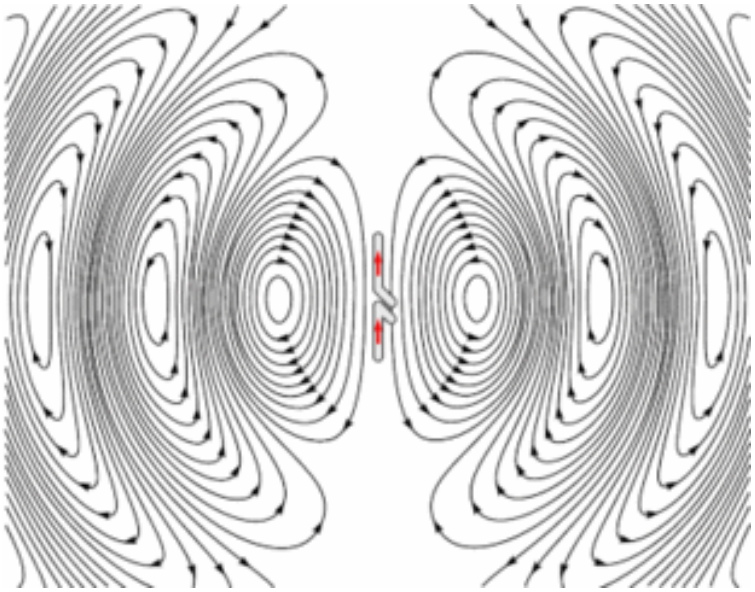
24. Antennas



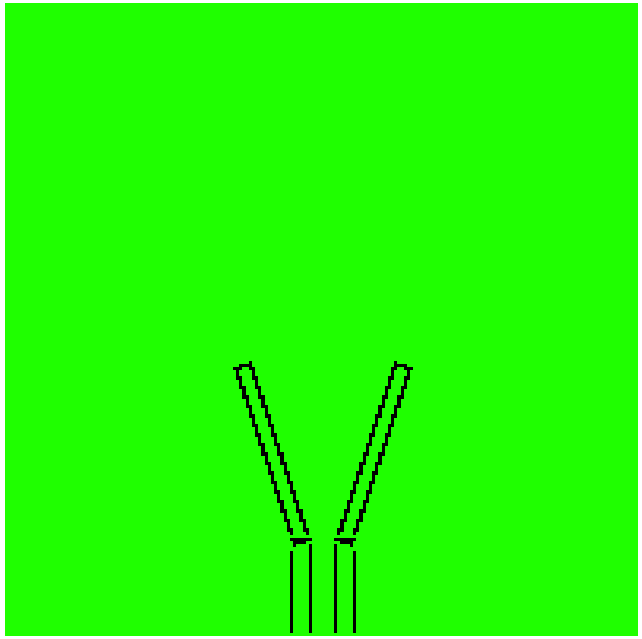
What is an antenna

Types of antennas

Reciprocity

Hertzian dipole

- near field
- far field: radiation zone
- radiation resistance
- radiation efficiency

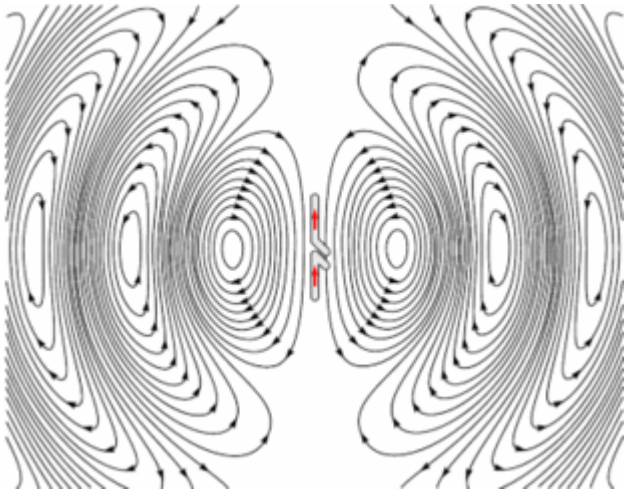


Antennas convert currents to waves

An antenna is a device that converts a time-varying electrical current into a propagating electromagnetic wave.

Since current has to flow in the antenna, it has to be made of a conductive material: a metal.

And, since EM waves have to propagate away from the antenna, it needs to be embedded in a transparent medium (e.g., air).



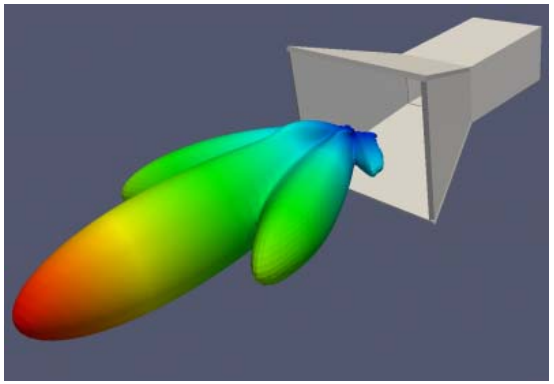
Antennas can also work in reverse: converting incoming EM waves into an AC current. That is, they can work in either transmit mode or receive mode.

Antennas need electrical circuits

- In order to drive an AC current in the antenna so that it can produce an outgoing EM wave,
- Or, in order to detect the AC current created in the antenna by an incoming wave,

…the antenna must be connected to an electrical circuit.

Often, people draw illustrations of antennas that are simply floating in space, unattached to anything.



Always remember that there needs to be a wire connecting the antenna to a circuit.

Bugs also have antennas.
But not the kind we care about.

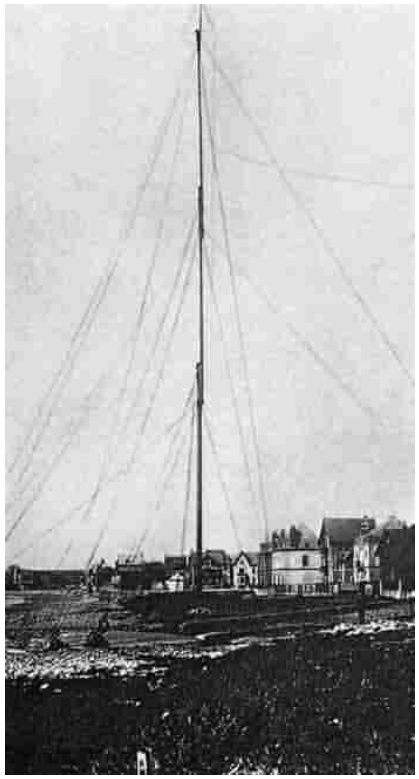
not made of metal

The word “antenna” comes from the Italian word for “pole”. Marconi used a long wire hanging from a tall pole to transmit and receive radio signals.

Guglielmo Marconi
1874-1937

His use of the term popularized it.

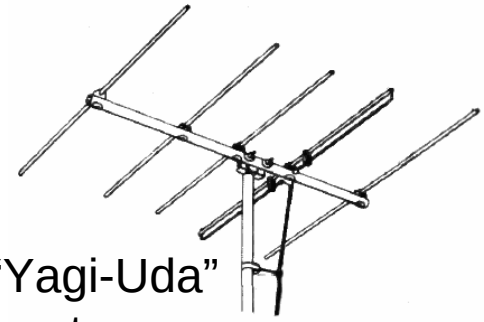
The antenna used by Marconi for the first trans-Atlantic radio broadcast (1902)



There are many different types of antennas

- Omnidirectional antennas – designed to receive or broadcast power more or less in all directions (although, **no** antenna broadcasts equally in every direction)
- Directional antennas – designed to broadcast mostly in one direction (or receive mostly from one direction)

Examples of omnidirectional antennas, typically used for receiving radio or TV broadcasts.



"Yagi-Uda" antenna

"rabbit ears"

automotive "whip" antenna

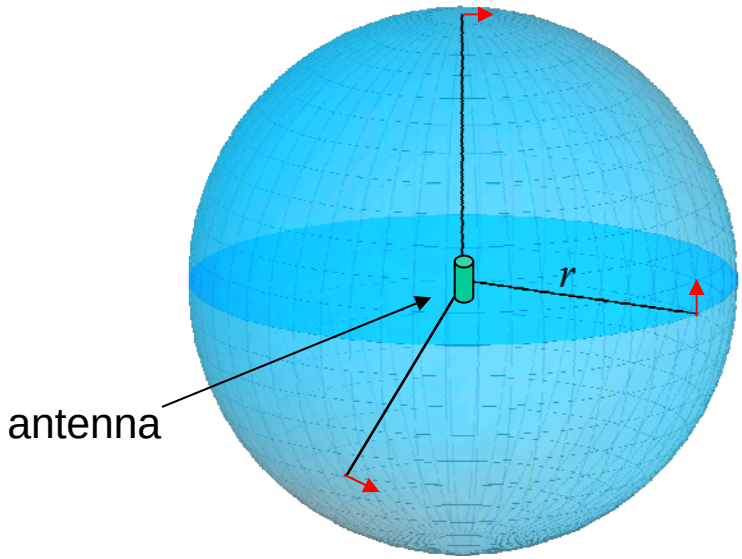
parabolic dish antenna

pyramidal horn antenna

Examples of directional antennas, typically used for higher frequencies, e.g., microwaves and millimeter waves

Why is there no such thing as a perfectly omnidirectional antenna?

Suppose there was one. Consider a large sphere centered on this hypothetical antenna.



Everywhere on the surface of this sphere, there would need to be a non-zero tangential electric field vector.

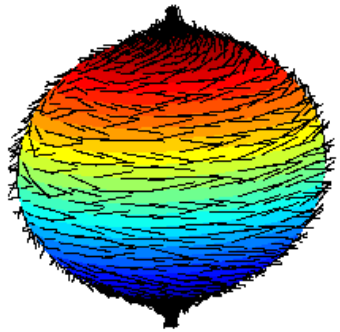
But this can't happen. It is impossible to completely cover the surface of a sphere with a continuous tangential vector field that has no zeros.

This is known as the Hairy Ball Theorem:

There is no nonvanishing continuous tangent vector field on a sphere.

OR:

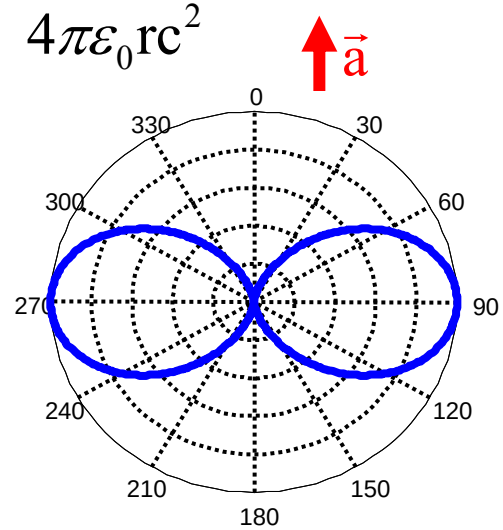
You can't comb a hairy ball flat without creating at least one cowlick.



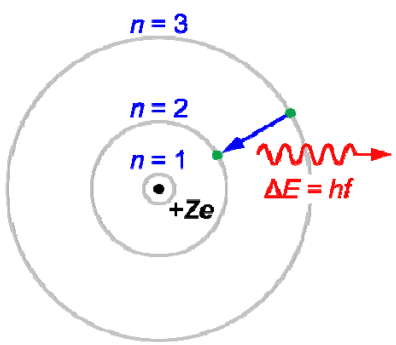
Reminder: accelerating charges

Accelerating charges radiate electromagnetic waves.

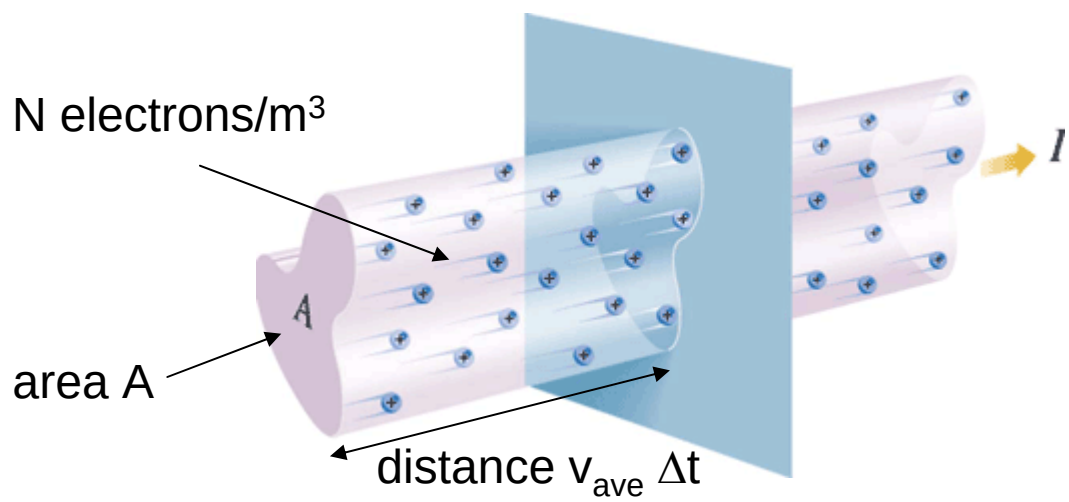
$$E(r,t) = \frac{q\mathrm{a}(t)\sin\theta}{4\pi\varepsilon_0 \mathrm{r}c^2}$$



This is true whether the charges are bound inside atoms:



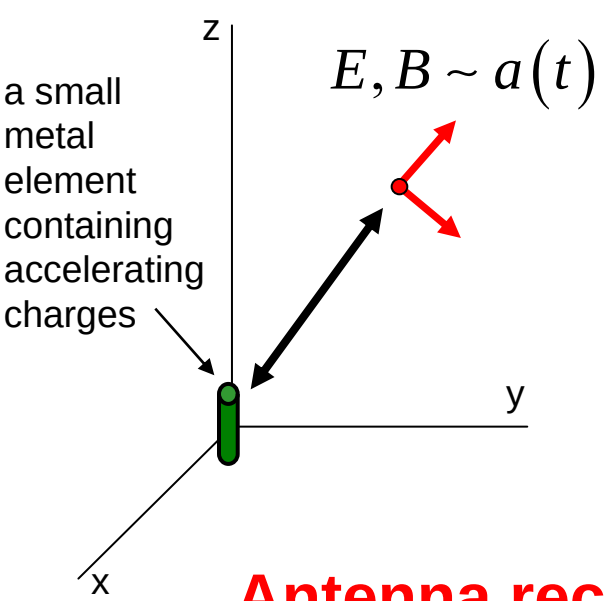
or they are freely flowing (like a time-varying current in a wire):



Reciprocity

$$E(r,t) = \frac{q\mathrm{a}(t)\sin\theta}{4\pi\varepsilon_0 \mathrm{r}c^2}$$

Note that the radiated electric field is proportional to the acceleration of the charge.



Since Maxwell's equations are linear: when a charge acceleration gives rise to a field, the same field can induce the same acceleration:

$$\mathrm{a}(t) = E(r,t)\frac{4\pi\varepsilon_0 \mathrm{r}c^2}{q\sin\theta}$$

cause ⟷ effect

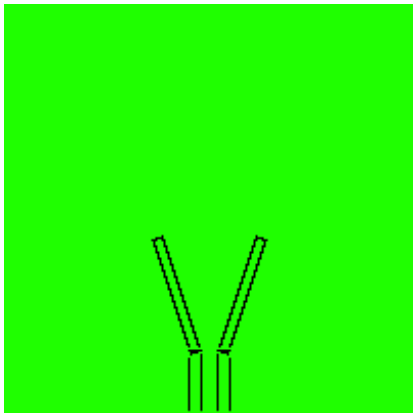
Antenna reciprocity: a receiving antenna's properties are identical to the transmitting properties of the same antenna when it is used as a transmitter.

Antenna reciprocity

Antenna reciprocity: a receiving antenna's properties are identical to the transmitting properties of the same antenna when it is used as a transmitter.

Properties: this includes <u>everything</u> about the antenna

- antenna gain
- radiation pattern
- radiation resistance
- polarization
- bandwidth



e.g. this antenna transmits mostly in one particular direction. It also receives mostly from that same direction.

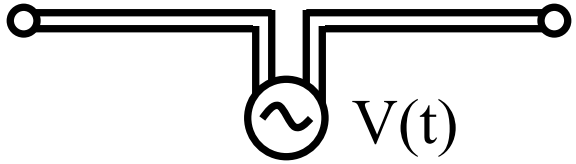
So we don't need to keep saying "transmit or receive". We can talk about just one mode, since the other one is the same in reverse.

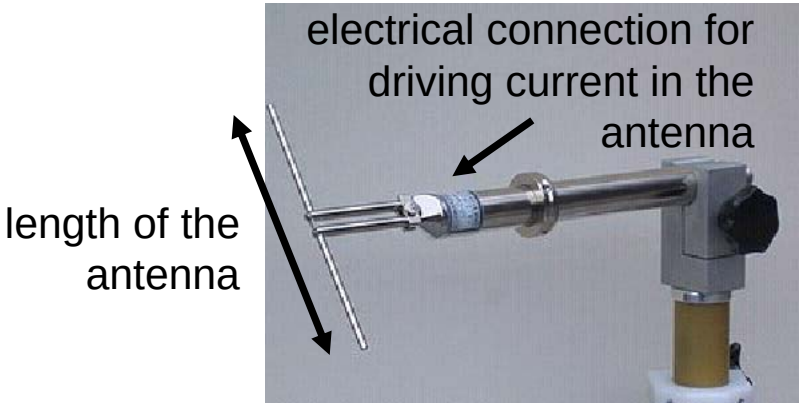
In an antenna, changing currents give rise to radiated electromagnetic waves

Obviously, the many different possible antenna geometries give rise to many different radiated fields, due to the different spatial distributions of changing currents.

We need to treat a simple case first:

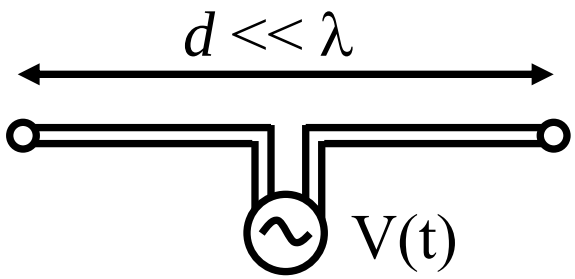
the “Hertzian dipole”





The Hertzian dipole antenna

The Hertzian dipole is a linear antenna which is much shorter than the free-space wavelength:



For our purposes, we can treat this as a wire of infinitesimal length d, carrying a current $I(\mathrm{t}) = I_0 \cos(\omega \mathrm{t}) \rightarrow I_0 \, \mathrm{e}^{j\omega \mathrm{t}}$.

In 1887, Hertz used a sub-wavelength antenna to generate (transmit) and detect (receive) radio waves – the first wireless broadcast. It is considered the first experimental proof of Maxwell's equations.

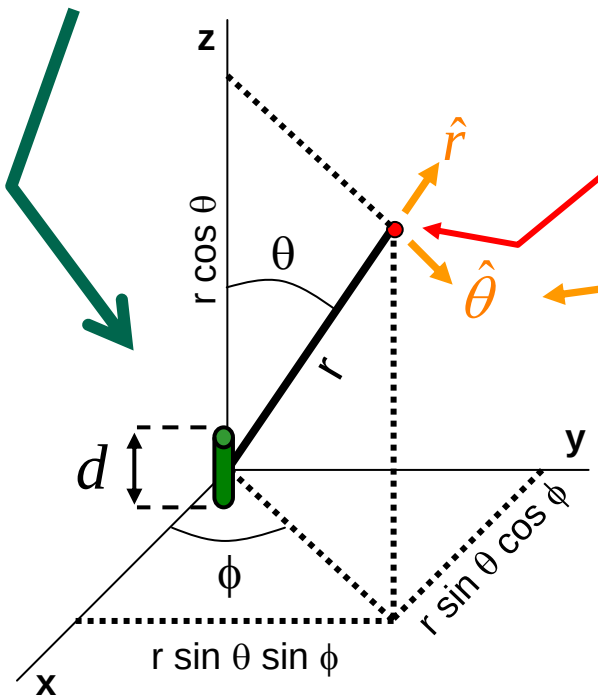
Heinrich Hertz
1857 - 1894

Asked about the ramifications of his discoveries, Hertz replied, "*Nothing, I guess.*"

Hertzian dipole – coordinate system

To treat this problem, we use spherical coordinates.

The small antenna is located at the origin, and oriented along the z axis.



Spherical coordinates: this point is both (x,y,z) and (r,θ,ϕ)

This shows 2 of the 3 unit vectors for spherical coordinates, pointing along the radial (r) and azimuthal (θ) directions. (The ϕ unit vector is not shown in this diagram.)

The field must obey Maxwell's equations in empty space everywhere outside of the wire.

The field of a Hertzian dipole antenna

The resulting electric and magnetic fields are:

$$\vec{E}(r,\theta,t) = \frac{\mu_0 \omega^2 I_0 d}{4\pi c_0}\left[2\cos\theta\left(\frac{1}{(k_0 r)^2} - \frac{j}{(k_0 r)^3}\right)\hat{r} + \sin\theta\left(\frac{j}{k_0 r} + \frac{1}{(k_0 r)^2} - \frac{j}{(k_0 r)^3}\right)\hat{\theta}\right] e^{-jk_0 r} e^{j\omega t}$$

$$\vec{B}(r,\theta,t) = \frac{\mu_0 \omega^2 I_0 d}{4\pi {c_0}^2}\sin\theta\left(\frac{j}{k_0 r} + \frac{1}{(k_0 r)^2}\right)\hat{\phi}\, e^{-jk_0 r} e^{j\omega t}$$

Complicated expressions! But we can gain physical insight by looking at limiting cases.

Note: even without any simplification, it is obvious that $\vec{E} \perp \vec{B}$.

The near field of a Hertzian dipole antenna

The properties of these fields are quite different if you are near the antenna or far from it.

First limiting case: very near the antenna. So: $k_0 r << 1$

In this case, the $1/r^3$ terms are the only ones that matter in the E-field expression. Also, $\exp(jk_0 r) \sim 1$.

$$\vec{E}(r,\theta,t) \approx \frac{I_0 d}{j\omega\varepsilon_0 \cdot 4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]e^{j\omega t}$$

To interpret the meaning of this, recall that the charge on a conductor is related to the current, according to:

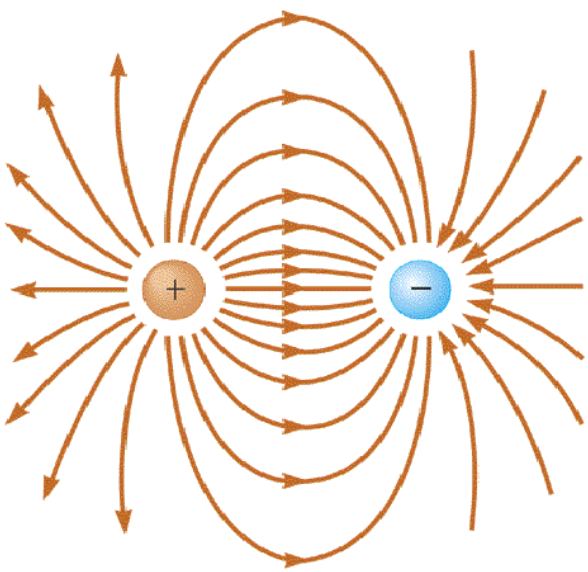
$$I(t) = \frac{dQ}{dt}$$

For our assumed current, this implies that $I_0 = j\omega Q$.

The near field of a Hertzian dipole antenna

$$\vec{E}(r,\theta,t) \approx \frac{Qd}{\varepsilon_0 \cdot 4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right] e^{j\omega t}$$

This is exactly the same result as one finds for the field of a simple static dipole (two charges of opposite sign separated by distance d), except for the time dependence.



For the magnetic field, with the same approximations, we find:

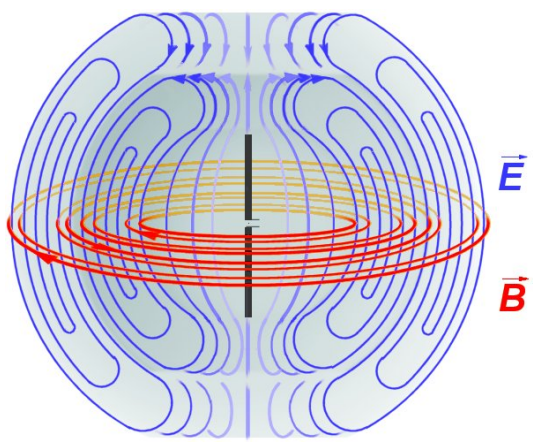
$$\vec{B}(r,\theta,t) \approx \hat{\phi}\,\frac{\mu_0 I_0 d}{4\pi r^2}\sin\theta\; e^{j\omega t}$$

which is the same result one finds for the static magnetic field of a current element $I_0 d$ (again except for the time dependence).

The near field is the quasi-static regime

Very close to the antenna, the electric and magnetic fields are the same as what one finds in electrostatics, except that they oscillate in magnitude according to $e^{j\omega t}$ (just as the current in the antenna does).

Thus the near fields are called “quasi-static” fields.

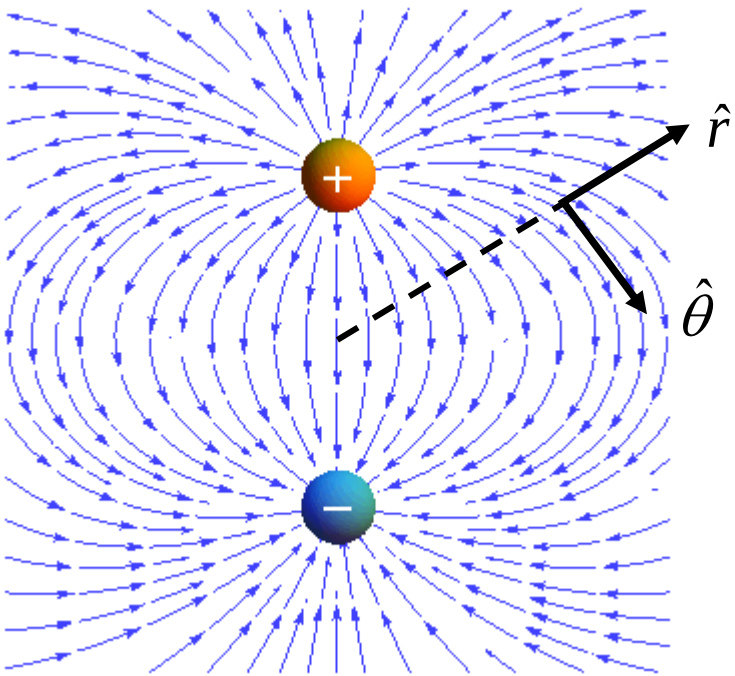


An illustration of the electric and magnetic near fields of a dipole antenna

Exploiting near-field interactions is important in proximity sensors, such as the touch screen on your phone or tablet.

The near field is not radiation

$$\vec{E}(r,\theta,t) \approx \frac{Qd}{\varepsilon_0 \cdot 4\pi r^3}\left[\underline{2\cos\theta\,\hat{r}} + \sin\theta\,\hat{\theta}\right]e^{j\omega t}$$



Note: this has a radially directed field component.

But we know that EM radiation cannot have a component pointing along the propagation direction:

$$\vec{E}_{radiation} \perp \vec{k}$$

Therefore this approximate near-field expression cannot be describing the radiated field. To describe the radiated field, we need to consider what is happening far away from the antenna.

The far field of a Hertzian dipole antenna

Let's consider **the other limiting case**: $k_0 r >> 1$ (far away)

In this case, the $1/r$ terms are the only ones that matter. Also, $\exp(jk_0 r)$ is no longer equal to 1.

The E and B fields are:

$$\vec{E}(r,\theta,t) = j\frac{\mu_0 \omega I_0 d}{4\pi}\sin\theta\left(\frac{e^{-jk_0 r + j\omega t}}{r}\right)\hat{\theta}$$

$$\vec{B}(r,\theta,t) = j\frac{\mu_0 \omega I_0 d}{4\pi c_0}\sin\theta\left(\frac{e^{-jk_0 r + j\omega t}}{r}\right)\hat{\phi}$$

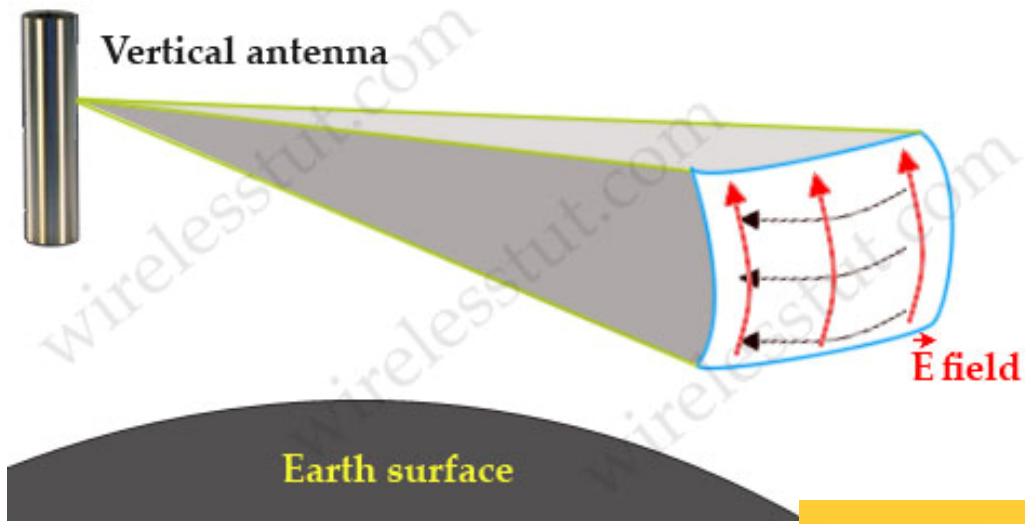
These have the form of propagating electromagnetic waves with spherical wave fronts (no more radially directed fields).

They satisfy $|E| = |B|/c$, and both are perpendicular to $\vec{k} = k_0\hat{r}$.

Antennas generally emit polarized radiation

Example: antennas for most wireless networks emit vertical ($\hat{\theta}$) polarization, which is perpendicular to the earth's surface.

$$\vec{E}(r,\theta,t) = j\frac{\mu_0 \omega I_0 d}{4\pi}\sin\theta\left(\frac{e^{-jk_0 r + j\omega t}}{r}\right)\hat{\theta}$$



If your cell phone had a simple linear antenna, then it wouldn't work if you try to use it while lying down.

Power radiated by a Hertzian dipole antenna

Calculate the Poynting vector $\vec{S} = \varepsilon_0 c_0^2 \vec{E} \times \vec{B}$, and then do a time average (which just gives a factor of ½):

angle brackets indicate time average

$$\left\langle \vec{S}(\theta,\phi) \right\rangle = \frac{\mu_0 \omega^2 I_0{}^2 d^2}{32\pi^2 c_0} \left(\frac{\sin^2\theta}{r^2} \right) \hat{r}$$

Integrate over all θ and ϕ to find the total power radiated:

$$P_{total} = \int_0^{2\pi} d\phi \int_0^{\pi} r^2 \sin\theta \, \left\langle S(\theta,\phi) \right\rangle \, d\theta = \frac{\mu_0 \omega^2 I_0{}^2 d^2}{12\pi c_0}$$

Use $\omega = 2\pi c_0/\lambda$ to write this in terms of the free-space wavelength:

$$P_{total} = \frac{\pi \mu_0 c_0 I_0{}^2}{3} \cdot \frac{d^2}{\lambda^2}$$

The total radiated power is proportional to the square of the current in the antenna, and the square of its length.

Radiation resistance

Since the power radiated by the antenna varies as the square of the current, it makes sense to draw an analogy to the power dissipated in a resistor due to Ohmic heating, which is given by the well-known formula:

$$P = I^2 R$$

Of course, for time-varying currents, we need to time-average, which contributes a factor of ½, making this:

$$P_{ave} = \frac{1}{2} I^2 R$$

"radiation resistance" = the hypothetical resistance that would dissipate the same time-averaged power (due to Ohmic heating) as that radiated by the antenna.

Radiation resistance

For the Hertzian dipole: $P_{total} = \frac{\pi \mu_0 c_0 I_0^{\,2}}{3} \cdot \frac{d^2}{\lambda^2}$

and therefore: $R_{rad} = \frac{2}{I_0^{\,2}} P_{total} = \frac{2\pi \mu_0 c_0}{3} \cdot \frac{d^2}{\lambda^2}$

Note, since $c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$, this can also be written:

$$R_{rad} = \frac{2\pi}{3} \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{d^2}{\lambda^2} = \frac{2\pi}{3} Z_0 \cdot \frac{d^2}{\lambda^2}$$

where $Z_0 \equiv \sqrt{\mu_0 / \varepsilon_0} \approx 377\Omega$ is the impedance of free space.

Since a Hertzian dipole antenna has $d << \lambda$, its radiation resistance is much smaller than 377Ω.

Radiation efficiency

When an antenna is driven by an AC current, it will dissipate power due to radiation, and also due to I^2R_{Ohmic} heating.

electrical power input to the antenna

$$P_{input} = P_{rad} + P_{Ohmic}$$

The radiation efficiency is the percentage of the input power that is converted to radiated power. Since the current is the same for both terms, the radiation efficiency is:

$$\eta = \frac{R_{rad}}{R_{rad} + R_{Ohmic}}$$

For a Hertzian dipole antenna ($d << \lambda$), this is usually a small number, which means that these antennas are not very efficient.

Radiation efficiency: example

A steel rod of length d = 1.5 meters, radius a = 1 mm is used as an antenna for radiation at f = 1 MHz (AM radio).

$$R_{rad} = \frac{2\pi}{3} Z_0 \cdot \frac{d^2}{\lambda^2} = \frac{2\pi}{3}(377\Omega)\left(\frac{1.5}{300}\right)^2 = 0.020\Omega$$

The resistance of a metal wire at high frequency (where the skin depth is much smaller than the wire's radius) is given by:

$$R_{Ohmic} = \sqrt{\frac{\pi f \mu}{\sigma}} \frac{d}{2\pi a}$$

For steel, $\mu = \mu_0$ and $\sigma = 2\times10^6\ \Omega^{-1}\ \mathrm{m}^{-1}$. Thus $R_{ohmic} = 0.34\Omega$

The radiation efficiency is therefore:

$$\eta = \frac{R_{rad}}{R_{rad} + R_{Ohmic}} = 0.055$$

(often this is expressed in decibels: $\eta = -12.6\ \mathrm{dB}$)