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Characterization of apparent superluminal effects in the focus of an axicon lens using terahertz time-domain spectroscopy

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Abstract

We describe time-resolved measurements of the propagating interference pattern at the focus of an axicon lens. The technique of terahertz time-domain spectroscopy permits a direct observation of the electric field within the line focus formed by a converging conical wave front. We extract the apparent superluminal velocity of this pulse from the peak of the time-domain waveform, and also in the frequency domain by Fourier transform. From these measurements, we conclude that pulse reshaping, arising from strong frequency-dependent propagation effects, does not play a substantial role. As a result, tracking the peak of the pulse in the time domain is a valid method for determining the average group velocity of the disturbance. We also elucidate a new mechanism which influences the value of the superluminal velocity, involving the curvature of the incident wave front.

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There has been much interest in recent years in the properties of 'diffractionless' beams, solutions to the wave equation which produce a line focus with invariant spot diameter over an extended depth of field. These solutions are known as Bessel beams, since the transverse intensity profile, within the depth of field, is a Bessel function in the radial coordinate [1]. A Bessel beam can be formed by the interference of wave fronts propagating at an angle to the optic axis, an effect which can be achieved in a number of different ways. One popular method utilizes a conical lens, known as an axicon [2]. Diffractionless beams have attracted attention for their use in imaging [3,4], dispersionless optical system design [5], and in the production of plasma waveguides [6–8].

A number of authors have noted that, due to the 'scissors effect', the intersection point of two or more crossing wave fronts need not obey the restrictions of relativity [9,10]. As a result, the interference maximum at the tip of an axicon may

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propagate at a velocity which exceeds the vacuum speed of light. Recent reports of the observation of this effect [11] have led to a great deal of discussion [9,10,12–14]. Although it is clear that the energy velocity is always subluminal, it is nevertheless possible to observe an effect that appears to propagate along the axis of the line focus with superluminal group velocity. This is possible because the event which occurs at a particular position $z = z_0$ on the optic axis is not causally related to the events which occurred at $z < z_0$, at earlier times.

Alexeev et al. [15] have recently described an observation of this apparent superluminal effect using femtosecond pulse ionization as a diagnostic for detecting the position of the intensity maximum. This work confirmed the superluminal effect, but left open the possibility that significant pulse distortion could play an important role. For example, it is well known that dramatic frequency-dependent features in the refractive index of a material through which a pulse propagates can lead to group velocities that greatly exceed the vacuum speed of light [16]. In the case at hand, both the dispersion of the axicon lens material and that of the ionized plasma could in principle lead to pulse distortions. If these effects are present, then a measurement of the velocity of the peak of the pulse, as reported in [15], would not be an accurate measure of the group velocity. In order to isolate the geometrical effects associated with the formation of the Bessel beam, it is necessary to perform the experiment using a dispersion-free optical arrangement. Furthermore, a direct measurement of the spectral dependence of the phase velocity of the propagating pulse would unambiguously determine the extent to which pulse distortion contributes to the superluminal effect.

Here, we describe measurements of the superluminal effect performed using terahertz time-domain spectroscopy [17,18]. This technique permits coherent detection of the electric field of the propagating wave, and therefore provides a direct measurement of the frequency-dependent phase velocity. The measurements are performed using axicons composed of teflon, which exhibits almost no dispersion within the spectral range of the terahertz radiation employed for the measurements [19]. This effectively eliminates material dispersion from consideration, so that only geometrical effects are relevant. In this work, we identify a new geometrical effect, involving the curvature of the incident wave front, which can modify the apparent superluminal velocity observed in experiments such as those described here. To our knowledge, this effect has not been discussed previously.

We consider a plane wave incident on a conical refracting surface, such as an axicon lens. Such a wave front is converted to a converging conical wave front, which approaches the optic axis at an angle of $\beta = \sin^{-1}(n \sin\gamma) - \gamma$, where γ is the axicon base angle and n is its refractive index. The radial distribution of the incident wave front is mapped onto the optic axis, defining the depth of field of the line focus [6]. A simple ray tracing argument demonstrates that the phase velocity of the intensity maximum, along the axis, is given by $c/\cos\beta$, which always exceeds c. If the lens material is dispersionless, then this phase velocity is equal to the group velocity of the wave, and the group velocity dispersion is zero [15].

A schematic of the experiment is shown in Fig. 1. A single-cycle pulse of terahertz radiation is generated by an optically gated photoconductive antenna coupled to a hyperhemispherical substrate lens, as described previously [20,21]. This beam is approximately collimated by a polyethylene lens, producing a ~2 cm diameter beam. The beam then passes through a teflon axicon lens, and is detected at a point on the optic axis with a photoconductive receiver. This receiver is optically gated by a train

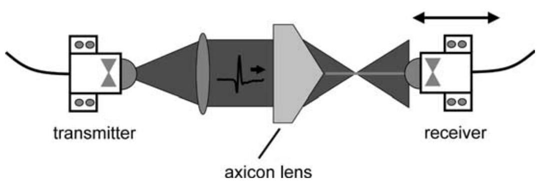

Fig. 1. A schematic of the experimental setup, using the time-domain spectrometer described in [22]. A fiber-coupled photoconductive transmitter generates a single cycle pulse of terahertz radiation. This beam is approximately collimated, then passes through a conical axicon lens, forming a line focus. The electric field of the interference maximum in this focus is detected by a fiber-coupled photoconductive receiver, which can be translated along the z axis (the propagation direction) without loss of temporal calibration.

of femtosecond pulses, delivered via fiber optic. As a result, the receiver can be moved along the optic axis in front of the axicon without loss of the absolute time reference [22]. The experiment consists of measuring the received terahertz waveform at a series of points along the optic axis, starting from the tip of the axicon. We have performed this measurement using three different axicons with base angles of $\gamma = 35°$, 40°, and 43°, and also with no axicon in the beam path, as a reference.

The results shown in Fig. 2 illustrate the apparent superluminal propagation. This compares time-domain waveforms measured with no axicon (solid curves) and with the 43° axicon (dashed curves), at two different detector positions. Because the finite thickness of the axicon introduces an overall temporal delay, the latter waveforms have been shifted to lower delays to facilitate the comparison. This temporal shift is chosen so that the peaks of the two upper waveforms coincide, at a detector position of $z_1 = 0.95$ cm. Here, $z = 0$

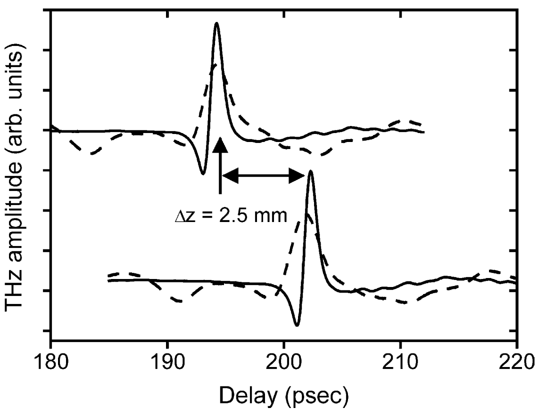

Fig. 2. Terahertz waveforms measured with and without a $\gamma = 43°$ axicon lens in the beam path, at two different detector locations along the line focus. In order to compare the propagation delays, the two waveforms measured with the lens (dashed curves) have both been shifted to smaller delays by 52.7 ps, so that the peaks of the two upper waveforms (measured at $z = 0.95$ cm from the axicon tip) coincide at the vertical arrow. When the detector is displaced by an additional 0.25 mm, the peaks of the two waveforms no longer coincide (lower curves). In particular, the curve corresponding to the reference measurement (i.e., with no axicon, solid curve) arrives ~0.44 ps later. Since this radiation propagates in air, it travels at $v \approx c$, so the other waveform, which arrives earlier, appears to have propagated at a speed exceeding c.

corresponds to the axicon tip. At a detector position of $z_2 = 1.20$ cm (lower curves), the focused pulse arrives at the detector at an earlier time. This is clear evidence of the superluminal effect. Further, it is evident from a comparison of the pulse shapes that only small changes in the pulse shape occur during the propagation from z_1 to z_2. The differences in the shapes of the focused and unfocused pulses are likely due to the nature of the photoconductive detector. Because the Bessel beam is formed by a superposition of waves traveling at relatively large angles to the optic axis, the detection of these pulses is subject to the complex angle- and frequency-dependent detector sensitivity. It has been established that this effect can lead to pulse reshaping in detected terahertz pulses, as a function of incident angle [23]. To the extent that the approach angle β does not depend on the distance z to the axicon tip, this angular sensitivity should also be independent of z. Below, we discuss the implications of this pulse reshaping further.

By tracking the pulse peak arrival time as a function of detector position, we can determine the spectrum-weighted average group velocity of the pulse. This result is shown in Fig. 3 for the case of the 43° axicon and for the reference situation, with no axicon. For the reference, we find a velocity of $0.9986c \pm 0.0008c$, which is approximately consistent with the expected value of $c/n_{\mathrm{air}} = 0.9997c$. The inset shows the velocities for the three axicons used in this study, all larger than c. The dashed curve shows the $1/\cos(\beta)$ dependence expected for an incident plane wave [11,15].

We note that there is marked disagreement between this prediction and the data. A simple explanation can be found in the fact that the incident beam is not a plane wave, but instead is a Gaussian beam with a spherical phase front. A curved wave front, like a plane wave, also forms a line focus after passing through an axicon, but the converging waves are no longer planar [24]. As a result, the approach angle β is no longer independent of z, the distance from the axicon tip. The effective superluminal velocity is still approximately independent of z, but is reduced from the expected value of $1/\cos(\beta)$. The implications of wave front curvature have previously been considered only for their effects on the depth of field

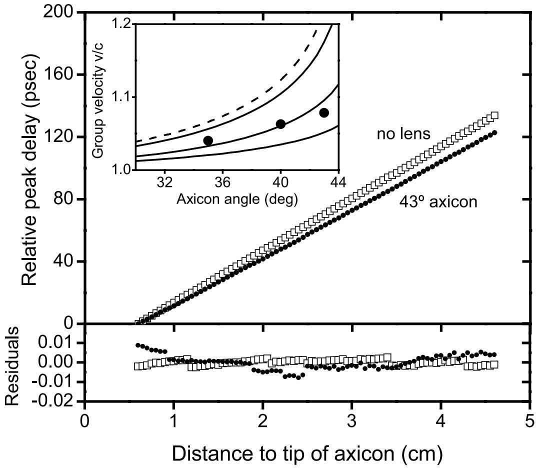

Fig. 3. Transit times for the peaks of the THz waveforms, as a function of the detector position. The data shown represent the reference (open squares), measured with no axicon lens, and the case of the 43° axicon (solid circles). Linear fits to these data give the effective propagation velocity. The lower panel shows the residuals of these linear fits, indicating that there is no appreciable nonlinearity in these results. The inset shows the extracted velocities for the three axicon lenses studied, as a function of axicon base angle. The dashed curve shows the expected $1/\cos(\beta)$ dependence for an incident plane wave. The three solid curves show the modified results, obtained with numerical ray tracing, for incident spherical waves with radii of curvature $R = 20$, 10, and 2 cm (upper to lower, respectively), as described in the text. The $R = 10$ cm curve gives a reasonable fit to the measured data points.

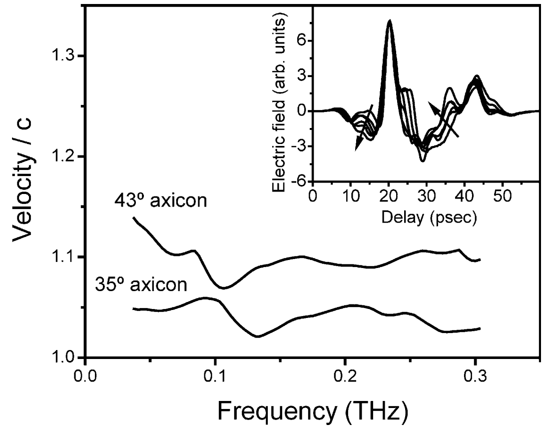

Fig. 4. Frequency-dependent phase velocities extracted by Fourier transform of the measured time-domain waveforms, as described in the text, for two of the axicon lenses studied. In both cases, the phase velocity is always greater than the vacuum speed of light. The inset shows a series of waveforms, measured with the 43° axicon. These waveforms have been acquired at position increments of 0.5 cm, starting from $z = 0.45$ cm. All of the waveforms have been normalized and shifted so that their peaks coincide, in order to highlight the degree of pulse reshaping. The arrows indicate the trends with increasing z. We note that the preponderance of the pulse reshaping occurs in the temporal wings of the pulse, and the central peak exhibits little change in shape.

[24]. The impact on the superluminal velocity has not previously been discussed. We employ numerical ray tracing to compute a series of revised curves for the dependence of the velocity on the axicon base angle γ. These are shown in the inset as solid curves, for several different values of the incident radius of curvature. The $R = 10$ cm curve gives a good agreement with the measured data points.

The measured waveforms contain a great deal more information than merely their arrival times. In particular, the spectral phase velocity can be extracted directly from the Fourier transforms of the measured fields $E_{\mathrm{THz}}(t)$. By comparing the spectral phases ϕ_1 and ϕ_2 of two waveforms measured at two different detector locations z_1 and z_2, we determine the phase velocity according to $v_{\mathrm{ph}}(\nu) = 2\pi\nu \cdot (z_2 - z_1)/(\phi_2 - \phi_1)$. Results are shown in Fig. 4 for two of the axicon lenses studied. These represent averages over several pairs of measurements. Although not dispersionless, it is clear from these data that dispersive effects are relatively minor, consistent with the minimal pulse reshaping effects discussed above. The inset shows a series of time-domain waveforms measured using the 43° axicon, at distances spaced by $\Delta z = 0.5$ cm, covering most of the depth of field of this lens. In order to compare these waveforms directly, and to highlight the pulse reshaping effects, these curves have all been normalized and shifted so that their maxima coincide. There is little reshaping of the central feature, consistent with the overall trend in the phase velocity. However, some reshaping does occur in the temporal wings of the pulse.

The pulse reshaping indicated in Fig. 4 could arise during the propagation of the pulse, as an intrinsic effect due to the geometry of a converging conical wave front. As noted above, it could also arise from a measurement artifact associated with the angular sensitivity of the receiver. Since a curved input wave front results in an approach angle β which depends on the propagation coordinate z, it is clear that the angle of incidence on the detector also varies with z. From these measurements, it is difficult to distinguish between these two possibilities. As a result, these data can only place an upper limit on the degree to which pulse reshaping plays a role in the measurement of apparent superluminal velocities. It should be clear, however, that the minimal amount of pulse reshaping observed here is not sufficient to give rise to misleading results in group velocity estimates which have been obtained by tracking the peak of the pulse in the time domain.

To be complete, we should consider one final possibility. It is conceivable that the pulse reshaping arising from the angular sensitivity of the receiver antenna could also give rise to apparent shifts in the arrival time of the peak of the pulse. This would constitute an additional contribution to the apparent superluminal velocity extracted from, e.g., the data of Fig. 3. However, we can be confident that this effect is of minimal importance, for two reasons. First, based on the known phase response function for the receiver used in these measurements [23] (a photoconductive antenna coupled to an aplanatic hyperhemispherical substrate lens), the manifestation of the pulse reshaping, in the time domain, exhibits only a very slight shift in the arrival time of the peak of the pulse. The pulse reshapes with changes in incident angle, but its peak occurs at more or less the same time. Second, if this effect were significant, then the z dependence of the approach angle β would lead to a nonlinear component in the observed arrival times in Fig. 3. However, there is little if any trend in the residuals of linear fits to the data of Fig. 3, indicating that any nonlinearity in the z dependence of the arrival time is below the noise level of the measurements. This implies that any peak shifts arising from the angular sensitivity of the receiver are also below the noise level of the measurement, and may therefore be neglected.

In conclusion, we have measured the pulse velocity at the focus of several dispersionless axicon lenses, confirming the apparent superluminal propagation. Through coherent detection of terahertz pulses, we are able to directly extract the frequency-dependent phase velocity of the propagating superluminal disturbance. We have elucidated a new effect which influences the magnitude of the apparent velocity, involving the wave front curvature of the incident beam. The residual pulse reshaping effects, observed in both the time and frequency domain, are probably a result of this wave front curvature, in conjunction with the angular sensitivity of the detectors used in these measurements. Our measurements seem to eliminate the possibility that the superluminal effects reported in [15] are due merely to pulse reshaping, since the central maximum of the propagating pulse does not experience substantial reshaping.

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References

[1] R.M. Herman, T.A. Wiggins, J. Opt. Soc. Am. A 8 (1991) 932.
[2] J.H. McLeod, J. Opt. Soc. Am. 44 (1954) 592.
[3] G. Häusler, W. Heckel, Appl. Opt. 27 (1988) 5165.
[4] Z. Ding et al., Opt. Lett. 27 (2002) 243.
[5] M. Porras, Opt. Lett. 26 (2001) 1364.
[6] C.G. Durfee, J. Lynch, H. Milchberg, Phys. Rev. E 51 (1995) 2368.
[7] S.P. Nikitin et al., Phys. Rev. E 59 (1999) 3839.
[8] J. Fan, E. Parra, H. Milchberg, Phys. Rev. Lett. 84 (2000) 3085.
[9] W.A. Rodrigues, D.S. Thober, A.L. Xavier, Phys. Lett. A 284 (2001) 217.
[10] T. Sauter, F. Paschke, Phys. Lett. A 285 (2001) 1.
[11] D. Mugnai, A. Ranfagni, R. Ruggeri, Phys. Rev. Lett. 84 (2000) 4830.
[12] J.T. Lunardi, Phys. Lett. A 291 (2001) 66.
[13] H. Ringermacher, L.R. Mead, Phys. Rev. Lett. 87 (2001) 059402.
[14] N.P. Bigelow, C.R. Hagen, Phys. Rev. Lett. 87 (2001) 059401.
[15] I. Alexeev, K.Y. Kim, H. Milchberg, Phys. Rev. Lett. 88 (2002) 073901.
[16] J.D. Jackson, Classical Electrodynamics, Wiley, New York, 1975.
[17] P.R. Smith, D.H. Auston, M.C. Nuss, IEEE J. Quantum Electron. 24 (1988) 255.
[18] M. van Exter, D. Grischkowsky, IEEE Trans. Microwave Theory Tech. 38 (1990) 1684.

[19] J.R. Birch, J.D. Dromey, J. Lesurf, Infrared Phys. 21 (1981) 225.
[20] C. Fattinger, D. Grischkowsky, Appl. Phys. Lett. 54 (1989) 490.
[21] D.M. Mittleman, R.H. Jacobsen, M.C. Nuss, IEEE J. Sel. Top. Quantum Electron. 2 (1996) 679.
[22] J.V. Rudd, D. Zimdars, M. Warmuth, Proc. SPIE 3934 (2000) 27.
[23] J.V. Rudd, D.M. Mittleman, J. Opt. Soc. Am. B 19 (2002) 319.
[24] O.G. Ivanov et al., Sov. Phys. Tech. Phys. 32 (1988) 1212.