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New Calculation of the Polarization Contribution to the Energy of a Negative Ion in Liquid Helium

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An electron injected into liquid helium-4 forms a bubble of radius approximately 19 Å. The size of the bubble is determined by a balance between the zero-point energy of the electron, the surface energy of the bubble wall, and the polarization energy of the helium in the electric field of the electron. We derive a modified result for the polarization energy of the bubble. We show that previous calculations in which the electron is treated as a point charge localized at the center of the bubble are inaccurate, and that it is essential to allow for the quantum fluctuations in the position of the electron.

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1. INTRODUCTION

It is well known that when an electron is injected into liquid helium, it forces open a spherical cavity that is essentially free of helium atoms. The energy E of this electron bubble has been considered to be given by the expression[1]

$$E_{\rm el} + 4\pi R^2\alpha + \frac{4\pi}{3}R^3 P - \frac{(\epsilon-1)e^2}{2R}, \tag{1}$$

where $E_{\rm el}$ is the ground state energy of the electron inside the spherical cavity of radius R, α is the energy per unit area of the liquid-vapor interface, P is the applied pressure, ϵ is the dielectric constant of liquid helium, and e is the charge on the electron. The first term is the zero-point energy of the electron that is confined in the bubble. The energy of the electron is obtained by solution of Schrödinger's equation with a potential that is zero inside the bubble, $(r < R)$ and equal to a constant V_0 outside the bubble $(r \geq R)^2$. The second and third terms represent the surface energy of the bubble, and the work done in forming the bubble against the pressure in the

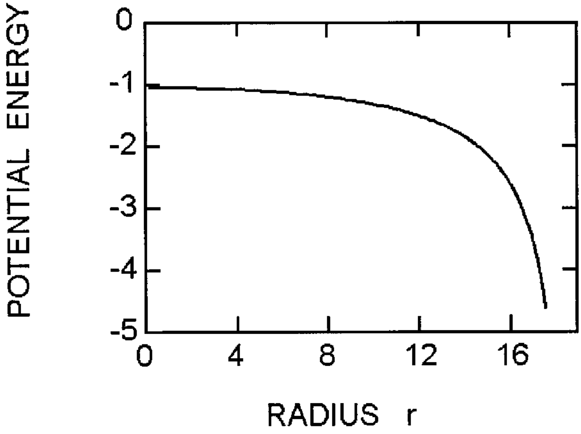

Fig. 1. Polarization potential for the electron inside the bubble.

liquid, respectively. The last term represents the polarization energy E_{pol} of the liquid under the application of the electric field of the electron. For zero pressure, the polarization energy contributes approximately 10% of the total energy of the bubble, and the remaining 90% is divided equally between the energy of the electron and surface energy. Equation 1 is based on a number of approximations[3]. For example, it has been assumed that the bubble has a sharp wall at which the density of the helium jumps discontinuously from zero to its value in bulk liquid, and the surface tension of the liquid has been taken to be independent of the curvature of the bubble surface. In this paper, we will not attempt to assess the accuracy of these approximations, but wish to consider the form of the polarization energy term.

2. CALCULATION OF THE POLARIZATION ENERGY

The dielectric constant ϵ for liquid helium is close to unity. As a result, the polarization energy can be calculated in terms of the integral of the square of the electric field over the volume of the liquid outside the bubble, i.e., as

$$E_{pol} = -\frac{\epsilon - 1}{8\pi} \int_{\vec{r}' \text{ liquid}} d^3\vec{r}' \, |\vec{E}(\vec{r}')|^2. \quad (2)$$

In the derivation of the form of the polarization energy as given in Eq.

1, it has been assumed implicitly that the electron is always located at the center of the bubble. The electric field at a distance $r' > R$ from the center of the bubble is then e/r'^2, (again the factor of ϵ can be neglected) and so the polarization energy is

$$E_{\text{pol}} = -\frac{(\epsilon - 1)e^2}{2R}. \tag{3}$$

However, this approach is incorrect because the position of the electron fluctuates throughout the volume of the bubble. The probability of finding the electron within the volume element $d^3\vec{r}$ is

$$\psi(\vec{r})^2 \, d^3\vec{r}. \tag{4}$$

Thus, the correct expression for the polarization energy is

$$E_{\text{pol}} = -\frac{(\epsilon - 1)}{8\pi} \int_{\vec{r}\ \text{inside}} d^3\vec{r}\ \psi(\vec{r})^2 \int_{\vec{r}'\ \text{outside}} d^3\vec{r}' \left(\frac{e}{|\vec{r}' - \vec{r}\,|}\right)^2, \tag{5}$$

where $E(\vec{r}, \vec{r}')$ is the magnitude of the electric field at $\vec{r}'$ when the electron is at $\vec{r}$. (Note that this again uses the fact that $\epsilon - 1 \ll 1$). If we neglect the penetration of the electron wave function into the helium, and take the potential inside the bubble to be zero, the wave function of the electron in the ground state is

$$\psi(\vec{r}) = \left(\frac{\pi}{2R^3}\right)^{1/2} \frac{\sin(\pi r/R)}{\pi r/R} \tag{6}$$

Evaluation of the integral in Eq. 5, gives the result

$$E_{\text{pol}} = -1.345 \frac{(\epsilon - 1)e^2}{2R}. \tag{7}$$

This result can be corrected to allow for the finite potential energy inside the bubble. When the electron is at $\vec{r}$, the potential energy due to the polarization of the helium is

$$U_{\text{pol}}(\vec{r}) = -\frac{(\epsilon - 1)}{8\pi} \int_{\vec{r}'\ \text{outside}} d^3\vec{r}' \left(\frac{e}{|\vec{r}' - \vec{r}|}\right)^2 \tag{8}$$

In Fig. 1 we show a plot of U_{pol} in units of $(\epsilon - 1)e^2/R$ as a function of the distance r from the center of the bubble. The effect of this potential on the form of the ground state wave function is small. In Fig. 2 we show the square of the wave function for zero potential inside the bubble (Eq. 6), together with the square of the wave function calculated for the potential

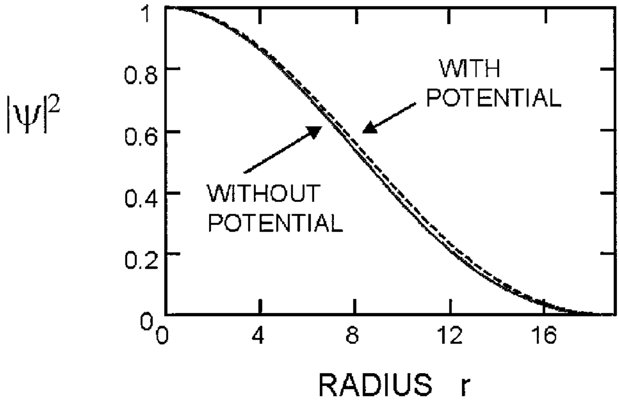

Fig. 2. Square of the electron wave function inside the bubble. The solid line is calculated from Eq. 6, and the dashed line allows for the effect of the poarization potential.

given by Eq. 7. In this figure the bubble radius has been taken to have the value 19 Å, and the two wave functions have been normalized to have unit magnitude at $r = 0$. Calculation of the polarization energy E_{pol} with this corrected wave function gives a result that is only 9% larger in magnitude than the result of Eq. 6.

Note that in all of the calculations in this paper, we have neglected the penetration of the electron wave function into the helium. In previous calculations of the polarization energy, the electron was implicitly considered to remain fixed at the center of the bubble, and so clearly any effect on the polarization energy that comes from the softness of the bubble wall was not included. It is known that the penetration has only a small effect on the wave function of the electron[3], and so it appears unlikely that the allowance for penetration will have a large effect on the result for E_{pol}.

3. TIME SCALES

In considering the polarization energy, it is important to recognize that a number of different time scales are involved. The physical processes associated with these times are as follows:

1) When an electric field is applied to a helium atom, the time for the polarization to develop is of the order of the inverse of the energy required to excite the first excited state. This time scale τ_{He} is thus of the order of 10^{-16} s.

2) The time scale τ_{el} of the fluctuations in position of the electron inside the bubble is of the order of $\hbar/\Delta E$, where ΔE is now the energy for the electron in the bubble to reach the first excited state. This energy is approximately $h^2/8mR^2$, and so τ_{el} is of order of 10^{-14} s.

3) The time scale τ_{bub} for the electron bubble to respond to a change in the position of the electron can be taken to be of the order of the reciprocal of the frequency of shape oscillations of the bubble. The lowest vibrational mode has a frequency of the order of 10^{11} s^{-1}, so $\tau_{\mathrm{bub}} \sim 10^{-11}$ s.

From these time scales, it can be seen that a fluctuation in the position of the electron results in a variation of the electric field in the helium on a time scale that is sufficiently slow that the helium atoms in the liquid will achieve a polarization given by the zero frequency dielectric constant ϵ. However, the fluctuations occur on a time scale that is very fast compared to the response time of the bubble wall. Thus, we are justified in treating the bubble wall as static, and the bubble remaining spherical at all times.

4. SUMMARY

In summary, we have shown that the form for the polarization energy of a negative ion in liquid helium that has been used previously is incorrect. The corrected value is larger by a value that is approximately 1.345.

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REFERENCES

1. A.L. Fetter, in *The Physics of Liquid and Solid Helium*, editor K.H. Benneman and J.B. Ketterson (Wiley, New York, 1976), chapter 3.
2. W.T. Sommer, Phys. Rev. Lett. **12**, 271 (1964).
3. J. Classen, C.-K. Su, M. Mohazzab, and H.J. Maris, Phys. Rev. **57**, 3000 (1998).