5. Light-matter interactions: Blackbody radiation

- The electromagnetic spectrum
- Sources of light
- Boltzmann's Law
- Blackbody radiation – why do hot things glow?
- The cosmic microwave background
The electromagnetic spectrum

All of these are electromagnetic waves. The amazing diversity is a result of the fact that the interaction of radiation with matter depends on the frequency of the wave.

The boundaries between regions are a bit arbitrary…
Sources of light

Linearly accelerating charge:

http://www.cco.caltech.edu/~phys1/java/phys1/MovingCharge/MovingCharge.html

Synchrotron radiation—light emitted by charged particles deflected by a magnetic field:

Bremsstrahlung (Braking radiation)—light emitted when charged particles collide with other charged particles:
But most light in the universe is emitted by atomic and molecular vibrations.

Electrons vibrate in their motion around nuclei.

High frequency:
\(~10^{14} - 10^{17}\) Hz

Nuclei in molecules vibrate with respect to each other.

Intermediate frequency:
\(~10^{11} - 10^{13}\) Hz

Nuclei in molecules rotate.

Low frequency:
\(~10^{9} - 10^{10}\) Hz
Three types of molecule-radiation interactions

**Absorption**
- Promotes molecule to a higher energy state
- Decreases the number of photons

**Spontaneous Emission**
- Molecule drops from a high energy state to a lower state
- Increases the number of photons

**Stimulated Emission**
- Molecule drops from a high energy state to a lower state
- The presence of one photon stimulates the emission of a second one
- This process has no analog in classical physics - it can only be understood with quantum mechanics!

Key idea: conservation of energy
Atoms and molecules have discrete energy levels and can interact with light only by making a transition from one level to another.

A typical molecule’s energy levels:

- **Ground electronic state**
  - Lowest vibrational and rotational level

- **1st excited electronic state**
  - Excited vibrational and rotational level

- **2nd excited electronic state**
  - Lowest vibrational and rotational level of this electronic “manifold”

Computing the energies of these levels is usually difficult, except for very simple atoms or molecules (e.g., H₂).
Different atoms emit light at different frequencies.

This is the emission spectrum from hydrogen. It is simple because hydrogen has only one electron.

Bigger atoms have much more complex spectra, because they are more complex objects. Molecules are even more complex.
In what energy levels do molecules reside?

In the absence of collisions, (low T) molecules tend to remain in the lowest energy state available.

Collisions can knock a molecule into a higher-energy state. The higher the temperature, the more this happens.

The ratio of the population densities of two states is:

\[ \frac{N_2}{N_1} = \exp\left(\frac{-\Delta E}{k_B T}\right) \],

where \( \Delta E = E_2 - E_1 = h\nu \)
Ni is the number density (also known as the population density) of molecules in state i (i.e., the number of molecules per cm³).

\[ N_i \propto \exp \left( -\frac{E_i}{k_B T} \right) \]

\( N_i \) is the number density (also known as the population density) of molecules in state \( i \) (i.e., the number of molecules per cm³).

\( T \) is the temperature, and \( k_B \) is Boltzmann’s constant, \( k_B = 1.38 \times 10^{-23} \) J/K.

Boltzmann population factor

Boltzmann didn't know quantum mechanics. But this result works equally well for quantum or classical systems.
Blackbody Radiation

Blackbody radiation is the radiation emitted from a hot body.

It results from a combination (balance) of spontaneous emission, absorption, and stimulated emission occurring in a medium at a given temperature.

Imagine a box, the inside of which is completely black.

Consider the radiation emitted from the small hole.

(This description is artificial, just designed to phrase the problem in a well-posed way. The result is much more general than just boxes with holes.)
The classical physics approach to the blackbody question

Lord Rayleigh figured out how to count the electromagnetic modes inside a box.

Basically, the result follows from requiring that the electric field must be zero at the internal surfaces of the box.

\( \text{Number of modes per unit frequency per unit volume} = \frac{8\pi v^2}{C^3} \)

For higher frequencies you can fit more modes into the cavity. For double the frequency, four times as many modes.

\( \# \text{ modes} \sim v^2 \)

And the energy per mode is \( k_B T \), according to Boltzmann (this is called the “equipartition theorem”)

John William Strutt, 3rd Baron Rayleigh (1842 - 1919)
The Rayleigh-Jeans Law

Rayleigh-Jeans law (circa 1900):

irradiance of a radiation field \( I_{RJ}(\nu) = \frac{8\pi\nu^2k_BT}{c^2} \)

Note: the units of this expression are correct. Strictly speaking, \( I(\nu) \) is power per unit area per unit bandwidth, such that the integral \( \int I(\nu) d\nu \) gives an answer with units of irradiance (power per unit area). The quantity \( I(\nu) \) is called the “spectral irradiance”.

Total energy radiated from a black body: \( \int I_{RJ}(\nu) d\nu = \infty \)

uh-oh… the "ultraviolet catastrophe"

Rayleigh (and others) knew this was wrong. They thought that it merely showed the inadequacy of the idea of equipartition (i.e., the idea that each mode should have the same average energy).

But something even more profound was going on…
Einstein A and B Coefficients

In 1916, Einstein considered the various transition rates between molecular states (say, 1 and 2) involving light of irradiance, $I$:

- Spontaneous emission rate = $A N_2$
- Absorption rate = $B_{12} N_1 I$
- Stimulated emission rate = $B_{21} N_2 I$

In equilibrium, the rate of upward transitions equals the rate of downward transitions:

$$A N_2 + B_{21} N_2 I = \text{Down} = \text{Up} = B_{12} N_1 I$$

Dividing by $N_1 (A + B_{21} I)$ yields $N_2/N_1$:

$$N_2 / N_1 = (B_{12} I) / (A + B_{21} I) = \exp(-h \nu/k_B T)$$

Recalling the Boltzmann distribution for $N_2 / N_1$ and using $\nu = E_2 - E_1$.
Solving for Blackbody Radiation

\[
N_2 / N_1 = \frac{(B_{12} I)}{(A + B_{21} I)} = \exp(-h \nu / k_B T)
\]

Solve this expression for the irradiance \( I \):

\[
I = \frac{(A/B_21)}{\left[ \left( \frac{B_{12}}{B_{21}} \right) \exp(h \nu / k_B T) - 1 \right]}
\]

Now, we expect that when \( T \to \infty \), \( I \) should also \( \to \infty \). But as \( T \to \infty \), the exponential \( \exp(h \nu / k_B T) \to 1 \).

So we must require: \( (B_{12} / B_{21}) = 1 \)

\[
\text{coefficient for “up”} = \text{coefficient for “down”!}
\]

\[
I = \frac{(A/B)}{\left[ \exp(h \nu / k_B T) - 1 \right]}
\]
The Planck Radiation Law

\[ I = \frac{A}{B} / \left[ \exp\left(\frac{h\nu}{k_BT}\right) - 1 \right] \]

We can determine \( A/B \) by requiring that, in the limit \( h\nu \ll k_BT \), this expression must give the same result as the Rayleigh-Jeans law.

Then, we find the irradiance per unit frequency, \( I_\nu \):

\[ I_\nu = \frac{8\pi h\nu^3 / c^2}{\exp(h\nu / k_BT) - 1} \]

This is the total irradiance per unit frequency (that is, in a range from \( \nu \) to \( \nu + \delta\nu \)) emitted by an arbitrary blackbody in equilibrium at temperature \( T \).

We considered only two levels, but our approach was general and so applies to any two levels and hence to any multi-level system.

This result was proposed by Max Planck in 1900, but he did not have any justification other than the excellent match to experimentally measured blackbody spectra. It was the origin of the introduction of “Planck’s constant”, which we now know to be one of the universe’s fundamental constants.
This solves the ultraviolet catastrophe.

\[ I(\nu) \sim \nu^2 \]

Rayleigh-Jeans: \( I(\nu) \sim \nu^2 \)

Planck / Einstein: \( I(\nu) \sim \frac{\nu^3}{\exp[\hbar \nu / k_B T] - 1} \)

At low frequencies, the two results are equivalent.

At high frequencies, Einstein’s result goes back down to zero, so the integral of \( I(\nu) \) is finite.
Blackbody Emission Spectrum

The higher the temperature, the more the emission at all wavelengths and the shorter the peak wavelength.

The sun’s surface is ~5800º K, so its blackbody spectrum peaks at ~500 nm—in the green. However, blackbody spectra are broad, so it contains red, yellow, green, and blue, too, and hence looks white.
We can tell how hot a star is by its emission spectrum.
Wien’s Law: The blackbody peak wavelength scales as $1/\text{temperature}$.

$\lambda_{\text{max}} = \frac{b}{T}$

$b = 2.90 \times 10^{-3}$ meter K

Wien’s Law constant
The earth is a blackbody, too.

![Black Body Emission Curves of the Sun and Earth](image)

- **Spectral irradiance** \([W/(m^2 \mu m)]\)
- **Wavelength** \([\mu m]\)

- **Sun** (scaled by a factor of \(10^{-6}\))
- **Earth**
So are most things. Including you.

Infrared vision goggles work by sensing the blackbody radiation from objects at or near room temperature.
Cosmic Microwave Background

The universe is also a blackbody.

Measurements of the uniformity of the microwave background tell us about the conditions in the early formation of the universe.

Theory and observation agree so well that you cannot distinguish them on this plot!

The stunningly good fit (and the amazing level of isotropy) are considered to be convincing proof of the Big Bang description of the birth of the universe.
Total Emitted Blackbody Irradiance: the Stefan-Boltzmann Law

To find the total emitted irradiance, we perform the integral of blackbody emitted light over all frequencies and (half of the) angles:

\[ I_{\text{total}} = \int_0^\infty dv \int_0^{2\pi} d\phi \int_0^{\pi/2} I_v d\theta \cos \theta d\theta \]

\[ = \frac{2\pi}{h^3 c^2} (k_B T)^4 \int_0^\infty \frac{x^3}{e^x - 1} dx \]

where \( x = \frac{h\nu}{k_B T} \)

This yields a very simple result for the total emitted blackbody irradiance of any object:

\[ I_{\text{total}} = \sigma_S T^4 \]

where: \( \sigma_S = \frac{2\pi^5 k_B^4}{15 h^3 c^2} = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \)

\( \sigma_S \) is called the Stefan-Boltzmann constant (not to be confused with Boltzmann’s constant, \( k_B \)).