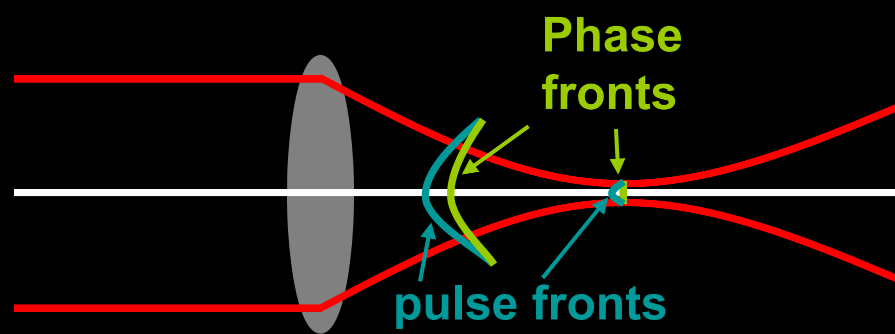
Focusing Ultrashort Pulses

Chromatic aberration

Radially varying group delay

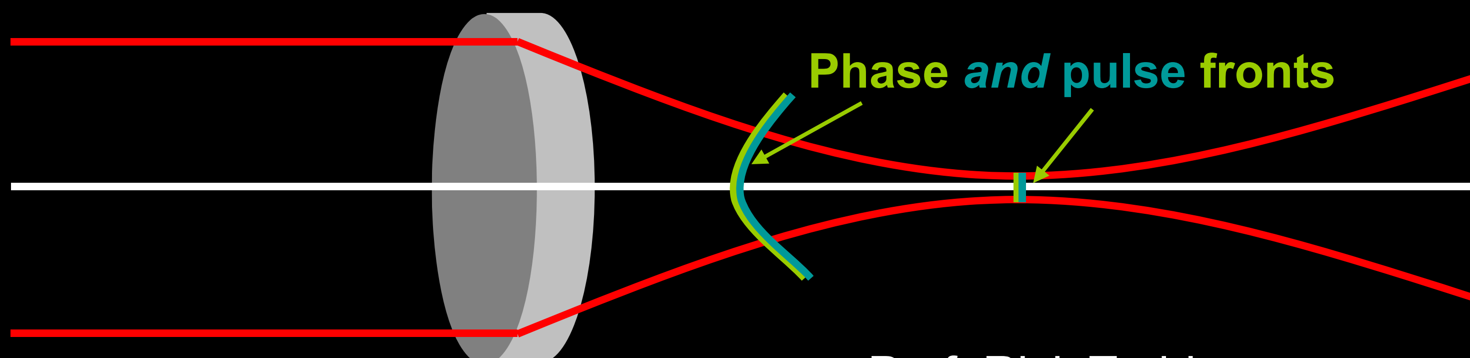
Achromatic lenses



Other aberrations

How to focus a pulse

Other issues



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Focusing Issues

Why focus an ultrashort pulse?

Ultrafast microscopy requires very a small spot size.

Focusing an ultrashort pulse can yield **ultrahigh intensity**.

typical Ti:sapphire output: 10nJ / (100fs) / $(10\mu m)^2$ ~ 10^{15} W/m^2

But ultrashort pulses are broadband, so a lens that focuses a single color well won't necessarily focus an ultrashort pulse well due to **chromatic aberration**.

And a lens that focuses white light well may not focus an ultrashort pulse well: the pulse will lengthen due to **group-velocity dispersion**.

We'll need to keep the pulse simultaneously short in time and small in diameter at the focus.

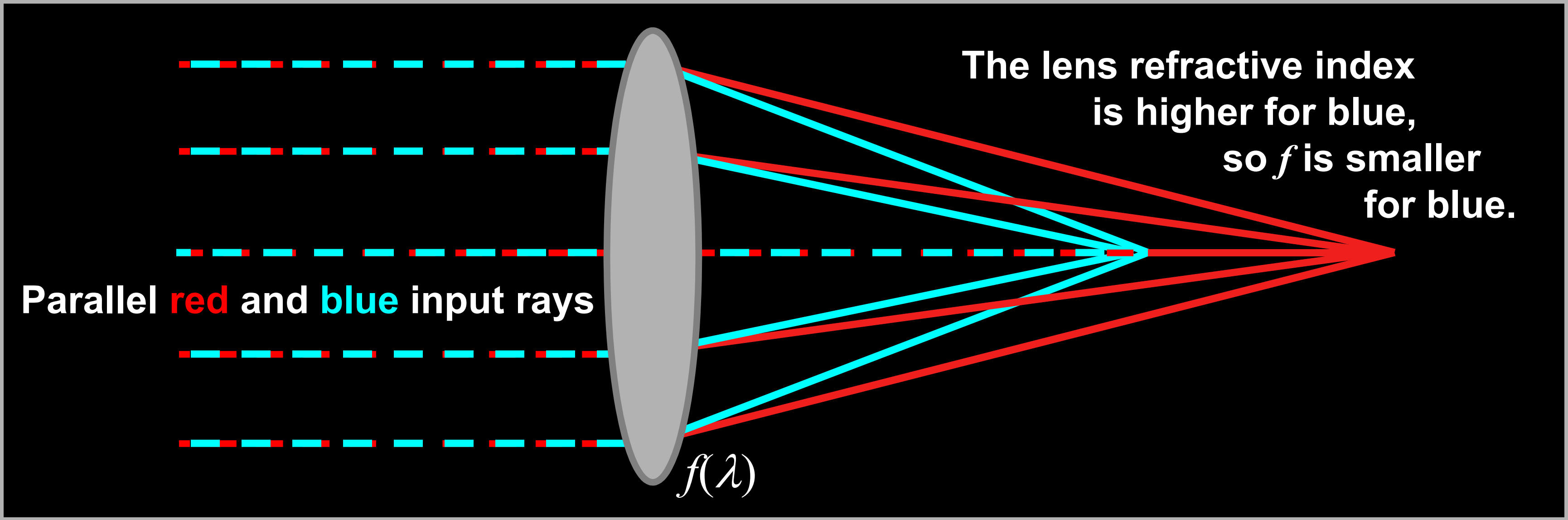
Nontrivial spatio-temporal distortions can occur.

Chromatic aberration distorts the pulse in space and time.

An ultrashort pulse is broadband, and different frequencies can focus at different points.

$$\frac{1}{f(\lambda)} = \left[n(\lambda) - 1\right]\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The lens-maker's formula



Chromatic aberration distributes the focused energy over a larger region than desired.

Chromatic aberration (cont'd)

Compute the variation in focal length Δf over the spectrum of a pulse of length Δt:

$$\frac{d(1/f)}{d\lambda} = \frac{-1}{f^2}\frac{df}{d\lambda} \approx \frac{-1}{f^2}\frac{\Delta f}{\Delta\lambda} \quad \Rightarrow \quad \boxed{\Delta f \approx -f^2 \frac{d(1/f)}{d\lambda}\Delta\lambda}$$

For a Gaussian pulse, the bandwidth, $\Delta\lambda$, is: $\Delta\lambda = 0.441\,\lambda^2 / c\,\Delta t$

Now: $$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \Rightarrow \quad \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{(n-1)f}$$

So: $$\frac{d(1/f)}{d\lambda} = \left(\frac{dn}{d\lambda}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \Rightarrow \quad \boxed{\frac{d(1/f)}{d\lambda} = \left(\frac{dn}{d\lambda}\right)\frac{1}{(n-1)f}}$$

Substi-tuting: $$\Delta f \approx -f^2 \frac{d(1/f)}{d\lambda}\Delta\lambda \approx -f^2\left[\left(\frac{dn}{d\lambda}\right)\frac{1}{(n-1)f}\right]\left[0.441\,\lambda^2 / c\,\Delta t\right]$$

Simplifying: $$\boxed{\Delta f \approx -0.441\,\frac{f\,\lambda^2}{(n-1)\,c\,\Delta t}\frac{dn}{d\lambda}}$$

Chromatic aberration: a numerical example

Fused silica lens focusing a 50fs ultraviolet pulse from a KrF laser

n = 1.51; $\lambda\, dn/d\lambda$ = –0.17

λ = 248nm; pulse length = 50fs

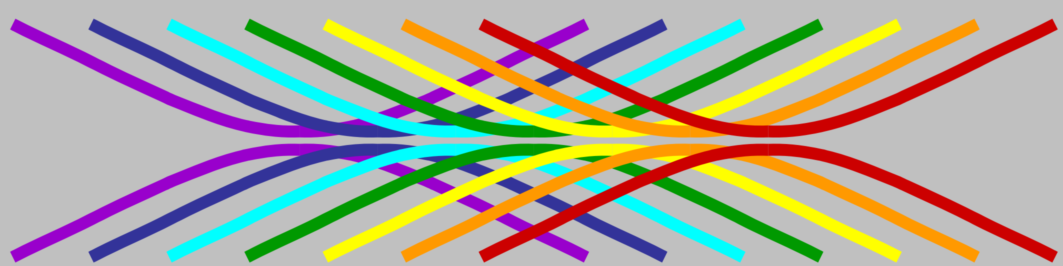
f = 30mm; desired focal spot size radius, w_0 = 0.6μm

$$\Delta f = -0.441 \frac{f\,\lambda^2}{(n-1)c\,\Delta t}\frac{dn}{d\lambda} = 60\mu\mathrm{m}$$

Is this a lot or a little?

Compare with the confocal parameter: $2\pi w_0^2/\lambda = 9\mu\mathrm{m}$ So it's a lot!

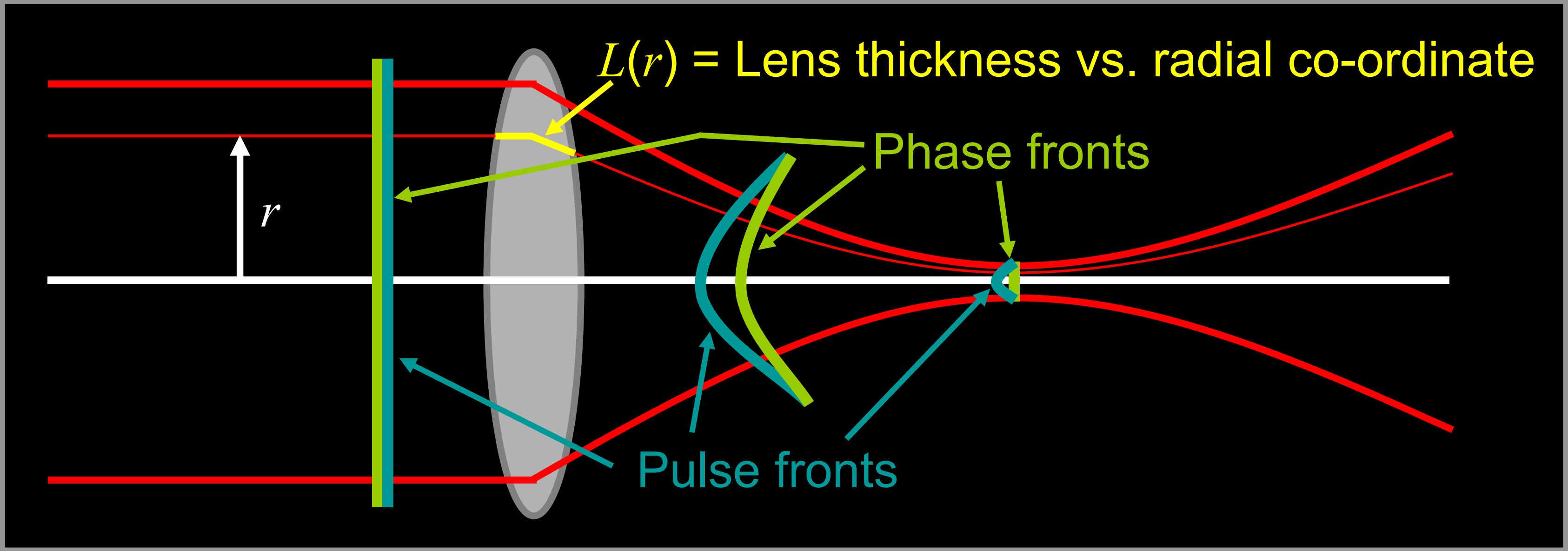
(2×Rayleigh range)



Note: this problem is not nearly as bad the IR. At λ = 800nm, the dispersion parameter $\lambda\, dn/d\lambda$ = –0.014, an order of magnitude smaller.

Radially varying group delay also affects the pulse focus.

For a lens, the phase delay at the focus is independent of input radial position (r) (if we ignore spherical aberration), so the phase fronts are flat there.

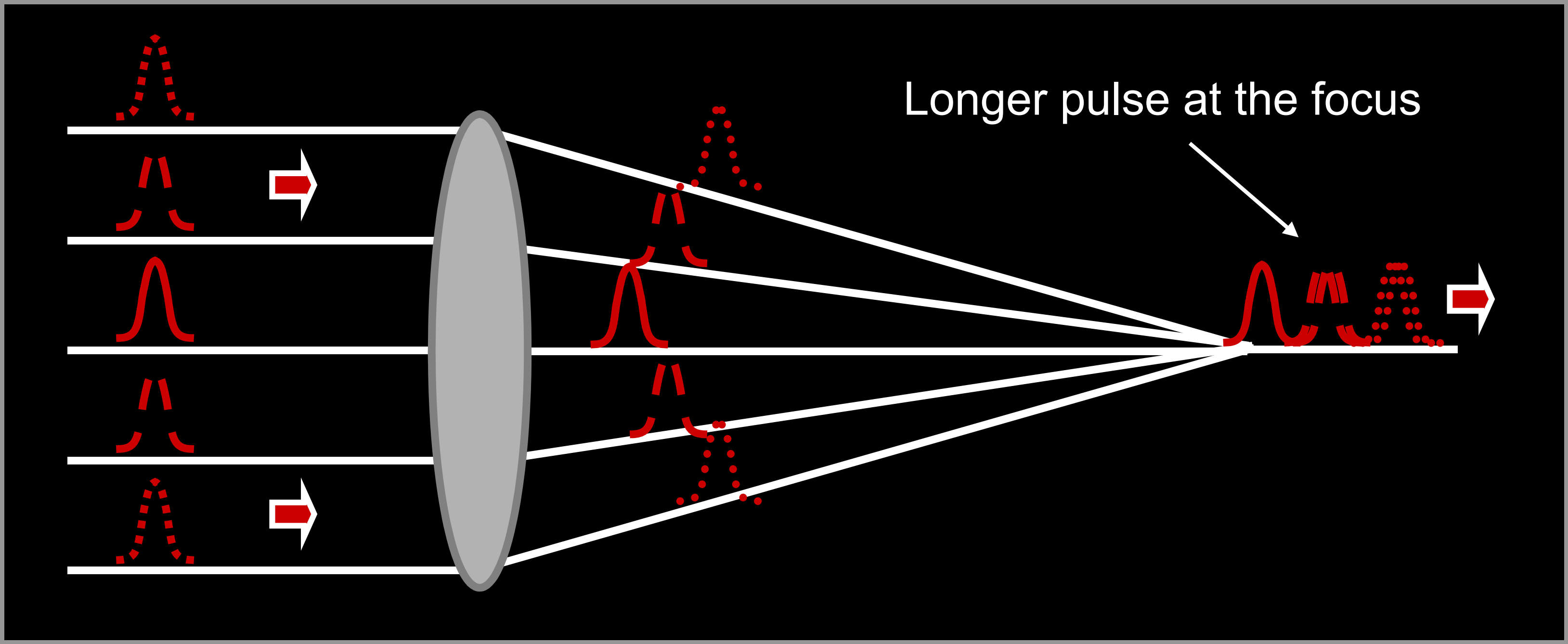


But the group velocity differs from the phase velocity, so the intensity fronts ("pulse fronts") will **not** be the same as the phase fronts and will lag behind them—the more glass in the path, the greater the lag.

Understanding the effects of radially varying group delay on the focus.

The group velocity is less than the phase velocity, so the more glass the later the pulse arrival time.

Radially varying group delay lengthens and distorts the focus.



Group vs. phase delay in a lens

The difference in propagation time between the phase and intensity:

$$\Delta t(r) = \left(\frac{1}{\mathrm{v}_\phi} - \frac{1}{\mathrm{v}_g}\right) L(r)$$

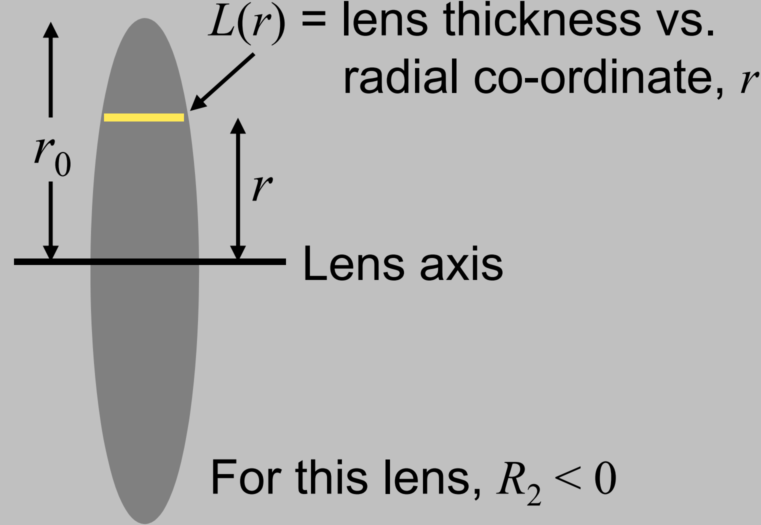
where v_ϕ is the phase velocity and v_g is the group velocity,

and where:

$$L(r) \approx \left(\frac{r_0^2 - r^2}{2}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\boxed{\approx \left(\frac{r_0^2 - r^2}{2}\right)\frac{1}{(n-1)f}}$$

R_1 = Front-surface radius of curvature
R_2 = Back-surface radius of curvature



For this lens, $R_2 < 0$

Group and phase delays in a lens (cont'd)

$$L(r) \approx \left(\frac{r_0^2 - r^2}{2}\right)\frac{1}{(n-1)f}$$

Expressions for the phase velocity, v_ϕ, and the group velocity, v_g:

$$v_\phi = \frac{c}{n} \qquad\qquad v_g = \left(\frac{dk}{d\omega}\right)^{-1} = \frac{c}{n - \lambda \dfrac{dn}{d\lambda}}$$

So:

$$\frac{1}{v_\phi} - \frac{1}{v_g} = \frac{n}{c} - \frac{n - \lambda \dfrac{dn}{d\lambda}}{c} = \frac{\lambda}{c}\frac{dn}{d\lambda}$$

Substituting for $L(r)$ and the inverse-velocity difference:

$$\boxed{\Delta t(r) = \left(\frac{1}{v_\phi} - \frac{1}{v_g}\right) L(r) \approx \frac{\lambda}{c}\frac{dn}{d\lambda}\left(\frac{r_0^2 - r^2}{2}\right)\frac{1}{(n-1)f}}$$

Practical example: focusing a UV pulse

$$\Delta t(r) \approx \frac{\lambda}{c}\frac{dn}{d\lambda}\left(\frac{r_0^2 - r^2}{2}\right)\frac{1}{(n-1)f}$$

So the **difference** in group delay at the lens edge, r_0, and on axis, 0, is:

$$\Delta t_g(r_0) \equiv \Delta t(r_0) - \Delta t(0) = \frac{-\lambda}{c}\frac{dn}{d\lambda}\left(\frac{r_0^2}{2}\right)\frac{1}{(n-1)f}$$

(The phase delays are equal and cancel out.)

Example: Fused silica lens focusing a 50fs pulse from a KrF laser

$\left.\begin{array}{l} n = 1.51; \quad \lambda\, dn/d\lambda = -0.17 \\ \lambda = 248\text{nm, pulse length} = 50\text{fs} \\ f = 30\text{mm, desired focal spot size radius} = 0.6\mu\text{m} \\ \text{input spot size radius} = 4\text{mm (required for } 0.6\mu\text{m focus)} \end{array}\right\} \Delta t_g = 300\text{fs}$

A big effect!

Radial group delay and chromaticity

Recall that: $$\frac{d}{d\lambda}\left(\frac{1}{f}\right) = \left(\frac{dn}{d\lambda}\right)\frac{1}{(n-1)f}$$

Substituting this result into the lens phase-minus-group time delay:

$$\Delta t(r) \approx \frac{\lambda}{c}\frac{dn}{d\lambda}\left(\frac{r_0^2 - r^2}{2}\right)\frac{1}{(n-1)f(\lambda)}$$

We find:

$$\boxed{\Delta t(r) \approx \frac{\lambda}{c}\left(\frac{r_0^2 - r^2}{2}\right)\frac{d}{d\lambda}\left(\frac{1}{f}\right)}$$

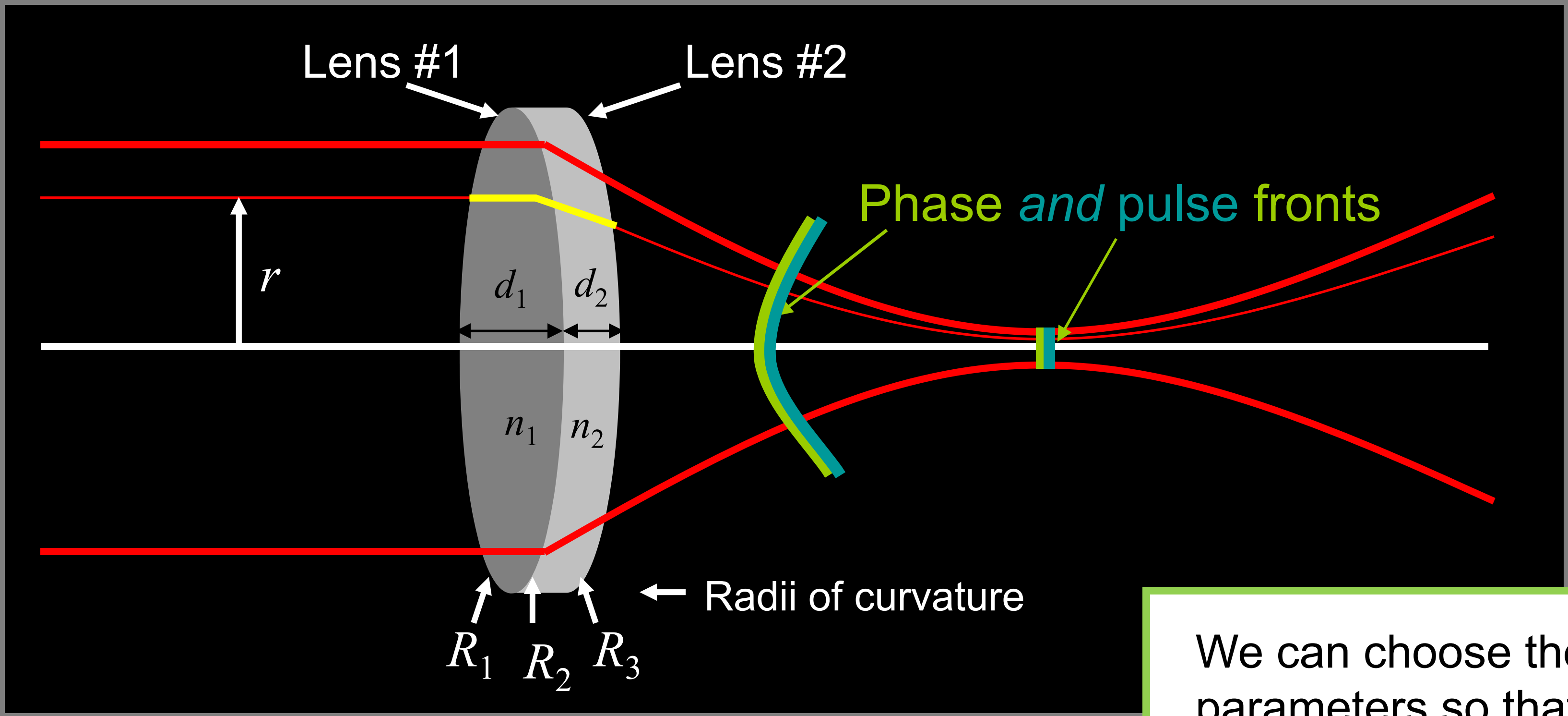
This result relates the difference between the group and phase delays to the **chromaticity** of the lens.

It says that an achromatic lens (for which f is independent of λ) has radially **independent** group delay and hence flat pulse fronts!!!

So an achromatic lens solves two problems!

Achromatic lenses solve two problems.

Combining two lenses into a **doublet** can create a lens that is **achromatic** (to first order) and that cancels out radially varying group delay.



$$\frac{1}{f} = (n_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + (n_2 - 1)\left(\frac{1}{R_2} - \frac{1}{R_3}\right)$$

We can choose the parameters so that:

$$\frac{d}{d\lambda}\left(\frac{1}{f(\lambda)}\right) = 0$$

Achromatic lenses solve two problems (cont'd)

The paths traveled through the lenses are:

$$L_1(r) \approx d_1 - \frac{r^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad L_2(r) \approx d_2 - \frac{r^2}{2}\left(\frac{1}{R_2} - \frac{1}{R_3}\right)$$

As a result, doublets have a phase-minus-group delay difference with additional terms:

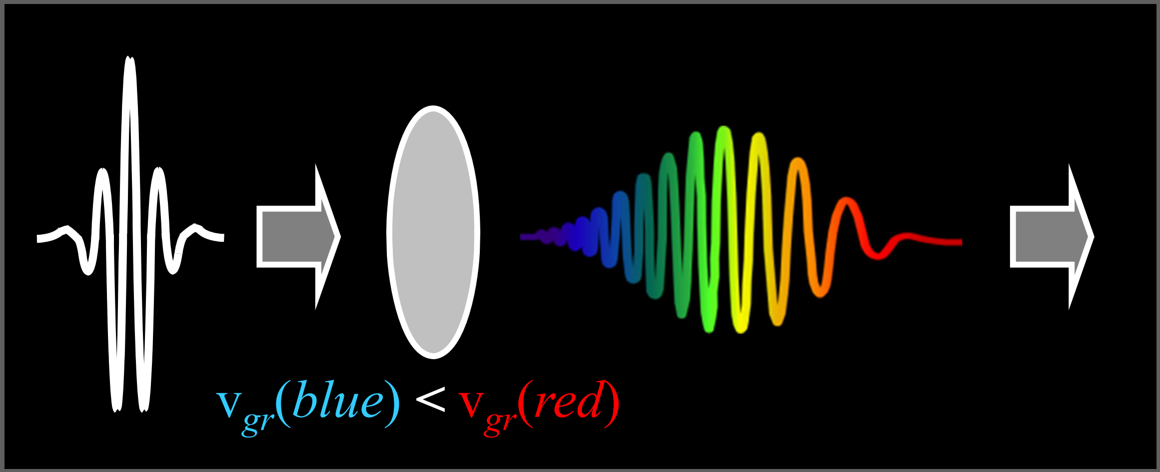
$$\Delta t_{doublet}(r) \approx \frac{d_1}{c}\left[n_1 - \lambda_1 \frac{dn}{d\lambda_1}\right] + \frac{d_2}{c}\left[n_2 - \lambda_2 \frac{dn}{d\lambda_2}\right] + \frac{\lambda}{c}\left(\frac{r^2}{2}\right)\frac{d}{d\lambda}\left(\frac{1}{f}\right)$$

Note that only the third term depends on the radial co-ordinate, r, and this term is zero for an achromatic lens.

Of course, an even more complex, highly aberration-corrected photography lens or microscope objective is even better.

Of course, GDD in the lens adds chirp and lengthens the focused pulse.

Aberration-corrected microscope objectives have several cm of glass in the several elements in them. This introduces GDD!



It may be necessary to **pre-compensate** for this glass using a pulse compressor (e.g., prism sequence) before the microscope objective.

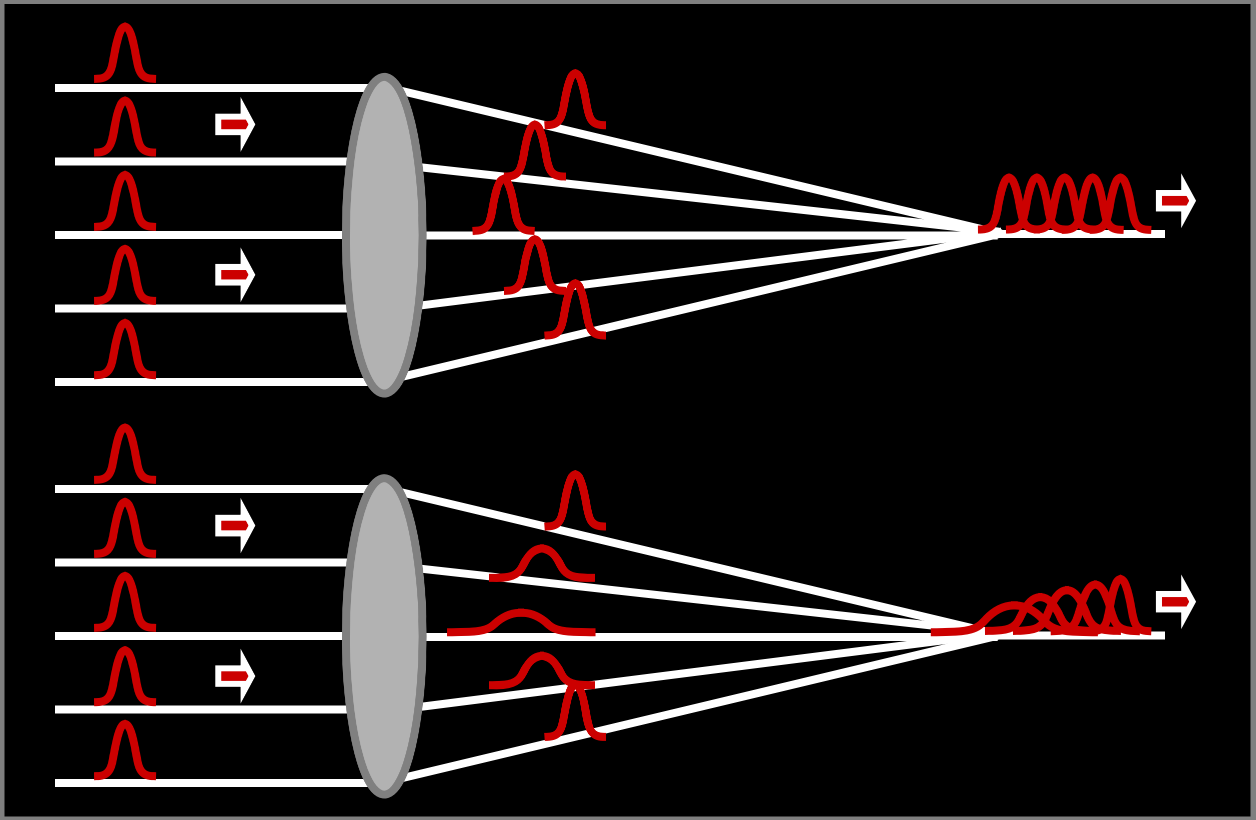
Unfortunately, achromatic, highly aberration-corrected lenses can be very thick, so they can require significant chirp compensation, and residual 3rd-order spectral phase can distort the pulse badly.

Worse, the GDD varies with lens radius.

The magnitude of the chirp depends on radial position.

Without GVD:

With GVD:

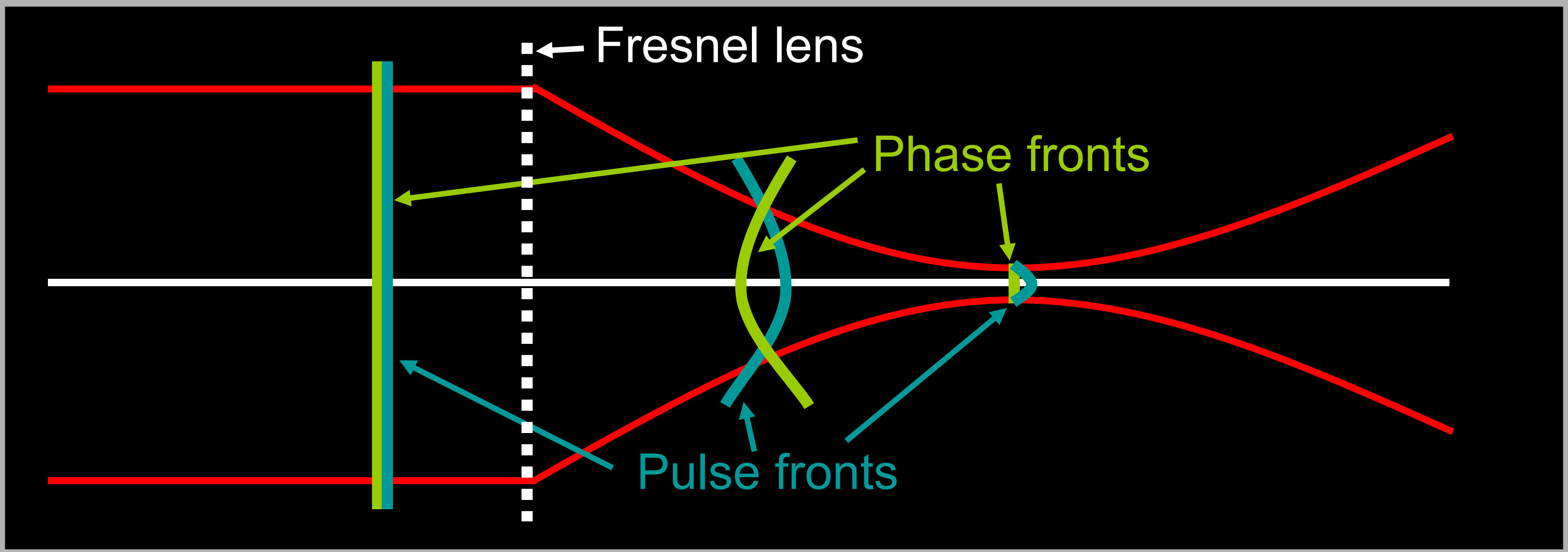


This effect is not well understood. But its magnitude is likely small.

Kempe, et al., JOSAB, 9, 1158 (1992)

Avoiding GDD in lenses (maybe…)

You might think that a Fresnel lens, which has no group-velocity component to the delay, would solve the problem...



But now the pulse fronts ***lead*** the phase fronts! You can't win!

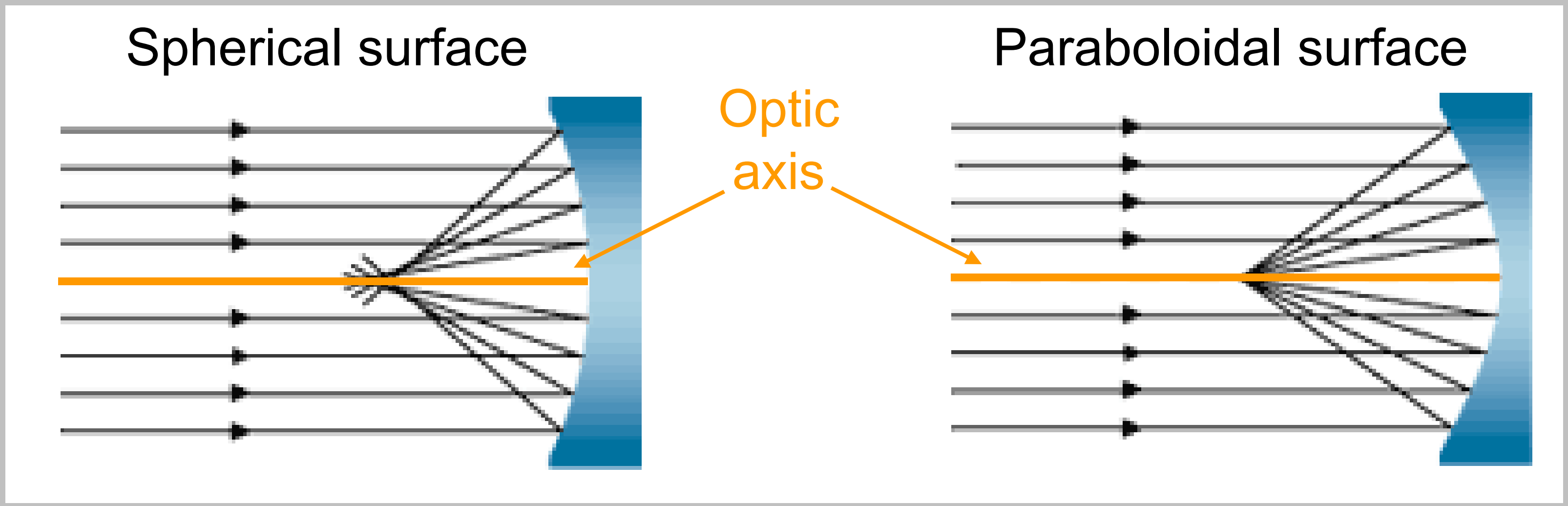
Maybe a combination of the two types of lenses would work…

Z. Bor, Opt. Lett., 14, 119 (1989)

How about using a mirror?

This nicely avoids GDD (and chromatic aberration) completely!

For all rays to converge to a point a distance f away from a curved mirror requires a **paraboloidal** surface.

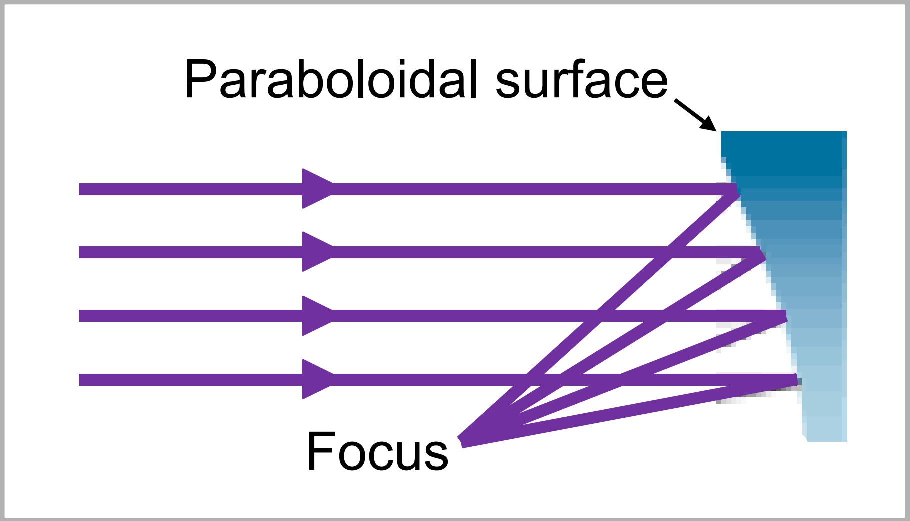


But this only works for on-axis rays. Alas, focused off-axis rays suffer from astigmatism.

Worse, the focus is in the middle of the incident beam.

The off-axis paraboloid

To keep the focus out of the beam, use only **part** of the paraboloidal mirror.

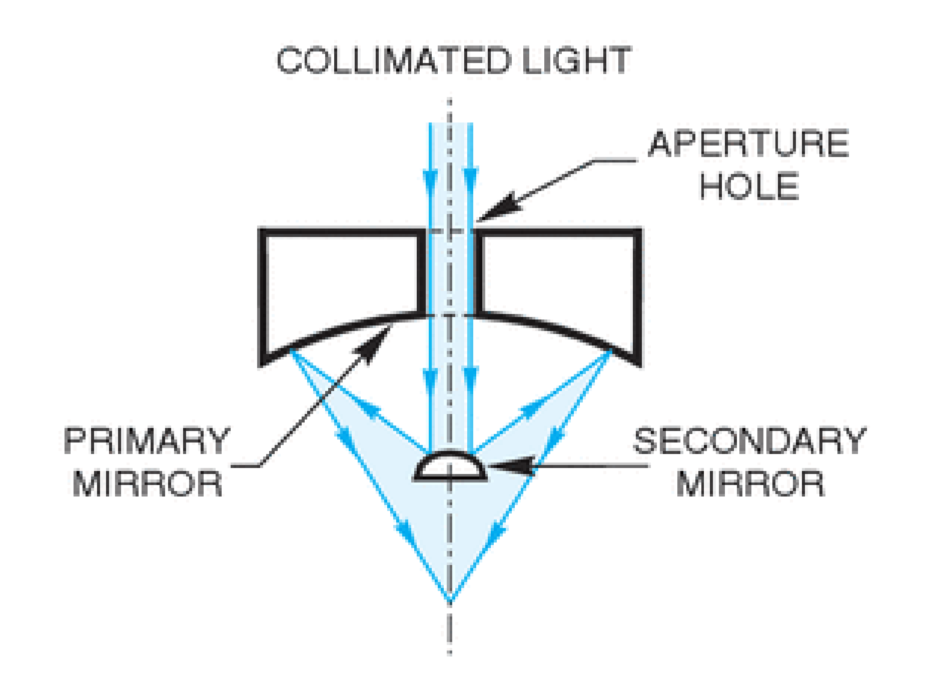


Alas, these focusing optics are difficult to align, and slight misalignments yield large distortions.

Reflective (Cassegrain) microscope objectives

You might think that the focus will have a hole in it, but no.

Away from the focus, there is a hole, however. And at the focus, some light is lost due to the hole.

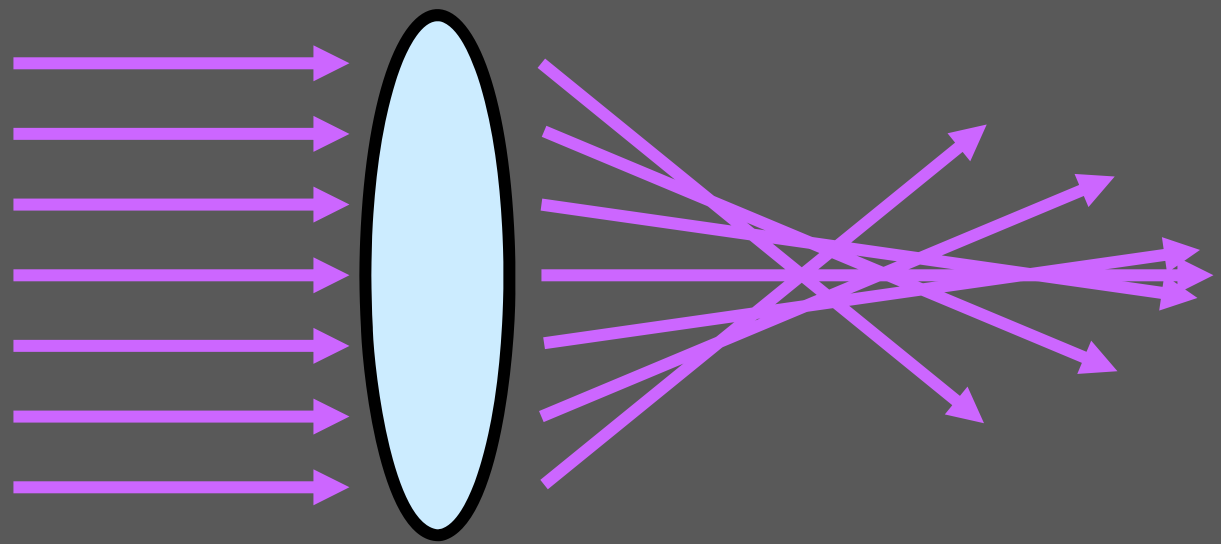


A photograph taken using a Cassegrain lens. Note that the in-focus foreground looks fine, but point-sources in the out-of-focus background are donut-shaped.

Spherical aberration in lenses

Usually we use spherical surfaces for lenses and mirrors, which work better for a wide range of input angles (avoiding astigmatism and coma).

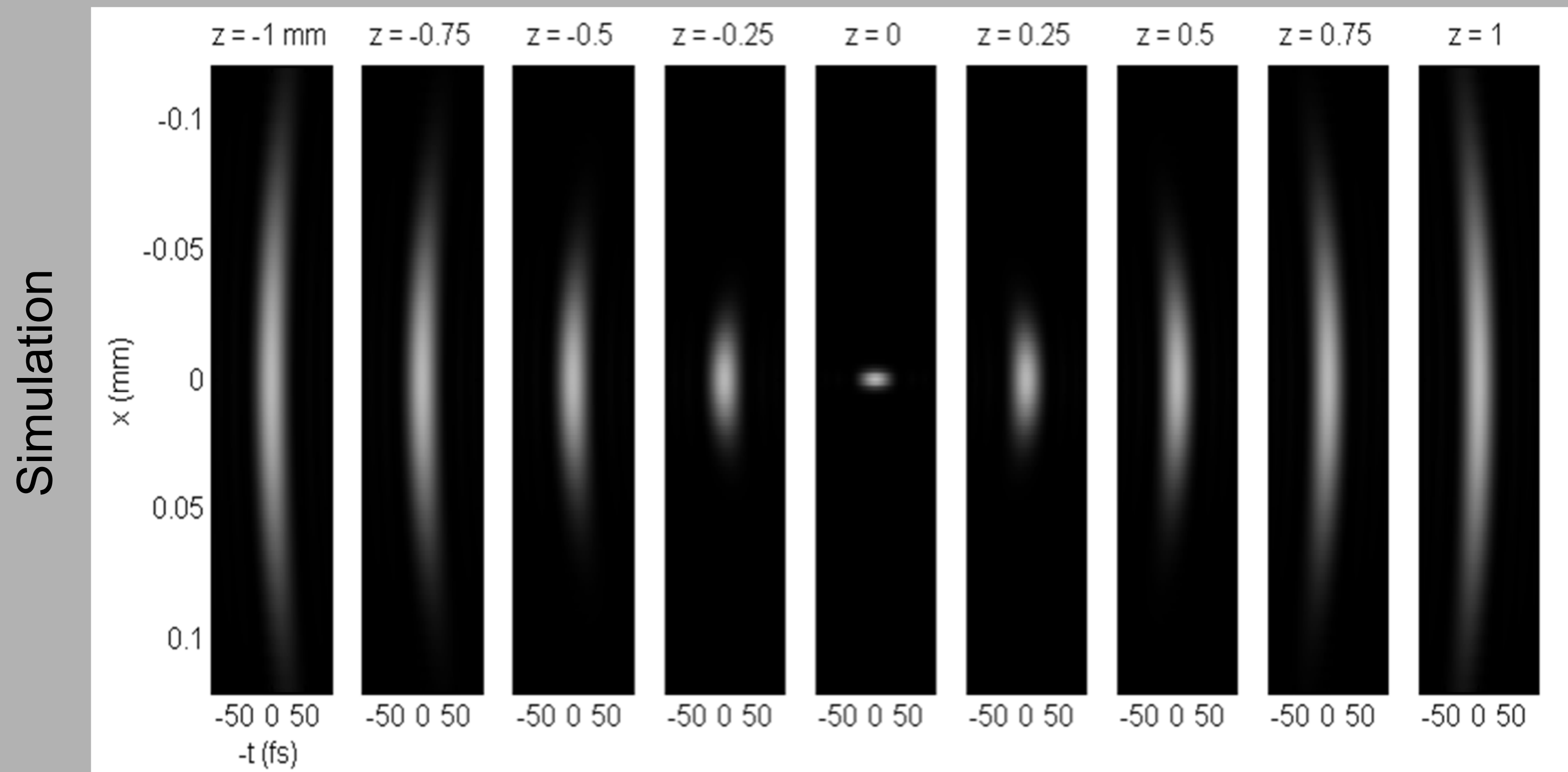
Nevertheless, off-axis rays see a different focal length, so lenses and curved mirrors have **spherical aberration**, too.

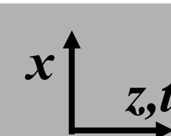


This yields spatial fringes before the focus due to the crossing of the beams.

A Theoretically Perfectly Focused Pulse

Side views: $E(x,z,t)$





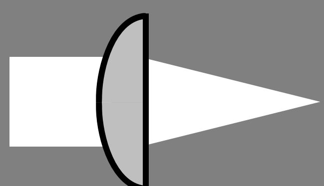
A white pulse remains white vs. x and t (that is, z).

Uniform color indicates a lack of phase distortions.

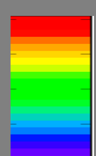
Spherical and chromatic aberration

Singlet BK-7 plano-convex lens with spherical and chromatic aberration and GDD.

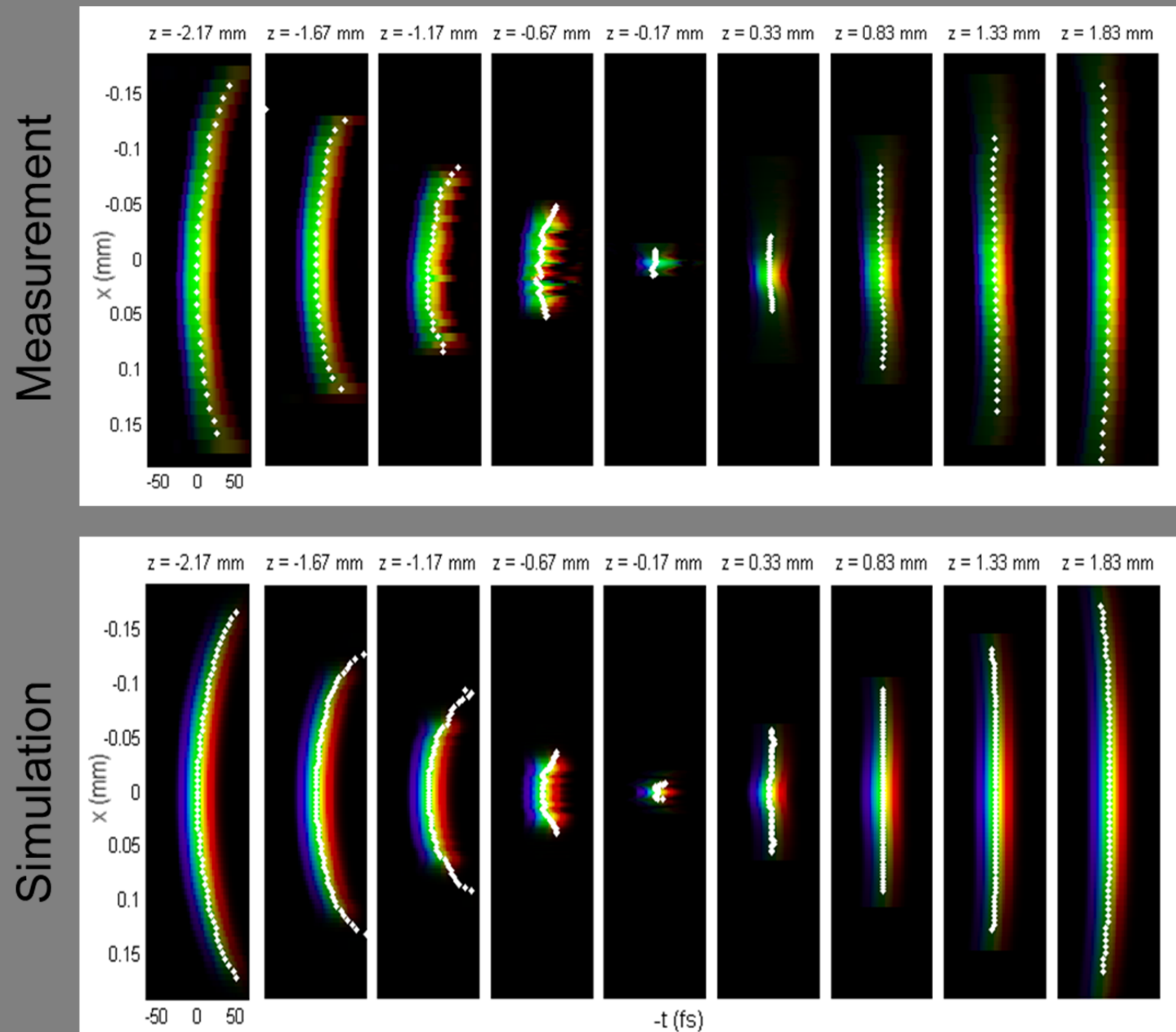
f = 50mm
NA = 0.03*



810 nm



790 nm



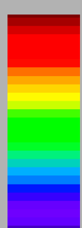
*reminder: NA = numerical aperture = 0.5/(f-number)

Distortions are more pronounced for a tighter focus.

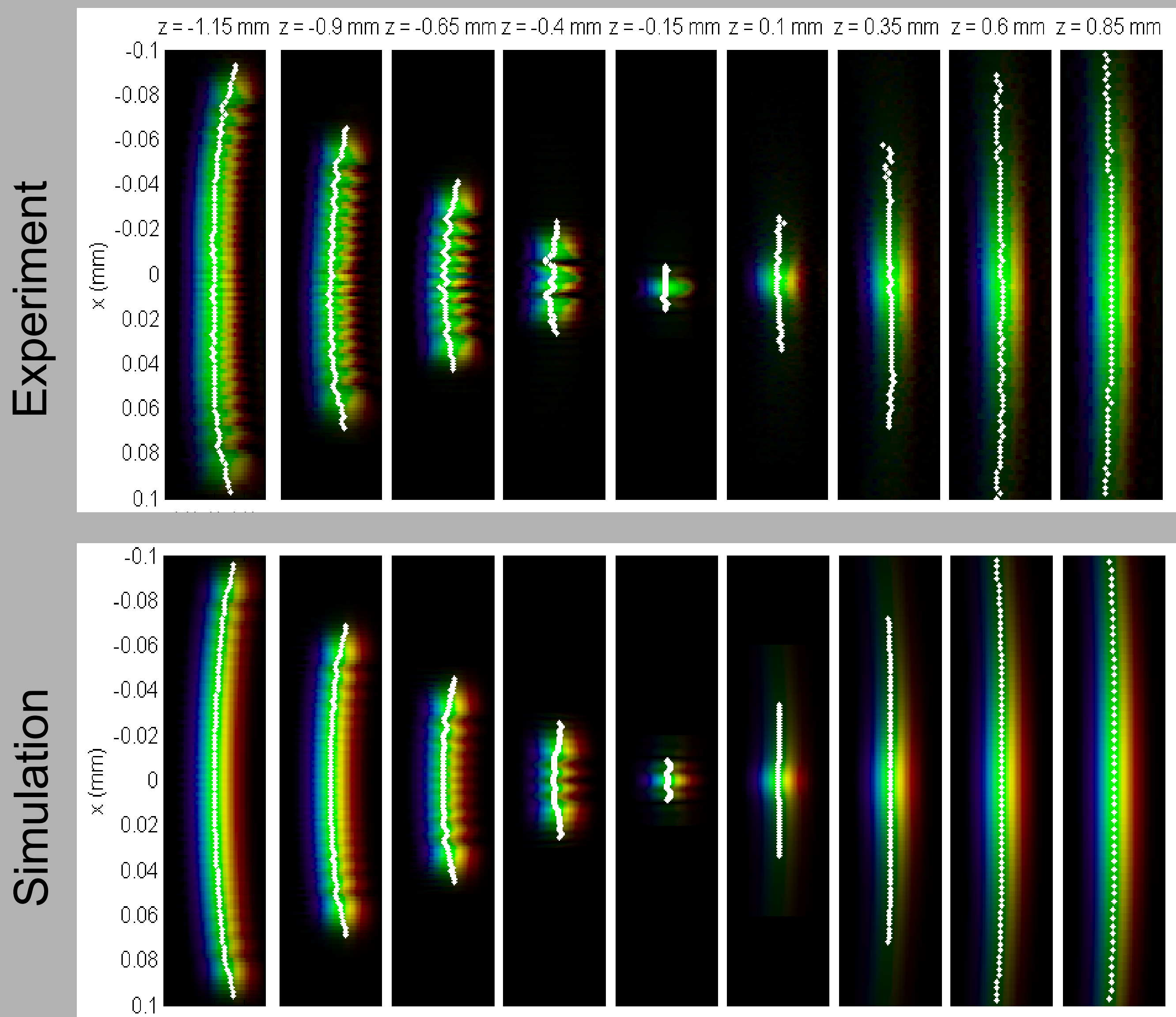
Singlet BK-7 plano-convex lens with a shorter focal length.

f = 25mm
NA = 0.06

814 nm



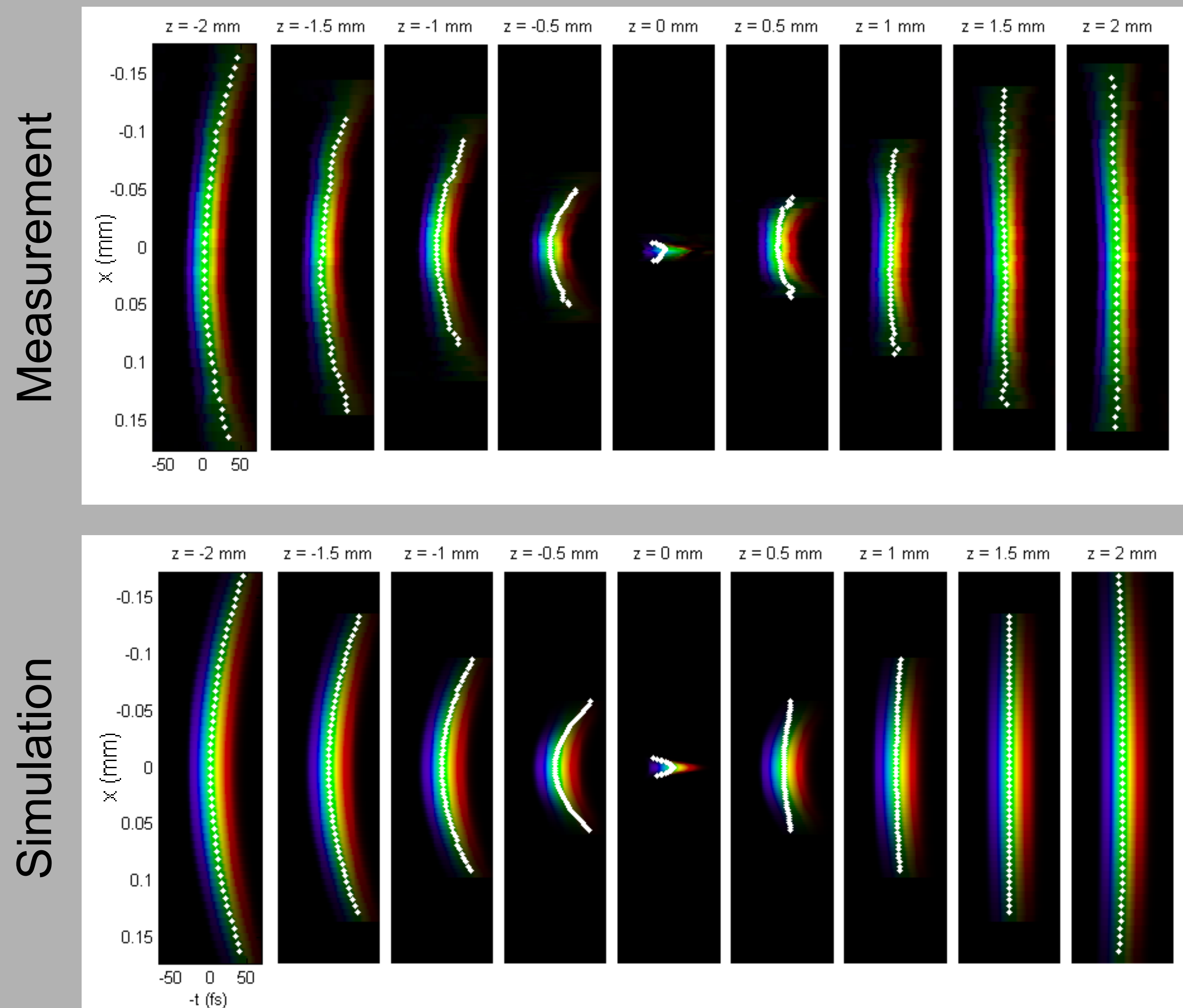
787 nm

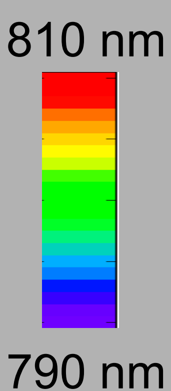


Measured $E(x,z,t)$ of a focused pulse

Aspheric PMMA lens with chromatic (but no spherical) aberration and GDD.

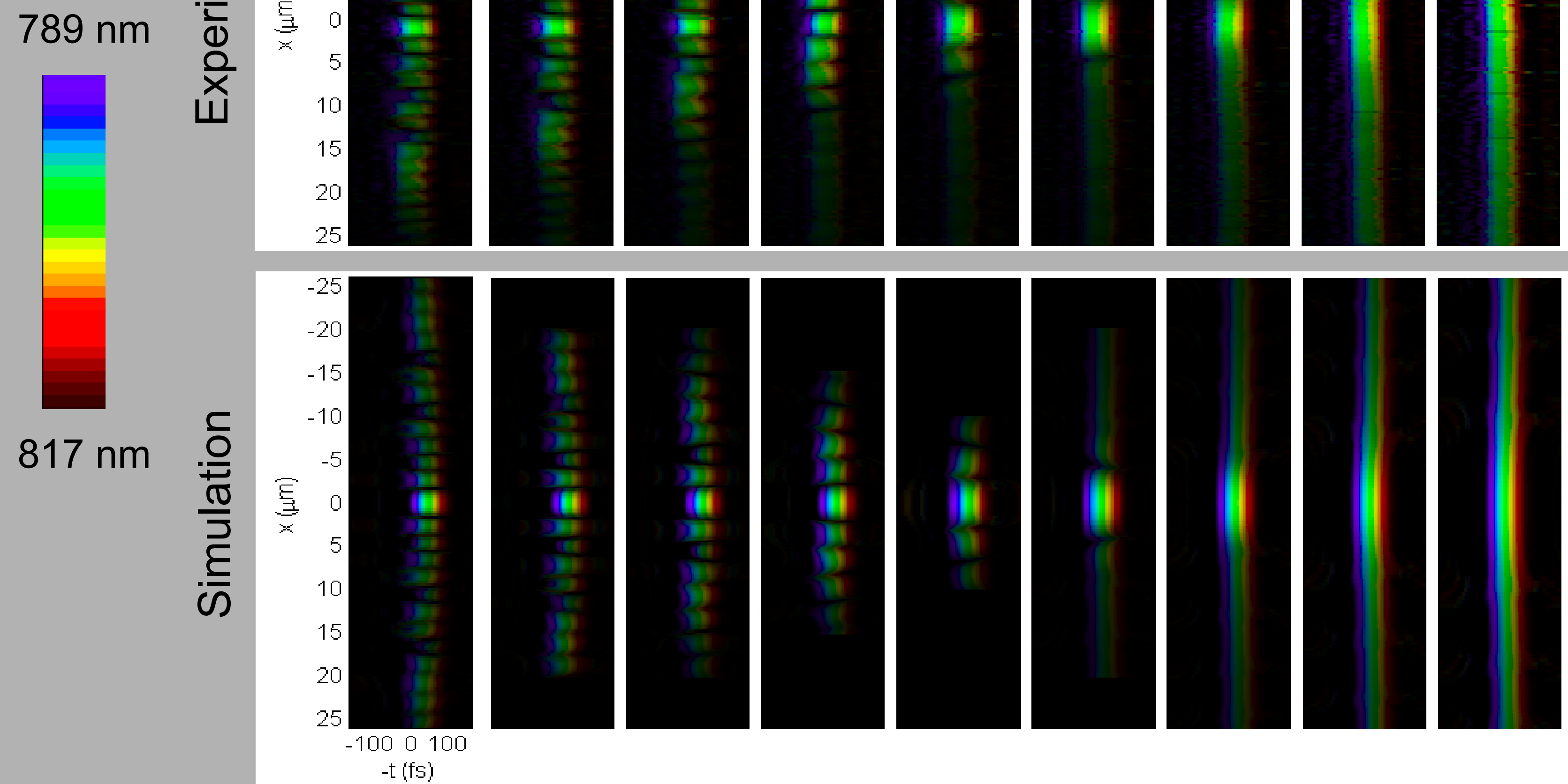
f = 50mm
NA = 0.03



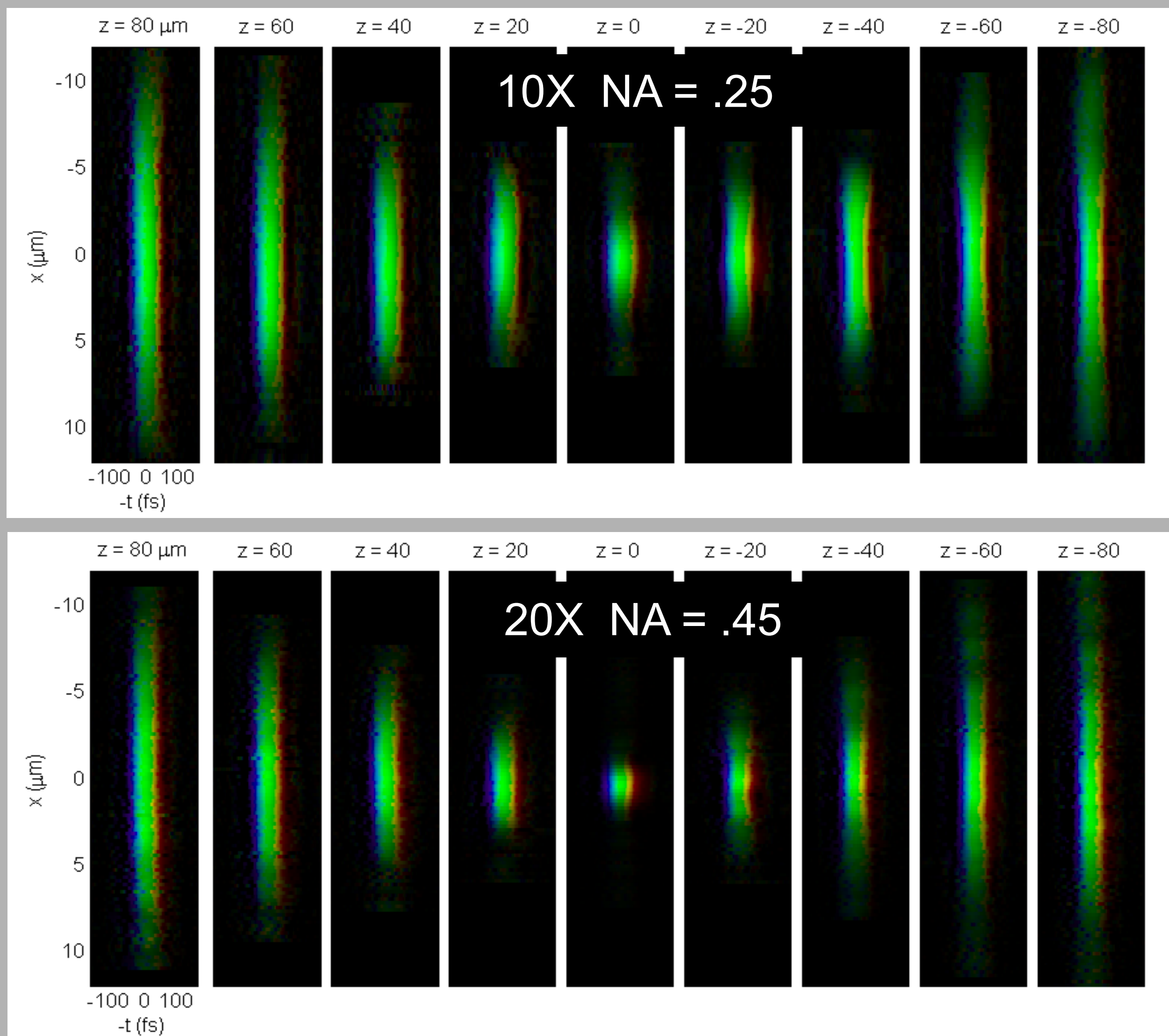


The focus of an SF11 plano-convex lens

NA = .28

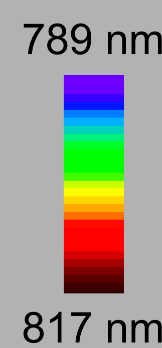


Measurements of microscope objectives



Minimal aberrations, but GVD is present.

The spot size at the focus is 4µm.



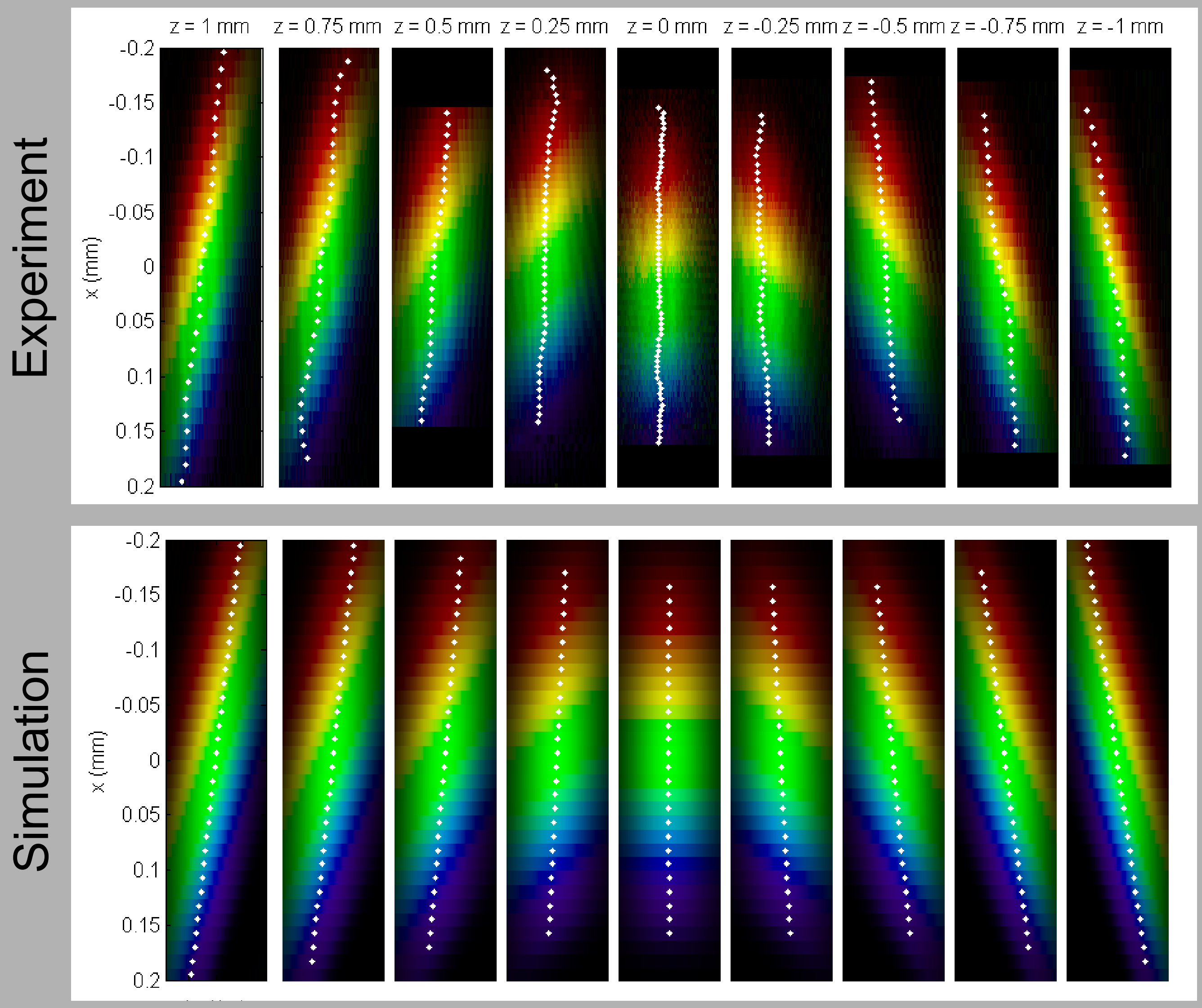
The spot size at the focus is 2µm.

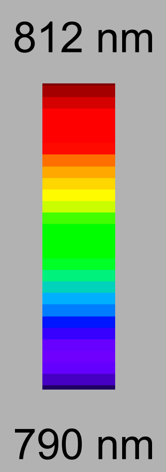
Some radially varying GDD is present.

Focusing a pulse with spatial chirp and pulse-front tilt.

Aspheric PMMA lens.

f = 50mm
NA = 0.03

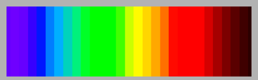




A “fore-runner” pulse due to edge effects

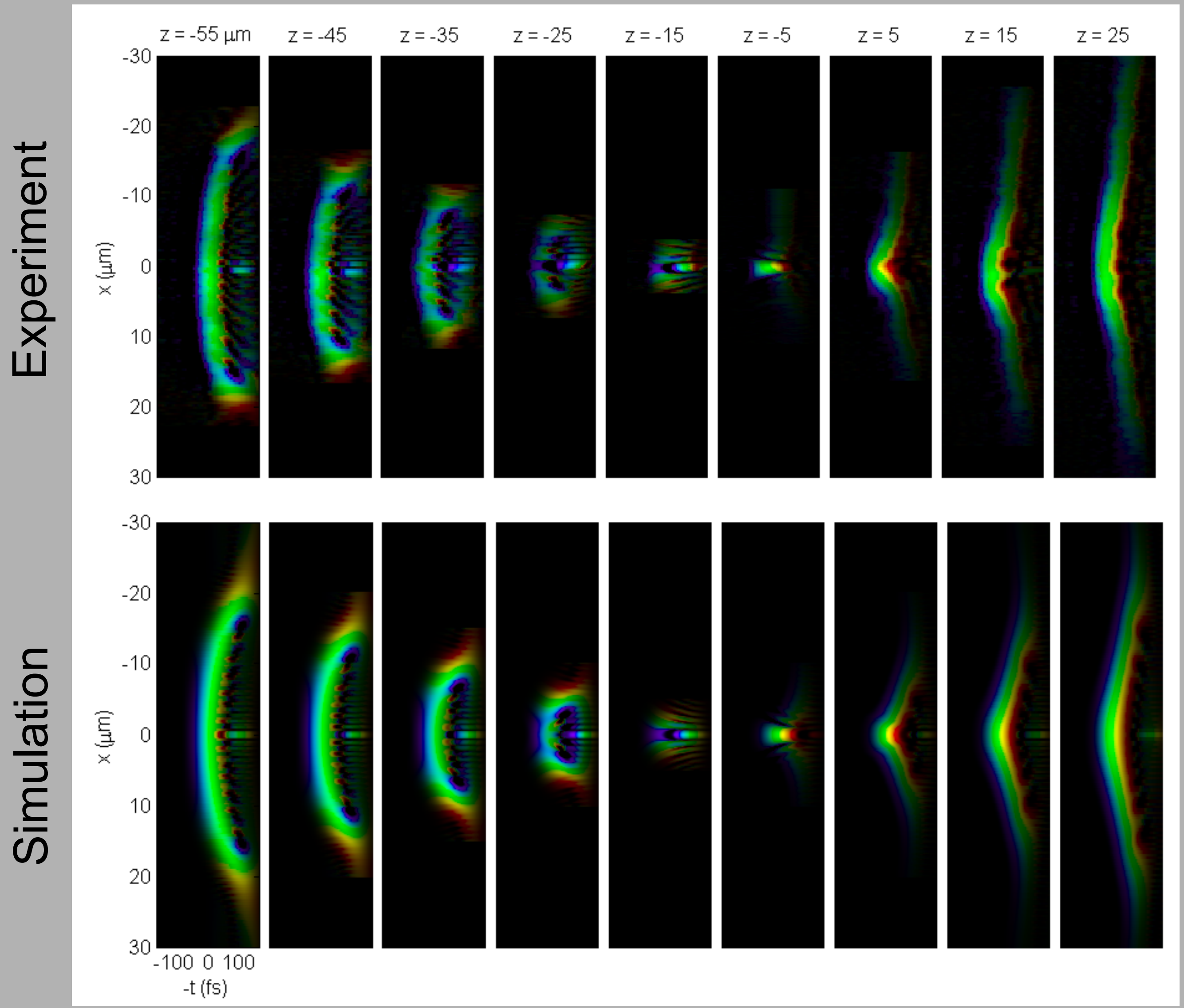
Overfilling of the lens and chromatic aberration cause an additional “fore-runner” pulse ahead of the main pulse.

789 nm



817 nm

NA = 0.4



How to focus an ultrashort pulse

You cannot do it perfectly and easily. Options:

1) Use an achromatic, highly corrected lens and **pre-compensate** for the average GDD.

   You can't really do this exactly (GDD varies with radius).
   Third-order spectral phase is also likely present.
   Some pulses have more bandwidth than any lens is designed for.

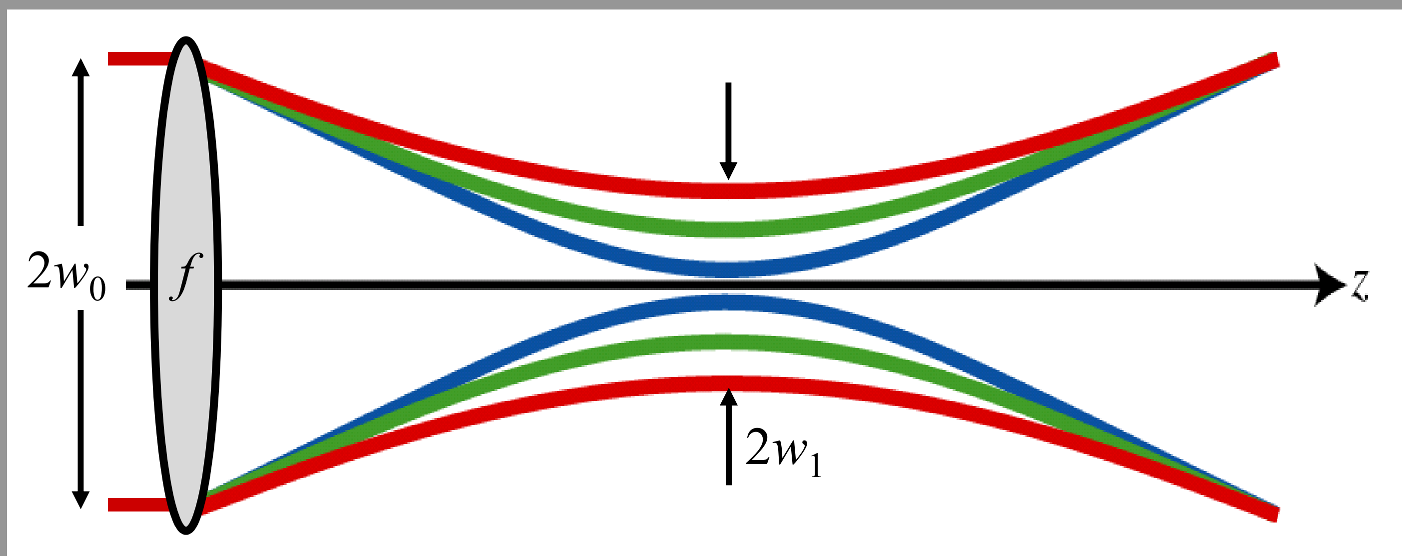
2) Use a curved **mirror**.

   On axis (Cassegrain design), the beam center is blocked.
   Off axis, an **off-axis paraboloid** is hard to align.

Clearly it's important to measure the pulse in space and time as well as possible to see if you've done what you hoped to do.

Even a perfectly achromatic lens with no GVD may not focus the way you'd like…

Different colors focus to different spot sizes: $w_1 = \dfrac{\lambda f}{\pi w_0} \propto \lambda$

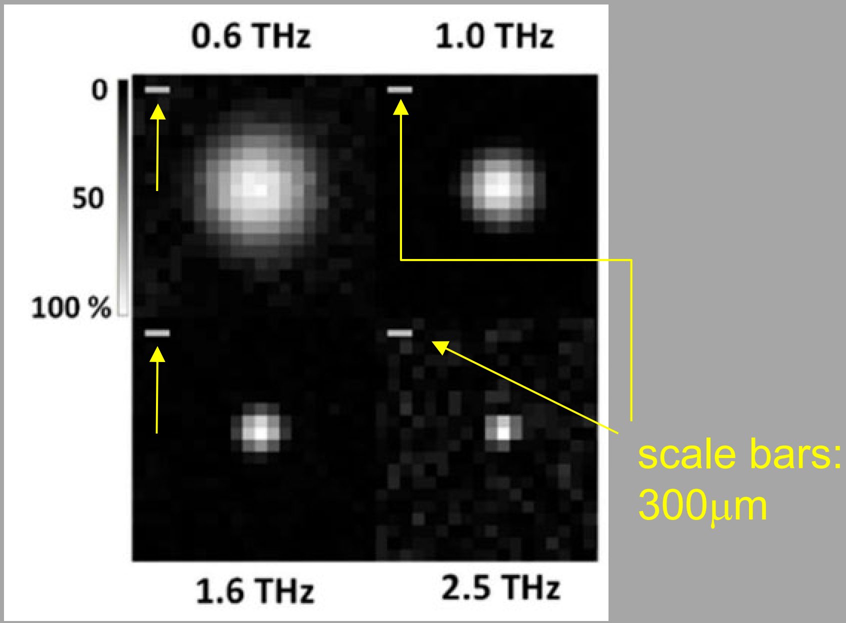


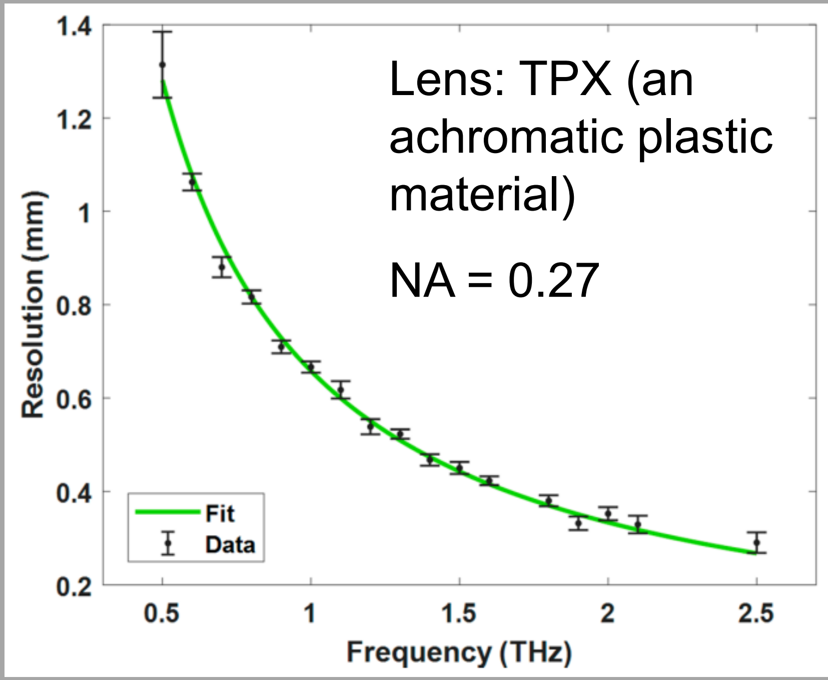
To avoid this, you would need to start with a beam with a color-dependent input spot size…

Again, good spatiotemporal pulse measurement is important.

Using an achromatic lens to focus a broadband terahertz pulse

Four images of a 200μm pinhole at four different frequencies, obtained by scanning the pinhole in the focus of a broadband THz pulse.





Resolution is given by the focusing of a Gaussian beam:

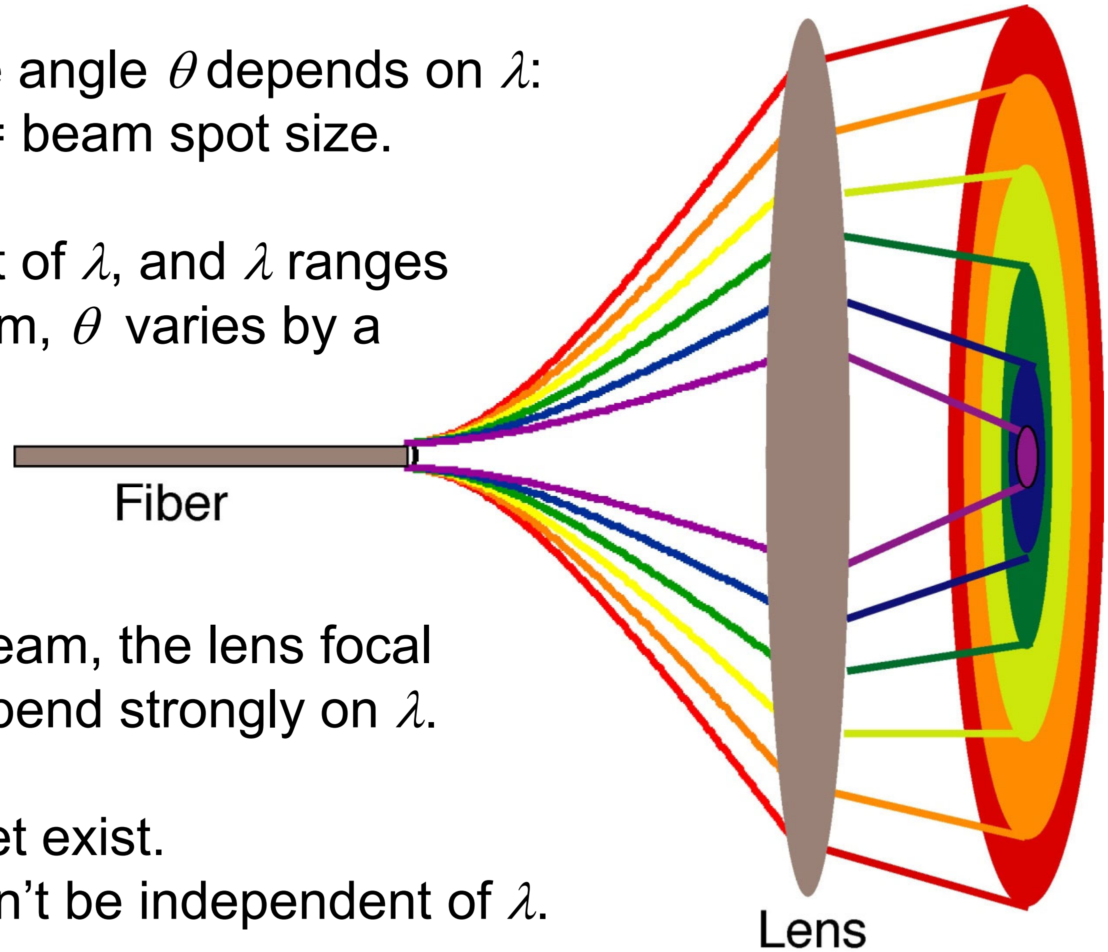
$$R = 1.22\lambda \cdot \frac{f}{D}$$

From: M. Di Fabrizio et al., *Appl. Sci.* **11**, 562 (2021)

Everything we've just said about focusing also applies to collimating a diverging pulse.

The beam divergence angle θ depends on λ:
$\theta = 2\lambda/\pi w$, where w = beam spot size.

So if w is independent of λ, and λ ranges from 400nm to 1600nm, θ varies by a factor of 4.



To collimate such a beam, the lens focal length will have to depend strongly on λ.

Such lenses do not yet exist.
Worse, w typically won't be independent of λ.