Invariant Subspace Theorems in Infinite-Dimensional Analysis

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Abstract

In a first course on linear algebra, one learns that if X is a finite-dimensional vector space over $\mathbb{C}$, then every linear map $T : X \to X$ has an eigenvalue and thus has a nontrivial invariant subspace. When X is an infinite-dimensional complex topological vector space and T is assumed to be a continuous linear transformation from X to itself, the problem of whether T has nontrivial closed invariant subspace is far more difficult and is for the most part unsolved. In 1975, Enflo gave an example of a bounded linear transformation T on a complex Banach space X that failed to have a nontrivial closed invariant subspace. Today, it is still unknown if every bounded linear transformation from a Hilbert space to itself possess such a subspace. In this paper, we present two of the known invariant subspace theorems: the Lomonosov theorem and the Iokvidov-Ky Fan theorem. As a necessary tool for proving both of these theorems, we establish the Tychonoff fixed-point theorem for continuous self-maps on a compact convex subset of a locally convex space. As a corollary of our derivation of the Tychonoff fixed-point theorem, we give a homology-free proof of the classical Brouwer fixed-point theorem.