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Bubbles in Liquid Helium Containing Electrons in Excited States

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When an electron enters liquid helium, it forces open a cavity within the liquid. We calculate the size and shape of these electron bubbles for different quantum states of the electron, and determine the negative pressure at which the different bubbles explode.

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1. INTRODUCTION

When an electron enters liquid helium and loses its kinetic energy, it forms a state known as an electron bubble[1]. The electron is confined to a cavity within the liquid from which the helium atoms are almost completely excluded. The size and shape of this cavity are such as to minimize the total energy E. This energy is, to a good approximation, given by the expression:

$$E = E_{\text{el}} + \alpha A + PV + E_{\text{pol}}, \tag{1}$$

where E_{el} is the energy of the electron state, α is the energy per unit area of the liquid-vapor interface, A is the surface area of the bubble, P is the applied pressure, V is the volume of the bubble, and E_{pol} is the energy associated with the polarization of the liquid resulting from the electric field of the electron.

A bubble in the ground 1S state can absorb light and make a transition to the 1P or 2P states. By the Franck-Condon principle, the energy at which these optical transitions occur is the difference in the energy of the 1S and the 1P or 2P states, calculated with the bubble in the same configuration, i.e., with the size that minimizes the energy of the 1S state. The energies E_{1S-1P} and E_{1S-2P} and the matrix elements for these transitions have been calculated [2–5], and the energies have been measured as a function of pressure[4–9].

One can also consider the size and shape of bubbles that contain electrons in excited states, such as the 1P or 2P, which have relaxed to a configuration of minimum energy. There have been some calculations of the characteristics of these states. Duvall and Ruvalds[3] used a perturbation approach in which it was assumed that the bubble shape had only a small distortion from sphericity. In a calculation by Fowler and Dexter[2], the bubble shape was approximated as a rectangular box. As far as we are aware, there have been no experimental studies of electrons in these excited states. Recently, we realized that it may be possible to investigate their properties through cavitation experiments[10]. When a negative pressure is applied to helium that contains an electron bubble, a bubble grows in size and at a critical pressure P_c becomes unstable and begins to grow without limit. This effect has studied in detail for 1S bubbles [11,12]. It is clear that the value of P_c must vary according to the state of the electron. As a consequence, there is the possibility of performing a new form of spectroscopy in which the quantum state of an electron in a bubble is determined through a measurement of the negative pressure at which the bubble explodes. In this paper, we present results of calculations of P_c for a number of different quantum states.

2. CALCULATION OF THE BUBBLE ENERGY AND SHAPE

The energy of an electron in helium is greater than the energy in vacuum by an amount V_0. This has been measured to be approximately 1 eV[13]. To calculate the energy of the electron state, we should solve Schrodinger's equation with a potential that is zero inside the bubble, and equal to V_0 outside. However, for the low lying electron states the value of $V_0 - E$ is sufficiently large that the wave function of the electron penetrates into the helium a distance that is small compared to the dimensions of the bubble. As a consequence, we will neglect this penetration and will require that the wavefunction goes to zero at the bubble wall. We will also neglect the polarization energy term in Eq. 1, which can be shown to be a fairly small correction [12]. We have shown [14] that with these simplifications and with the choice of the value 0.341 erg cm^{-2} for α, the calculated values of E_{1S-1P} and E_{1S-2P} are in good agreement with experimental data. To find the equilibrium shape of a relaxed electron bubble, it is necessary to carry out the following steps. The bubble shape is first specified by writing the radius r at an angle θ in the form

$$r(\theta) = \sum_{L=0,2,\ldots L_{\max}} a_L P_L(\theta), \tag{2}$$

where a_L are some coefficients, and P_L are Legendre polynomials. In this paper we restrict attention to bubble shapes that have axial symmetry, and which have reflection symmetry in the plane $z = 0$. For a given choice of the a_L coefficients, we then calculate $E_{\rm el}$. This is done by expanding the wave function as a sum of the form

$$\psi(r, \theta, \phi) = \sum_{\ell=0,2,\ldots \ell_{\max}} A_\ell P_\ell(\theta) j_\ell(kr), \tag{3}$$

where $j_\ell(kr)$ is a spherical Bessel function, and $k = (2mE_{\rm el})^{1/2}/\hbar$. We are here considering only states with azimuthal quantum number $m = 0$. The energy eigenvalues are those values of $E_{\rm el}$ for which it is possible to choose the A_ℓ in a way such that the wave function is very close to zero at all points on the bubble surface. Once the energy $E_{\rm el}$ is found for a given choice of the a_L coefficients, the total energy of the bubble can be calculated. Finally, the a_L coefficients are varied to give the lowest energy for the particular quantum state that is being considered. In deriving the results given below, it was found that it was sufficient to take $L_{\max} = 4$ and $\ell_{\max} = 12$.

3. RESULTS

Except for when the electron is in an S-state, the bubbles do not have a spherical shape. For bubbles that are not spherical, L is not a good quantum number, i.e., the wave function of the electron inside the bubble is a linear combination of states with different values of L. For bubbles of the form considered here, i.e., for shapes with axial symmetry, the z-component of the angular momentum is a constant of the motion. Thus, the wave function of the electron has a definite azimuthal quantum number m. Although L is not a good quantum number, it is convinient to refer to the bubbles as 1P, 1D, etc. Thus, 1P refers to a bubble containing an electron that would be in the 1P state, if the bubble were adiabatically deformed into a spherical shape.

In Figs. 1 and 2 we show the shape of the bubbles for selected pressures. The pressures have been chosen so that for each quantum state, the last shape plotted is for a pressure close to the pressure at which the bubble becomes unstable. The shape of the surface is determined by a balance between the outward pressure exerted by the electron, by the applied pressure in the liquid, and by the surface tension. For $P = 0$, the force due to the electron must be balanced entirely by the surface tension. The outward

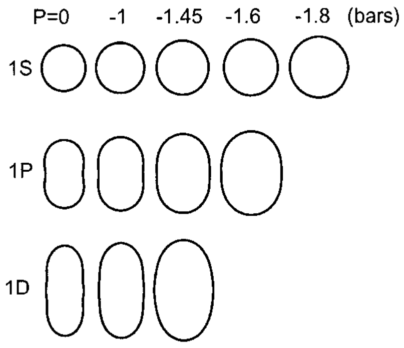

Fig. 1. Shape of the electron bubble for the 1S, 1P, and 1D states for selected pressures. The scale is such that the radius of the 1S bubble is 19.4 Å.

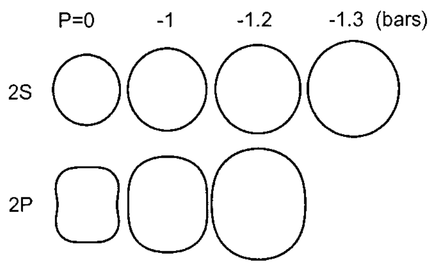

Fig. 2. Shape of the electron bubble for the 2S and 2P states for selected pressures. The scale is such that the radius of the 2S bubble is 27.4 Å.

force due to the electron is proportional to the value of $|\nabla\psi|^2$ at the surface. As a consequence, the bubble shape reflects the location of the lines on the bubble surface along which the wave function is zero. For example, the 1P state has a pronounced waist in the plane $z = 0$. As the pressure is made more negative, the features arising from nodes in the wave function become less pronounced, and for all of the states the bubble shapes become more spherical.

For negative pressures, the size of the bubble increases. Beyond the critical pressure P_c, it is not possible to find a stable minimum energy configuration for the bubble. We have found the value of P_c for a number of quantum states, and the results are as listed in Table I. As expected, the magnitude of P_c decreases as either the radial quantum number or the angular momentum increase.

State	P_c (bars)
1S	-1.89
2S	-1.33
1P	-1.63
2P	-1.22
1D	-1.49

Table I. Pressures at which the different electron bubbles explode.

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