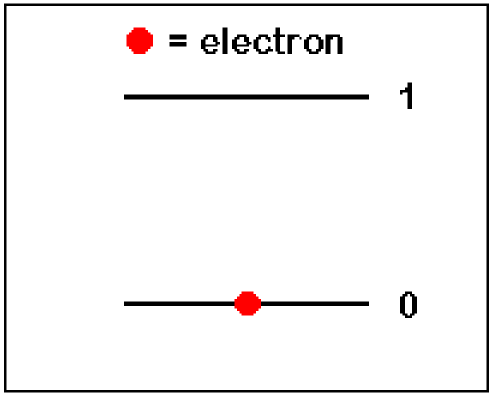
2. Laser physics - basics

Spontaneous and stimulated processes

Einstein A and B coefficients

Rate equation analysis

Gain saturation



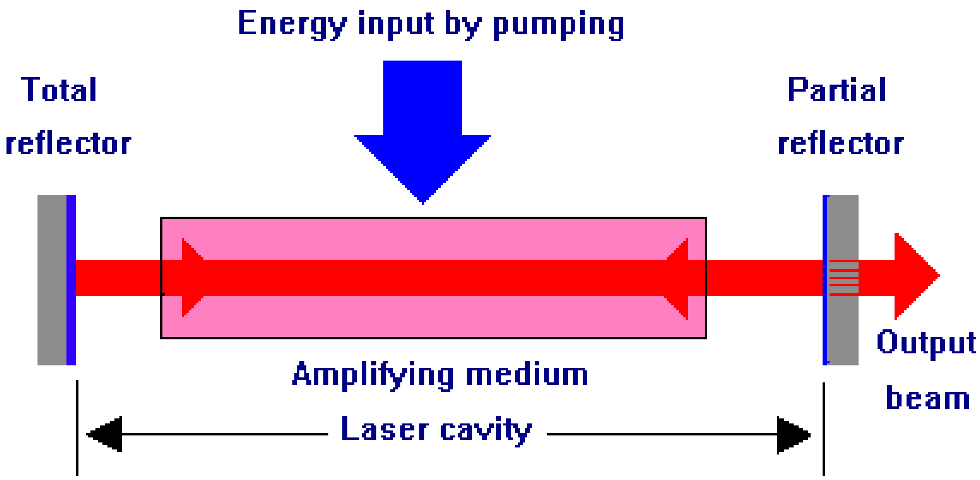
What is a laser?

LASER: Light Amplification by Stimulated Emission of Radiation

"light" could mean anything from microwaves to x-rays

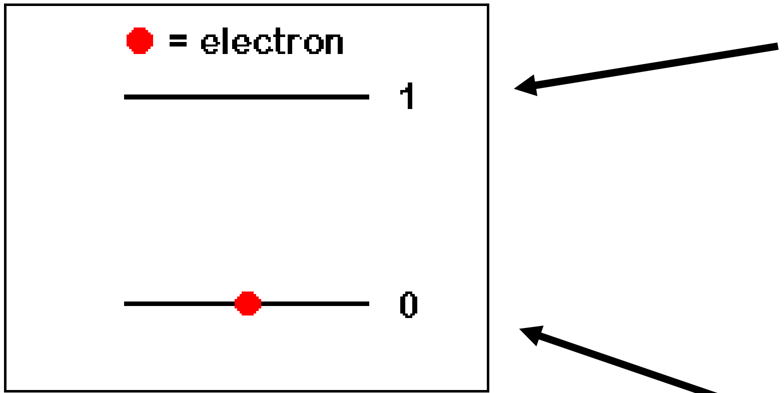
Essential elements:

1. A laser medium - a collection of atoms, molecules, etc.
2. A pumping process - puts energy into the laser medium
3. Optical feedback - provides a mechanism for the light to interact (possibly many times) with the laser medium



The two-level atom

Quantum energy levels



excited state

ground state

Absorption: promotes an electron from the ground to the excited state
Emission: drops the electron back to the ground state

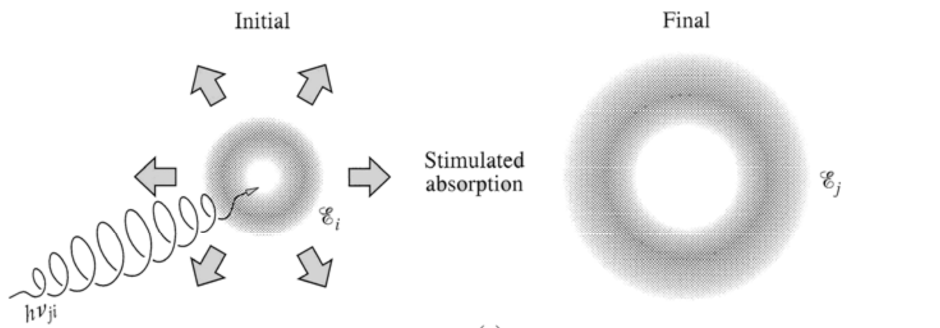
"spontaneous emission" - the decay of an excited state to the ground state with the corresponding emission of a photon

Conservation of energy: $E_{excited} - E_{ground} = E_{photon}$

Three things can occur

Absorption

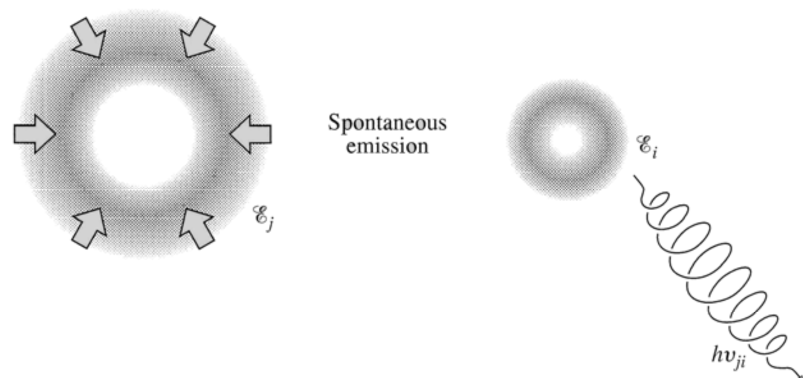
•Promotes molecule to a higher energy state
•Decreases the number of photons



(a)

Spontaneous Emission

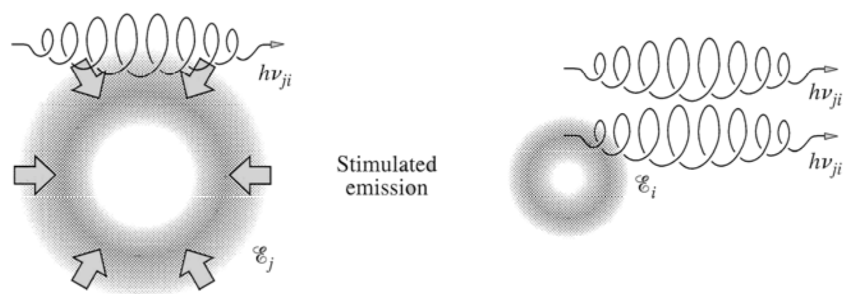
•Molecule drops from a high energy state to a lower state
•Increases the number of photons
•This is the only one that does NOT require a photon in the initial state



(b)

Stimulated Emission

•Molecule drops from a high energy state to a lower state
•The presence of one photon stimulates the emission of a second one



Relaxation of the two-level atom

An atom in the excited state can relax to the ground state by:

- spontaneous emission: rate is γ_{rad}
- any of a variety of non-radiative pathways: rate = γ_{nr}

All of these processes are single-atom processes;
each atom acts independently of all the others.

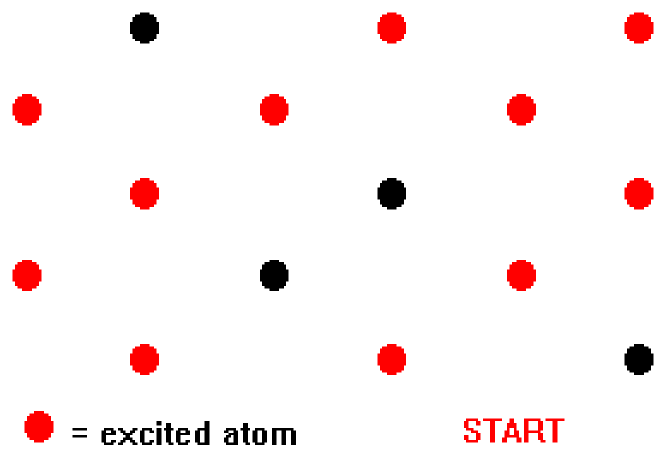
Thus, evolution of the excited state population only depends on the number of atoms in the excited state:

$$\frac{dN_e}{dt} = -\gamma_{rad} N_e - \gamma_{nr} N_e = -\gamma_{10} N_e$$

γ_{10} = total spontaneous relaxation rate from state 1 to state 0

A collection of two-level atoms

"Stimulated transitions" - a collective process involving many two-level atoms



stimulated absorption: light induces a transition from 0 to 1

stimulated emission: light induces a transition from 1 to 0

In the emission process, the emitted photon is identical to the photon that caused the emission!

Stimulated transitions: likelihood depends on the number of photons around

How did it all begin?

Rayleigh-Jeans law (circa 1900):

energy density of a radiation field $u(\nu) = 8\pi\nu^2 kT/c^3$

Note: the units of this expression are correct. Strictly speaking, $u(\nu)$ is an energy density per unit bandwidth, such that the integral $\int u(\nu)\,d\nu$ gives an answer with units of energy/volume.

Total energy radiated from a black body: $$\int u(\nu)\,d\nu = \infty$$

uh-oh… the "ultraviolet catastrophe"

Solution: quantum mechanics

<u>Time-dependent perturbation theory</u>

As a result of a perturbation h(t), a system in quantum state 1 makes a transition to quantum state 2 with probability given by:

$$P_{1\to 2} = \frac{1}{\hbar^2}\left|\int_{-\infty}^{t} e^{iE_{21}t'/\hbar} h(t')\,dt'\right|^2$$

Notation: $\omega_{21} = \frac{E_{21}}{\hbar}$

Harmonic perturbation

Key example: suppose we subject a two-level system, initially in state 1, to a harmonic perturbation, of the form:

$$h(t) = \begin{cases} 0 & t < 0 \\ 2A_0 \sin \omega t & t > 0 \end{cases}$$

(and suppose that the frequency of the perturbation, ω, is close to ω_{21})

Transition probability to state 2 is:

$$P_{1\to 2} = \frac{A_0^2}{\hbar^2} \left| \int_0^t e^{i\omega_{21}t'} \left(e^{i\omega t'} - e^{-i\omega t'} \right) dt' \right|^2$$

$$= \frac{4A_0^2}{\hbar^2} \left| \left(\frac{e^{i(\omega_{21}-\omega)t/2} \sin\left(\omega_{21} - \omega \right) t/2}{\left(\omega_{21} - \omega \right)} \right) - \cancelto{\text{RWA}}{\left(\frac{e^{i(\omega_{21}+\omega)t/2} \sin\left(\omega_{21} + \omega \right) t/2}{\left(\omega_{21} + \omega \right)} \right)} \right|^2$$

$$\approx \frac{4A_0^2}{\hbar^2} \cdot \frac{\sin^2 \left[\left(\omega_{21} - \omega \right) t/2 \right]}{\left(\omega_{21} - \omega \right)^2}$$

Note that $P_{1\to 2} = P_{2\to 1}$

Absorption and stimulated emission are equally likely!

Einstein A and B coefficients

Consider a radiation field and a collection of two-level systems, in thermal equilibrium with each other.

<u>stimulated emission probability:</u> proportional to the number of atoms in upper state N_2, and also to the number of photons

<u>spontaneous emission probability:</u> proportional to N_2, but does not depend on the photon density!

$$W_{2\to 1} = \boxed{A} \cdot N_2 + \boxed{B} \cdot u(\nu) N_2$$

Note: this is the same as γ_{rad}

<u>stimulated absorption probability:</u> proportional to the number of atoms in lower state N_1, and also to the number of photons

<u>spontaneous absorption:</u> there is no such thing

$$W_{1\to 2} = \boxed{B} \cdot u(\nu) N_1$$

Quantum mechanics says that these two coefficients must be equal!

But: in thermal equilibrium, the upward and downward transition rates must balance: $W_{1\to 2} = W_{2\to 1}$

Einstein A and B coefficients

Equate these two rates:

$$\frac{N_1}{N_2} = \frac{A + B \cdot u(\nu)}{B \cdot u(\nu)}$$

But Boltzmann's Law tells us that
(in equilibrium)

$$\frac{N_1}{N_2} = e^{(E_2 - E_1)/kT}$$

Recognizing that $E_2 - E_1 = h\nu$, we solve for $u(\nu)$:

$$u(\nu) = \frac{A/B}{e^{h\nu/kT} - 1}$$

This must correspond to the Rayleigh-Jeans result in the classical limit ($h \to 0$), which implies:

$$\frac{A}{B} = \frac{8\pi h \nu^3}{c^3}$$

Also has units of energy density per unit bandwidth

Since $A = \gamma_{rad}$, we can now solve for B also:

$$B = \frac{\gamma_{rad} c^3}{8\pi h \nu^3}$$

Transition rates

Our expression for the downward transition rate is now:

$$W_{2\to1} = A\cdot N_2 + B\cdot u(\nu)N_2$$
$$= A\cdot N_2\left(1+\frac{B}{A}u(\nu)\right)$$

Bose-Einstein distribution

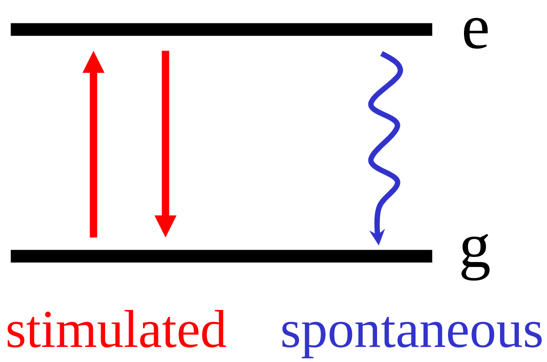
But since $u(\nu)=\dfrac{A/B}{e^{h\nu/kT}-1}$ we therefore have $W_{2\to1} = A\cdot N_2\left(1+\dfrac{1}{e^{h\nu/kT}-1}\right)$

In other words, W_{21} is proportional to: 1 + the number of photons.

It is easy to see that the upward transition rate, W_{12}, is proportional to the number of photons:

$$W_{1\to2} = B\cdot u(\nu)N_1 = AN_1\left(\frac{1}{e^{h\nu/kT}-1}\right)$$

Rate equation analysis



spontaneous emission:
- proportional to initial state population

stimulated transitions:
- proportional to initial state population
- proportional to photon density n_p
- the same for upwards, downwards transitions

$$\frac{dN_e}{dt} = -\gamma_{eg} N_e - K n_P N_e + K n_P N_g = -\frac{dN_g}{dt}$$

$$\frac{dN_e}{dt} = -\gamma_{eg} N_e + K n_P \left(N_g - N_e \right)$$

Note: the constant *K* is simply given by hν·B, where B is the Einstein B coefficient

Rate equation analysis, continued

$$\frac{dN_e}{dt} = -\gamma_{eg} N_e + K n_P \left(N_g - N_e \right)$$

emitted photons go in all directions

emitted photons go only into the direction of the incident light

So, photon number varies according to:

$$\frac{dn_P}{dt} = -K n_P \left(N_g - N_e \right) = -K n_P \cdot \Delta N$$

Number of photons grows exponentially if $N_e > N_g$

A LASER!

Rate equation analysis, part 3

In thermal equilibrium:

$$\boxed{N_e = e^{-\Delta E/kT} \cdot N_g < N_g}$$

$\longrightarrow$ Population inversion is impossible in equilibrium.

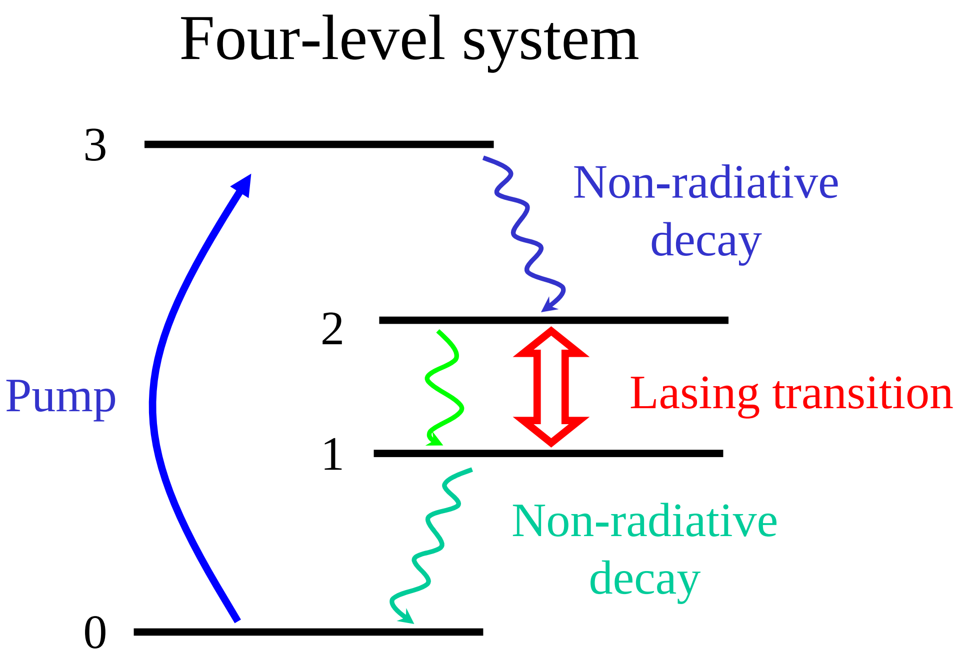
In a steady-state situation:

$$\frac{dN_e}{dt} = -\gamma_{eg} N_e + K n_P \left(N_g - N_e \right) = 0$$

$$\boxed{N_e = \frac{K n_P}{\gamma_{eg} + K n_P} N_g < N_g}$$

$\longrightarrow$ Population inversion is impossible in steady-state.

So how do you make a laser?



Steps 1 and 2:
Combine to give an *effective* pumping rate for level 2: R_p

Step 3:
stimulated transitions due to n_p
spontaneous decay rate: γ_{21}

Step 4:
spontaneous decay rate: γ_{10}

$$\frac{dN_2}{dt} = R_p - \gamma_{21} N_2 + K n_p (N_1 - N_2)$$

$$\frac{dN_1}{dt} = \gamma_{21} N_2 - K n_p (N_1 - N_2) - \gamma_{10} N_1$$

$$\frac{dN_0}{dt} = \gamma_{10} N_1 - R_p$$

The four-level model

Steady-state solution:

$$\Delta N = N_1 - N_2 = \frac{R_P}{\gamma_{10}}\left[\frac{\gamma_{21}-\gamma_{10}}{\gamma_{21}+Kn_P}\right]$$

Population inversion (i.e., $\Delta N < 0$) is assured if $\gamma_{10} > \gamma_{21}$

(even if $n_p = 0$, and even if R_p is small)

A necessary condition for lasing

Other necessary conditions:

- a resonant cavity - provides feedback
- net gain per round trip > net loss per round trip - “threshold”

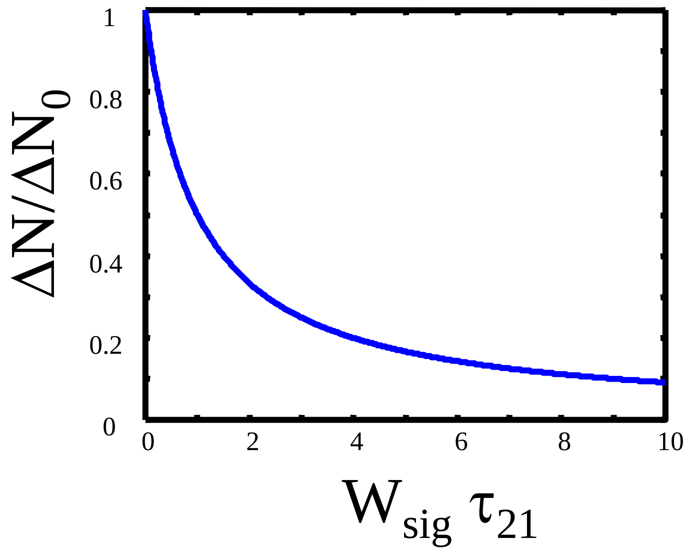
Saturation in the four-level atom

$$\Delta N = \frac{R_P}{\gamma_{10}}\left[\frac{\gamma_{21}-\gamma_{10}}{\gamma_{21}+Kn_P}\right] = \boxed{\frac{\gamma_{21}-\gamma_{10}}{\gamma_{21}\gamma_{10}}\cdot R_P}\left[\frac{1}{1+W_{sig}\tau_{21}}\right]$$

$\uparrow$ ΔN_0 $\qquad$ $\uparrow$ Kn_p $\qquad$ $\uparrow$ $1/\gamma_{21}$

- population inversion when $\tau_2 > \tau_1$
- small signal inversion is proportional to the pump rate
- inversion level drops when $W_{sig} > \gamma_{21}$
- the characteristic intensity for this effect is independent of pump rate R_p

→ "gain saturation"



Note: W_{sig} is proportional to n_P and therefore to the intensity of the light in the medium. Thus, we can define a saturation intensity I_{sat} such that:

$$W_{sig}\tau_{21} = I/I_{sat}$$