4. Energy, Power, and Photons

Energy in a light wave

Why we can often neglect the magnetic field

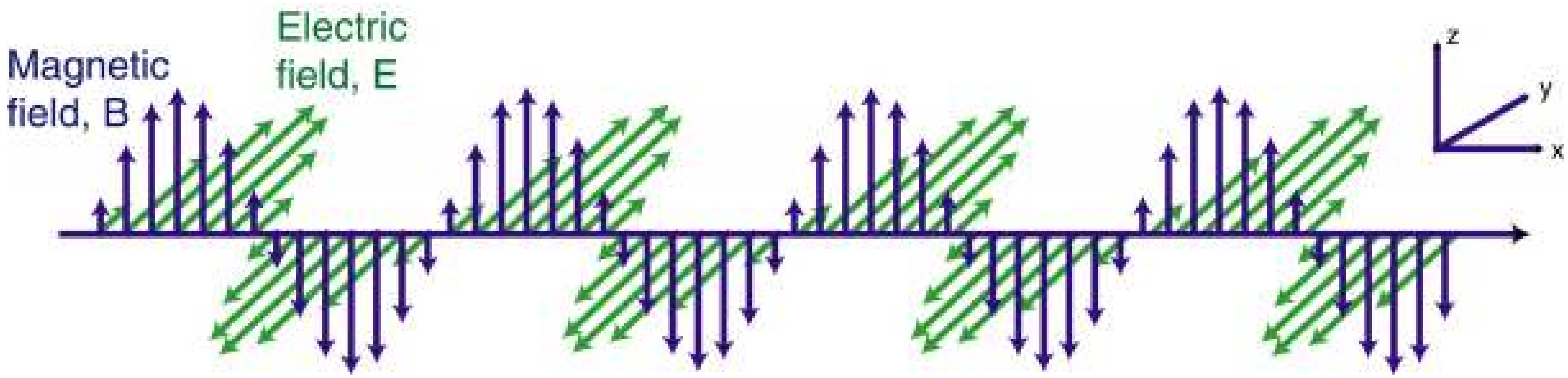
Poynting vector and irradiance

The quantum nature of light

Photon energy and
photon momentum

An electromagnetic wave in empty space

The electric and magnetic fields are **in phase**.



The electric field, the magnetic field, and the k-vector are all **perpendicular**:

$$\vec{E} \times \vec{B} \propto \vec{k}$$

And the magnitude of the B field is **simply related** to the magnitude of the E field:

$$\left|\vec{B}_0\right| = \left|\vec{E}_0\right| / c_0$$

where the speed of the wave is given by: $c_0 = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}} \approx 3 \times 10^8 \text{ m/s}$

Why we often ignore the magnetic field

In some situations, we ignore the effects of the magnetic field component of the EM wave. How can we justify this?

The force on a charge, q, is:

$$\vec{F} = \underbrace{q\vec{E}}_{F_{electrical}} + \underbrace{q\vec{v}\times\vec{B}}_{F_{magnetic}}$$

where $\vec{v}$ is the velocity of the charge

so:

$$\frac{\left|F_{magnetic}\right|}{\left|F_{electrical}\right|} \le \frac{qvB}{qE}$$

Since $B = E/c$:

$$\frac{\left|F_{magnetic}\right|}{\left|F_{electrical}\right|} \le \frac{v}{c}$$

So, as long as a charge's velocity is much less than the speed of light, we can neglect the light's magnetic force compared to its electric force.

The Energy density of a light wave

The energy density of an electric field is: $U_E = \frac{1}{2}\varepsilon E^2$

The energy density of a magnetic field is: $U_B = \frac{1}{2}\frac{1}{\mu}B^2$

Units check:

In empty space: $\varepsilon_0 = 8.854 \times 10^{-12}$ C^2/Nm^2

Electric field: units of V/m

$$[U_E] = \frac{C^2}{Nm^2}\frac{V^2}{m^2}$$

Using: $C = Nm/V$

$$[U_E] = \frac{Nm}{m^3} = \frac{Joule}{m^3} = \frac{energy}{volume}$$

The Energy density of a light wave (cont.)

Using $B = E/c$, and $c = \dfrac{1}{\sqrt{\varepsilon\mu}}$, which together imply that $B = E\sqrt{\varepsilon\mu}$

we have:

$$U_B = \frac{1}{2}\frac{1}{\mu}\left(E^2\varepsilon\mu\right) = \frac{1}{2}\varepsilon E^2 = U_E$$

So the electrical and magnetic energy densities in light are equal. The electric and magnetic fields each carry half of the total energy of the wave.

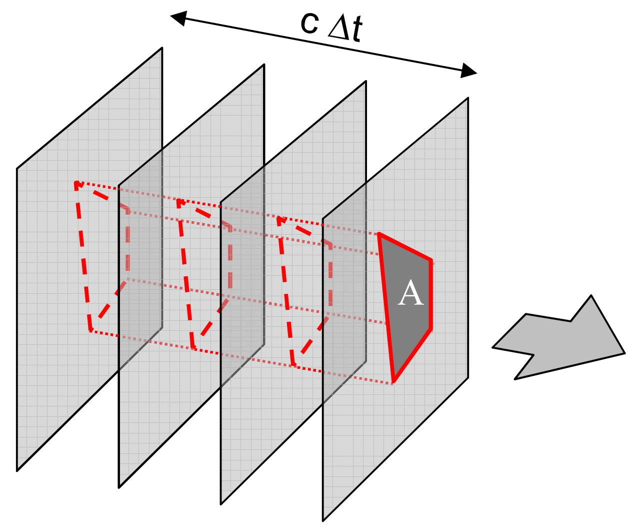
Total energy density: $U = U_E + U_B = \varepsilon E^2$

We can **never** ignore the magnetic field's contribution to the total energy of a light wave.

The Poynting vector

The power per unit area in a beam is given by the Poynting vector:

$$\vec{S} = \varepsilon c^2 \; \vec{E} \times \vec{B}$$



Justification (but not a proof):

Energy passing through area A in time Δt:

$$= \quad U \; V \quad = \quad U \; A \; c \; \Delta t$$

Volume V = A c Δt

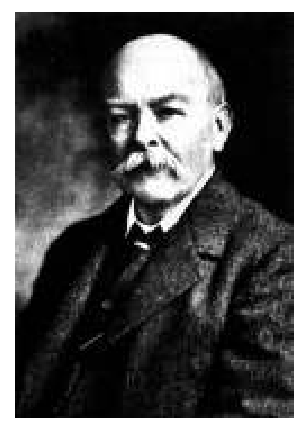
So the energy per unit time per unit area:

$$= \quad U \, V \,/\, (\, A \, \Delta t \,)$$

$$= U \; c \; = \; c \, \varepsilon \, E^2$$

$$= \quad c^2 \, \varepsilon \, E \, B \; = \; c^2 \, \varepsilon \, |\vec{E} \times \vec{B}|$$

And the direction $\vec{E} \times \vec{B} \propto \vec{k}$ is reasonable.



John Henry Poynting
(1852-1914)

The Irradiance (often called the Intensity)

A light wave's *average* power per unit area is called its "irradiance" or "intensity".

$$\left\langle \vec{S}(\vec{r},t) \right\rangle_T = \frac{1}{T} \int_{t-T/2}^{t+T/2} \vec{S}(\vec{r},t')\, dt'$$

Let's compute the irradiance of a light wave, like the ones we've been discussing so far.

$$\vec{E} = \vec{E}_0 \cos\left(\vec{k}\cdot\vec{r} - \omega t + \theta\right) \qquad \vec{B} = \vec{B}_0 \cos\left(\vec{k}\cdot\vec{r} - \omega t + \theta\right)$$

real amplitudes

Substituting these into the expression for the Poynting vector, $\vec{S} = c^2 \varepsilon\, \vec{E} \times \vec{B}$, yields:

$$\vec{S}(\vec{r},t) = c^2\, \varepsilon\, \vec{E}_0 \times \vec{B}_0 \cos^2\left(\vec{k}\cdot\vec{r} - \omega t + \theta\right)$$

The Irradiance (continued)

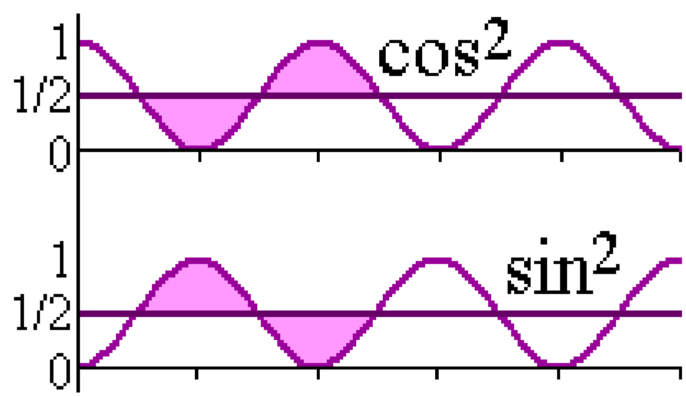
$$\vec{S}(\vec{r},t) \quad = \quad c^2 \, \varepsilon \, \vec{E}_0 \times \vec{B}_0 \, \cos^2\left(\vec{k}\cdot\vec{r} - \omega t + \theta\right)$$

The irradiance (or intensity) is the magnitude of the time average of the Poynting vector:

$$\Rightarrow \; I(\vec{r},t) \; = \left| \left\langle \vec{S}(\vec{r},t) \right\rangle_T \right|$$

Since the time-dependence of S is given by a $\cos^2$, we must take the time average of this function.

The average of $\cos^2(x)$ is ½:



$$I\left(\vec{r},t\right) = \left| c^2 \, \varepsilon \, \vec{E}_0 \times \vec{B}_0 \cdot \frac{1}{2} \right|$$

The Irradiance (continued)

Since the electric and magnetic fields are perpendicular and $B_0 = E_0 / c$,

$$I = \tfrac{1}{2} c^2 \varepsilon \left| \vec{E}_0 \times \vec{B}_0 \right|$$ becomes:

$$I = \frac{1}{2} c \, \varepsilon \left| \vec{E}_0 \right|^2$$

where $$\left| \vec{E}_0 \right|^2 = E_{0x}^{\;2} + E_{0y}^{\;2} + E_{0z}^{\;2}$$

Note: this formula still holds even if we had used complex amplitudes $\underset{\sim}{\vec{E}}_0$

except in that case: $$\left| \underset{\sim}{\vec{E}}_0 \right|^2 = \left| \underset{\sim}{E}_{0x} \right|^2 + \left| \underset{\sim}{E}_{0y} \right|^2 + \left| \underset{\sim}{E}_{0z} \right|^2$$

Units of irradiance

$$I = \tfrac{1}{2} c\,\varepsilon \left|\vec{E}_0\right|^2$$

We've already seen that εE^2 has units of Joules/m^3.

$$[\,I\,] = \frac{m}{s} \cdot \frac{J}{m^3}$$

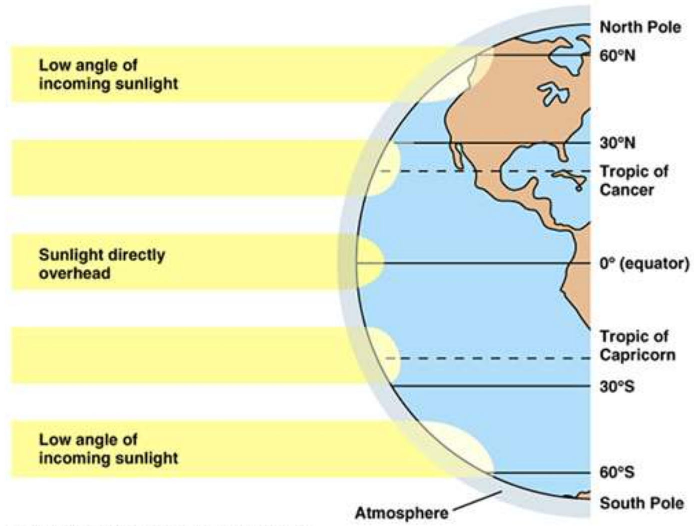
$$= \frac{Watts}{m^2}$$

Irradiance is power per unit area.

Irradiance: examples

The irradiance of sunlight, at the top of the atmosphere:

I ~ 1360 W/m^2
(about 1000 W/m^2 at sea level)



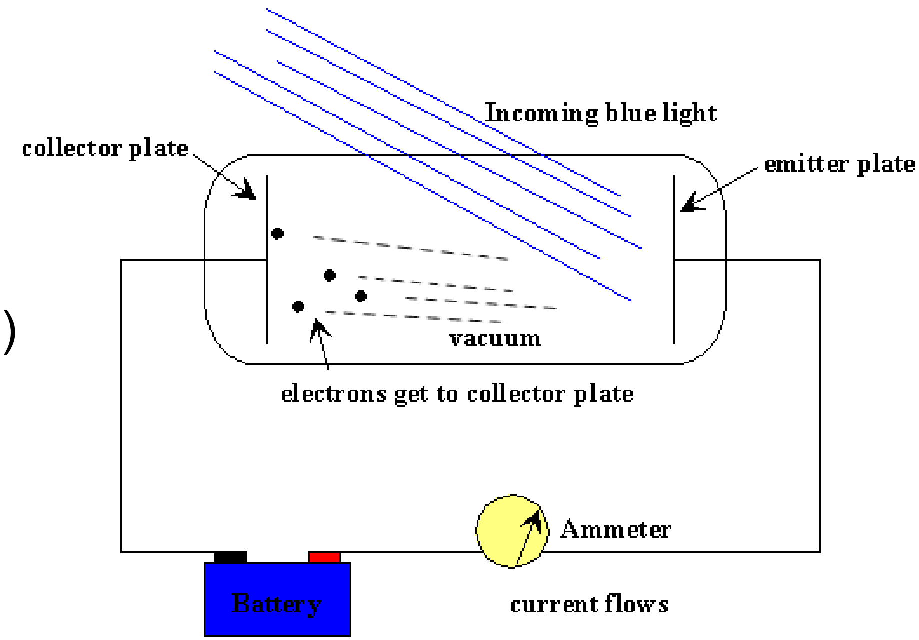
The irradiance of the Trident laser (Los Alamos National Lab):

I ~ 2,000,000,000,000,000,000,000,000 W/m^2

The photoelectric effect

It was observed* that unusual things happen when we shine light on a piece of metal in a vacuum:

1. The light can cause electrons to be ejected from the metal surface (that's not so unusual).



2. The energy of these electrons (i.e., their velocity once they leave the metal) does not depend on the light intensity, although the total number of electrons does depend on intensity (rather unusual).

3. If the light frequency is lowered below a certain value (which depends on the type of metal used), then NO electrons are observed (very very unusual!).

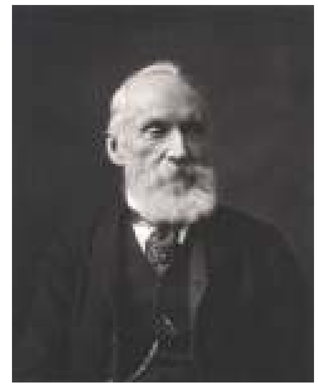
* Hertz (1887), Lenard (1890's)

A few “dark clouds”

In 1900, the prevailing view was that all the important problems in physics were solved.

"The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote.... Our future discoveries must be looked for in the sixth place of decimals." - Albert A. Michelson, 1894

"There is nothing new to be discovered in physics now. All that remains is more and more precise measurement" - Lord Kelvin, 1900



William Thomson, 1st Baron Kelvin (1824-1907)

In 1900, Kelvin described two “dark clouds” that were not yet explained using the classical theories of Newton (for particles) and Maxwell (for electromagnetic waves). He was confident that they soon would be explained.

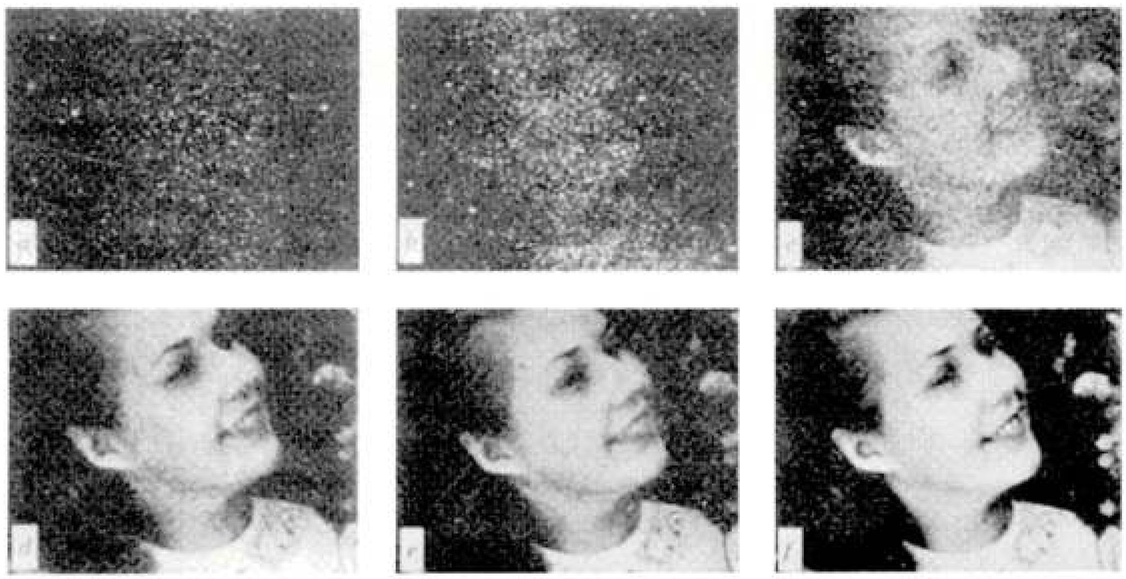
Robert Millikan (1868-1953)

One such unexplained effect (although not one of the ones that Kelvin had in mind) was the photoelectric effect. It cannot be explained using a wave theory of light. In 1905, Einstein offered an explanation which required light to be a particle, not a wave. This explained all of the observations, and was confirmed by experiments in 1915 by Robert Millikan.

Light is not only a wave, but also a particle.

Photographs taken in dimmer light look grainier.

Very very dim (about 3000 photons) | Very dim | Dim



Getting brighter | Bright | Very bright (over 30,000,000 photons)

When we detect very weak light, we find that it is made up of particles.
We call them photons.

Photons

Note: $\hbar = \frac{h}{2\pi}$

The energy of a single photon is: $h\nu$ or $\hbar\omega$

where h is Planck's constant: 6.626 x 10^{-34} Joule-sec.

One photon of visible light contains about 10^{-19} Joules (not much!)

Φ is the "photon flux," or the number of photons per second in a beam.

$$\Phi = P / h\nu$$

where P is the beam power (in watts).

TABLE 3.1 The Mean Photon Flux Density for a Sampling of Common Sources

Light Source	Mean Photon Flux Density Φ/A in units of (photons/s·m^2)
Laserbeam (10 mW, He-Ne, focused to 20 μm)	10^{26}
Laserbeam (1 mW, He-Ne)	10^{21}
Bright sunlight	10^{18}
Indoor light level	10^{16}
Twilight	10^{14}
Moonlight	10^{12}
Starlight	10^{10}

What is the energy in a light wave?

Is there a contradiction? Earlier we found that the energy density of an electromagnetic wave is:

$$U_{wave} = U_E + U_B = \varepsilon E^2$$

which does NOT depend on the frequency of the wave.

But now we're saying that each photon has energy hν, which means that the total energy density of an EM wave is:

$$\boldsymbol{U_{particle} = N \cdot h\,\nu}$$

where N is the number of photons per m^3. This explicitly *does* depends on the frequency ν of the wave.

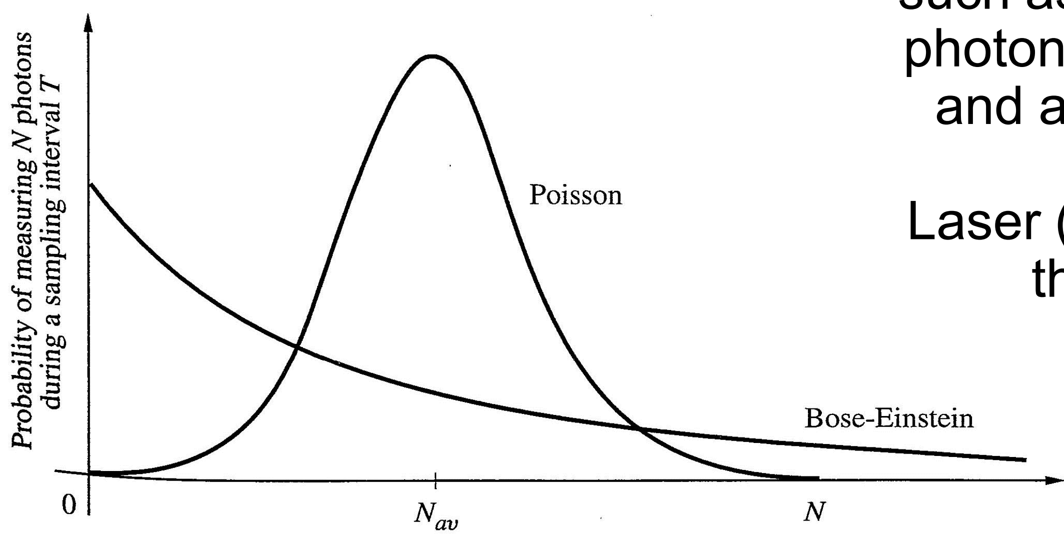
These two formulations always give the same answer: $\boldsymbol{U_{particle} = U_{wave}}$

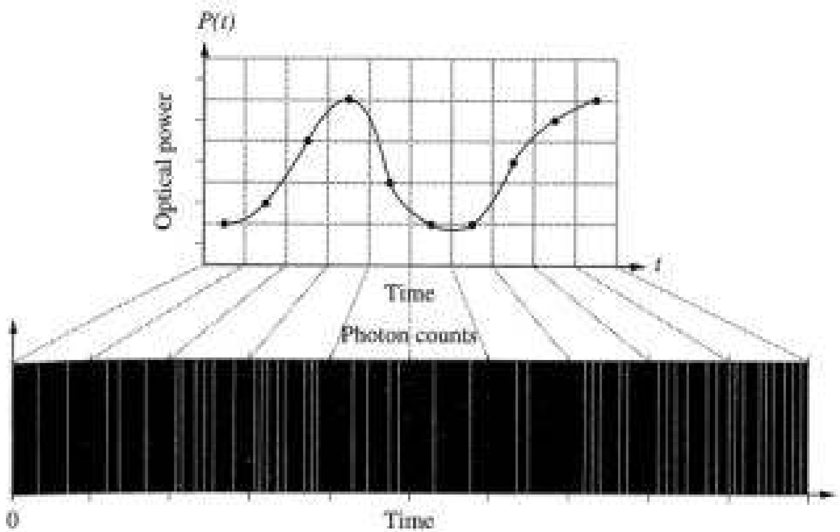
Thus, the number of photons (per unit volume) must be proportional to E^2, and the proportionality constant must depend on the frequency of the wave.

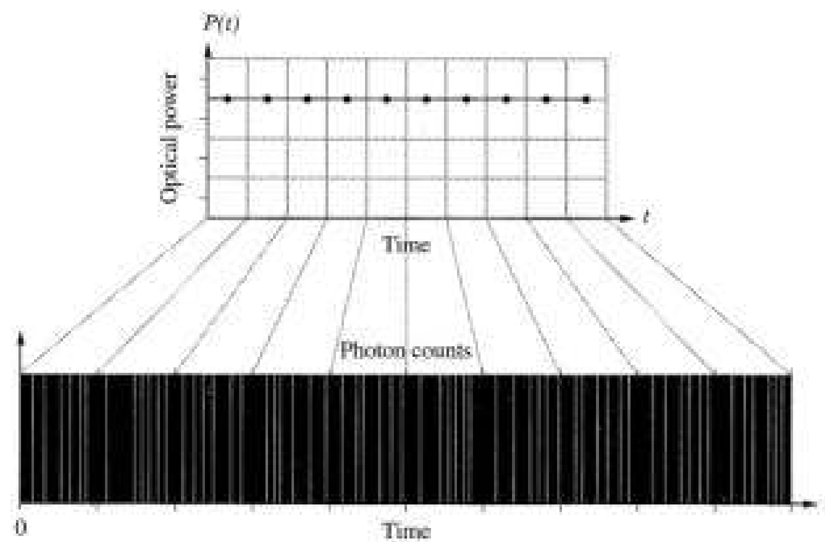
Counting photons tells us a lot about a light source.

Random (incoherent) light sources, such as stars and light bulbs, emit photons with random arrival times and a Bose-Einstein distribution.

Laser (coherent) light sources, on the other hand, have a more uniform (but still random) distribution: Poisson.



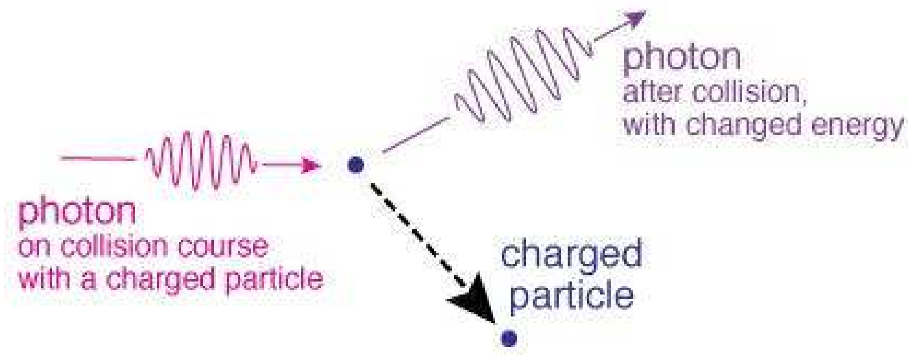




Photons have momentum: the Compton effect

Photons have no mass and always travel at the speed of light, so we can't use $p = mV$ to determine their momentum.

Instead, the momentum of a single photon is: h/λ, or $\hbar k$ since $k = 2\pi/\lambda$



Compton scattering: the photon transfers some of its energy to a particle (causing the particle to accelerate).

This causes a change in the photon's energy, and therefore in its wavelength (since $E = h\nu = hc/\lambda$).

Compton's discovery of this effect (1923) was an important proof of the particle nature of light.

Arthur H. Compton
(1892 – 1962)

Photons—radiation pressure

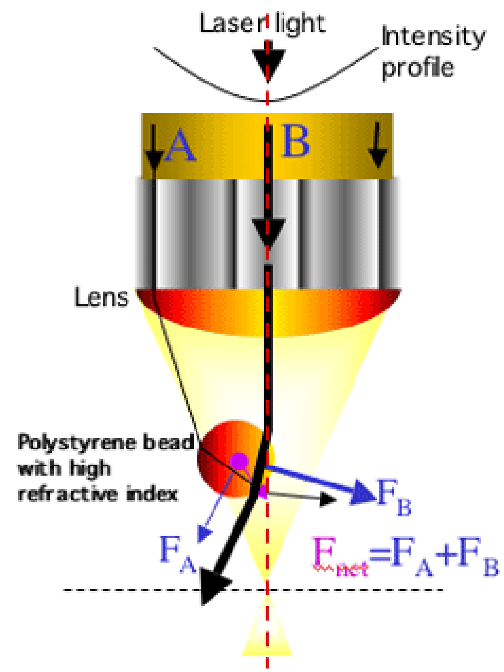
Since photons carry momentum, they can therefore exert pressure. This is known as radiation pressure.

Note that energy per unit volume is the same as pressure (which is force per unit area):

$$1\ J/m^3 = 1\ N \cdot m/m^3 = 1\ N/m^2$$

Radiation pressure is usually very small.

A few situations when radiation pressure cannot be neglected:
- Comet tails (other forces are small)
- Spacecraft trajectories (small force over a long time)
- Stellar interiors (resists collapse due to gravity)
- Really big lasers (10^{15} Watts or more!)
- Optical trapping of tiny objects by a focused laser:

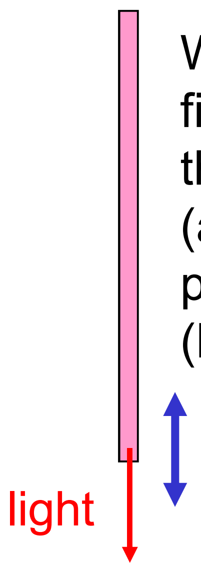


Photon momentum - current research

From *Physical Review Letters*[*], December 2008:

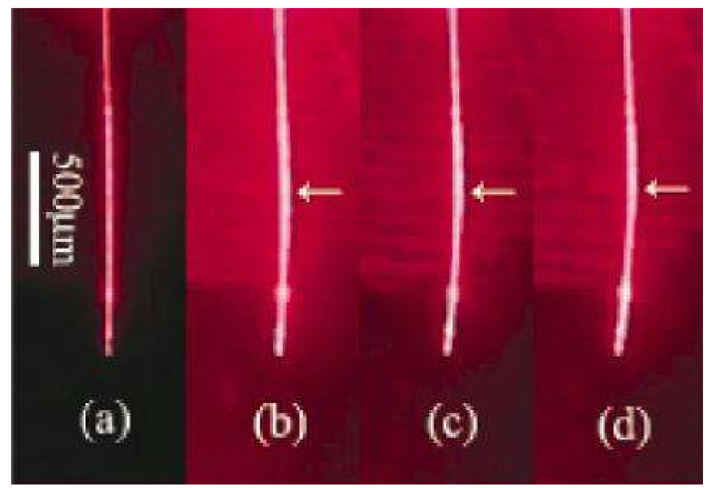
> There are two different proposals for the momentum of light in a transparent dielectric… Despite many tests and debates over nearly a century, momentum of light in a transparent dielectric remains controversial.

The two proposals:



When light exits out of an optical fiber, the momentum transferred to the fiber is:
(a) in the direction of the light propagation (1908)
(b) in the opposite direction (1909)

The experiment (2008):



[*] W. She *et al., Phys. Rev. Lett.*, 101, 243601 (2008)

Photons

"What is known of [photons] comes from observing the results of their being created or annihilated."

Eugene Hecht

What is known of nearly *everything* comes from observing the results of photons being created or annihilated.