NOTE: All students must sign up for an exam time slot in order to take exam 1. You must do this by email, before the beginning of class on Friday Feb. 22. Check the web page for details.

1. A laser beam is directed at one facet of a glass prism. The beam refracts twice: once as it enters the prism, and a second time as it exits. As illustrated in the figure, we may define the angle of incidence at the input facet as $\theta_{\text{in}}$. Obviously, the direction of the output beam depends on this input angle, and also on the apex angle of the prism (defined in the figure as $\beta$) and on the refractive index of the glass $n$. If we keep the input beam fixed, but rotate the prism around an axis that points out of the page, then we can change the value of $\theta_{\text{in}}$, and therefore also the direction of the output beam. Since the input beam is fixed, it makes sense to define the direction of the output beam relative to that input beam’s direction (i.e., relative to the red dashed line in the figure, which shows where the input beam would have gone if the prism weren’t there). In the figure, $\theta_{\text{dev}}$ defines this angle, known as the ‘angle of deviation’.

(a) Derive an analytic expression for $\theta_{\text{dev}}$ as a function of $\theta_{\text{in}}$, $n$, and $\beta$. Use $n_{\text{air}} = 1$.

(b) Using Matlab, make a plot of the relationship between the input angle and the angle of deviation. For this plot, assume that the prism is equilateral, with a refractive index of $n = 3/2$. You should find that the output beam does not move continuously in one direction as the input angle changes, but rather that the deviation angle reaches a minimum at a certain point. Estimate from the plot the minimum deviation angle, and the input angle at which minimum deviation occurs.

(c) Using your result from (a), prove the following statement: when $\theta_{\text{in}}$ is chosen so that the output beam experiences minimum deviation, the beam propagation direction inside the glass is parallel to the bottom face of the prism.

2. A beam of white light is directed towards an isosceles right angle prism, as shown in the figure. Assume that the light beam is incident on the vertical edge at normal incidence, as the figure illustrates. The incident beam is also assumed to be collimated (i.e., not spreading as it propagates). After the beam exits the prism, this is no longer true – the beam spreads out as it propagates away from the prism.

(a) Determine the angular spread of this beam. You may assume that the white light beam is composed of wavelengths spanning the visible portion of the spectrum from 400 nm to 700 nm.
The refractive index of this glass prism can be adequately described using the Cauchy formula: 
\[ n = A + \left( \frac{B}{\lambda^2} \right), \] 
with \( A = 1.3450 \) and \( B = 3780 \text{ nm}^2 \).

(b) Suppose we consider the same problem again, but this time we invert things. The incident beam is propagating in glass, not in empty space. And the prism is composed of empty space, not of glass. So the ‘prism’ is actually an empty triangular region of space embedded in a very large block of glass. Now, describe what happens when the light beam emerges from the prism-shaped void and re-enters the solid glass. Is the angular spread the same, more, or less? Is anything else changed?

3. The following expression specifies the electric field component of an outgoing spherical wave, (emerging from the origin) at any point along the positive z axis:
\[ \vec{E}(0,0,z,t) = \hat{x}(90 \text{ V}) e^{j \left[ (2.79 \times 10^8 \text{ m}^{-1}) z - (6.28 \times 10^4 \text{ rad/sec}) t \right]} \]

(a) What is the electric field at the point \((x = 0, y = z = 3 \text{ meters})\), at time \(t = 0\)? Express your answer as a real-valued vector quantity. Be sure to specify both the magnitude and direction of the field.

(b) What is the irradiance of this spherical wave at a distance of 10 centimeters from the origin?

4. A cyclotron is a machine that sends charged particles (like electrons) round and round in a circular orbit. Assuming that the electrons accelerated in this device are moving at essentially the speed of light (in fact, they move at a tiny bit less than \( c_0 \), but the difference is insignificant for this problem), determine how much power each electron radiates. Assume that the radius of the circular orbit is 3 meters. In which direction is there no radiation? This power is lost as the accelerating electron radiates electromagnetic waves, so the electrons need to be re-accelerated continuously to keep them moving in a circle. You may be interested to know that there are several MUCH larger and more sophisticated versions of this device (now known as synchrotrons) in use throughout the world. They provide one of the brightest sources available for x-ray radiation, and are used to study all sorts of different types of materials. They can also be used to collide high energy particles, which is how the LHC works. See here for more information: https://en.wikipedia.org/wiki/Synchrotron

5. A beam of light (in air) is directed towards a slab of plastic with a refractive index of 1.55, and at an angle of 20° to the normal. The incident light has both parallel and perpendicular polarization components: the parallel component has an amplitude of 10 V/m, while the perpendicular component has an amplitude of 20 V/m. Determine the amplitudes of the reflected wave’s parallel and perpendicular components. Do they still have a magnitude ratio of \( E_\perp : E_\parallel = 2 : 1 \)? If not, what is the ratio for the reflected wave?