11. Dispersion and Ultrashort Pulses II

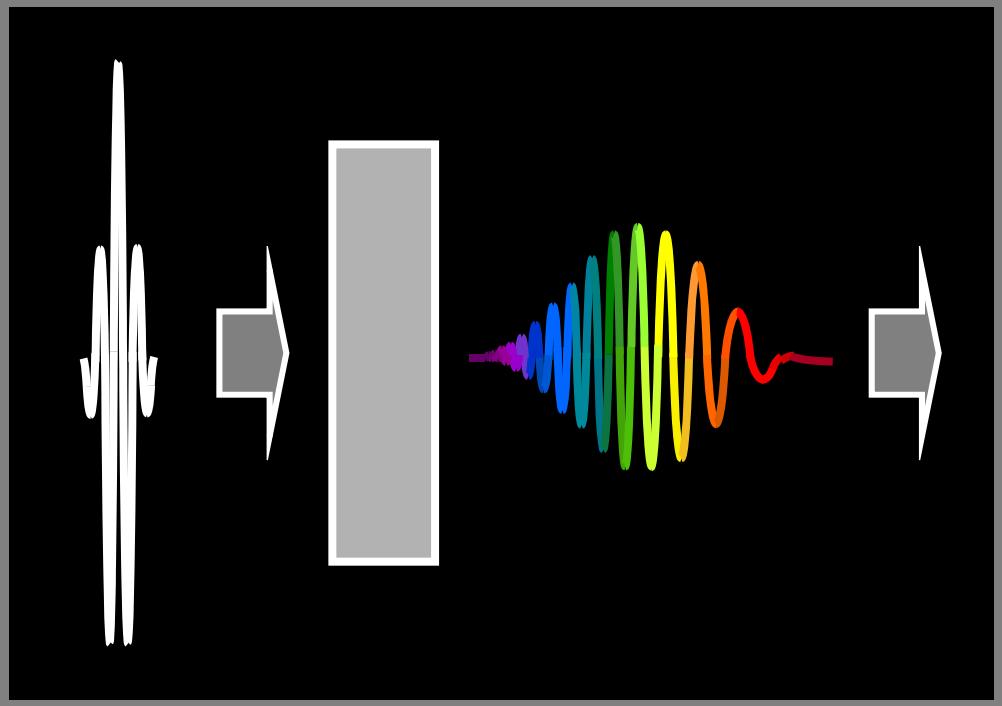
Generating negative group-
delay dispersion – angular dispersion

Pulse compression

Prisms

Gratings

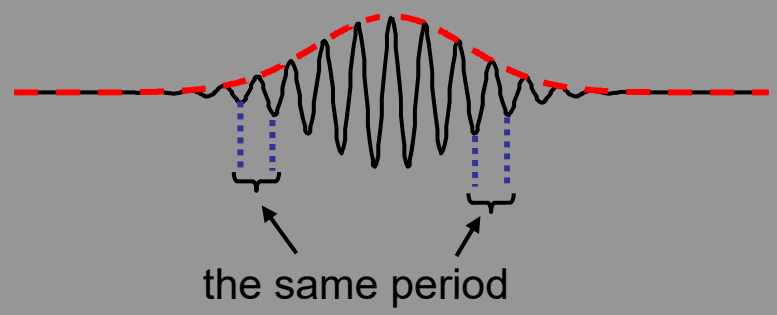
Chirped mirrors



Chirped vs. transform-limited

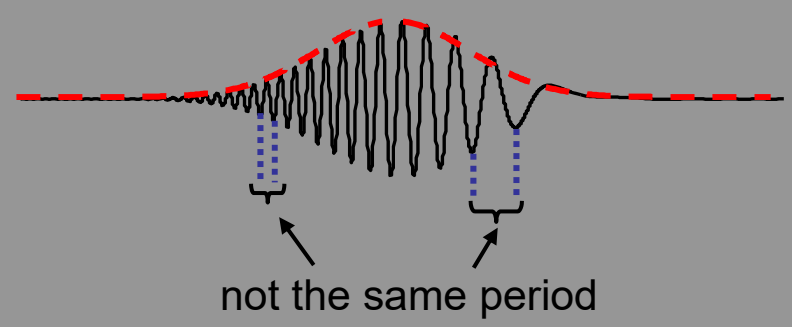
A transform-limited pulse:

- satisfies the 'equal' sign in the uncertainty relation $\Delta\nu\ \Delta t \geq C$
- is as short as it could possibly be, given the spectral bandwidth
- has an envelope function which is REAL (spectral phase $\Phi(\omega) = 0$)
- has an electric field that can be computed directly from $S(\omega)$
- exhibits zero chirp:



A chirped pulse:

- satisfies the 'greater than' sign in the relation $\Delta\nu\ \Delta t \geq C$
- is longer than it could be, given the spectral bandwidth
- has an envelope function which is COMPLEX (spectral phase $\Phi(\omega) \neq 0$)
- requires knowledge of more than just $S(\omega)$ in order to determine $E(t)$
- exhibits non-zero chirp:



Pulse propagation and broadening

After propagating a distance z, an (initially) unchirped Gaussian of initial duration t_G^{in} becomes:

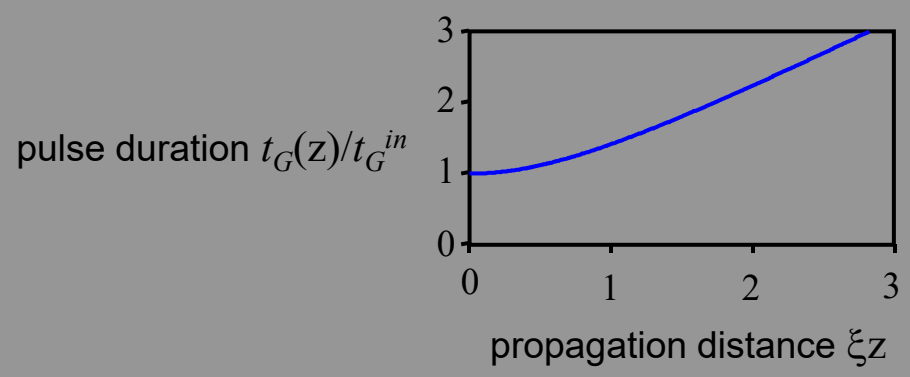
$$E(t,z) \propto \exp\left[-\frac{\left(t - z/\mathrm{V}_g\left(\omega_p\right)\right)^2}{\left(t_G^{in}\right)^2 + 2ik''z}\right]$$

$$= \exp\left[-\frac{\tau^2}{\left(t_G^{in}\right)^2\left(1+\xi^2 z^2\right)}\left(1 - i\xi z\right)\right]$$

dimensionless chirp parameter $\gamma \propto z$

where $\xi = \dfrac{2k''}{\left(t_G^{in}\right)^2}$

pulse duration increases with z



Most of the time, γ is positive (normal dispersion, k'' > 0), thus generating positive GDD (lower frequencies arrive earlier than higher ones).

So how can we generate negative GDD?

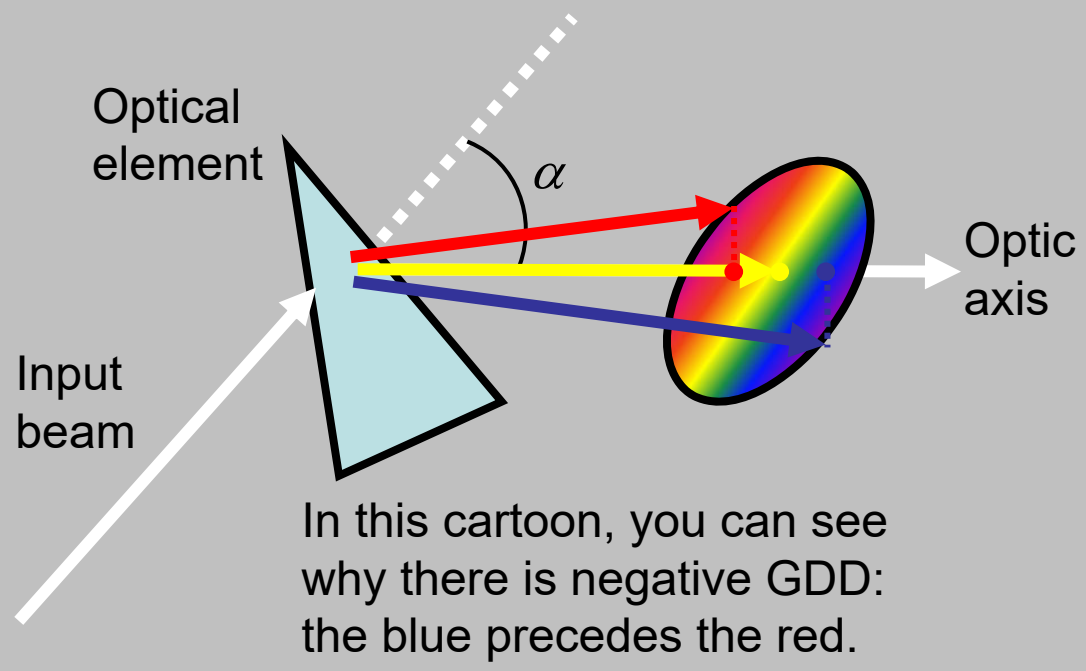
This is a big issue because pulses spread further and further as they propagate through materials.

We need a way of generating negative GDD to compensate.

Negative GDD
Device

Angular dispersion produces GDD

Suppose that an optical element introduces angular dispersion.



In this cartoon, you can see why there is negative GDD: the blue precedes the red.

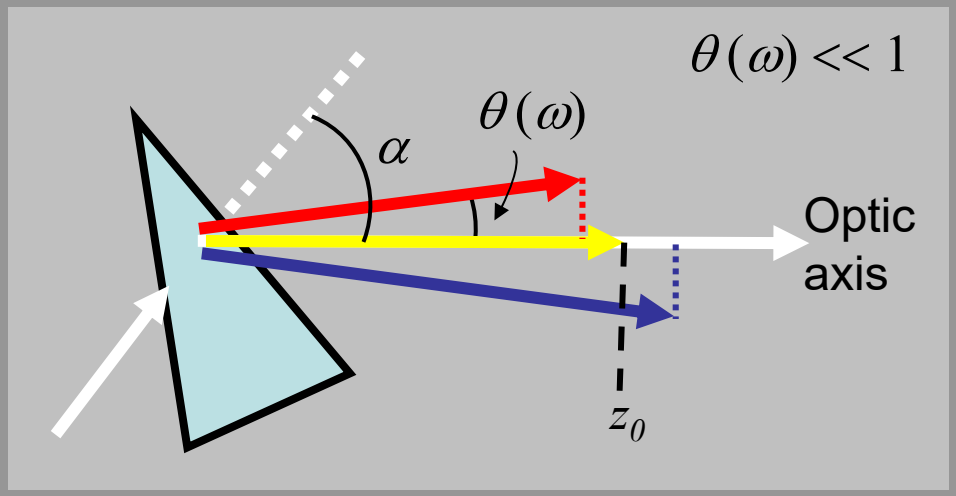
We'll need to compute the projection onto the optic axis (the propagation direction of the center frequency of the pulse).

GDD from angular dispersion

Taking the projection of $\vec{k}(\omega)$ onto the optic axis, a given frequency ω sees a phase delay of:

z axis

$$\varphi(\omega) = \vec{k}(\omega) \cdot \vec{r}_{optic\ axis}$$
$$= k(\omega)\, z \cos[\theta(\omega)]$$
$$= (\omega / c_0)\, z \cos[\theta(\omega)]$$



We're considering only the GDD due to the angular dispersion $\theta(\omega)$ and *not* that of the prism material. Also we assume that $n_{air} = 1$.

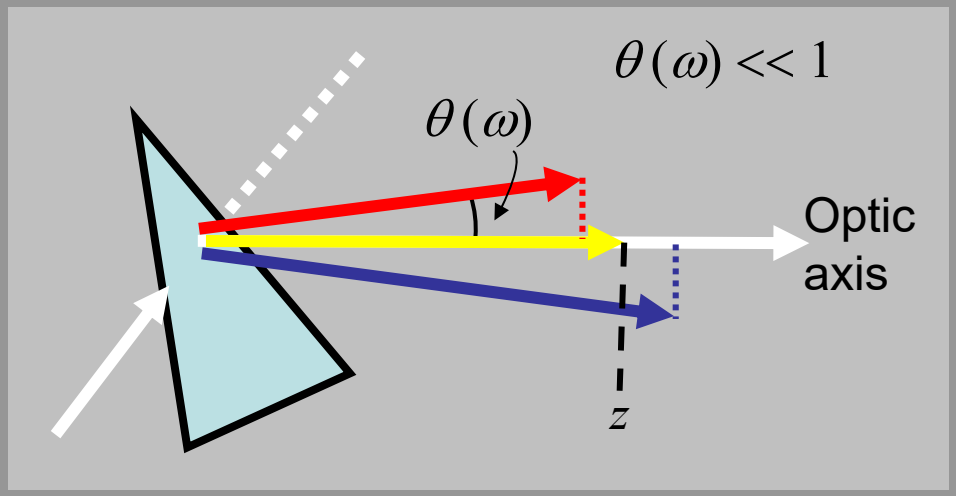
In fact, this is only an approximate result. You can tell because $\varphi(\omega)$ should be an odd function of ω but in this expression it isn't. The more accurate result is:

$$\varphi(\omega) = (\omega / c_0)\, z \cos[\theta(\omega)] \left(\frac{\cos\alpha}{\cos\left[\alpha - \theta(\omega)\right]} \right)$$

But since $\alpha >> \theta$, we are going to ignore the factor in parentheses.

GDD from angular dispersion

Now, we need the 2nd derivative of $\varphi(\omega)$ in order to calculate the GDD that results from this frequency-dependent difference in optical path lengths.



$$\frac{d\varphi}{d\omega} = \frac{z}{c_0}\cos(\theta) - \frac{\omega z}{c_0}\sin(\theta)\frac{d\theta}{d\omega}$$

$$\frac{d^2\varphi}{d\omega^2} = -\frac{z}{c_0}\sin(\theta)\frac{d\theta}{d\omega} - \frac{z}{c_0}\sin(\theta)\frac{d\theta}{d\omega} - \omega\frac{z}{c_0}\cos(\theta)\left(\frac{d\theta}{d\omega}\right)^2 - \omega\frac{z}{c_0}\sin(\theta)\frac{d^2\theta}{d\omega^2}$$

But $\theta << 1$, so the sine terms can be neglected, and $\cos(\theta) \sim 1$.

Angular dispersion yields negative GDD.

Copying the result from the previous slide:

$$\Rightarrow \quad \boxed{\left.\frac{d^2\varphi}{d\omega^2}\right|_{\omega_0} \approx -\frac{\omega_0 z}{c_0}\left(\left.\frac{d\theta}{d\omega}\right|_{\omega_0}\right)^2}$$

The GDD due to angular dispersion is ***always negative***!

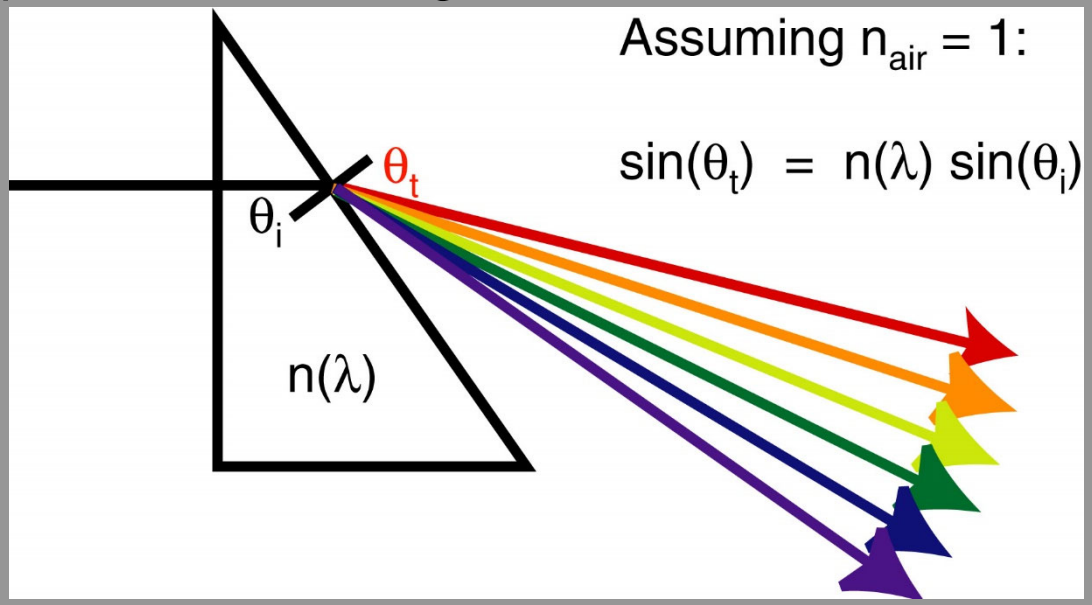
Also, note that **we did not assume anything at all** about $\theta(\omega)$ (other than that it is a smoothly varying function of ω).

Thus, it doesn't matter what kind of device gave rise to the angular dispersion (i.e., whether it was a prism or a diffraction grating or whatever). Angular dispersion always produces negative GDD.

Let's consider prisms first.

Prisms disperse light due to refraction

Because the refractive index depends on wavelength, the refraction angle also depends on wavelength.



Because n generally decreases with wave-length ($dn/d\lambda < 0$), smaller wavelengths experience greater refraction angles.

Differentiating implicitly with respect to λ:

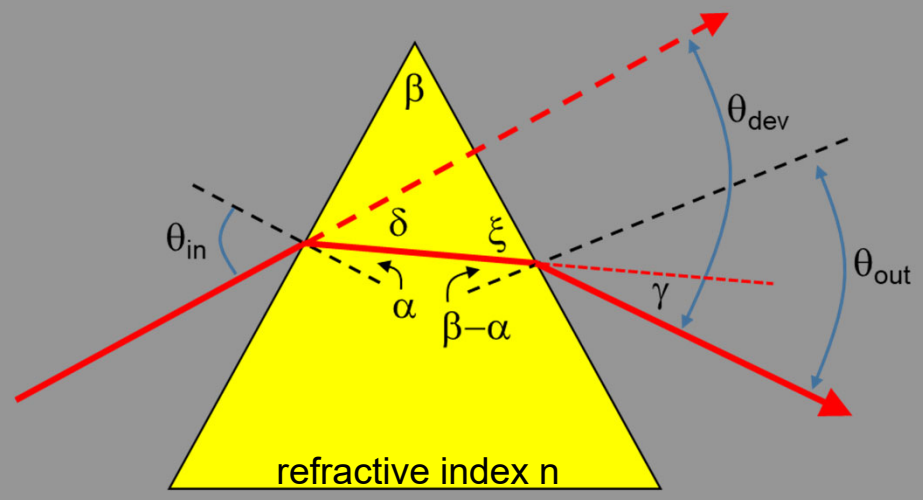
$$\cos(\theta_t)\frac{d\theta_t}{d\lambda} = \frac{dn}{d\lambda}\sin(\theta_i)$$

We obtain the "prism angular dispersion:"

$$\frac{d\theta_t}{d\lambda} = \frac{dn}{d\lambda}\frac{\sin(\theta_i)}{\cos(\theta_t)}$$

Diffraction by a prism is not so intuitive

The angle at which a ray emerges after refracting twice (measured relative to the original propagation direction) is known as the **angle of deviation**, θ_{dev}.



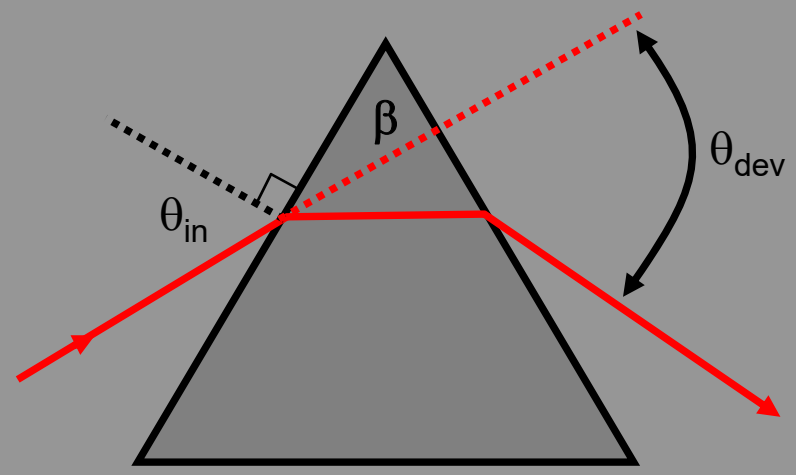
Using geometry and Snell's Law (twice), one can compute the relation between the input angle and the angle of deviation:

$$\theta_{dev} = \theta_{in} - \beta + \sin^{-1}\left[n \sin\left(\beta - \sin^{-1}\left(\frac{\sin\theta_{in}}{n} \right) \right) \right]$$

This is a non-monotonic function! As θ_{in} increases, θ_{dev} passes through a minimum.

One can also prove that, when θ_{in} is chosen such that θ_{dev} is minimized, this is the condition for symmetric propagation: the ray inside the prism is parallel to the base of the prism (so $\alpha = \beta - \alpha$).

Prism refraction

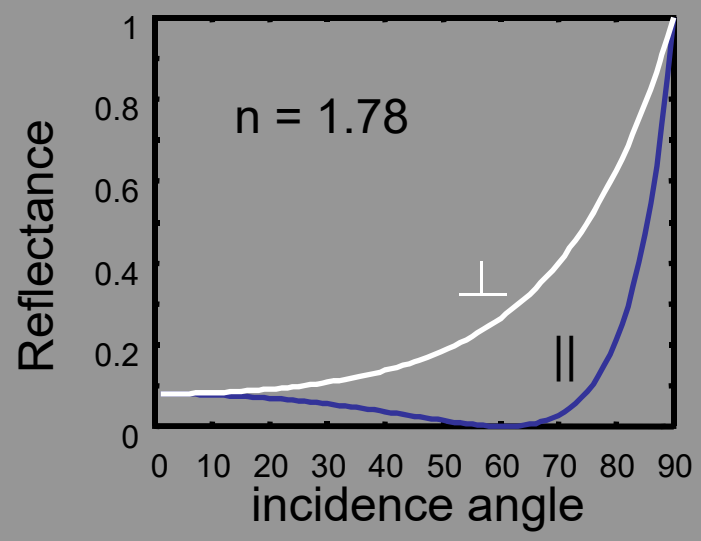


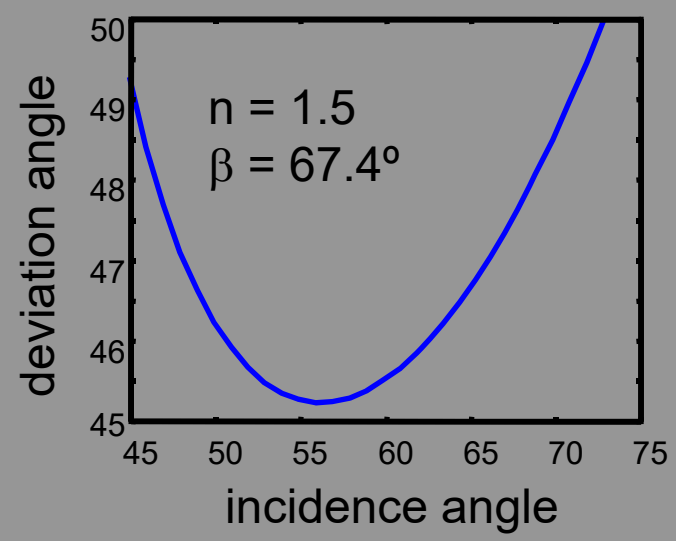
In principle, we are free to specify:

- the apex angle β
- the angle of incidence θ_{in}

These are chosen using two conditions:

- Brewster condition for minimum reflection loss (|| polarization)
- minimum deviation condition (symmetric propagation)

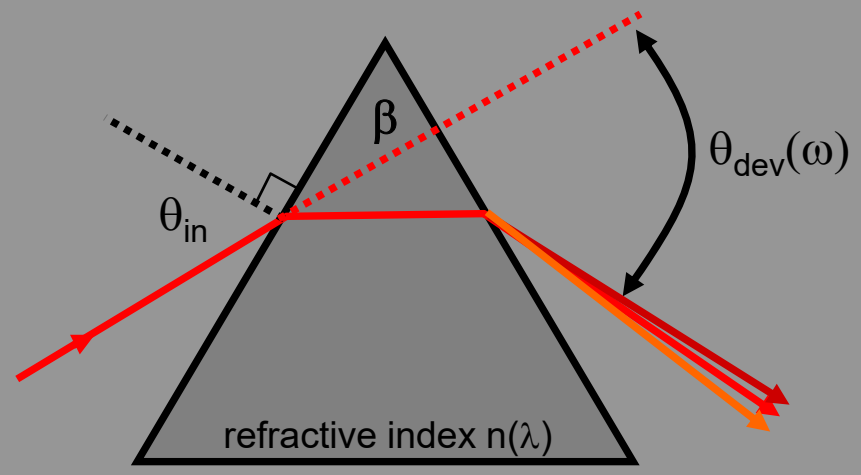




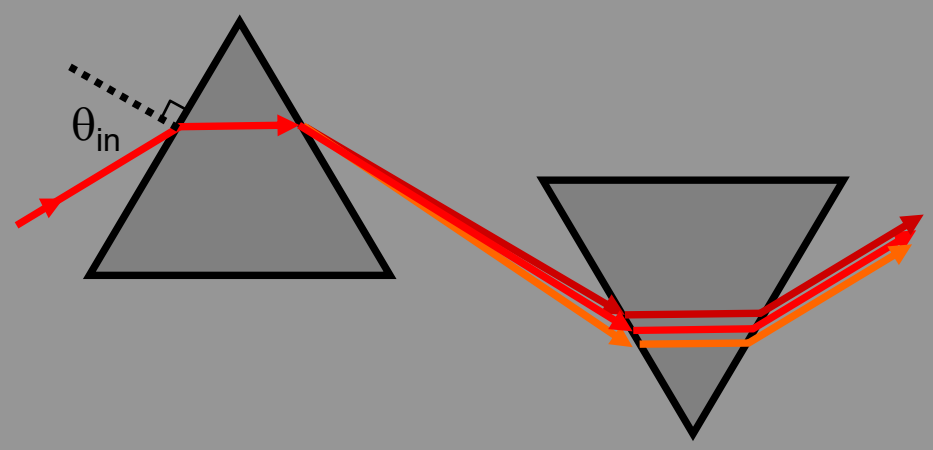
Prism: beam divergence

One problem: the broadband beam emerging from a prism is diverging.

If you want to use the beam after the prism, that's inconvenient.



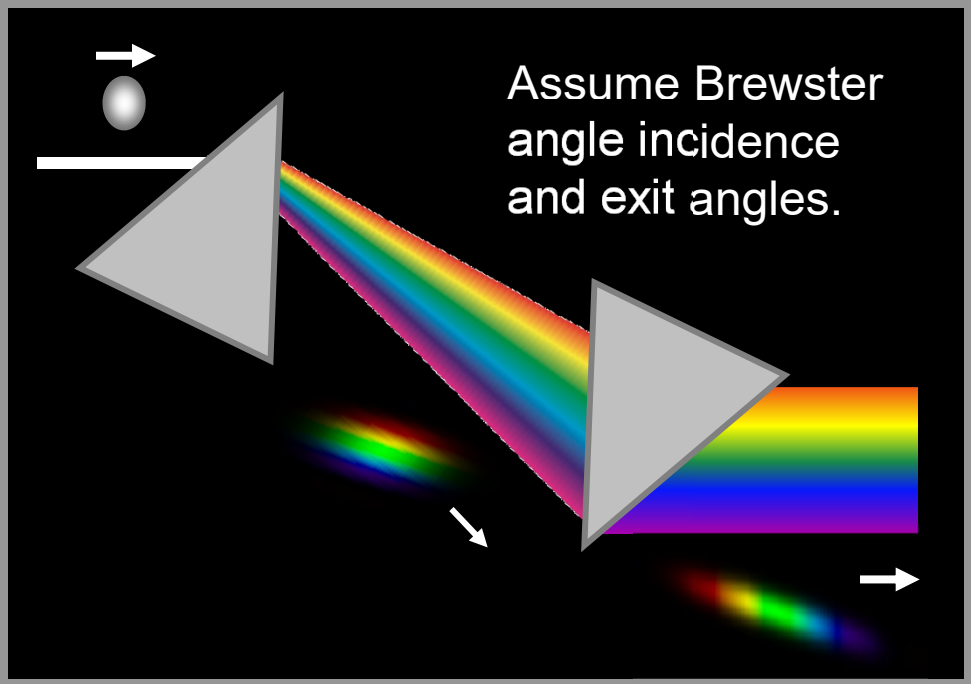
Solution: use a 2nd prism



No longer diverging.
But spectrally dispersed.

Output beam || input beam

A prism pair has negative GDD.



Let's write the GDD including both the angular dispersion and the material dispersion.

Let L_{prism} be the length of the path inside the prisms and L_{sep} be the prism separation.

$$\left.\frac{d^2\varphi}{d\omega^2}\right|_{\omega_0} \approx -4L_{sep}\frac{\lambda_0^3}{2\pi c_0^{\,2}}\left(\left.\frac{dn}{d\lambda}\right|_{\lambda_0}\right)^2 + L_{prism}\frac{\lambda_0^3}{2\pi c_0^{\,2}}\left.\frac{d^2n}{d\lambda^2}\right|_{\lambda_0}$$

Always negative!

This term assumes that the beam grazes the tip of each prism

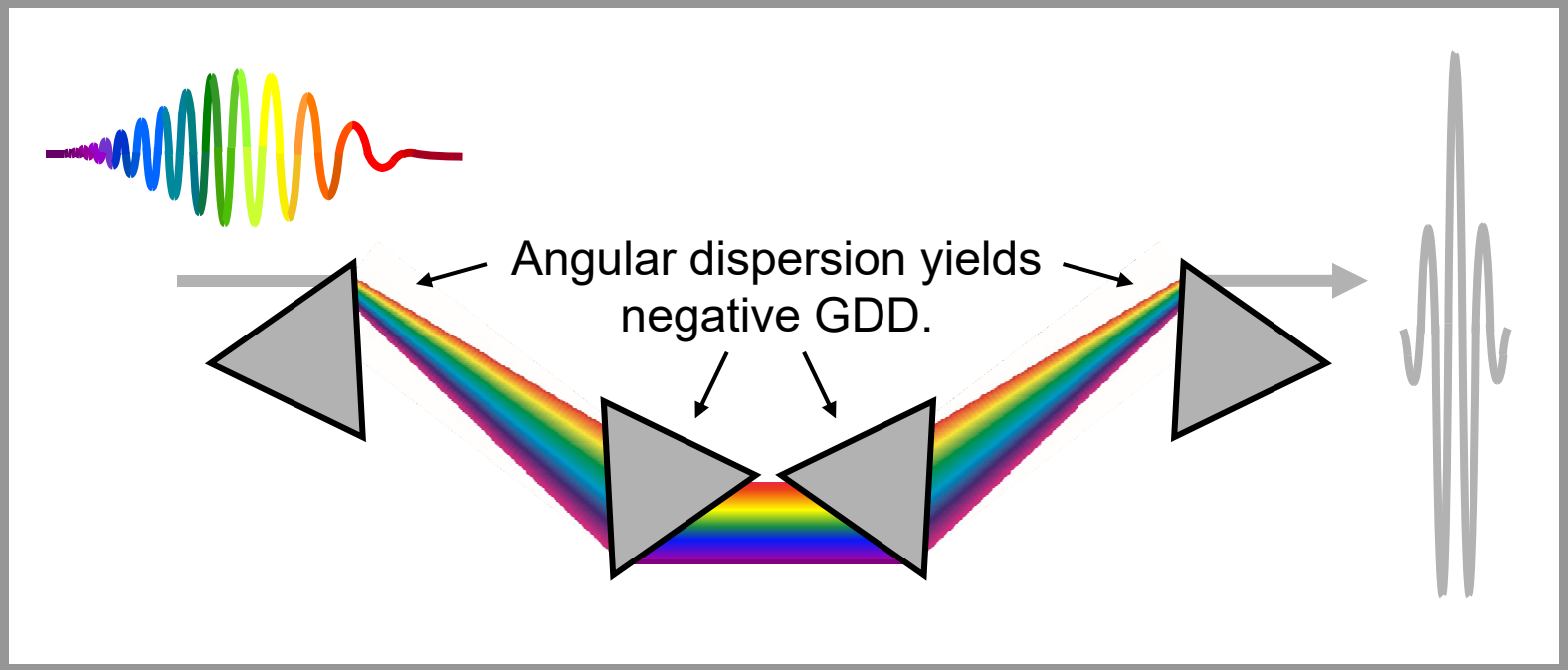
This term allows the beam to pass through an additional length, L_{prism}, of prism material.

Always positive (in visible and near-IR)

We can independently vary L_{sep} or L_{prism} to tune the GDD!

Four prism Pulse Compressor

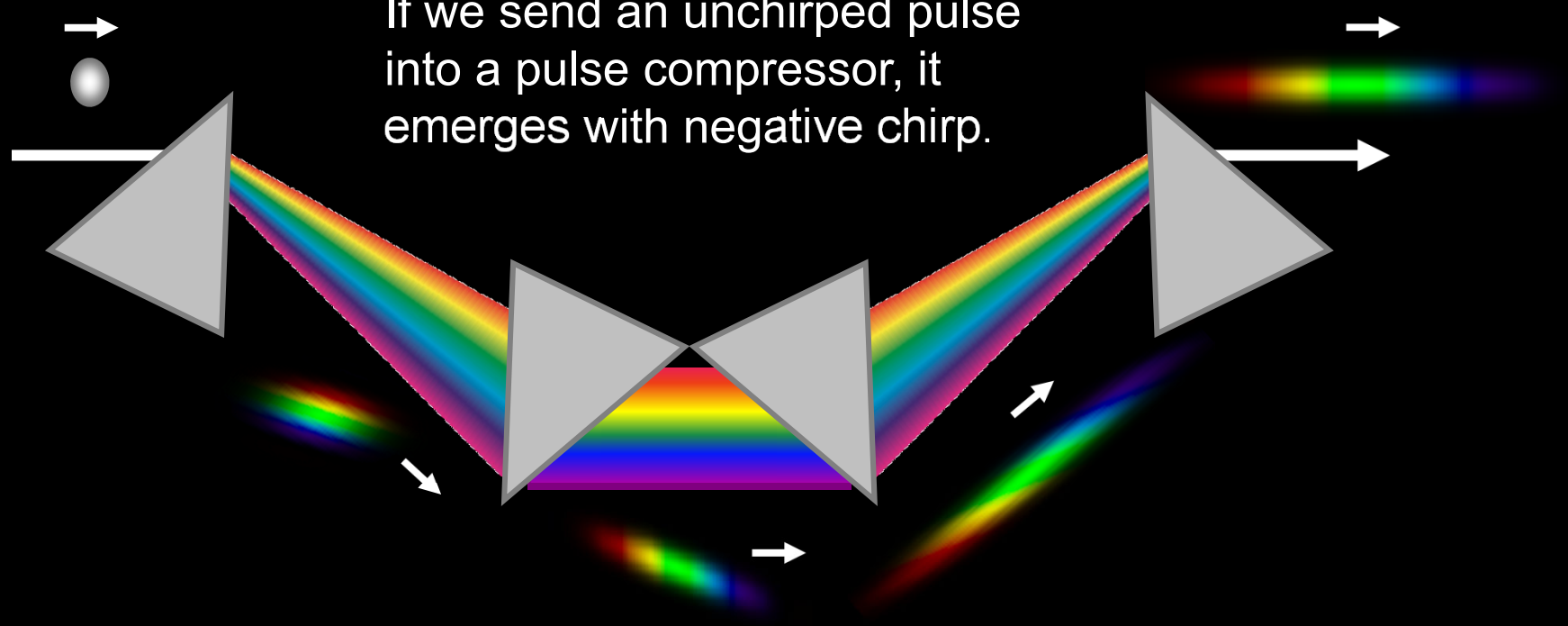
This device, which also puts the pulse back together, has **negative** group-delay dispersion and hence can compensate for propagation through materials (i.e., for positive chirp due to material dispersion).



It's routine to stretch and then compress ultrashort pulses by factors of >1000.

What does the pulse look like inside a pulse compressor?

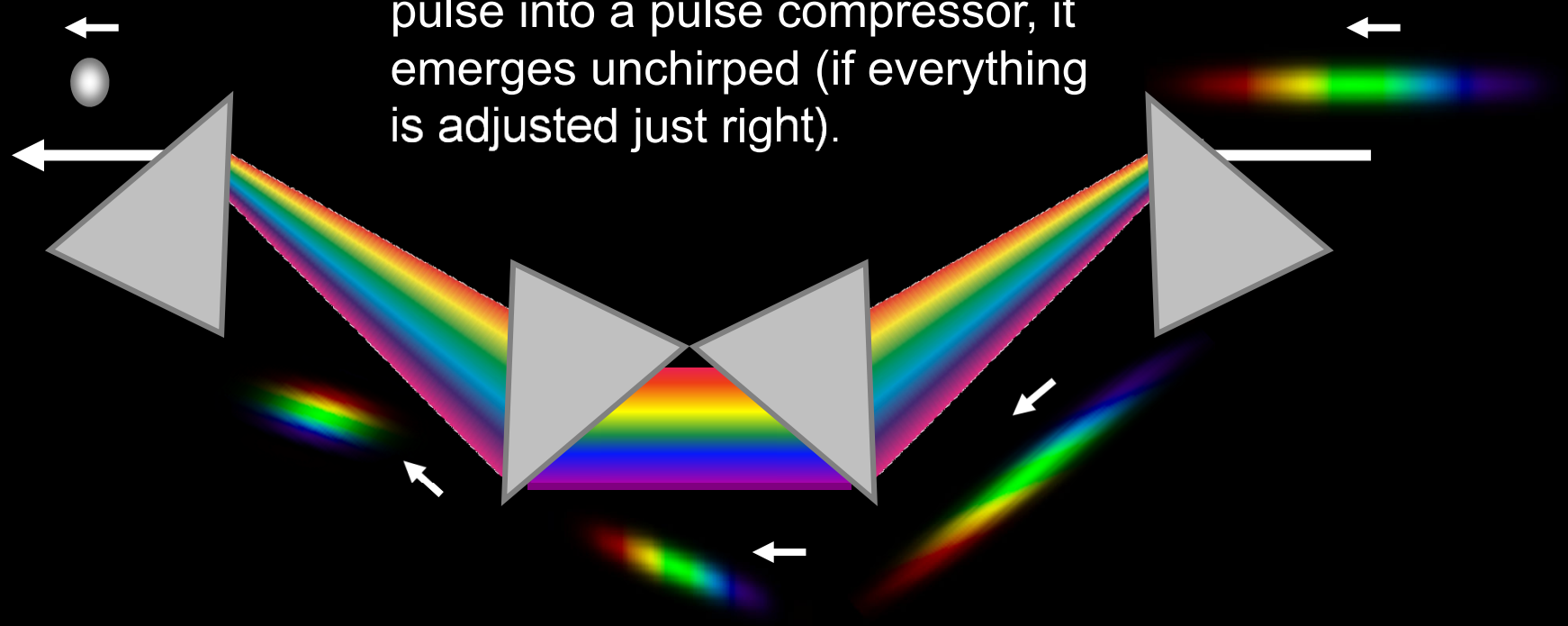
If we send an unchirped pulse into a pulse compressor, it emerges with negative chirp.



Note all the spatio-temporal distortions.

What does the pulse look like inside a pulse compressor?

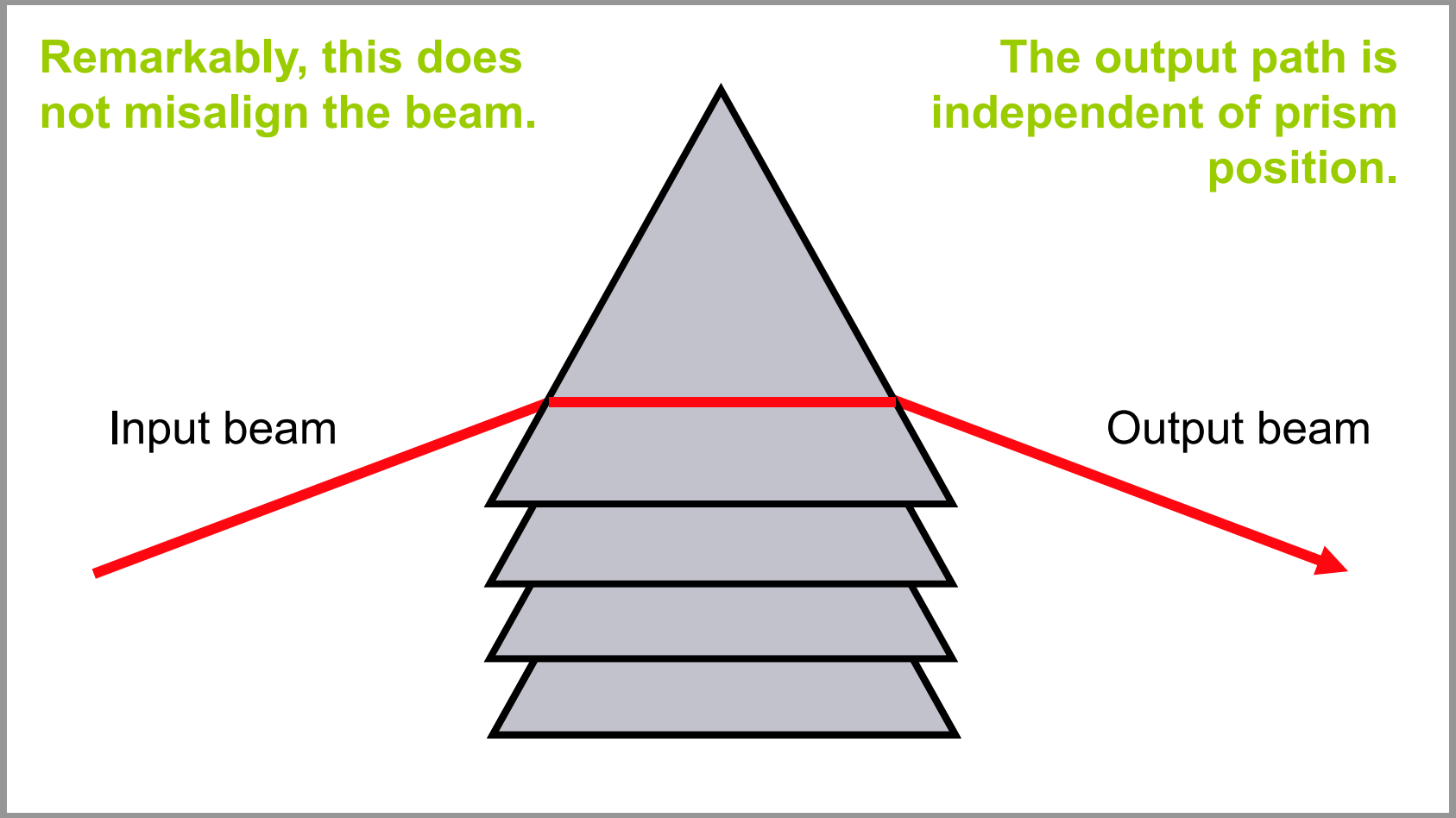
If we send a positively chirped pulse into a pulse compressor, it emerges unchirped (if everything is adjusted just right).



Note all the spatio-temporal distortions.

Adjusting the GDD maintains alignment.

Any prism in the compressor can be translated perpendicular to the beam path to add glass and reduce the magnitude of negative GDD.



Incorporating a four-prism pulse compressor into the laser cavity was a revolutionary advance.

112 IEEE JOURNAL OF QUANTUM ELECTRONICS, VOL. QE-22, NO. 1, JANUARY 1986

Design Considerations for a Femtosecond Pulse Laser Balancing Self Phase Modulation, Group Velocity Dispersion, Saturable Absorption, and Saturable Gain

JANIS A. VALDMANIS AND R. L. FORK

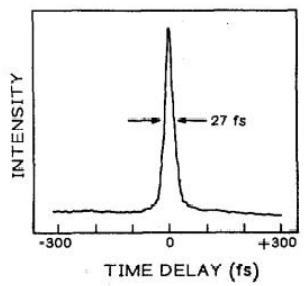


Fig. 1. Autocorrelation trace of pulses having 27 fs duration assuming $sech^2$ pulse shape.

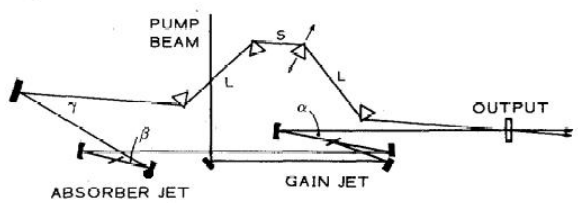


Fig. 2. Schematic diagram of the six-mirror ring cavity incorporating a four-prism sequence for the adjustment of intracavity group velocity dispersion.

Generation of optical pulses shorter than 0.1 psec by colliding pulse mode locking

R. L. Fork, B. I. Greene, and C. V. Shank
Bell Telephone Laboratories, Holmdel, New Jersey 07733

Appl. Phys. Lett. **38**, 671 (1981)

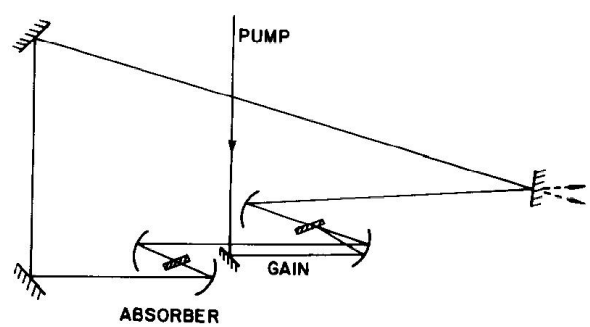
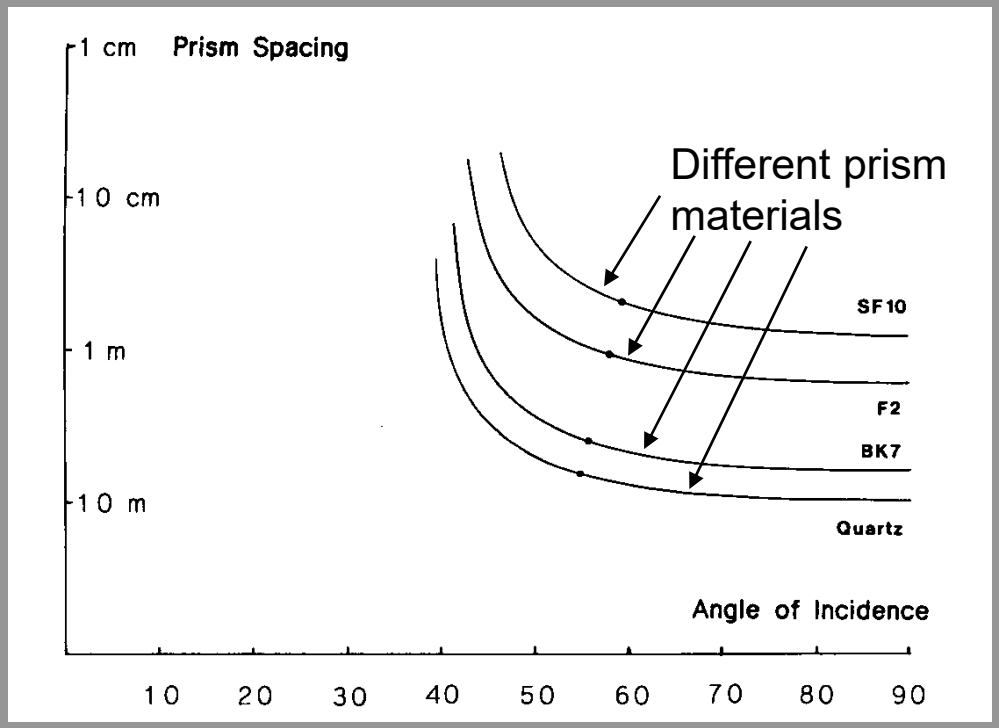


FIG. 1. Schematic diagram of ring laser used for CPM. The focusing mirrors for the gain region have a 10-cm radius and those for the absorber have a 5-cm radius. Cavity roung-trip time was 10 nsec.

The required separation between prisms in a pulse compressor can be large.

The GDD ∝ the prism separation and the square of the dispersion.



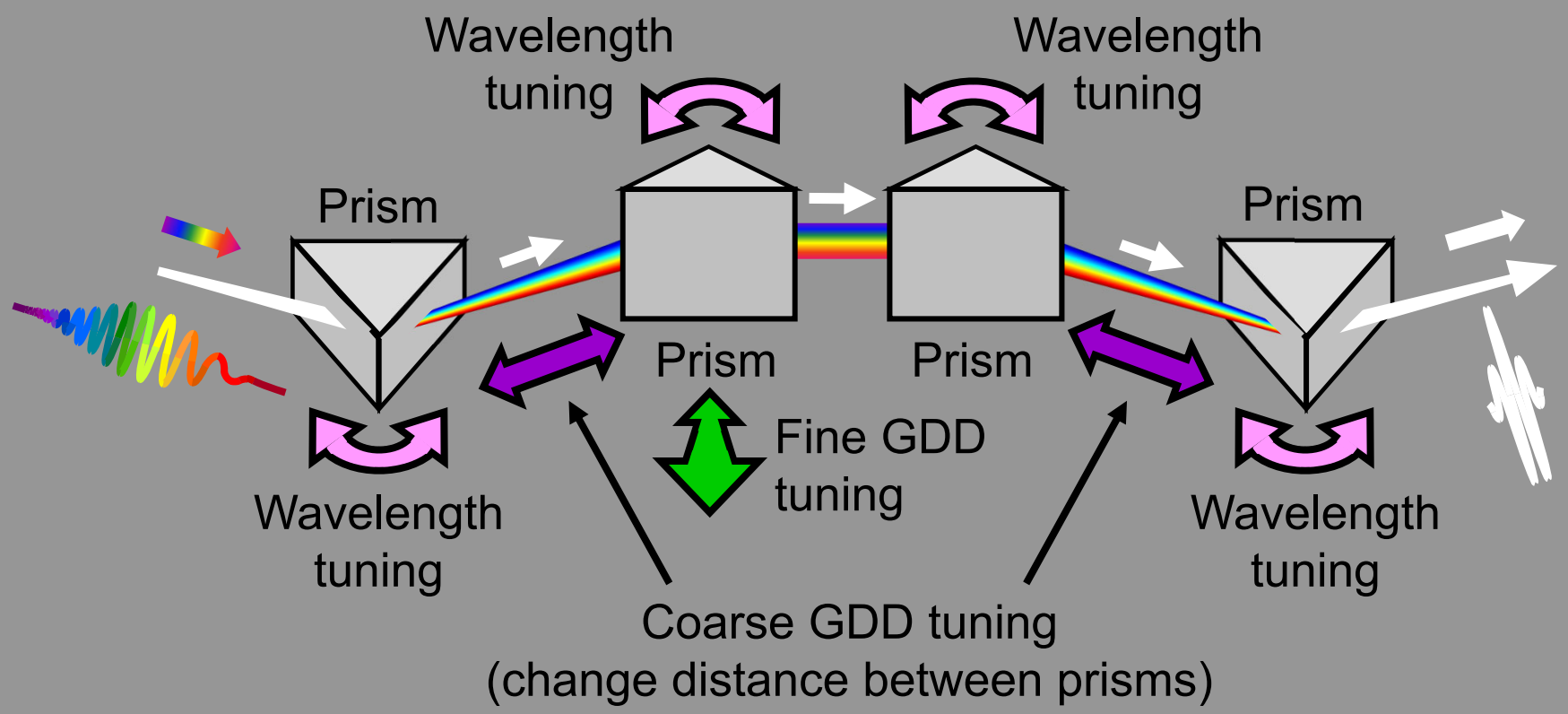
Compression of a 1-ps, 600-nm pulse with 10 nm of bandwidth (to about 50 fs).

Kafka and Baer, Opt. Lett., 12, 401 (1987)

It's best to use highly dispersive glass, like SF10, or gratings.
But compressors can still be > 1 m long.

Four-prism pulse compressor

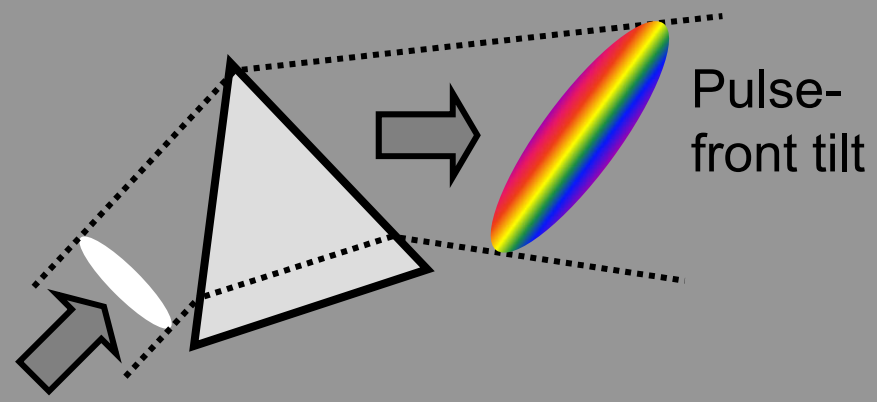
Also, alignment is critical, and many knobs must be tuned.

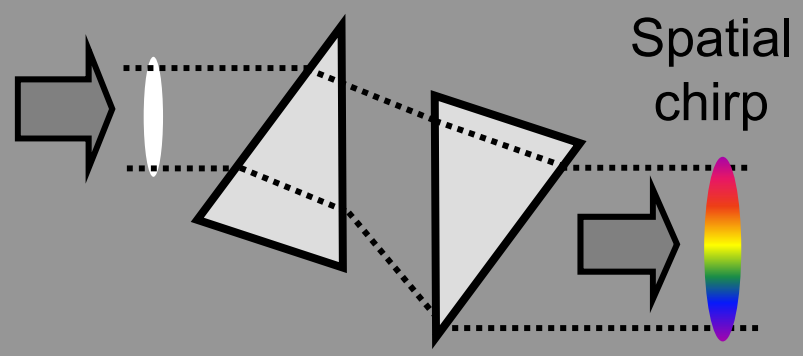


All prisms and their incidence angles must be identical.

Pulse-compressors can have alignment issues.

Pulse compressors are notorious for their large size, alignment complexity, and spatio-temporal distortions.



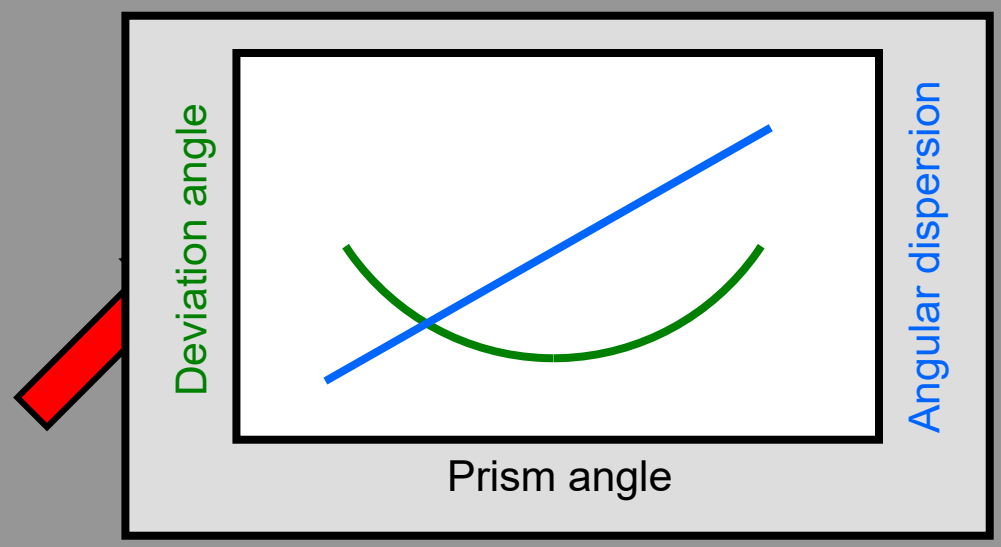


Unless the compressor is aligned perfectly, the output pulse has significant:

1. **1D beam magnification**
2. **Angular dispersion**
3. **Spatial chirp**
4. **Pulse-front tilt**

Why is it difficult to align a pulse compressor?

The prisms are usually aligned using the **minimum deviation** condition.

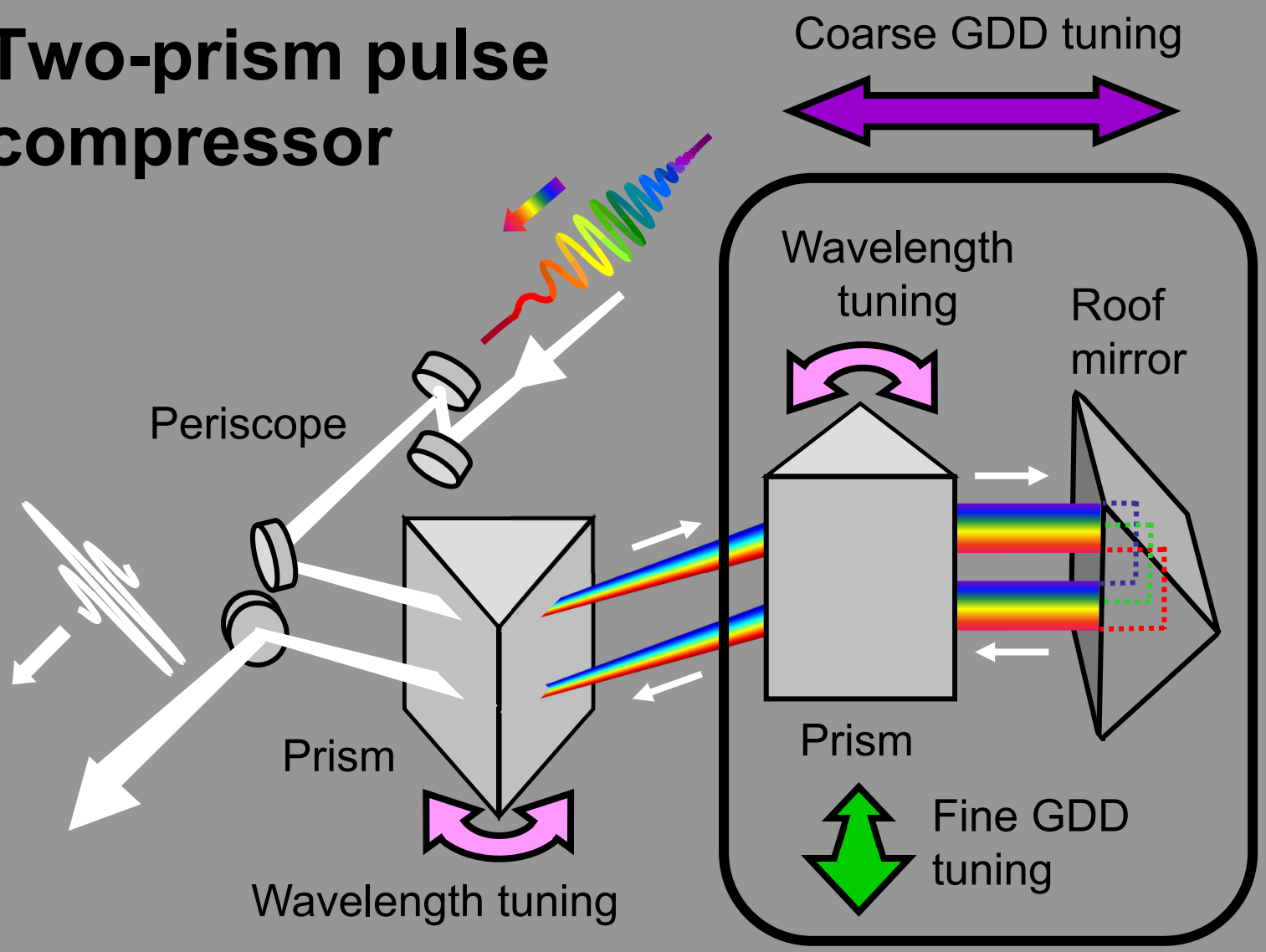


The variation of the **deviation angle** is **2nd order** in the prism angle.

But what matters is the prism **angular dispersion**, which is **1st order**!

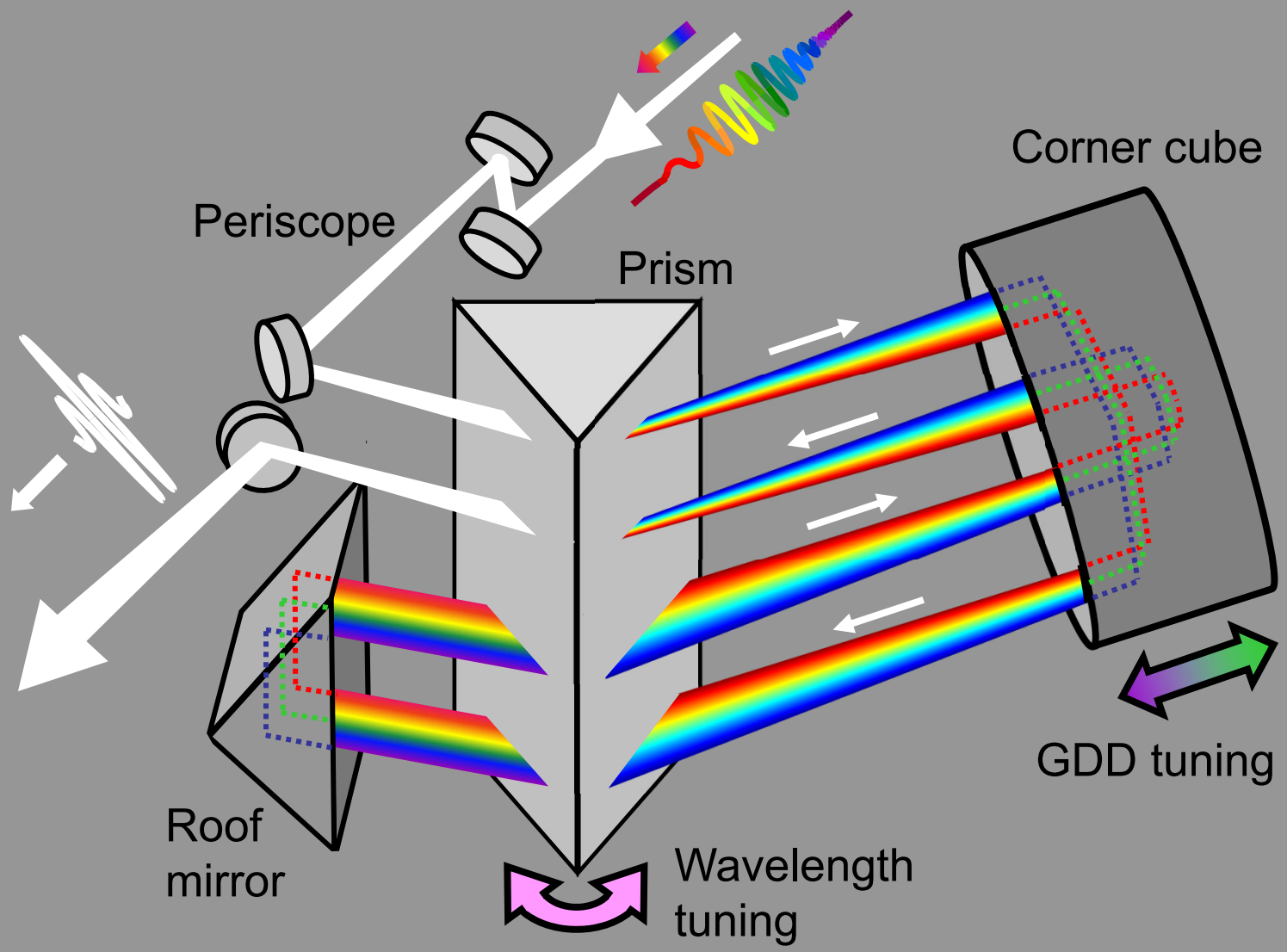
Using a 2nd-order effect to align a 1st-order effect is challenging.

Two-prism pulse compressor

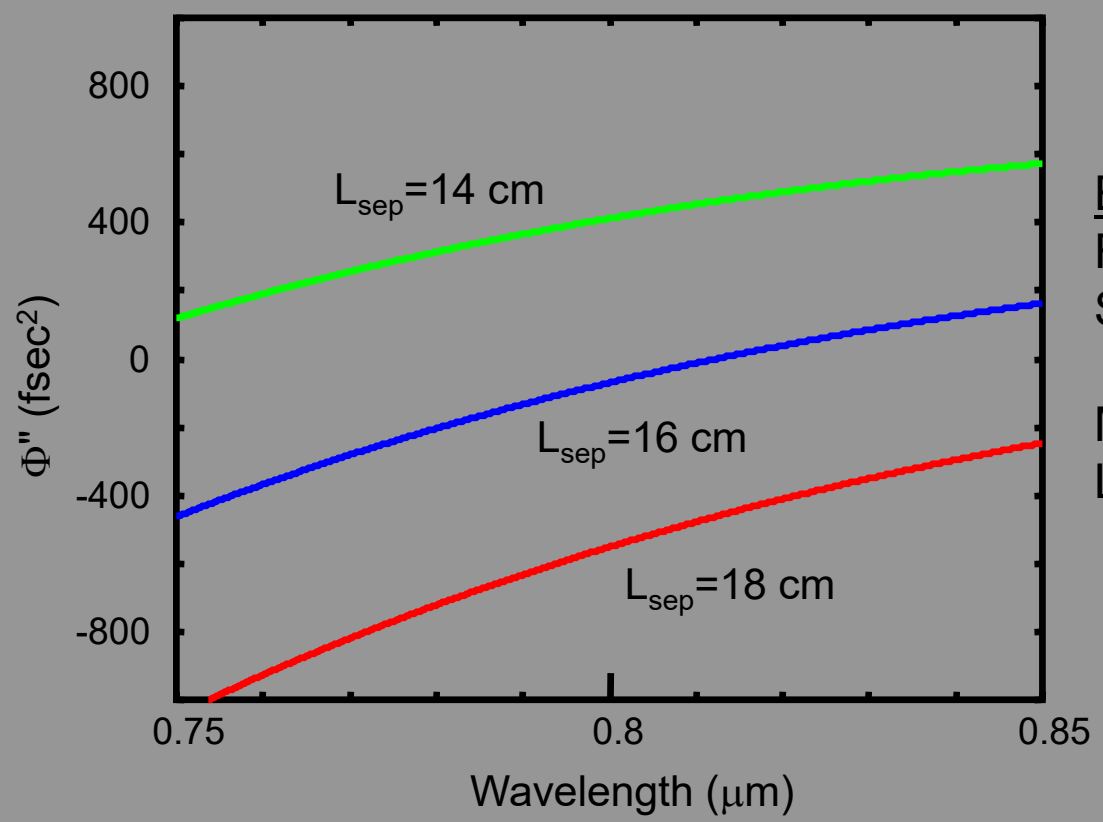


This design cuts the size and alignment issues in half.

Single-prism pulse compressor



Angular dispersion from a sequence of four prisms



Example:
Four perfectly aligned SF11 prisms

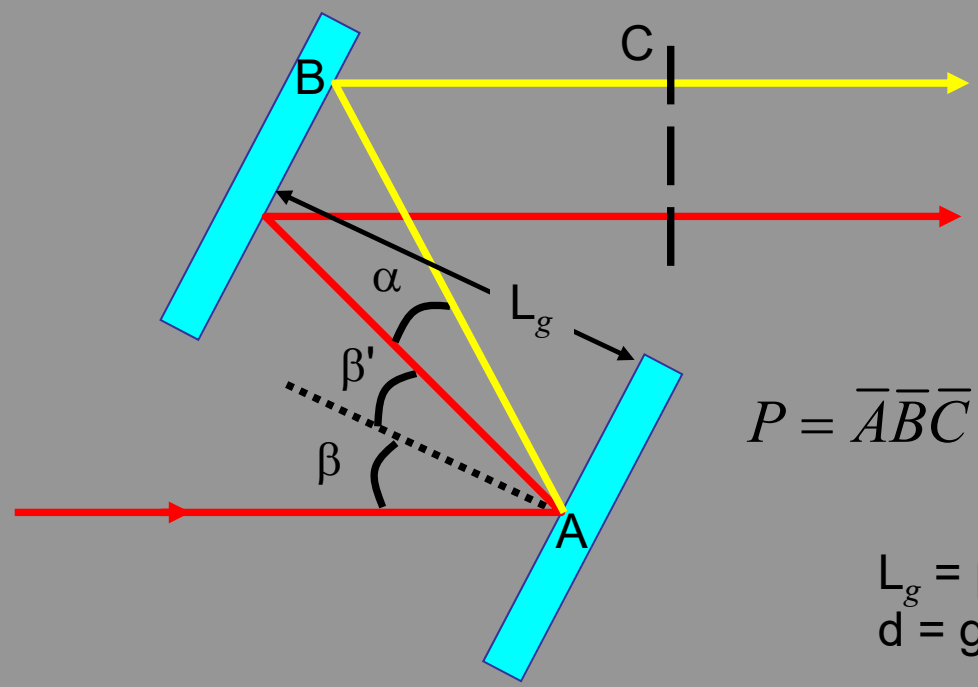
Material length:
L_{prism}=1 cm

Note that Φ" depends on λ! ⟶ For very short pulses, third order dispersion can be important.

$$\Phi''' = \frac{\lambda^4}{2\pi^2 c_0^{\,3}}\left[12L_{sep}\left(n'^2\left(1-\lambda n'\left(n^{-3}-2n\right)\right)+\lambda n' n''\right)-L_{prism}\left(3n''+\lambda n'''\right)\right]$$

Diffraction-grating pulse compressor

The two-grating pulse compressor also provides negative GDD.



grating equation for first-order diffracted beam:

$$\sin\beta - \sin(\beta' + \alpha) = -\frac{2\pi c_0}{\omega d}$$

$$P = \overline{A}\overline{B}\overline{C} = \frac{L_g}{\cos(\beta' + \alpha)}\left[1 + \cos(\beta + \beta' + \alpha)\right]$$

L_g = perpendicular grating separation
d = grating constant (distance/groove)

Assuming a double pass through the grating pair:

$$\Phi'' = -\frac{\lambda^3 L_g}{\pi c_0^{\,2} d^2}\left[1 - \left(\frac{\lambda}{d} - \sin\beta\right)^2\right]^{-3/2} = -\frac{\lambda^3 L_g}{\pi c_0^{\,2} d^2}\Lambda^{-3/2}$$

$$\Phi''' = -\Phi'' \cdot \frac{6\pi\lambda}{c_0\Lambda}\left[\Lambda + \frac{\lambda}{d}\left(\frac{\lambda}{d} - \sin\beta\right)\right]$$

Fork et al., Opt. Lett., 12, 483 (1987)

2nd- and 3rd-order phase terms for prism and grating pulse compressors

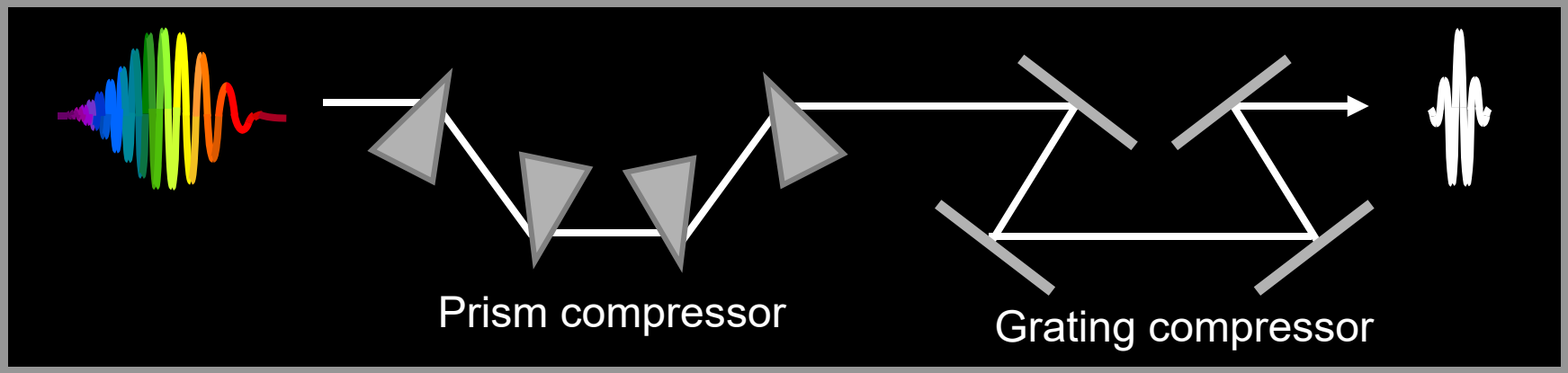
Grating compressors yield more compression than prism compressors.

Device	λ_ℓ [nm]	φ'' [fs^{-2}]	φ''' [fs^{-3}]
SQ1 ($L = 1$ cm)	620	550	240
Piece of glass	800	362	280
Brewster prism pair, SQ1	620	-760	-1300
$\ell = 50$ cm	800	-523	-612
grating pair	620	$-8.2\ 10^4$	$1.1\ 10^5$
$b = 20$ cm; $\beta = 0°$			
$d = 1.2\ \mu$m	800	$-3\ 10^6$	$6.8\ 10^6$

Note that the relative signs of the 2nd and 3rd-order terms are opposite for prism compressors and grating compressors.

Compensating 2nd and 3rd-order spectral phase

Use both a prism and a grating compressor. Since they have 3rd-order terms with opposite signs, they can be used to achieve almost arbitrary amounts of both second- and third-order phase.



Given the 2nd- and 3rd-order phases of the input pulse, φ_{input2} and φ_{input3}, one can solve two simultaneous equations:

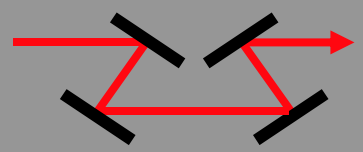
$$\varphi_{input\,2} + \varphi_{prism\,2} + \varphi_{grating\,2} = 0$$

$$\varphi_{input\,3} + \varphi_{prism\,3} + \varphi_{grating\,3} = 0$$

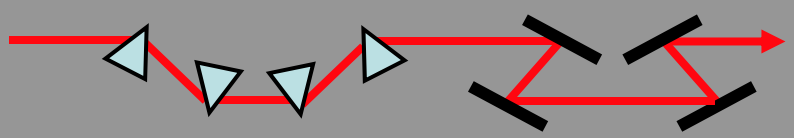
Pulse Compression: Simulation

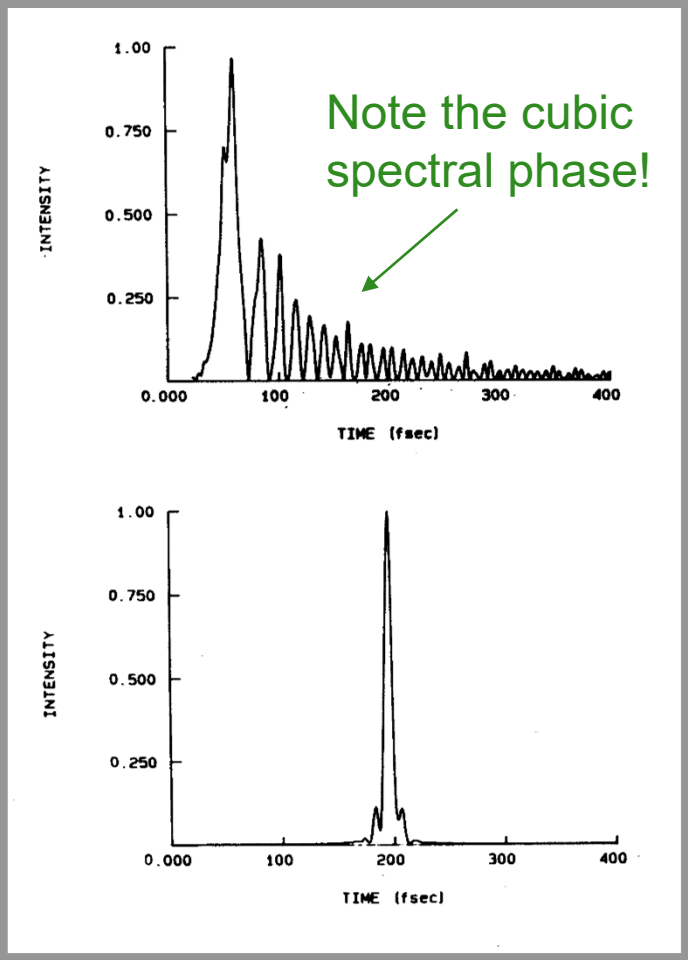
Using prism and grating pulse compressors vs. only a grating compressor

Resulting intensity vs. time with only a grating compressor:



Resulting intensity vs. time with a grating compressor and a prism compressor:

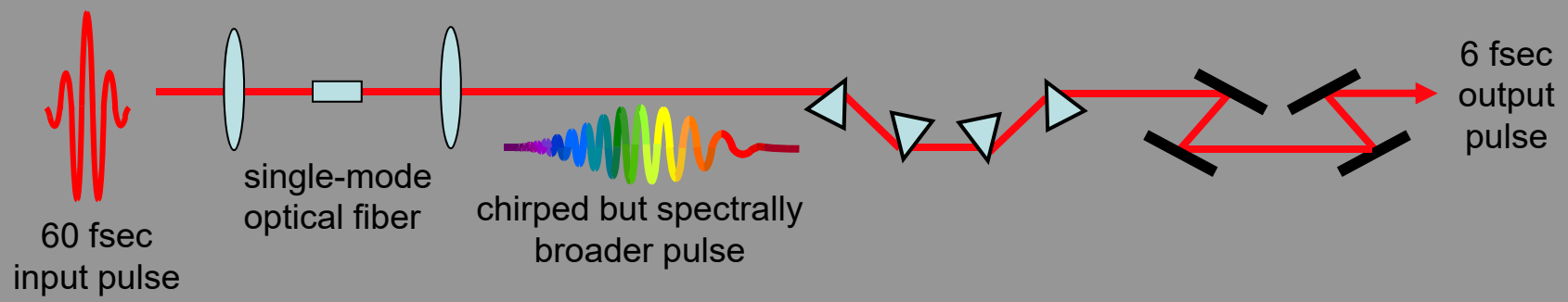




Brito Cruz, et al., Opt. Lett., 13, 123 (1988).

Pulse Compression: Results

The ‘prisms + gratings’ pulse compressor design was used by Fork and Shank in 1987 to compress pulses to six femtoseconds – a record that stood for over a decade.

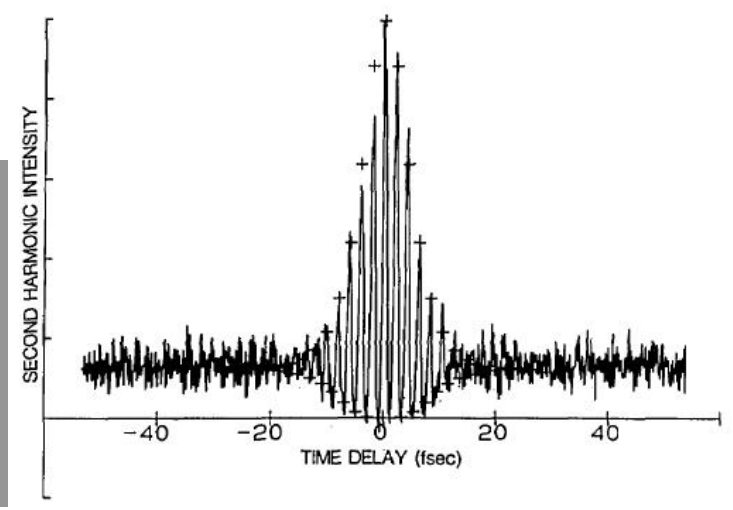


July 1987 / Vol. 12, No. 7 / OPTICS LETTERS 483

Compression of optical pulses to six femtoseconds by using cubic phase compensation

R. L. Fork, C. H. Brito Cruz, P. C. Becker, and C. V. Shank

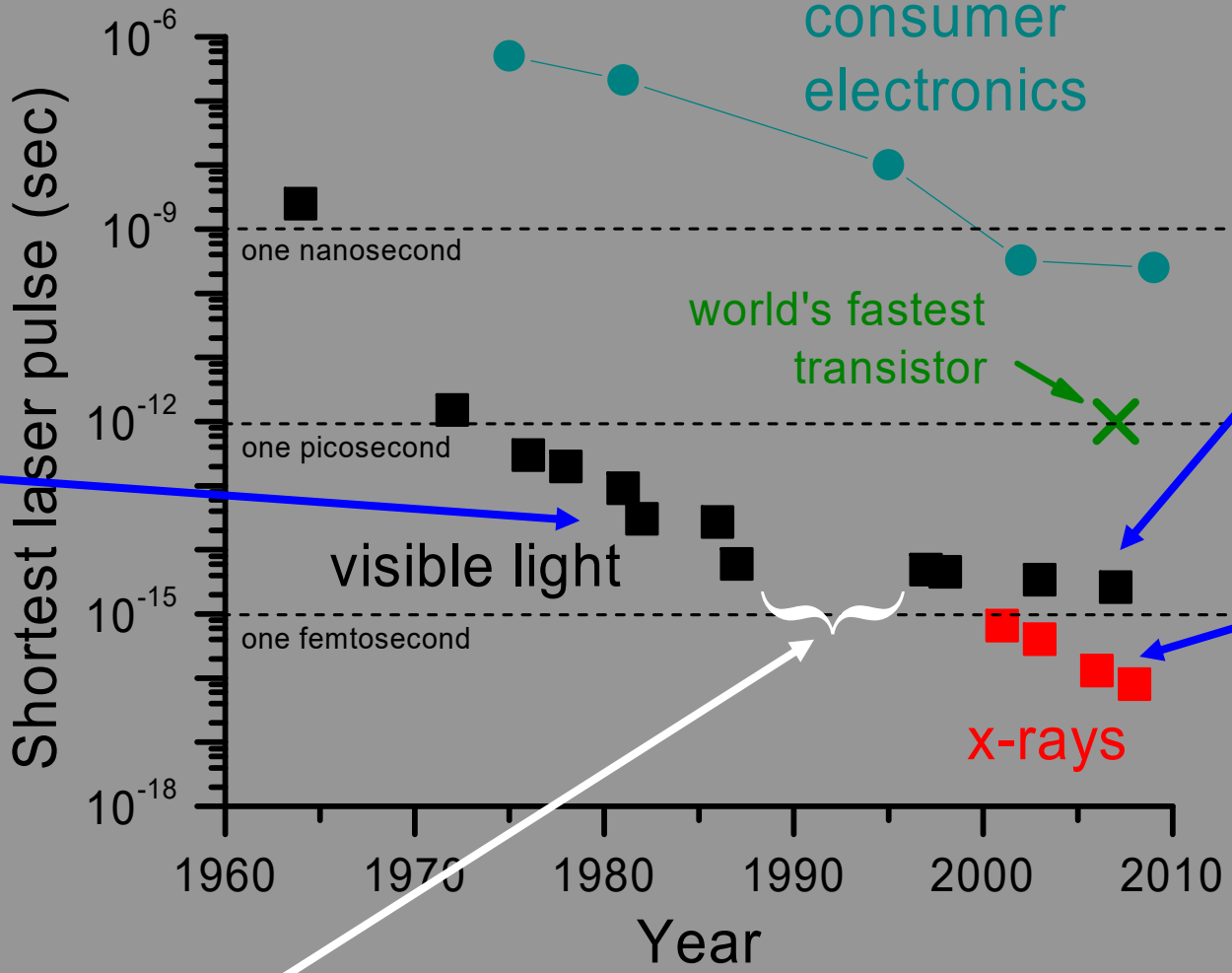
AT&T Bell Laboratories, Crawfords Corner Road, Holmdel, New Jersey 07733-1988



Short optical pulses – progress is amazing

Charles Shank

Erich Ippen



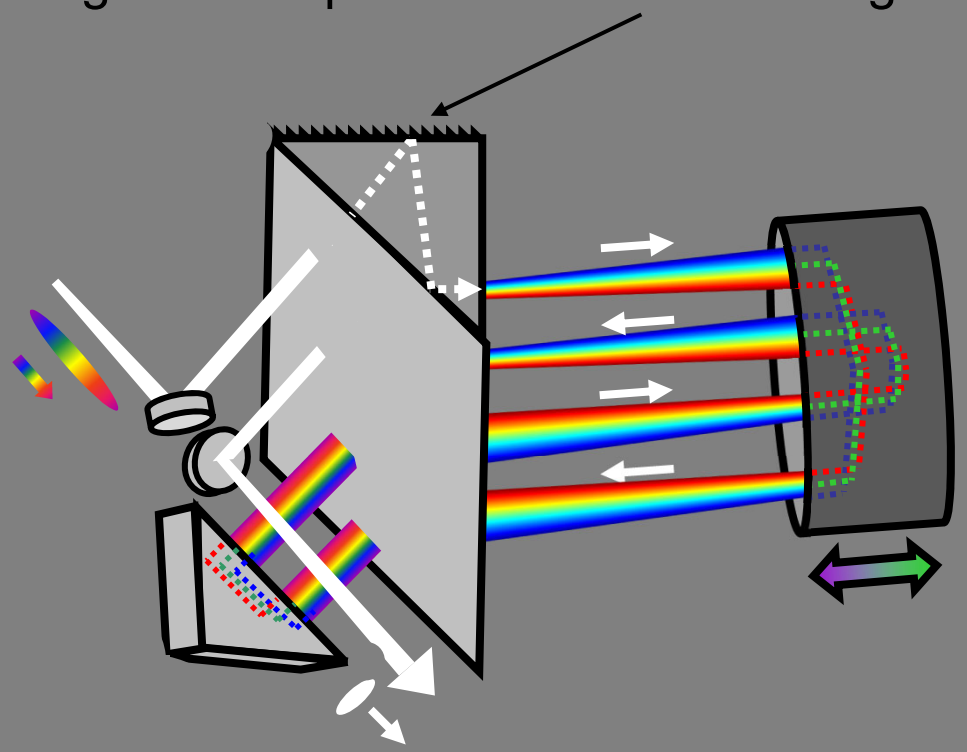
Mikio Yamashita

Ferenc Krausz

This ten-year gap happened (in part) because it did not occur to anybody to try to figure out a way to compensate the fourth-order phase.

The grism pulse compressor has tunable third-order dispersion.

A grism is a prism with a diffraction grating etched onto it.



The (transmission) grism equation is:

$$a\,[\sin(\theta_m) - n\,\sin(\theta_i)] = m\lambda$$

Note the factor of n, which does not occur for a diffraction grating.

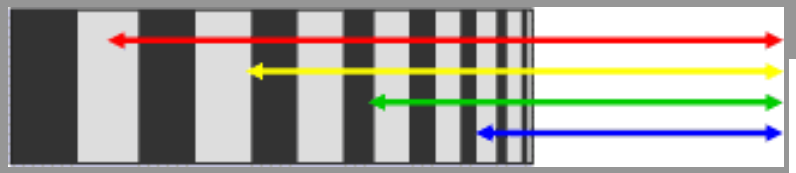
A grism compressor can compensate for both 2nd and 3rd-order dispersion due even to many meters of fiber.

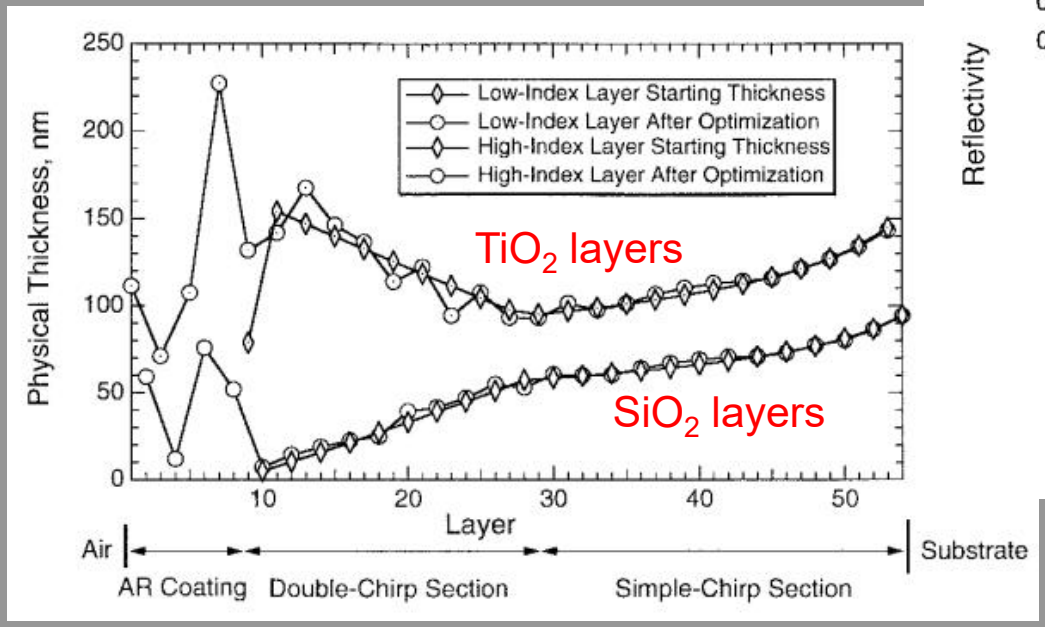
Chirped mirrors

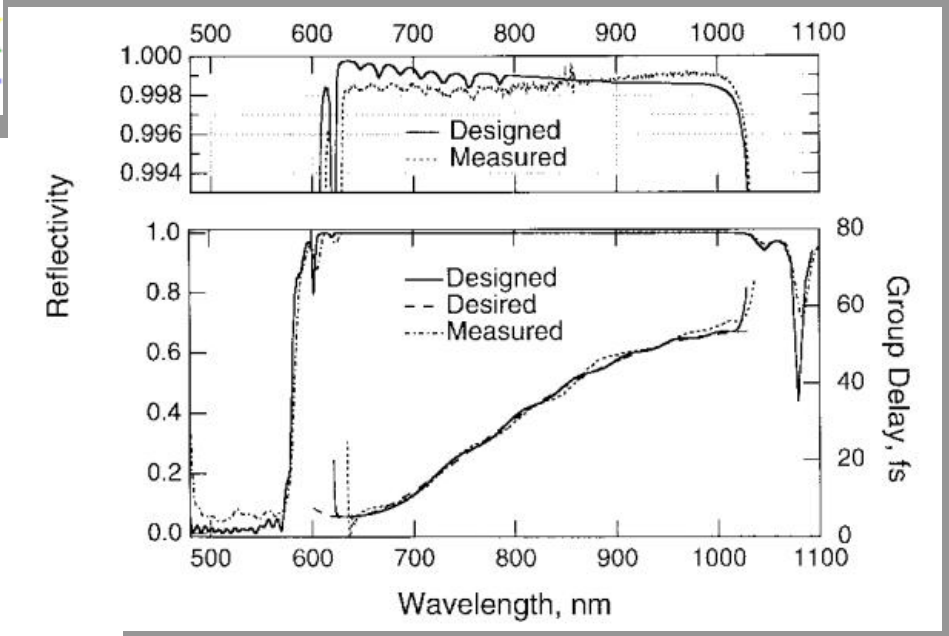
A mirror whose reflection coefficient is engineered so that it has the form:

$$r(\omega) = e^{i\phi(\omega)}$$

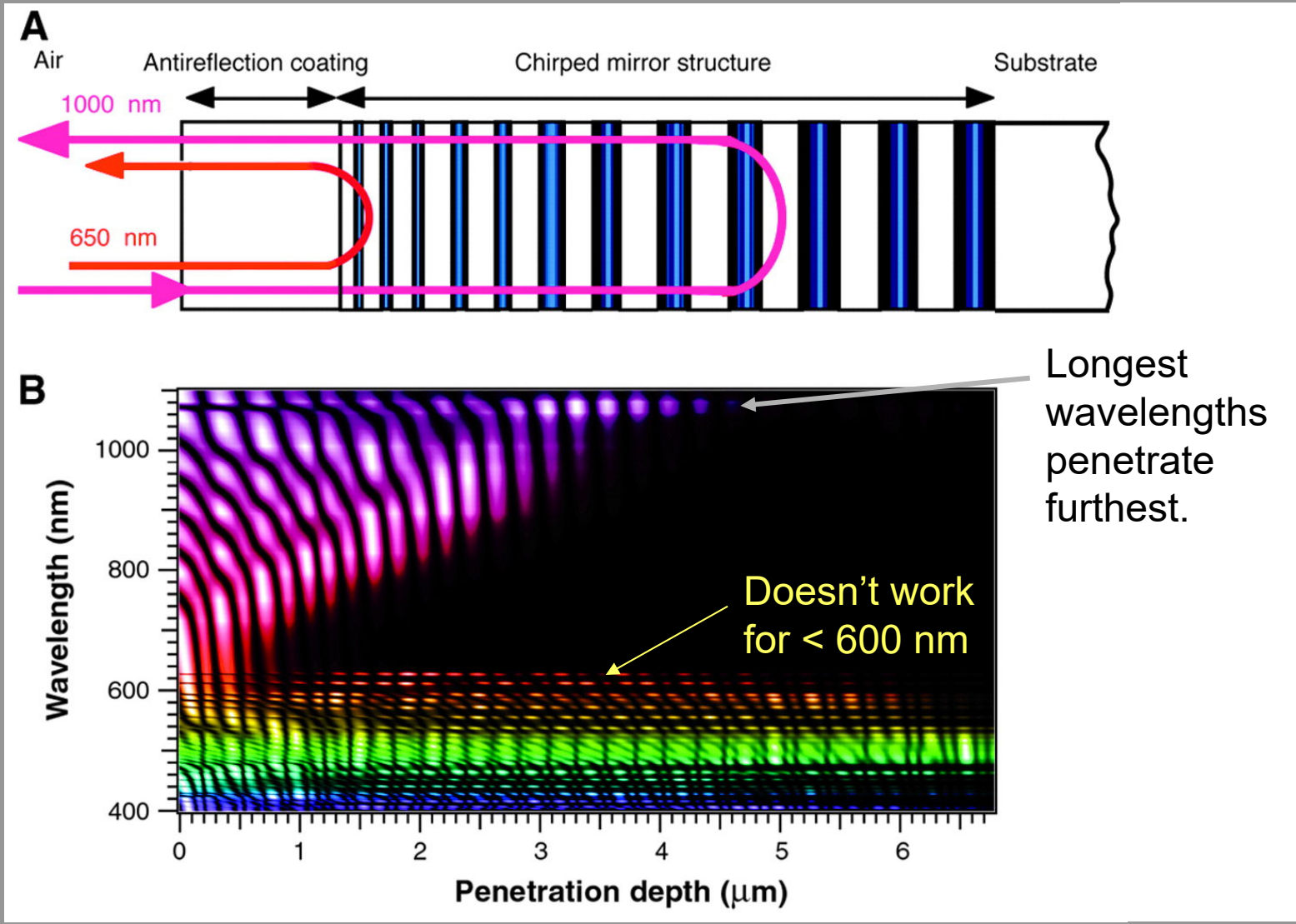
so that $|r(\omega)| = 1$ and $\phi(\omega)$ is chosen to cancel out the phase of the incident pulse.





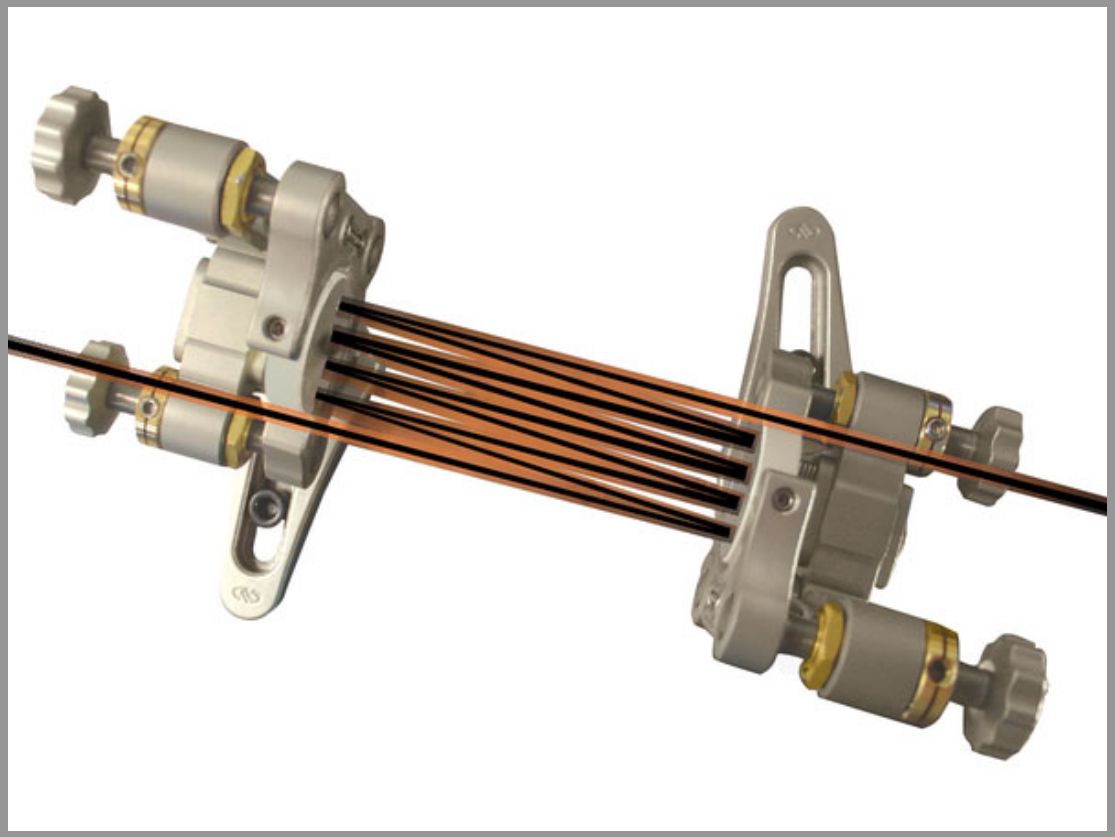


Chirped mirror coatings



Chirped mirrors for extra-cavity dispersion control

Each bounce off of a mirror adds the chosen spectral phase $\phi(\omega)$ to the pulse. One can accumulate large changes in $\phi(\omega)$ through multiple bounces.



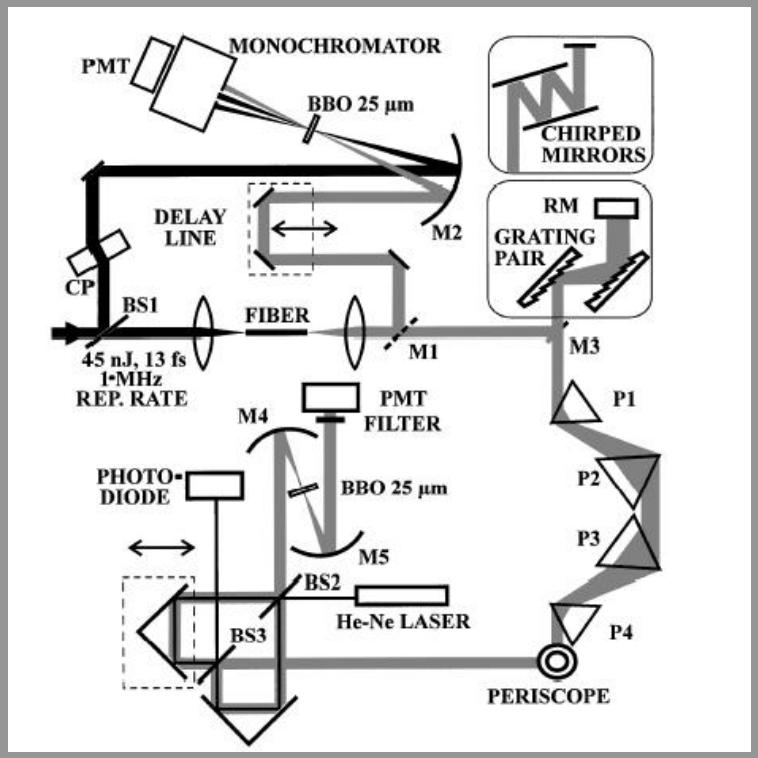
Chirped mirrors for extra-cavity dispersion control

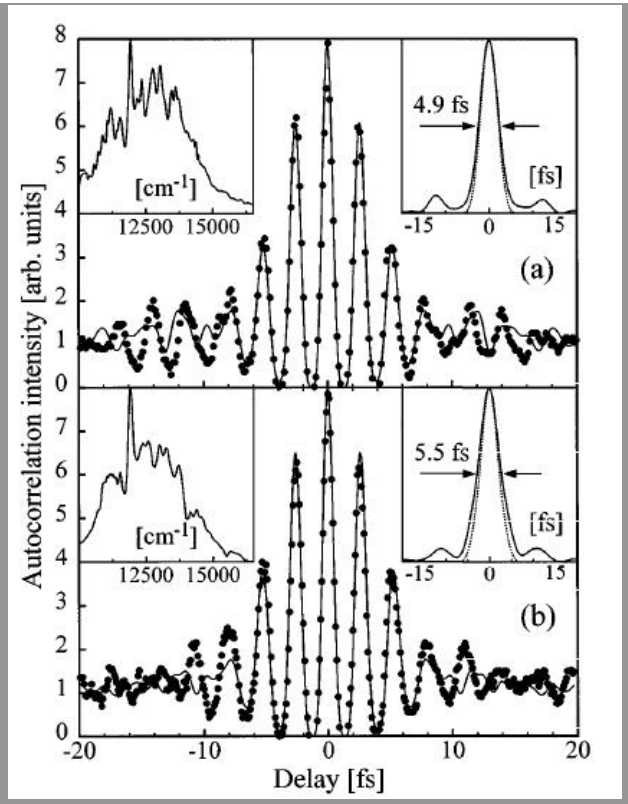
102 OPTICS LETTERS / Vol. 22, No. 2 / January 15, 1997

Optical pulse compression to 5 fs at a 1-MHz repetition rate

Andrius Baltuška, Zhiyi Wei, Maxim S. Pshenichnikov, and Douwe A. Wiersma

Ultrafast Laser and Spectroscopy Laboratory, Department of Chemistry, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands





Chirped mirrors – intra-cavity

July 1, 1997 / Vol. 22, No. 13 / OPTICS LETTERS 1009

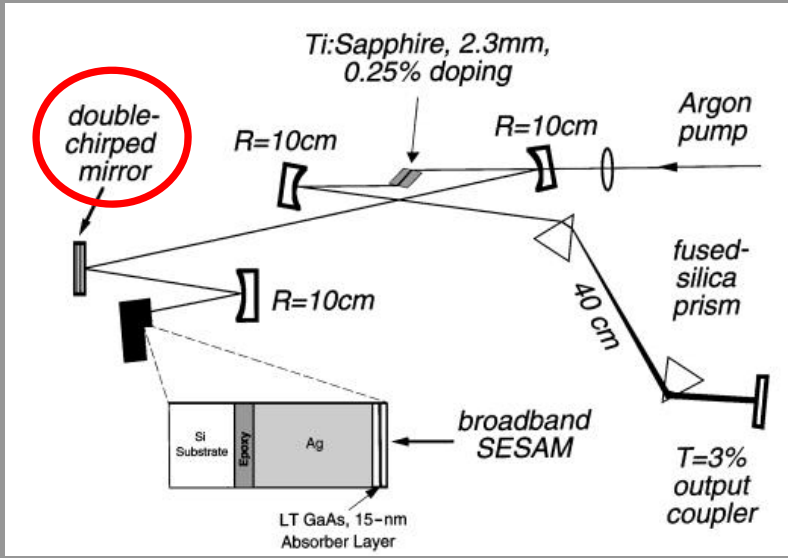
Self-starting 6.5-fs pulses from a Ti:sapphire laser

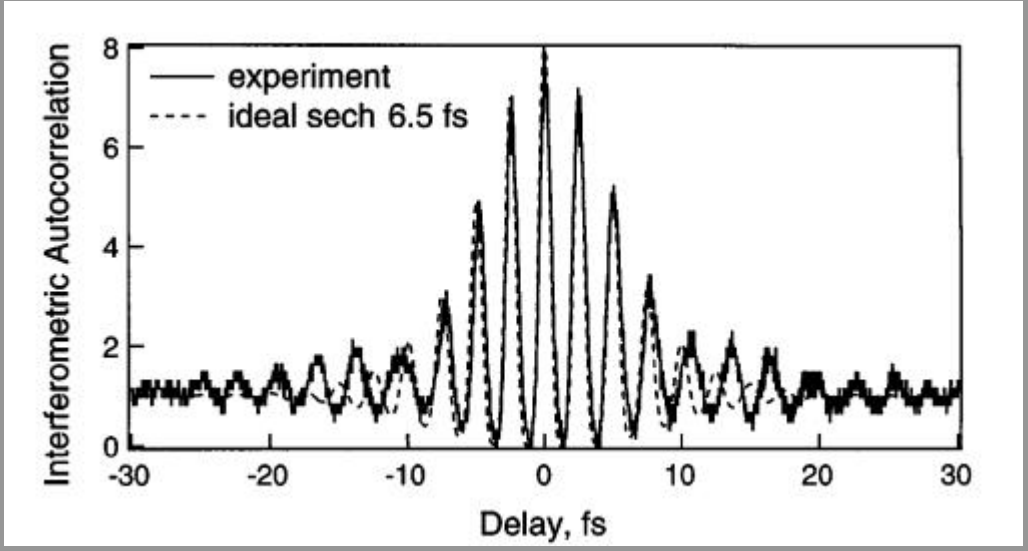
I. D. Jung, F. X. Kärtner, N. Matuschek, D. H. Sutter, F. Morier-Genoud, G. Zhang, and U. Keller

Ultrafast Laser Physics, Institute of Quantum Electronics, Swiss Federal Institute of Technology, ETH Hönggerberg-HPT, CH-8093 Zürich, Switzerland

V. Scheuer, M. Tilsch, and T. Tschudi

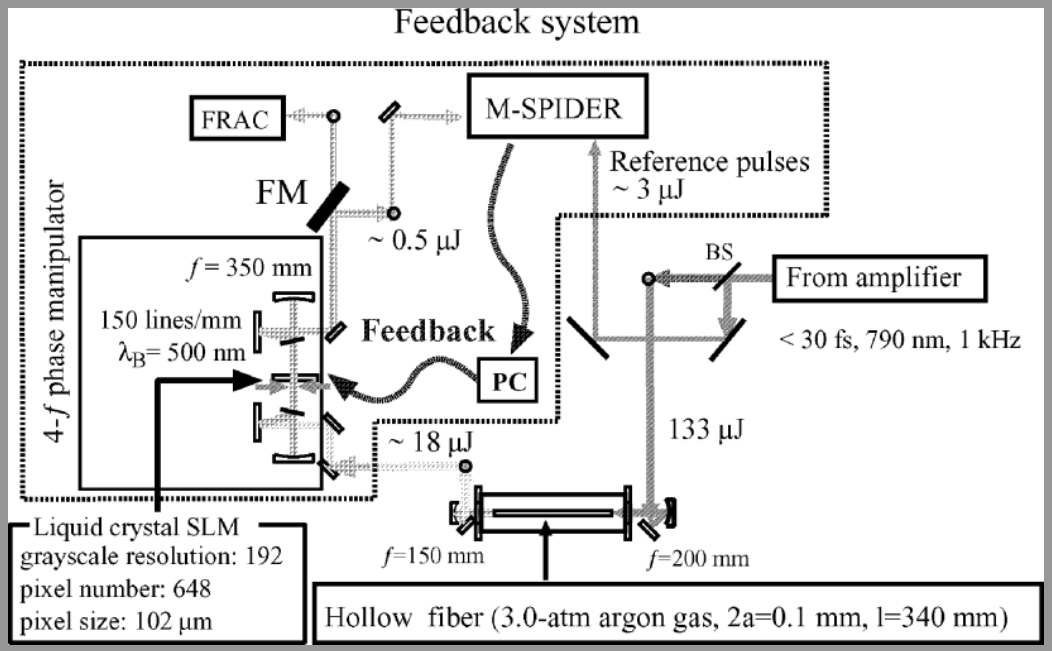
Institute for Applied Physics, TH Darmstadt, D-64289 Germany

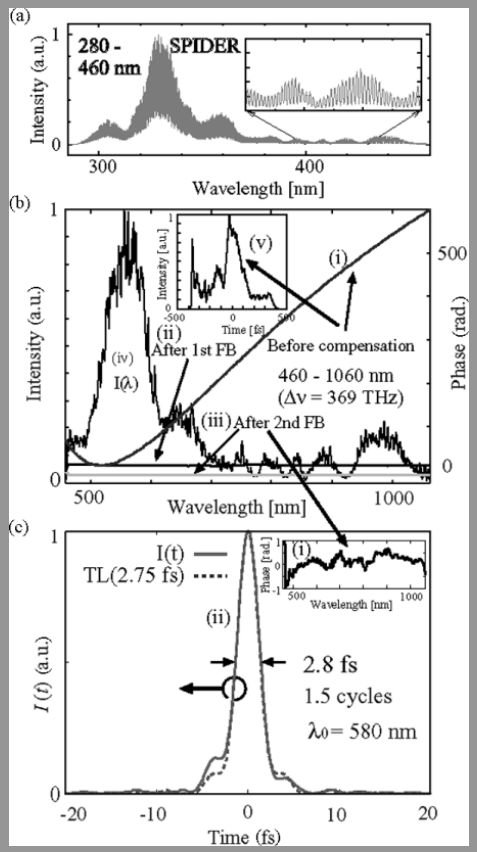




The shortest possible optical pulses

M. Yamashita, et al., IEEE Journal of Selected Topics in Quantum Electronics, 12, 213-222, 2006





Duration: 2.8 fs (1.5 cycles)
Spectrum: > 500nm width

This requires:

- Starting with intense (>100 μJ) 30 fs pulses
- Spectrally broadening them in a gas-filled fiber
- Using a 648-pixel liquid-crystal modulator to dynamically adjust the spectral phases
- And a feedback system which iterates the LCM phases until the pulse is as short as possible.