Properties of Moving Electron Bubbles in Superfluid Helium

Wei Guo and Humphrey J. Maris

Department of Physics, Brown University, Providence, Rhode Island 02912, USA

Abstract. It is well known that the Bernoulli effect modifies the shape of gas bubbles moving through a liquid. In this paper we investigate the influence of the Bernoulli pressure on the shape of electron bubbles moving through superfluid helium. We show that an electron bubble moving through liquid at zero pressure becomes unstable when its velocity reaches approximately 47 m s^{-1}. In addition, the change in shape contributes significantly to the variation of the bubble mobility with velocity.

Keywords: superfluid helium, electrons, Bernoulli.
PACS: 67.40, 78.20c

When a bubble moves through a liquid of low viscosity, the Bernoulli effect results in a variation of the liquid pressure over the surface of the bubble. If the liquid is treated as incompressible and the bubble is, for the moment, taken to be spherical it is straightforward to show that the liquid velocity is

$$\vec{v} = \frac{R^3}{2r^3}[3\hat{r}(\vec{V}\cdot\hat{r}) - \vec{V}] \qquad (1)$$

where $\vec{V}$ is the velocity of the bubble, R is the bubble radius, and $\vec{r}$ is the distance from the center of the bubble. As a result of this flow of the liquid there is a pressure distribution over the surface of the bubble given by

$$P = P_0 + \frac{1}{8}\rho V^2(9\cos^2\theta - 5) \qquad (2)$$

where θ is the angle between the velocity $\vec{V}$ and the location of a point on the bubble surface, ρ is the liquid density and P_0 is the pressure in liquid that is at rest and far away from the bubble. As a result of this pressure variation, the bubble undergoes a change in shape such that the waist increases. The shape is determined by the Weber number We defined as

$$We = \frac{2\rho V^2 R}{\alpha} \qquad (3)$$

where α is the surface tension. At a critical Weber number We_c of 3.37, a gas bubble becomes unstable against growth of the waist.[1]

Consider now an electron bubble moving through superfluid helium at low temperatures where the drag on the bubble due to the normal fluid is small. It is important to recognize that the elastic properties of an electron bubble are different from those of a gas bubble. To a very good approximation a gas bubble of macroscopic size (e.g., ~ 1 cm) deforms in a way such that its volume remains constant. The extent to which the shape of the bubble changes due to the variation of the Bernoulli pressure over the bubble surface is determined by a balance between the pressure and the surface tension. For an electron bubble, on the other hand, the energy of the electron inside the bubble is a function of both the volume and the shape of the bubble.

For small velocity, the shape change can be calculated by perturbation theory. If the bubble is nearly spherical, the pressure is as given by Eq. 2, and the change in the shape can readily be calculated from the results for the stiffness of the bubble against deformation given in ref. 2. For larger velocity, we have developed a numerical method. We start with a guess at the bubble shape and then calculate the electron wave function and determine the pressure the electron exerts on the bubble surface. The flow of the liquid outside the bubble is next calculated and the variation of the Bernoulli pressure over the

CP850, *Low Temperature Physics: 24th International Conference on Low Temperature Physics;*
edited by Y. Takano, S. P. Hershfield, S. O. Hill, P. J. Hirschfeld, and A. M. Goldman

surface is determined. We next add in the effect of surface tension and determine the net pressure imbalance ΔP at different locations on the surface. We then move the surface by a distance which is proportional to ΔP, and repeat this until we find the equilibrium shape. Note that in this calculation the liquid is treated as incompressible.

In Fig. 1 we show the pressure at which bubbles explode as a function of their velocity. At zero pressure bubbles become unstable and explode at a critical velocity V_{ex} of 47 m s^{-1}. Figure 2 shows the distance from the center of the bubble to the pole and to the equator for a bubble moving in liquid at zero pressure.

When electron bubbles move through the liquid at a sufficiently high velocity, a vortex can be nucleated and the bubble is then trapped on the ring. The critical velocity V_v for vortex nucleation [3] is approximately 44 m s^{-1}. Thus, by coincidence, V_{ex} and V_v are almost identical. From Fig. 2 it can be seen that when the velocity of the bubble is 44 m s^{-1}, there is a very significant change in the size (and shape) of the bubble compared to the size at zero velocity. This change is not taken into account in the Muirhead-Vinen-Donnelly [4] theory of vortex nucleation and it would be interesting to know how inclusion of this change in size would modify the predictions of this theory.

Finally, we mention that the increase in the radius of the waist of the bubble will increase the drag on the bubble due to scattering of thermal excitations. Thus, the change in shape should lead to a mobility that decreases with velocity. At 30 m s^{-1}, the change in the area of the equator is 14 %, corresponding to a decrease in mobility relative to the low field value of 12 %. The measured decrease in mobility[5] at this velocity is 18%, in surprisingly good agreement, considering that there may be other factors that contribute to the change in mobility with velocity.

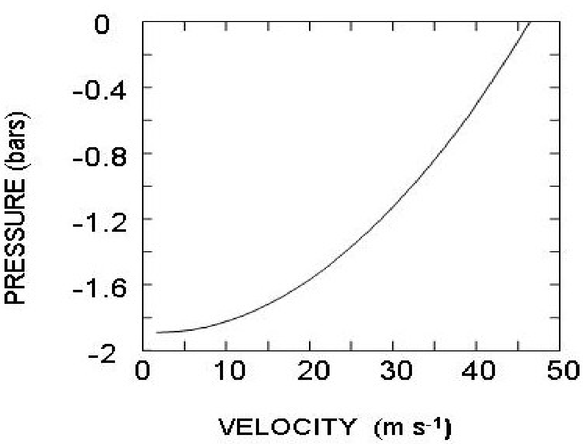

FIGURE 1. Pressure at which an electron bubble explodes as a function of the bubble velocity.

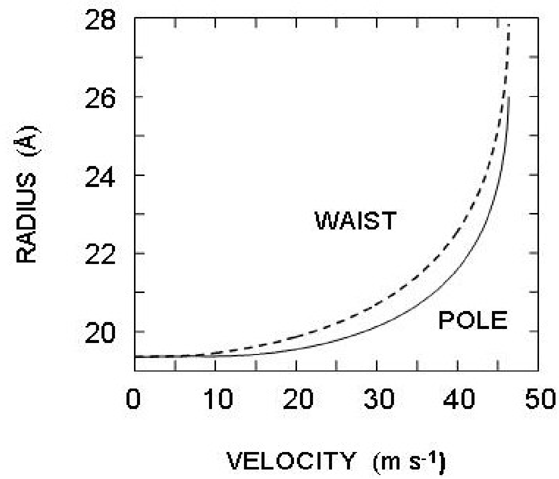

FIGURE 2. Radius to the poles and to the waist of an electron bubble as a function of velocity. Calculations are for $P = 0$.

This work was supported in part by the National Science Foundation through Grant No. DMR-0305115.

REFERENCES

1. V.I. Kusch, A.S. Sangani, P.D.M. Spelt and D.L. Koch, *J. Fluid. Mech.* **460**, 241 (2002).
2. H.J. Maris and W. Guo, *J. Low Temp. Phys.* **137**, 491 (2004).
3. The cited velocity is for isotopically pure ^{4}He. For a discussion, see R.J. Donnelly, *Quantized Vortices in Helium II* (Cambridge, Cambridge, 1991), pp 291-292. The estimate of 44 m s^{-1} is obtained by extrapolation of the data at higher pressure (R.M. Bowley, P.V.E. McClintock, F.E. Moss and P.C.E. Stamp, (*Phys. Rev. Lett.* **44**, 161 (1980)) to $P = 0$.
4. C.M. Muirhead, W.F. Vinen and R.J. Donnelly, *Phil. Trans. Roy. Soc.* **A311**, 433 (1984).
5. V.L. Eden and P.V.E. McClintock, *Phys. Lett.* **102A**, 197 (1984).