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Observations of the Motion of Single Electrons in Liquid Helium

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Abstract We have developed an apparatus which can be used to monitor the motion of individual electrons in liquid helium. A sound wave is used to explode an electron bubble for a fraction of a microsecond. While the bubble is expanded, it is illuminated by light from a flash lamp. We describe the details of the experiment and show results obtained. In some cases, an electron is seen to follow a snakelike path. Our tentative explanation is that the electron is sliding along a quantized vortex line. If this interpretation is correct, the recorded track of the electron provides an image of the path of the vortex.

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1 Introduction

In several recent experiments the cavitation in liquid helium that results from nucleation at an electron bubble has been studied [1–4]. The energy E of an electron bubble of radius R in helium is given by the approximate expression

$$E = \frac{\hbar^2}{8mR^2} + 4\pi R^2\alpha + \frac{4\pi}{3}R^3P, \tag{1}$$

where the three terms represent the zero-point energy of the electron confined in a spherical cavity, the surface energy, and the work done against the applied pressure P in forming the cavity. α is the surface tension and m is the mass of the electron. The variation of the energy E with radius is shown in Fig. 1.[1] At zero pressure the mini-

[1] This figure is based on a surface tension of 0.341 erg cm^{-2}. This value differs from the low temperature experimental value of 0.375 erg cm^{-2}, but is chosen because it leads to a bubble size that is consistent with optical absorption measurements.

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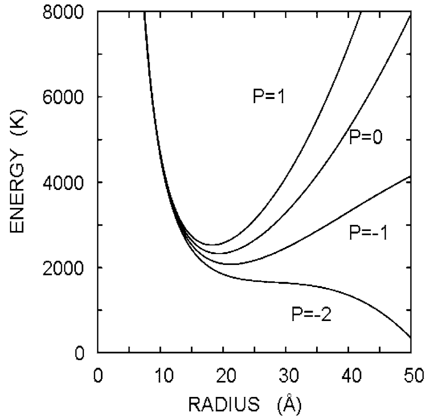

Fig. 1 Energy of an electron bubble as a function of radius. The curves are labeled by the pressure in bars

mum energy is for a radius of 19.4 Å. For negative pressures, the radius increases and at a critical pressure P_c, the bubble becomes unstable against isotropic radial expansion and "explodes". P_c has the value -1.9 bars in the low temperature limit and has a smaller magnitude at higher temperatures due to the temperature dependence of α. This critical pressure is considerably smaller in magnitude than the pressure required to cause homogeneous nucleation of bubbles in helium. Consequently, cavitation due to electron bubbles is readily distinguished from cavitation due to homogeneous nucleation.

In the experiments performed so far [1–4], a hemispherical transducer has been used to focus sound to a region in the liquid with a volume of the order 10^{-5} cm^3. If the pressure swing due to the sound pulse is large enough *and* if an electron bubble is in the focal region, cavitation will occur. In this type of measurement, a series of pulses is applied and the number of times that cavitation occurs is recorded. From an analysis of how the probability of cavitation varies with the driving voltage applied to the transducer, it is possible to deduce the pressure threshold P_c for cavitation and also the number density of the electron bubbles.

In the work reported here instead of using focused sound we employ a planar ultrasonic transducer so as to produce a transient negative pressure over a large volume ($\sim$1 cm^3). In this way, we explode all electron bubbles within the volume. By choosing a suitable ultrasonic frequency, we can make each bubble expand to a size that is sufficiently large that we can determine its position. Through the application of a series of sound pulses, we can then make a record of the track of individual electrons.

2 Experiment

An electron bubble at zero pressure cannot be detected optically because the scattering cross-section is extremely small; the total cross-section due to the presence of the empty spherical cavity in the helium is of the order of 10^{-23} cm^2 for red light. We therefore need to increase the size of the bubble so that it can be detected, but at the same time we do not want it grow to a size comparable to the dimensions of the experimental cell. The dynamics of a bubble in an oscillating pressure field can be

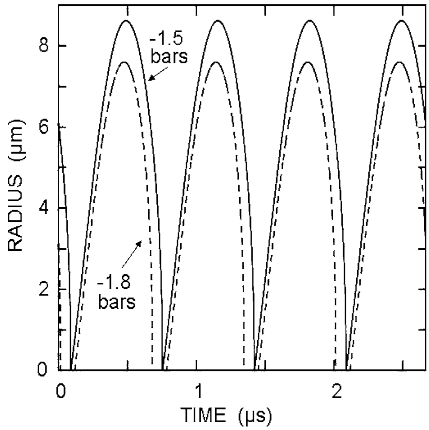

Fig. 2 Calculated radius of a bubble as a function of time based on (2). The two curves show results for when the electron bubble is unstable at −1.8 bars (*dashed line*) and −1.5 bars (*solid line*)

treated using the Rayleigh-Plesset equation which we write in the form [5]

$$\ddot{R} = -\frac{P(t)}{\rho_{\mathrm{He}} R} - \frac{3\dot{R}^2}{2R}. \tag{2}$$

In Fig. 2 we show the bubble radius as a function of time when the pressure has the form $P(t) = -P_0 \sin(2\pi f t)$, with P_0 equal to 2 bars and $f = 1.5$ MHz, and for two different values of P_c.

It is difficult to produce a large amplitude sound wave in helium using a conventional piezoelectric ultrasonic transducer because the acoustic impedance of liquid helium is much smaller than that of most solid materials. Nevertheless, in the present experiment after considering various options, we decided to use a standard 1.5 MHz lithium niobate piezoelectric transducer. The transducer has dimensions 1.2 × 1.2 × 0.22 cm. The amplitude δP_{He} of the pressure wave in the helium is related to the velocity v_{surface} at the surface of the transducer by

$$\delta P_{\mathrm{He}} = \rho_{\mathrm{He}} c_{\mathrm{He}} v_{\mathrm{surface}}, \tag{3}$$

where ρ_{He} and c_{He} are the density and sound velocity in helium, respectively. In order to achieve a pressure swing of 2 bars, this requires $v_{\mathrm{surface}} = 600\ \mathrm{cm\,s^{-1}}$, corresponding to a maximum strain amplitude in the interior of the transducer of around 10^{-4}. Because of this large strain, we have broken several transducer crystals and this has limited the results we have been able to obtain so far. To achieve the required amplitude, we typically drove the transducer at resonance for 35 μs. Only the late part of the sound pulse has sufficient amplitude to explode a bubble. With the given waveform of the sound, we expect that each electron bubble within the volume of the sound field will be exploded and collapse a few times, probably 1 to 5, for each sound pulse. We have not investigated this point experimentally.

In designing the experiment we were concerned that heterogeneous nucleation of bubbles might take place on the surface of the transducer [6]. If a large number of bubbles were to form this would greatly reduce the amplitude of the sound wave that would propagate away into the liquid. We do not know if some bubbles were produced but there was no evidence that they interfered with the experiment.

To detect the exploded bubble we illuminated it with a flash lamp[2] that was designed for use in industry as a strobe. It takes the sound pulse approximately 40 μs to cross the part of our helium cell that we can observe. As a result, the time that an electron bubble explodes will vary according to its position in the cell. To help make sure that the bubble is illuminated by the flash lamp regardless of its location, we selected a lamp that has a pulse duration of approximately 30 μs. The output energy of the lamp was 0.22 J. A mirror was used to direct the light towards the cell which was at a distance of 20 cm. We estimate that the flux of photons through the cell was of the order of $2 \times 10^{20}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$.

The flash lamp was triggered so that the flash began at the time at which the amplitude of vibration of the transducer had built up to its maximum value. The repetition rate of the sound (and light) pulse could be adjusted but was normally 20 per second.

The helium cell was a cube with exterior side length of 6.8 cm. The cell was attached to a continuously operating 1 K pot and measurements could be made at temperatures down to 1.3 K. The cell was cooled at the bottom. The transducer was mounted at the top of the cell so that the sound was directed down. Two side surfaces of the cell had windows of diameter 1.4 cm, one for the light from the flash lamp and the other for viewing. The interior of the cell was painted black to minimize scattered light. To detect the light scattered from the bubble we used a home-style camcorder[3] with its lens 15 cm from the center of the cell. Based on geometrical optics, the differential scattering cross-section for scattering unpolarized light at 90° from a bubble in helium is calculated to be

$$\frac{d\sigma}{d\Omega} \approx 2 \times 10^{-4} R^2. \tag{4}$$

Thus, for an 8 μm bubble the rate at which photons are scattered per unit solid angle in our experiment is $2.6 \times 10^{11}\ \mathrm{s}^{-1}$. More generally, the total number of photons scattered into unit solid angle at 90° will be

$$N = 2 \times 10^{20} \times 2 \times 10^{-4} \int R^2 dt, \tag{5}$$

where the integral extends over the duration of the sound pulse. We do not know the precise value of the integral, but a rough estimate gives the value $10^{-12}\ \mathrm{cm}^2\ \mathrm{s}$. The solid angle subtended by the camcorder lens is 0.056 steradians. Based on these values, the total number of photons entering the camcorder is found to be 2000. With the available camera, we were able to detect these photons provided we used the camera in "super night mode" running at 4 frames per second. Because this rate was less than the rate at which sound pulses were generated, on each frame recorded by the camcorder there would typically be several images of the electron bubble.

In the experiments that we report here no source was used to inject electrons into the liquid. We discuss this point later.

[2] Digital Nova-Strobe from Monarch Industries, Inc.

[3] Sony model TRV740 camcorder.

3 Results

3.1 Image of electrons

In Fig. 3, we show two frames, each capturing the motion of a single electron. The electron is moving from the top to the bottom of the cell.

When we left the ultrasonic pulse generator and camcorder running, we found that approximately 1% of the frames showed an image of an electron somewhere within the part of the volume of the cell that could be viewed. It was natural to ask whether the objects that were recorded were, in fact, electrons. We were concerned that the light that was detected could come from scattering at dust particles, i.e., particles other than electrons. These particles could be carried around the cell by flow induced in the helium due to the sound pulses. However, strong evidence against a dust theory was provided by an investigation of how the probability of seeing scattering varied with the amplitude of the voltage applied to the transducer. It was found that no scattering was detected when the transducer voltage was below 200 V. In a 10 V range above 200 V, the number of frames showing scattering increased rapidly. There is no reason to expect that for dust particles there would be sharp threshold for cavitation and so the observation of a sharp threshold is strong evidence that we are seeing electrons. For higher voltages there is a slow increase in the probability of seeing electrons. We presume this is because the sound field falls off at the edge of the transducer and so when the transducer amplitude is increased, the volume within which electrons can be exploded increases.

The electrons shown in Fig. 3 have a downward velocity of roughly 5 $\mathrm{cm\,s^{-1}}$. All of the electrons that we have seen have a similar velocity. We believe that the electrons are dragged along by the normal fluid flowing down the cell. The driving of the transducer results in a heat input $\dot{Q}$ at the top of the cell and this heat is extracted at the bottom of the cell. If we consider this heat transfer to occur over an effective area A, the velocity v_n of the normal fluid will be

$$v_n = \frac{\dot{Q}}{ATS}, \tag{6}$$

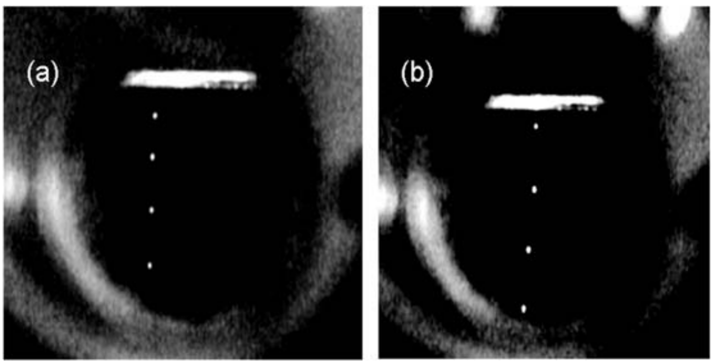

Fig. 3 Images of an electron moving down the cell. The temperatures are 1.5 K

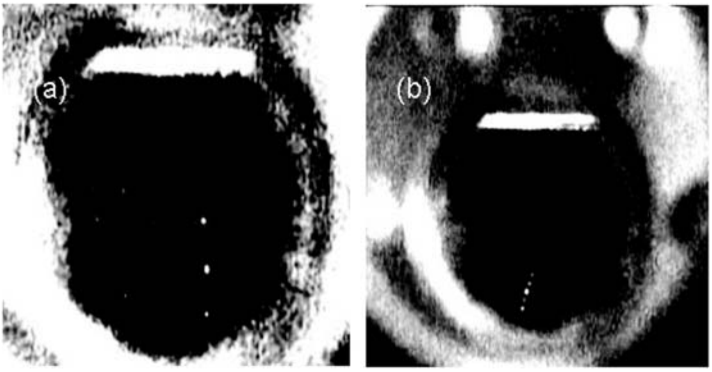

Fig. 4 Images in which an electron bubble is first seen at a point in the interior of the cell rather than at the top

where S is the entropy per unit volume. This gives reasonable agreement with experiment. For example, Fig. 3a was obtained at 1.5 K, with an average power input of $\sim$250 mW. The measured electron velocity is 6 cm s^{-1} and this is consistent with (6) if the area is taken to be 1 cm^2. Of course, an accurate calculation of the velocity of the normal fluid requires detailed allowance for the shape of the cell, the heat flow in the walls, and the Kapitza resistance. There may also be a contribution from acoustic streaming.

As already mentioned, the cell does not contain any source of electrons. At the present time we do not know where the electrons that we see come from. One possibility is cosmic rays. A high energy muon passing through the cell will cause ionization along its track. Most of the electrons knocked off of helium atoms will quickly recombine with the resulting positive ions, and only a small fraction of the electrons will escape. These electrons will be distributed uniformly throughout the cell. However, these cannot be the electrons that we are seeing since most of the electrons appear to start at the very top of the cell, rather than being distributed throughout the liquid. When the cosmic rays excite or ionize the helium, UV photons will be produced. The energy of these photons is around 16 eV. It is possible that when the UV photons reach the cell wall electrons are ejected into the helium by the photoelectric effect. At the bottom wall of the cell these electrons would be swept back into the wall by the normal fluid.

A small fraction of the electrons that were seen first appeared at a point within the helium. Examples are shown in Fig. 4. There are at least two possible origins of these electrons. They could result from cosmic rays, or other charged particles, ionizing helium atoms and the resulting positive and negative ions not recombining. A second possibility is that the electron bubbles appear when a gamma from outside the helium, i.e., from the cell wall or other part of the cryostat, undergoes Compton scattering or photoelectric conversion within the helium. In Fig. 4b, the electron is moving much slower than it is in Fig. 4a. We assume this is because the downward component of the normal fluid velocity is different in different parts of the cell.

In some of the images, a second electron is also detected. Of course, this could just be a coincidence that two electrons enter the cell at the same time, but the number of times that this happens appears to be larger than would be expected on this basis.

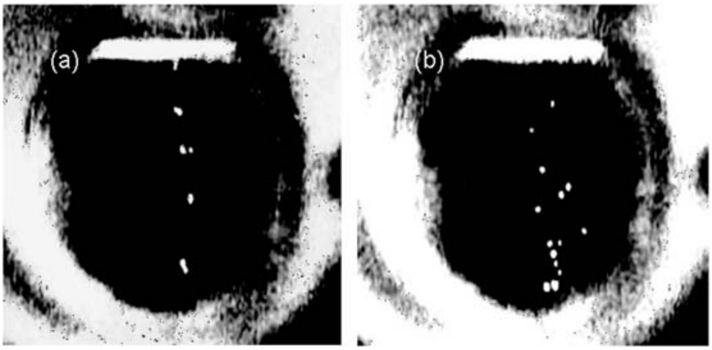

Fig. 5 Images in which more than one electron are seen

Two examples are shown in Fig. 5. In one case, see Fig. 5a, it appears that the two electrons originate a close distance apart and at the top of the cell. We do not know the origin of these events. We have also seen events in which more than two electrons appear in the interior of the cell (Fig. 5b). If a Compton scattering event occurs, the recoiling electron will have some probability of ionizing a helium atom (or atoms), and it is possible that this mechanism is responsible for these events.

3.2 Image of Vortices

When planning this experiment, our idea was to extend it so that images of quantized vortices could be obtained. The plan was to use a radioactive source to introduce a large number of electrons into the helium. By going to a temperature of around 1 K, many of these electrons would become attached to vortex lines. We would then send in a single sound pulse that would explode all of the electrons along the lines and thus provide an image of the path of the line. We have not done this experiment yet but have accidentally detected vortices in another way.

Most of the electrons that we see follow a nearly straight path from the top to the bottom of the cell, presumably following the streamlines of the normal fluid. However, occasionally,[4] we see an electron that follows a very different path as shown in Fig. 6. Typically these electrons follow a path resembling a snake. This cannot be due to normal fluid turbulence because these paths appear in the same volume of liquid as the other events. Electrons seen immediately before and after the electrons that follow snake paths move through the cell in nearly straight paths. We consider it likely that an electron is trapped on a vortex line and then follows the line from the top of the cell to the bottom. If this is correct, this simple experiment provides a way to visualize the geometry of quantized vortices [7–9]. At the present time we do not know whether the electron follows an almost motionless line from the top of the cell to the bottom or if the vortex line moves significantly as the electron passes down the cell.

We are currently trying to perform the experiment in the way originally envisaged, and hope this will make it possible to obtain photographs of quantized vortex rings.

[4] A rough estimate is that this occurs 10% of the time.

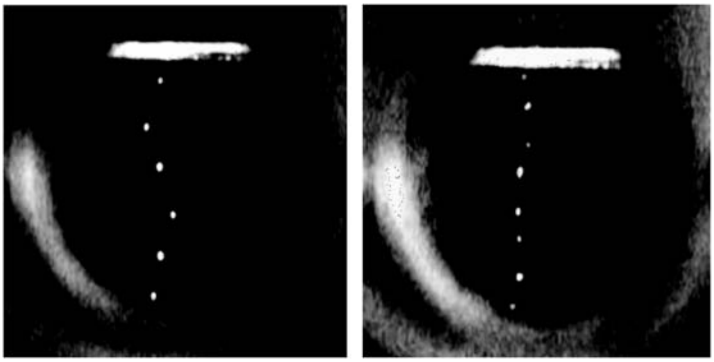

Fig. 6 Images of an electron sliding along a vortex line

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