of the model and the estimation method. To estimate the proposed model, we need to make three sets of functional form assumptions: assumptions for the computation of the probability that deputy $j$ wins the election; assumptions on the welfare function $w_m$; and assumptions on the distributions of the shocks.

To characterize the probability of winning an election, we make the following four assumptions. First, we assume that the expected allocation function $f(q^i,m, q^{-j,m}, X^j)$ is linear in its arguments and does not depend on the opponents’ allocations $q^{-j,m}$. Second, $f(q^i,m, q^{-j,m}, X^j)$ depends on deputy $j$’s characteristics in the following simple way. To measure the significance of the incumbency effect, we allow the constant in the linear form of $f(q^i,m, q^{-j,m}, X^j)$ to vary between incumbents and challengers, but we restrict this constant to be identical within the group of incumbents and within the group of challengers. As a third assumption, we allow the coefficient on the past allocation $q^i,m$ to vary across municipalities. Therefore, conditional on $q^i,m$, voters have different expectations about future allocations depending on which municipality they live in. These three assumptions imply that $f(q^i,m, q^{-j,m}, X^j)$ takes the following form:

$$f(q^i,m, q^{-j,m}, X^j) = \gamma_{0,j} + \gamma_{1,m}q^i,m$$

Notice that each deputy’s allocation to the $M$ municipalities has to satisfy the feasibility constraint $\sum_{m=1}^{M} Q^m = \bar{Q}$. This implies that only $M - 1$ of the coefficients $\gamma_{1,m}$ can be identified. We will therefore normalize one of them. Finally, we assume that the voters’ shocks $\xi^{i,j,m}$ are drawn from a uniform distribution.

Given these assumptions, we can compute the probability that a deputy wins the election in a way that approximates the D’hondt open-list proportional representation election system.

\footnote{To capture the effect of $q^{-j,m}$, we have also tried to estimate the model with the additional term $q^{-j,m}_{\text{max}} - q^i,m$, where $q^{-j,m}_{\text{max}}$ is the maximum amount of resources allocated to region $m$ among all the opponents. Our results indicate that, once one controls for $q^i,m$, the introduction of $q^{-j,m}_{\text{max}}$ has no significant effect on the results.}
used in Brazil. Consider a particular allocation of resources by all deputies $q = \{q^1, \ldots, q^J\}$, a set of allocations voters believe will be selected in the future term by the challengers, and a particular realization of the voters’ shocks. Using equation (1), which determines the decision of each voter, we first compute the total number of votes received in the district by each deputy. We then rank the candidates starting with the one who was awarded the largest number of votes. Deputy $j$ wins the election if he is ranked in the top $S$ positions, where $S$ is the number of available seats. We repeat these steps for 2000 different choices of the voters’ shocks and compute the probability that deputy $j$ wins the election conditional on the allocations as the average over the 2000 simulations.

One potential limitation of the way we model electoral outcomes is the lack of party politics. The data, however, seem to support this omission, as approximately only 1 out of 10 legislators in a given election is elected through his party, and as we previously noted, a large number of deputies change party during a term. This is consistent with decades of research on Brazil arguing for its relatively weak party system.

We will now discuss the functional form chosen for the welfare function. Its functional form was chosen with the following objectives in mind. First, it should have some concavity to capture decreasing returns to public funds. Second, the projects financed by funds allocated by deputies are not the only public projects that affect an individual’s welfare. Transfers from other public offices and public goods generated by private organizations also enter the welfare function. Our specification must be able to account for these other sources of public projects. Finally, the welfare function must be able to account for the possibility that the allocated funds are more productive and valued more in some municipalities than in others. They therefore generate higher welfare in those municipalities. To capture these features, we employ the following welfare function:

$$w_m = \rho_m \log \left( y_m + \sum_{j=1}^{J} q_{j,m} \right).$$

where $y_m$ is per-capita income of municipality $m$. The logarithm enables us to capture decreasing returns. The municipality per-capita income $y_m$ accounts for the existence of other public projects and for the possibility that their values vary by municipality. Finally, the coefficient $\rho_m$ plays two roles. First, it allows us to introduce productivity differences across municipalities. Second, by means of that parameter, we can capture possible differences in the weights a deputy assigns to the welfare of different municipalities. We can therefore only identify the joint effect of these two features of the politicians’ decision process. This identification
result does not depend on our functional form assumption. To identify separately the effect of differences in productivity and preferences, data on productivity of projects located in different municipalities must be observed.

We now discuss one point related to the identification of the parameters of the welfare function. Notice that in principle all productivity parameters $\rho_m, m = 1, \ldots, M$, can be identified using variation in per-capita income $y^m$. In the data, there is some geographical variation in $y^m$, but it is not large. For this reason, we normalize the sum of the productivity parameters to be equal to 1.

In addition to the ideology shock to voters, which was described earlier, our model is characterized by two other shocks: the shock to the deputy’s preferences that depends on the decision to run for reelection, $\nu_R$; and the shock to the deputy’s preferences that depends on the chosen allocation, $\varepsilon_j (q^j)$. The preference shock dependent on the decision to run is drawn from a normal distribution with mean 0 and variance $\sigma_\nu$. To add flexibility to the model we allow the allocation-dependent preference shock to have a different distribution depending on whether the deputy chooses to compete in the next election. Specifically, we assume that the shock is drawn from a normal distribution with mean 0 and variance $\sigma_\varepsilon,R$ if the deputy chooses to run for reelection and $\sigma_\varepsilon,NR$ if she decides not to compete in the election.

In addition to the functional form assumptions just described, we impose other assumptions to make the estimation computationally more tractable. First, we assume that there are two types of deputies: deputies with low $\beta$, the egoistic types, and deputies with high $\beta$, the altruistic types. Second, we discretize the provision of public goods into four choices. Specifically, a deputy can choose to give 0 percent, 33.33 percent, 66.66 percent, or 100 percent of the budget to a given municipality subject to the constraint that the allocations must add up to BRL$1.5 million. Third, as it is standard in the estimation of games, we will assume that only one equilibrium is observed in the data (Draganska et al. 2008). Fourth, in the model, the probability of winning is determined under the assumption that challengers will choose each possible allocation with equal probability.

The model will be estimated for the state of Roraima. This state is comprised of 15 municipalities and is representative of a group of states that are poorer and less populated, and for which understanding how resources are allocated and how to reduce the effect of political distortions should have large and positive effects on welfare. In the estimation, we aggregate the 15 municipalities into 4 macro-regions. We do this for two reasons. The first one is that public projects assigned to one municipality are likely to benefit the surrounding municipalities.
as well. Typical examples are hospitals and schools. The aggregation of municipalities into macro-regions allows us to mitigate the effects of these spillovers. The second reason is that computationally the estimation becomes manageable. Each term Roraima elects 8 deputies to the Chamber. Therefore, with four regions and four possible choices for each region, everyone of the 8 deputies can select among 20 feasible allocations. To construct the deputy’s expected utility for one of the allocations he may choose, we have to consider all possible combinations that can be selected by the deputy’s rivals. This implies that to solve the deputy’s problem we have to consider $20^8 = 2.56e^{10}$ combinations. Even with the use of Message Passing Interface (MPI), which allows us to use simultaneously multiple processors in the estimation, and the aggregation of municipalities into macro regions, our model is computationally demanding. Without the aggregation, the estimation would be extremely difficult.

The next Proposition proves that, under the assumptions discussed in this section, the model estimated in this paper has a Bayesian Nash Equilibrium.

**Proposition 1** The model estimated in this paper has a Bayesian Nash equilibrium in mixed strategies. Moreover, for every $\epsilon > 0$, the model is consistent with a pure-strategy $\epsilon$-equilibrium.

**Proof.** In the Appendix. ■

Given our assumptions, there are 14 parameters that need to be estimated. They can be divided into five sets. The first set includes the three parameters related to the type of politician: $\beta_L$, the degree of altruism of the egoistic type, $\beta_H$, the degree of altruism of the altruistic type, and $\pi$, the fraction of altruistic types in the candidate pool. The second set is composed of the four parameters that characterize the voting decision. The first parameters is $\gamma_0$, which measures the incumbency effect. We then have the three parameters that determine the effect of the transfers on voters’ decisions. Specifically, without loss of generality, we normalize the coefficient for the region that experiences the lowest share of transfers, $\gamma_{1,3}$, to 0.01 and estimate $\gamma_{1,1}$, $\gamma_{1,2}$, and $\gamma_{1,4}$. The third set comprises the 3 parameters of the welfare function, $\rho_2$, $\rho_3$, and $\rho_4$ which measure the productivity of the funds in those regions relative to region 1, whose coefficient is normalized to be equal to 1. The fourth set includes the cost of competing in the elections, $\nu$, and the variances of the preference shocks $\sigma_\nu$, $\sigma_{\epsilon,R}$, and $\sigma_{\epsilon,\bar{R}}$.

The parameters of interest are estimating using the SMM and data on allocation choices by deputies, on decisions on whether to run for reelection, and on electoral outcomes. Specifically, for one set of parameters we consider an initial set of beliefs and simulate the deputies’ decisions. Given the deputies’ decisions, we compute the beliefs generated by the model and compare them
with the initial beliefs. If the distance between the two sets of beliefs is large, we simulate the model one more time using as initial beliefs the beliefs generated by the model. If the distance is sufficiently small, we compute the simulated moments used in the estimation and compare them with the corresponding data moments. We will discuss the choice of the moments used in the estimation in the next subsection where we describe the variation in the data required to identify the different parameters. The standard errors are computed using the asymptotic distribution of the estimated parameters.

6 Identification Discussion and Moments Selection

In this section, we discuss identification of our model’s parameters, and provide a heuristic argument for how they are being identified by the data. Given the model’s complexity, it is not possible to provide a rigorous proof of identification.

Welfare Parameters: Identification Discussion. Three parameters describe the welfare function: $\rho_2$, $\rho_3$, and $\rho_4$. In our model, deputies who choose not to run for reelection allocate resources based only on welfare considerations. Thus, we can use the allocation decisions of deputies who do not run for reelection to help identify these parameters of the welfare function. Specifically, the difference between the average allocation to region $i$ and the average allocation to region 1 helps us to identify the value of $\rho_i$, for each $i$.

Our assumption that deputies who do not run for reelection only care about welfare is clearly a simplifying assumption: Deputies who do not run for reelection, may choose to run for a different elected position, in which case they may still have electoral motives when allocating their public funds. Theoretically, one could relax this assumption by simply allowing for different altruism parameters $\beta^j$ among deputies who run for reelection and those that don’t. This formulation is isomorphic to allowing deputies who do not run for reelection to also care about the possibility of being elected to a different office. Of course, the main limitation of this modeling choice is that the source of identification of our welfare parameters then becomes less clear.

In addition, empirically, the data appear to support our simplifying assumption. Among the deputies from the state of Roraima who did not run for reelection, over 65 percent did not run for an elected office. Thus, the electoral motives for these deputies are likely to be nonexistent, or at best minimal. Of the remaining 35 percent, 5 percent ran for the Senate or the State Legislature, whereas the rest sought an elected office based in the capital city,
such as vice mayor or vice governor. The electoral motives for these politicians are likely to be stronger, and given their career choices they have an incentive to target region 1, which is where the capital city resides. In this case, the estimated size of the distortion will represent a lower bound.

Another possible threat to our identifying assumption is that deputies who choose not to run for reelection allocate more resources to their hometown for personal reasons such as funding projects that will benefit themselves and their neighbors. In the state of Roraima, all the deputies come from region 1. The estimation will therefore interpret these allocations as electoral distortion. Once again, this implies that the estimated size of our distortion will be a lower bound.

Welfare Parameters: Moment Selection. To estimate the welfare parameters $\rho_1$, $\rho_2$, $\rho_3$, and $\rho_4$, we follow the previous discussion and use as moments the average share of resources allocated to regions 1, 2, and 3 by incumbents who choose not to participate in the elections. The corresponding share for region 4 is equal to one minus the share for the other regions. It is not, therefore, an independent moment. Since, the sum of the welfare parameters is normalized to be equal to 1, the three moments provide the necessary variation to identify the three welfare parameters.

Altruism Parameters: Identification Discussion. If the parameters of the welfare function are known, the altruism parameters $\beta_L$ and $\beta_H$ and the probability that a deputy has low altruism $\pi$ can also be identified. To see this, consider first the case in which there is only one type of deputy. In this case, the difference in allocations between deputies who choose to run for reelection and deputies who choose not to run identifies the parameter $\beta$. If there is no difference, the degree of altruism $\beta$ will be identified to be 1. If deputies who compete in the election transfer a larger fraction of resources to municipalities with higher political gains, $\beta$ will be identified to be lower than 1, with a coefficient that will be closer to 0 the larger the difference. The generalization to two types allows us to consider situations in the data in which there are two distinct groups of deputies who run for reelection. The first group includes deputies who allocate a much larger fraction of their funds to regions with high political gains compared to deputies who do not run for reelection. The second group is comprised of deputies whose allocations differ less from the allocations of politicians who do not compete in the election. If in the data there is only one group, $\beta_L$ and $\beta_H$ will be estimated to be statistically equal. We experimented with more than two types, but the data do not support this extension.

Altruism Parameters: Moment Selection. The previous discussion implies that the
three altruism parameters can be estimated using information on the differences in allocations
between incumbents who choose to compete in the elections and incumbents who choose not to
compete. For this reason, we add to the four moments used to identify the welfare parameters,
the average share of resources allocated to regions 1, 2, and 3 by incumbents who decide to run
for reelection.

Voting Function Parameters: Identification Discussion. The parameters of the voting
function can be identified using data on the probability that a deputy is elected. Specifically,
the incumbency effect, $\gamma_0$, can be identified by comparing the average probability with which an
incumbent is elected versus the average probability for a challenger. To understand what type
of variation is needed to identify the parameters that measure the influence of public funds
on the number of votes, $\gamma_{1,m}$, $m = 1, \ldots, 4$, notice that each deputy allocation of resources
must satisfy the budget constraint: the amount allocated to different regions must add up to
1.5 million Reals. The parameter $\gamma_{1,m}$ can therefore be identified by measuring the effect of
reallocating to municipality $m$ a small amount of the funds given by a deputy to all other
municipalities on the deputy’s probability of reelection.

Voting Function Parameters: Moment Selection. To capture the insight provided
in the previous paragraph, to estimate the voting function parameters we add to the set of
moments (i) the difference between the average probability with which an incumbent is elected
and the same probability for a challenger and (ii) the average probability with which an incum-
бent is reelected conditional an transferring to municipality $m$ at least $2/3$ of the resources for
$m = 1, \ldots, 4$. Since one of the $\gamma_{1,m}$ is normalized, we have four parameters and five moments.

Cost of Running Parameter and Variances: Identification Discussion. The last
four parameters of the model are the cost of running, $\nu$, the variance of shocks to the decision to
run $\sigma_\nu$, and the variances of the preference shocks $\sigma_{\varepsilon,R}$ and $\sigma_{\varepsilon,NR}$. The probability with which
a deputy chooses to run for reelection helps us to identify the value of the cost of running.
The identification of the $\sigma_\nu$ is based on the following idea. If the variance of the shock to the
decision to participate in the elections is negligible, only incumbents for which the benefits
of being reelected outweigh the cost of participation will chose to run in the elections. This
group is composed of deputies with a probability of being reelected that is sufficiently large:
if the probability is low, it is not worth paying the cost of participation. Incumbents with a
sufficiently large probability of reelection are the deputies that allocated most of the resources
to the region with the largest electoral gains. All this implies that, if $\sigma_\nu$ is low, the probability
of running for reelection conditional on having allocated a large fraction of the budget to the
region with the largest electoral gains will be high and the corresponding probability conditional on large transfers to the region with the smallest electoral gains will be low. Using the same line of reasoning, if $\sigma_\nu$ is high, the decision to run is mostly based on individual shocks. As a consequence, the probability of participating in the elections conditional on transferring a large fraction of resources to the region with high political gains and the same probability conditional on a large allocation to the region with low political gains will be similar. We can therefore identify $\sigma_\nu$ using the difference between those two probabilities. Finally, $\sigma_\varepsilon,R$ and $\sigma_\varepsilon,NR$ can be identified using the variance of the allocations chosen by deputies who ran for reelection, and similarly $\sigma_\varepsilon,NR$ can be identified based on the variance for deputies who decided not to compete in the elections.

Cost of Running Parameter and Variances: Moment Selection. Following the previous discussion, the cost of running parameter and the variance of the shock to the decision to run are estimated by using the probability with which an incumbent chooses to participate in the elections and the difference between the probability of participating in the elections conditional on transferring a large fraction of resources to the region with high political gains and the same probability conditional on a large allocation to the region with low political gains. The variances of the preference shocks are estimated by using the following two moments: the variance across regions in the allocation of resources conditional on running and the same variance conditional on not running.

7 Results

In this section, we present the estimates of our structural model and our policy simulations. As mentioned above, the model is estimated for the state of Roraima, which is among the poorest of states in Brazil. Fifty-six percent of households live below the poverty line, and average per-capita income is only R$133 per month. A reduction in the distortionary effects of the electoral incentives could therefore have larger effects.

The state is comprised on 15 municipalities, where the average population is around 18,000 inhabitants and the average municipal budget is R$5.5 million. Residents and therefore votes are geographically skewed with 71,691 individuals living in the capital city and 2,826 people living on average in the other municipalities.

Elections in the state of Roraima are highly competitive. In 1998, for instance, 42 candidates competed in the average municipality with an average vote share of 8%, a minimum vote share
of 0%, and a maximum vote share of 29%. The average difference in votes between the top two candidates in a municipality was 6%. The characteristics of the politicians running for election in Roraima are similar to the national average. For example, in 1998 in Roraima 100% of politicians were males and 75% had a college degree, whereas at the national average 95% of politicians were males and 78% had a college degree.

7.1 Parameter Estimates

The parameters estimated are presented in Table 4. Two types of candidates characterize the data: an egoist type who places little weight on aggregate welfare ($\beta = 0.282$) and an altruist type who weights almost exclusively aggregate welfare ($\beta = 0.998$). The egoistic types represent about 47 percent of the candidate pool. To put these numbers into context, we measure the deputies’ willingness to substitute welfare considerations for political returns by computing the corresponding marginal rate of substitution. Given our estimates of the $\beta$ parameters, we find that the two types are at the opposite extremes of the altruistic range. The egoistic types are virtually unwilling to substitute political returns for welfare improvements, whereas the altruistic types must be rewarded with the almost certainty of being reelected to give up negligible welfare gains.

A direct interpretation of the coefficients describing the voting function is difficult for two reasons. First, because of the budget constraint, a deputy can increase the allocation to a region only by reducing the transfer to other regions. Second, the reallocation of resources from one region to another triggers general equilibrium responses by other deputies. We therefore provide an economic interpretation of these parameters by calculating how the probability of getting re-elected changes as an incumbent shifts his resources out of one region and into another. We find that if an incumbent spends all of his funds in region 2 as opposed to region 3, his probability of getting re-elected increases by 28 percentage points, compared to an increase of only 12 percentage points if he were to target only region 4. Shifting one’s resources from region 3 to region 1 raises one’s probability of re-election by 57 percentage points. We can therefore conclude that region 1 is by far the most attractive region in terms of electoral returns, followed by region 2, and then by region 4. Region 3 is clearly at the bottom of this ranking.

The constant in the voting function $\gamma_0$ provides a value for the incumbency advantage. To provide a measure of its economic meaning we compare the average probability with which an incumbent wins the elections for the base case with the corresponding average probability
when the parameter \( \gamma_0 \) is set to 0, where the average is taken over all possible allocations. Our results indicate that the incumbency effect is substantial. The average probability of winning the elections increases from 26 percent without incumbency advantage to 65 percentage with the advantage, an increase of 39 percentage points. Although numerous studies have measured incumbency advantage in other settings (e.g. Gelman and King (1990), Ansolabehere and Snyder (2002)) and have estimated similar results to ours, we are not aware of any other study that have done so for federal legislators in Brazil.\(^{13}\)

Four parameters characterize the welfare function. These coefficients explain why regions with few votes and hence limited political gains receive a significant share of resources. To understand our estimation results, it is important to remember that deputies base their choice of where to allocate their budget on the number of votes they can receive in exchange for the transfer and on the amount of welfare they can generate with the transfer. If political incentives dominate the deputies’ decisions, then we would observe the majority of resources being allocated to region 1, which contains the capital of Roraima and is the richest and most populated of the four geographical areas. Instead, we observe a significant fraction of the allocations given to poor and smaller regions with limited political gains, suggesting that welfare considerations are important. However, for the welfare considerations to explain the transfers to less populated areas, it must be that resources transferred to less populated regions generate a larger increase in welfare than if the same resources were transferred to region 1. In our model this is the case if \( \rho_1 \), the coefficient on the logarithm of the total amount of resources allocated to the region with the largest political gains, is smaller than \( \rho_m \), the corresponding coefficient for the other regions. If this condition is satisfied, then deputies have limited incentives to transfer funds to region 1 for welfare reasons because the resources will not be as productive as in the other regions. Our estimates of the \( \rho \)'s suggest the welfare returns to an extra dollar of public funds are much higher in regions 2, 3 and 4 relative to region 1. The welfare coefficients therefore help explain why in the data region 4 despite its low population count receives close to 30 percent of the funds (as we depict below). To provide a quantitative measure of the importance of the welfare parameters, we have computed the change in welfare if one Real is reallocated from region 1 to one of the other three regions. In all three cases, the reallocation has a positive effect on welfare with the largest effect being for region 4, followed by region 2, and region 3. Specifically, the welfare effect of the reallocation is 5.5 times larger for region 4.

\(^{13}\)Titimnik (2009) uses a regression discontinuity approach to estimate the incumbency advantage of Brazilian mayors. She finds that mayors in Brazil suffer from a negative incumbency advantage.
than for region 2 and 5.1 times larger for region 4 than for region 3.

The cost of running parameter and the variances are all estimated as a fraction of the value of running to simplify their interpretation. As discussed in the model section, the cost of running parameter includes the real cost of running and utility value of not running for reelection. Our estimation results suggest that the utility value of not running for reelection is larger than the cost of participating in the next elections. The variance of the shock to the decision to run is estimated to be about 53 percent of the utility value of deputy that chooses to participate in the elections. The variance of the preference shocks conditional on running and not running for reelection are relatively small and estimated to be equal to, respectively, 0.0097 and 0.0098 percent of the utility value of running and not running for reelection.

7.2 Insights from the model

To investigate the implications of the model, we begin by simulating each deputy’s allocation decisions given our parameter estimates. From these allocation choices, we can compute the share of funds each region receives. We plot the average (across simulations) of these shares in Figure 7, along with the shares corresponding to the actual data. On average, deputies allocate most of their funds to region 1 and region 4, with region 1 receiving 26.3 percent of their funds and region 4 receiving 32.1 percent of the funds. Our model, although parsimonious, matches the data quite well. In fact, the biggest difference between our model’s predictions and the data occurs in region 2, where our model under-predicts the share by only 2.5 percentage points. Consequently, we slightly over-predict the allocations to regions 1 and 3, while in region 2 we match the data exactly.

In Figure 8, we plot the allocation of public funds distinguishing between incumbents who run and don’t run. As the figures depict, the model also fits these patterns well, with again the biggest difference occurring in region 2. Overall, this is an important test of our model since as we discussed above, the variation in allocations among politicians who do not run for re-election identifies our social welfare function. It is also evident from the figure that those who chose not run behave quite differently than those who do. Among those who do not seek re-election, region 4 receives 40 percent of the public funds and region 1 only 11 percent. In contrast, politicians with re-election incentives allocate 32 percent to region 1 and 29 percent to region 4. The fact that our model can capture these stark differences in the allocation of public funds across these two types of politician is quite reassuring.
In Figure 9 we examine the allocations of incumbents who won reelection, and those who lost their reelection bid. The incumbents who won reelection targeted region 1 the most (with 40 percent of their funds), and region 3 the least (with less than 15 percent of their funds). As the figure in Panel A depicts, our model is able to match these patterns quite well.

In Panel B of Figure 9, we see that incumbents who sought reelection but lost, spent only 15 percent of their funds in region 1, compared to almost 37 percent of their funds to region 4. The fraction given by these deputies to regions 2 and 3 is 22 and 25 percent, respectively. The allocation to region 2 of deputies who run for reelection is lower than the 27 percent observed in the data. This is the moment the model has more difficulty matching. To match this moment better, the model would have to reduce the coefficient in the electoral rule for region 2. But this change would generate a decline in the share of resources allocated by deputies who seek reelection to region 2, which is already lower than in the data. Notwithstanding this last moment, our model, even in its parsimony, does remarkably well in matching several key features of the data.

One advantage of our approach is that we can investigate how unobserved heterogeneity affects the allocation public funds. In Figure 16, we recompute our expenditure shares distinguishing between our two types of politicians. The differences across politicians are quite stark: deputies with low levels of altruism allocate close to 55 percent of their funds to region 1, where the electoral incentives are the strongest. Alternatively, altruistic deputies target region 4 relatively more, where the returns to an extra dollar are highest. Altruistic deputies also give a significant share of their funds to regions 2 and 3, which are the poorest regions in the state.

Having estimated our model, we can address the first aim of the paper: To what extent do political incentives affect the allocation of public funds. To address this question, we compare the distribution of public funds generated by the model to a social planner allocation. In Figure 17, we plot the allocation of public funds relative to the social planner allocation. Electoral incentives distort 26% of the public funds, with all of the deviation occurring towards region 1, and away from less populated and poorer regions. Given our estimates of the welfare function, aggregate welfare is maximized by allocating relatively more funds to regions 2, 3, and particularly region 4, which is according to our estimates the most productive region. The political distortions arise because politicians cannot afford to ignore region 1, which is the region with the most votes. Given a large fraction of the public funds are being allocated based on electoral motives, the question remains as to whether we can change the electoral rules to reduce these distortions. This leads us to our second aim of the paper.
7.3 Policy Evaluations

In this section, we investigate how different electoral rules will affect the distribution of these public funds. We focus on an important class of voting rules called rank-score voting. Rank-score voting allows voters to rank all or a subset of the candidates, where the ranking is associated with a given number of points. Specially, let $J$ be the set of candidates, $J = \{x_1, \ldots, x_J\}$. Then a scoring rule is defined by a vector of scores $(s_1, \ldots, s_J)$, with $s_1 \geq \ldots \geq s_J$ and $s_1 > s_J$, where for each voter’s ranking, $s_1$ points is assigned to the top ranked alternative, $s_2$ points is assigned to the second-ranked alternative, and so forth. The $n$ seats allocated in the election are won by the politicians with the $n$ highest total number of points.

Given the current system in Brazil, voters can only vote for a single candidate, which corresponds to the following scoring rule $(1,0,\ldots,0)$ and is commonly referred to as plurality voting. A natural alternative to this current system would be to allow voters to rank a subset of the candidates, which scholars often argue is a much more expressive form of voting, and encourages voters to vote sincerely. In particular we consider the following set of scoring rules:

**2-person scoring rules** $\{(0.85, 0.15), (0.75, 0.25), (0.65, 0.35), (0.65, 0.35)\}$

**N-person scoring rules** $\{(1,1), (1,1,1), (1,1,1,1), (1,1,1,1,1), (1,1,1,1,1,1,1,1)\}$

In order to simulate the effects of these alternative voting rules on the distribution of public funds, we need to recalculate the set of beliefs deputy have since the ones estimated using the observed allocations correspond to the equilibrium that characterizes the data. In each counterfactual, we use the estimated parameters and find the deputies’ equilibrium beliefs that match these parameters using fixed point iterations.

Figure 18 presents the results from simulating the 2-person scoring rules, along with the original results (the base case). As before, we plot the allocation of public funds across each region relative to the allocation that maximizes aggregate municipal welfare. Based on these simulations, we find that the impact of 2-person scoring rules on the allocation of these public funds is small. As the weights placed on the top two candidates becomes more even, politicians have less of an incentive to target region 1, which is the region that offers the highest electoral returns. But given the parameters of the model, we find that in practice the change in incentives are small, hence the limited reduction in distortions.

But as we see from Figure 19, our N-person voting rules provide larger changes in electoral incentives. As we move from a two-person approval voting rule to a 8-person rule, the level
of distortion reduces monotonically. Under the 5-person rule, the level of misallocation is 23.2 percent compared to 26.0 percent under the 2-person rule. This represents an 11 percent reduction in the level of misallocation. When we move to an 8-person rule the level of distortion reduces even further to 22.5 percent, which represents a 14 percent reduction.

The reason why these N-person rules are so effective is straightforward. Because a majority of the voters reside in region 1 and the winner-take-all nature of the original voting rule, politicians have a strong political incentive to target region 1. But as citizens can vote for more and more candidates, the necessity to come in first diminishes. As a result, politicians can afford to target region 1 less and put relatively more weight on welfare considerations. In sum, a policy increasing the number of candidates a citizen can vote for can be an effective approach to reducing the political distortion in the allocation of public funds.

Electoral reform is one practical approach to changing the incentives politicians face. There are, of course, other types of policies aimed at fostering political competition that may also affect how politicians allocate resources. Although we did not design our model to examine one these specific types of policies, we can nevertheless explore the effects of increasing the number of challengers that incumbents face on the allocation of these public funds.

In Figure 20, we plot the results of a policy simulation in which we doubled the number of challengers to 40 candidates. As we see from the figure, the effects of these types of policies on how incumbents distribute their funds are minimal. The intuition for this is simple. Incumbents have an incumbency advantage. So even though incumbents are receiving fewer voters as a result of the increased political competition, their probability of getting re-elected still remains quite high and thus their incentives to adjust their allocations are weak.

8 Conclusions

A central question in redistributive politics is how do politicians target public funds. In this paper, we present a novel approach to the empirical analysis of how politicians allocate public funds in an environment in which other politicians are behaving strategically. This approach allows us, among other things, to compute the extent to which political incentives distort the allocation of these public funds from a social planner’s problem, and to investigate the effects of electoral rules and political competition in exacerbating this distortion.

14Political reservations and quotas are examples of policies that lower the entry costs for certain under-represented groups, and in process foster more political competition.
Using data from Brazil’s federal legislature, we find that at least 26 percent of these budget amendments are distorted from a social planner’s allocation. This distortion is driven by the behavior of non-altruistic politicians, who put little weight on the welfare of the municipalities. According to our estimates, non-altruistic politicians represent 47 percent of the candidate pool. We also explore the effects of score voting, a widely endorsed form of voting, on the allocation of public goods. We find that the allocation of public goods becomes less distorted since it reduces the incentives politicians have to target more populous regions.

Although our model fits the data well, it is quite parsimonious and can be extended and generalized in several directions that represent exciting possibilities of future research. One possible extension would be to make the game dynamic. As Diermeier, Keane, and Merlo (2005) correctly emphasize, politicians are forward-looking agents who career choices are dynamic in nature. Although our model in some respects captures this behavior in the decision to run for reelection, it would be interesting to model these decisions more explicitly, such as the decision to enter higher offices. Another extension would be to add political parties into the model. Although we do not think that this is an important feature of Brazilian politics, one could potentially exploit a nested structure to extend the model in this direction. Other directions of future research will ultimately depend on the collection of new data. For instance, with data on campaign spending, one could easily extend our model to examine whether budgetary amendment complement or substitute campaigning. One could then investigate the impact of campaigning financing laws on not only electoral performance but also public goods allocation.
References


9 Tables and Figures

Table 1: Budgetary Amendments

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>Average per Deputy</th>
<th>Average Amount per Deputy</th>
<th>Standard Deviation</th>
</tr>
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<tbody>
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<td>1996</td>
<td>3859</td>
<td>16.82</td>
<td>1,370,380</td>
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<td>1,335,346</td>
<td>200,642.7</td>
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<tr>
<td>1998</td>
<td>6106</td>
<td>14.35</td>
<td>1,300,812</td>
<td>194,080.3</td>
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<tr>
<td>1999</td>
<td>5275</td>
<td>13.28</td>
<td>1,266,827</td>
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<tr>
<td>Total</td>
<td>20195</td>
<td>15.12</td>
<td>1,313,702</td>
<td>253048.100</td>
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Notes: These data are available: http://www2.camara.leg.br/
Table 2: Summary Statistics the 1998 Elections for Federal Deputies

<table>
<thead>
<tr>
<th>State</th>
<th>Number of legislators</th>
<th>Number of Candidates</th>
<th>Number of Parties</th>
<th>Reelection Rates</th>
<th>Share ran for reelection</th>
<th>Average Votes Among Non-elected</th>
<th>Average Votes Among Elected</th>
<th>Share male</th>
<th>Share college degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acre</td>
<td>8</td>
<td>51</td>
<td>13</td>
<td>0.33</td>
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<td>11,185</td>
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<td>20</td>
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<td>49,648</td>
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<td>0.58</td>
<td>0.57</td>
<td>6,538</td>
<td>62,900</td>
<td>0.90</td>
<td>0.54</td>
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<td>8</td>
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<td>17</td>
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<td>0.38</td>
<td>1,690</td>
<td>8,180</td>
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<td>0.52</td>
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<td>8,293</td>
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<td>0.55</td>
</tr>
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<td>Paraíba</td>
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<td>0.64</td>
<td>8,024</td>
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<td>0.65</td>
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<td>62</td>
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<td>3,223</td>
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<td>70,444</td>
<td>0.95</td>
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<tr>
<td>Rio de Janeiro</td>
<td>46</td>
<td>434</td>
<td>29</td>
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<td>0.59</td>
<td>5,600</td>
<td>78,009</td>
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<td>0.57</td>
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<tr>
<td>Rio Grande do Norte</td>
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<td>47</td>
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<td>0.64</td>
<td>0.71</td>
<td>6,783</td>
<td>81,350</td>
<td>0.89</td>
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<td>Rondônia e</td>
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<td>11,934</td>
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<td>0.68</td>
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<td>17</td>
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<td>12,724</td>
<td>62,713</td>
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<td>Sergipe</td>
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<td>0.45</td>
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<td>0.96</td>
<td>0.42</td>
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<td>São Paulo</td>
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<td>0.77</td>
<td>0.59</td>
<td>8,885</td>
<td>105,326</td>
<td>0.90</td>
<td>0.60</td>
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<tr>
<td>Tocantins</td>
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<td>0.75</td>
<td>3,878</td>
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<td>0.61</td>
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<td>0.65</td>
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<td>0.59</td>
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Notes: Data are available: http://www.tse.jus.br/
### Table 3: Relationship between Electoral Performance and Allocation of Budget Amendments

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<thead>
<tr>
<th>Dependent variable</th>
<th>Number of votes per municipality</th>
<th>Vote share per municipality</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Amount of the public work ($100,000s)</td>
<td>677.491</td>
<td>679.28</td>
</tr>
<tr>
<td>Rank within the municipality</td>
<td>-452.763</td>
<td>-0.027</td>
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<tr>
<td>Municipal intercepts</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>Deputy intercepts</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>Observations</td>
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<td>154,139</td>
</tr>
<tr>
<td>R-squared</td>
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<td>0.47</td>
</tr>
</tbody>
</table>

Notes: Column 1 reports the unadjusted correlation between the number of votes a deputy received and the amount of his budgetary amendment approved in the municipality. Column 2 reports the same relationship as in Column 1 but adjusts for both deputy and municipal fixed-effects. Column 3 reports the relationship between the number of votes a deputy received and his ranking in the municipality with respect to the amount of public goods he provided in his budgetary amendment. Columns 4-6 replicate the regressions in columns 1-3 but use the deputy’s vote share in the municipality as the dependent variable. The estimation has been restricted to only those incumbents that ran for re-election. Robust standard errors in brackets.
Table 4: Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std Err</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Altruism</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proportion of Altruistic Deputies</td>
<td>$\pi$</td>
<td>0.531</td>
</tr>
<tr>
<td>Altruistic</td>
<td>$\beta_H$</td>
<td>0.998</td>
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<tr>
<td>Non-altruistic</td>
<td>$\beta_L$</td>
<td>0.282</td>
</tr>
<tr>
<td><strong>Voting Function</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public funds on vote shares, region 1</td>
<td>$\gamma_{1,1}$</td>
<td>0.0044</td>
</tr>
<tr>
<td>Public funds on vote shares, region 2</td>
<td>$\gamma_{1,2}$</td>
<td>0.0100</td>
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<td>Public funds on vote shares, region 3</td>
<td>$\gamma_{1,3}$</td>
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<td>Public funds on vote shares, region 4</td>
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<tr>
<td>Incumbency Advantage</td>
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<td><strong>Welfare</strong></td>
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<td></td>
</tr>
<tr>
<td>Productivity in region 1</td>
<td>$\rho_1$</td>
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</tr>
<tr>
<td>Productivity in region 2</td>
<td>$\rho_2$</td>
<td>0.203</td>
</tr>
<tr>
<td>Productivity in region 3</td>
<td>$\rho_3$</td>
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<tr>
<td>Productivity in region 4</td>
<td>$\rho_4$</td>
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<td><strong>Decision to Run and Shocks</strong></td>
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<td></td>
</tr>
<tr>
<td>Cost of running</td>
<td>$\nu_L$</td>
<td>-0.267</td>
</tr>
<tr>
<td>Var. of running preference shocks</td>
<td>$\sigma_\nu$</td>
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<tr>
<td>Var. of allocation preference shocks if running</td>
<td>$\sigma_{\epsilon,R}$</td>
<td>0.0097</td>
</tr>
<tr>
<td>Var. of allocation preference shocks if not running</td>
<td>$\sigma_{\epsilon,NR}$</td>
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Table 5: Moments Used in the Estimation

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<th>Moments</th>
<th>Model</th>
<th>Data</th>
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<tr>
<td><strong>Welfare</strong></td>
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<td></td>
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<tr>
<td>Average share allocated if not running for reelection, region 1</td>
<td>0.115</td>
<td>0.104</td>
</tr>
<tr>
<td>Average share allocated if not running for reelection, region 2</td>
<td>0.230</td>
<td>0.229</td>
</tr>
<tr>
<td>Average share allocated if not running for reelection, region 3</td>
<td>0.256</td>
<td>0.250</td>
</tr>
<tr>
<td>Average share allocated if not running for reelection, region 4</td>
<td>0.398</td>
<td>0.417</td>
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<tr>
<td><strong>Altruism</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average share allocated if running for reelection, region 1</td>
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<td>0.300</td>
</tr>
<tr>
<td>Average share allocated if running for reelection, region 2</td>
<td>0.207</td>
<td>0.242</td>
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<tr>
<td>Average share allocated if running for reelection, region 3</td>
<td>0.180</td>
<td>0.167</td>
</tr>
<tr>
<td>Average share allocated if running for reelection, region 4</td>
<td>0.290</td>
<td>0.292</td>
</tr>
<tr>
<td><strong>Voting Function</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference in prob. of election bet. incumbents and challengers</td>
<td>0.503</td>
<td>0.508</td>
</tr>
<tr>
<td>Prob. of winning elections if at least 2/3 of budget given to region 1</td>
<td>0.901</td>
<td>0.857</td>
</tr>
<tr>
<td>Prob. of winning elections if at least 2/3 of budget given to region 2</td>
<td>0.655</td>
<td>0.625</td>
</tr>
<tr>
<td>Prob. of winning elections if at least 2/3 of budget given to region 3</td>
<td>0.488</td>
<td>0.500</td>
</tr>
<tr>
<td>Prob. of winning elections if at least 2/3 of budget given to region 4</td>
<td>0.561</td>
<td>0.546</td>
</tr>
<tr>
<td>Average share allocated if running for reelection and losing, region 1</td>
<td>0.157</td>
<td>0.118</td>
</tr>
<tr>
<td>Average share allocated if running for reelection and losing, region 2</td>
<td>0.219</td>
<td>0.275</td>
</tr>
<tr>
<td>Average share allocated if running for reelection and losing, region 3</td>
<td>0.250</td>
<td>0.235</td>
</tr>
<tr>
<td>Average share allocated if running for reelection and losing, region 4</td>
<td>0.374</td>
<td>0.373</td>
</tr>
<tr>
<td><strong>Cost of Running and Variances</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of participating in the elections</td>
<td>0.708</td>
<td>0.714</td>
</tr>
<tr>
<td>Probability of participating if at least 2/3 given to region 1 minus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of participating if at least 2/3 given to region 3</td>
<td>0.374</td>
<td>0.373</td>
</tr>
<tr>
<td>Variance of allocations across regions if running for reelection</td>
<td>0.0026</td>
<td>0.0028</td>
</tr>
<tr>
<td>Variance of allocations across regions if not running for reelection</td>
<td>0.0089</td>
<td>0.0124</td>
</tr>
</tbody>
</table>
Figure 1: Distribution of Budgetary Amendments Per Capita

Notes: The map depicts the distribution of public funds per capita during the 1996-1999 term by municipality.
Figure 2: Budgetary Amendments and Electoral Performance

Notes:
Figure 3: Budgetary Amendments and Vote Share

Notes:
**Figure 4**: Distribution of Budgetary Amendments By Population

Notes:
Figure 5: Vote Share and Rank of Budgetary Amendment Amount

Notes:
Notes: The figure in Panel A depicts kernel density plots of the allocation public funds by poverty level of the municipality. The density plots are estimated separately for incumbents who ran for re-election and those that didn’t. The figure in Panel B depicts kernel density plots of the allocation public funds by the municipality’s Human Development Index. The figure also distinguishes between those incumbents that ran for re-election and those that didn’t. These figures were computed based on a sample of 5,550 municipalities.
Notes: This figure compares the allocation of public funds as predicted from our model to the actual data. The share of public funds is computed by region and averaged over 5,000 simulations.
Figure 8: Comparison Between Model’s Prediction and Actual Allocations

Notes: This figure compares the allocation of public funds as predicted from our model to the actual data. In the top panel, we compute the allocations for incumbents who sought re-election. In the bottom panel, we compute the allocations for incumbents who did not seek re-election. The share of public funds is computed by region and averaged over 5,000 simulations.
Figure 9: Comparison Between Model’s Prediction and Actual Allocations

Notes: This figure compares the allocation of public funds as predicted from our model to the actual data. In the top panel, we compute the allocations for incumbents who won re-election. In the bottom panel, we compute the allocations for incumbents who did not win re-election. The share of public funds is computed by region and averaged over 5,000 simulations.
Notes: This figure depicts the probability that an incumbent wins conditional on allocating at 2/3 of his resources in a particular region. The figure compares our model’s predictions to the actual data. The share of public funds is computed by region and averaged over 5,000 simulations.
Figure 11: Comparison Between Model’s Prediction and Actual Allocations

Notes: This figure depicts the probability that an incumbent runs for office conditional on allocating at 1/3 of his resources in a particular region. The figure compares our model’s predictions to the actual data. The share of public funds is computed by region and averaged over 5,000 simulations.
**Figure 12:** Comparison Between Model’s Prediction and Actual Allocations

Notes: This figure depicts the probability that an incumbent do not run for office conditional on allocating at 1/3 of his resources in a particular region. The figure compares our model’s predictions to the actual data. The share of public funds is computed by region and averaged over 5,000 simulations.
Figure 13: Comparison Between Model’s Prediction and Actual Allocations

Notes: This figure depicts the probability that an incumbent wins conditional on allocating at 1/3 of his resources in a particular region. The figure compares our model’s predictions to the actual data. The share of public funds is computed by region and averaged over 5,000 simulations.
Notes: This figure depicts the probability that an incumbent runs for office conditional on allocating at 2/3 of his resources in a particular region. The figure compares our model’s predictions to the actual data. The share of public funds is computed by region and averaged over 5,000 simulations.
Figure 15: Comparison Between Model’s Prediction and Actual Allocations

Notes: This figure depicts the probability that an incumbent do not run for office conditional on allocating at 2/3 of his resources in a particular region. The figure compares our model’s predictions to the actual data. The share of public funds is computed by region and averaged over 5,000 simulations.
**Figure 16:** Distribution of Allocations By Politician Type

*Notes: This figure plots the allocation of public funds by politician type. The share of public funds is computed by region and averaged over 5,000 simulations.*
**Figure 17:** Deviation from the Social Planner

Notes: This figure plots the allocation of public funds relative to the social planner allocation for each of the policy simulations. The share of public funds is computed by region and averaged over 5,000 simulations.
Notes: This figure plots the allocation of public funds relative to the social planner allocation for each of the policy simulations. The share of public funds is computed by region and averaged over 5,000 simulations.
Figure 19: Policy Simulations: N-person Electoral Rules

Notes: This figure plots the allocation of public funds relative to the social planner allocation for each of the policy simulations. The share of public funds is computed by region and averaged over 5,000 simulations. The 2-person policy refers to the scoring rule: \((1, 1, 0, \ldots, 0)\). The 3-person policy refers to the scoring rule: \((1, 1, 1, 0, \ldots, 0)\). The 4-person policy refers to the scoring rule: \((1, 1, 1, 1, 0, \ldots, 0)\). The 5-person policy refers to the scoring rule: \((1, 1, 1, 1, 1, 0, \ldots, 0)\), etc. The base case policy refers to our original results.
Figure 20: Policy Simulations: Challenger Competition

Notes: This figure plots the allocation of public funds relative to the social planner allocation for each of the policy simulations. The share of public funds is computed by region and averaged over 5,000 simulations. Challengers refers to a policy where the number of challengers were doubled. The base case policy refers to our original results.
A Appendix: Proof of Proposition 1

The proof is based on the existence results established in Milgrom and Weber (1985). To use their results is helpful to rewrite our model in the following way. Let

$$U^j_R (q^j, d^j_R, q^{-j}, d^{-j}_R; \beta^j, \beta^{-j}) = (1 - \beta^j) p^j (q, d_R) + \beta^j \sum_{m=1}^{M} W_m$$

and

$$U^j_{NR} (q^j, d^j_R, q^{-j}, d^{-j}_R; \beta^j, \beta^{-j}) = \bar{v}^j_{np} + \beta^j \sum_{m=1}^{M} W_m.$$ 

Then, deputy $j$ chooses the optimal allocation and whether to run according to the following problem:

$$\max_{d_R} \left\{ \max_{q^j} \int \left[ U^j_R (q^j, d^j_R, q^{-j}, d^{-j}_R; \beta^j, \beta^{-j}) \sigma (d\beta^{-j}) \right] \right. \left. \max_{q^j} \int U^j_{NR} (q^j, d^j_R, q^{-j}, d^{-j}_R; \beta^j, \beta^{-j}) \sigma (d\beta^{-j}) \right\}$$

The problem can alternatively be written in the following form:

$$\max_{q^j, d^j_R} \int U^j_R (q^j, d^j_R, q^{-j}, d^{-j}_R; \beta^j, \beta^{-j}) \sigma (d\beta^{-j}) + (1 - d^j_R) \int U^j_{NR} (q^j, d^j_R, q^{-j}, d^{-j}_R; \beta^j, \beta^{-j}) \sigma (d\beta^{-j})$$

Or equivalently,

$$\max_{q^j, d^j_R} \int d^j_R U^j_R (q^j, d^j_R, q^{-j}, d^{-j}_R; \beta^j, \beta^{-j}) + (1 - d^j_R) U^j_{NR} (q^j, d^j_R, q^{-j}, d^{-j}_R; \beta^j, \beta^{-j}) d\sigma (\beta^{-j})$$

We can therefore redefine the utility of deputy $j$ as

$$U^j (q^j, d^j_R, q^{-j}, d^{-j}_R; \beta^j, \beta^{-j}) = d^j_R U^j_R (q^j, d^j_R, q^{-j}, d^{-j}_R; \beta^j, \beta^{-j}) + (1 - d^j_R) U^j_{NR} (q^j, d^j_R, q^{-j}, d^{-j}_R; \beta^j, \beta^{-j})$$

Deputy $j$’s problem can therefore be written as follows:

$$\max_{q^j, d^j_R} \int U^j (q^j, d^j_R, q^{-j}, d^{-j}_R; \beta^j, \beta^{-j}) \sigma (d\beta^{-j})$$

We can now define a pure-strategy and a mixed-strategy Bayesian Nash equilibrium for this
Definition 2 A pure-strategy Bayesian Nash equilibrium is a vector of strategies \( s = (s^1, \ldots, s^J) \) such that for every \( j \in J \):

\[
    s^j (\beta^j) = \arg \max_{q^j, d_R^j} \int U^j (q^j, d_R^j, s^{-j} (\beta^{-j}); \beta^j, \beta^{-j}) \sigma (d\beta^{-j}).
\]

To define a mixed-strategy Bayesian Nash equilibrium, for every type \( \beta^j \), let \( m^j (s^j; \beta^j) \) be a probability measure over the strategy space \( S^j \), and \( M^j \) player \( j \)'s set of such mixed strategies. Then, we can extend the deputy \( j \)'s utility to the set of mixed strategies by an expected utility calculation:

\[
    U^j (m^j, m^{-j}; \beta^j, \beta^{-j}) = \int_{S^j} \ldots \int_{S^j} U^j (s^j, s^{-j}; \beta^j, \beta^{-j}) m^1 (ds^1; \beta^1) \ldots m^J (ds^J; \beta^J).
\]

We can now introduce the mixed extension of the initial game in pure strategy \( G = (S^j, U^j)_{j=1}^J \) as \( \bar{G} = (M^j, U^j)_{j=1}^J \).

Definition 3 A mixed-strategy \( m^* \) is a mixed-strategy Bayesian Nash equilibrium of the initial game \( G \) if \( m^* \) is a pure-strategy Bayesian Nash equilibrium of the extended game \( \bar{G} \).

Theorem 1, Proposition 1, and Proposition 3 in Milgrom and Weber (1985) establish that a game of incomplete information of the type considered in this paper has a mixed-strategy Bayesian Nash equilibrium if two conditions are satisfied: (i) the set of actions available to each player \( S^j \) is finite and (ii) the types of the players, \( \beta^1, \ldots, \beta^J \), are drawn from independent distributions. In the model we estimate, each player has a finite set of actions since she can choose among four possible allocations of resources. Moreover, by assumption, types are drawn independently from the same distribution. Hence, a mixed-strategy Bayesian Nash equilibrium exists.

Let a pure-strategy \( \epsilon \)-equilibrium is defined as a vector of strategies \( m = (m^1, \ldots, m^J) \) such
that for every player $j$ and every alternative pure strategy $m^j$,

$$\int U^j \left( m^j, m^{-j}; \beta^j, \beta^{-j} \right) \sigma \left( d\beta^{-j} \right) + \epsilon \geq \int U^j \left( m^{j^*}, m^{-j}; \beta^j, \beta^{-j} \right) \sigma \left( d\beta^{-j} \right).$$

A Corollary to Theorem 1 in Milgrom and Weber (1985) establishes that a pure-strategy $\epsilon$-equilibrium exists in a game of incomplete information if, in addition to the two conditions required for the existence of a mixed-strategy Bayesian Nash equilibrium, the players’ beliefs over types are atomless. In the model we estimate, we do not need to make specific assumptions on the beliefs over types. Moreover, in the data we only observe the actions played by the deputies. Hence, we cannot make inference over the distribution of types. Our estimated model is therefore consistent with distributions over types that are atomless and distributions that are not atomless. It is therefore also consistent with a pure-strategy $\epsilon$-equilibrium.