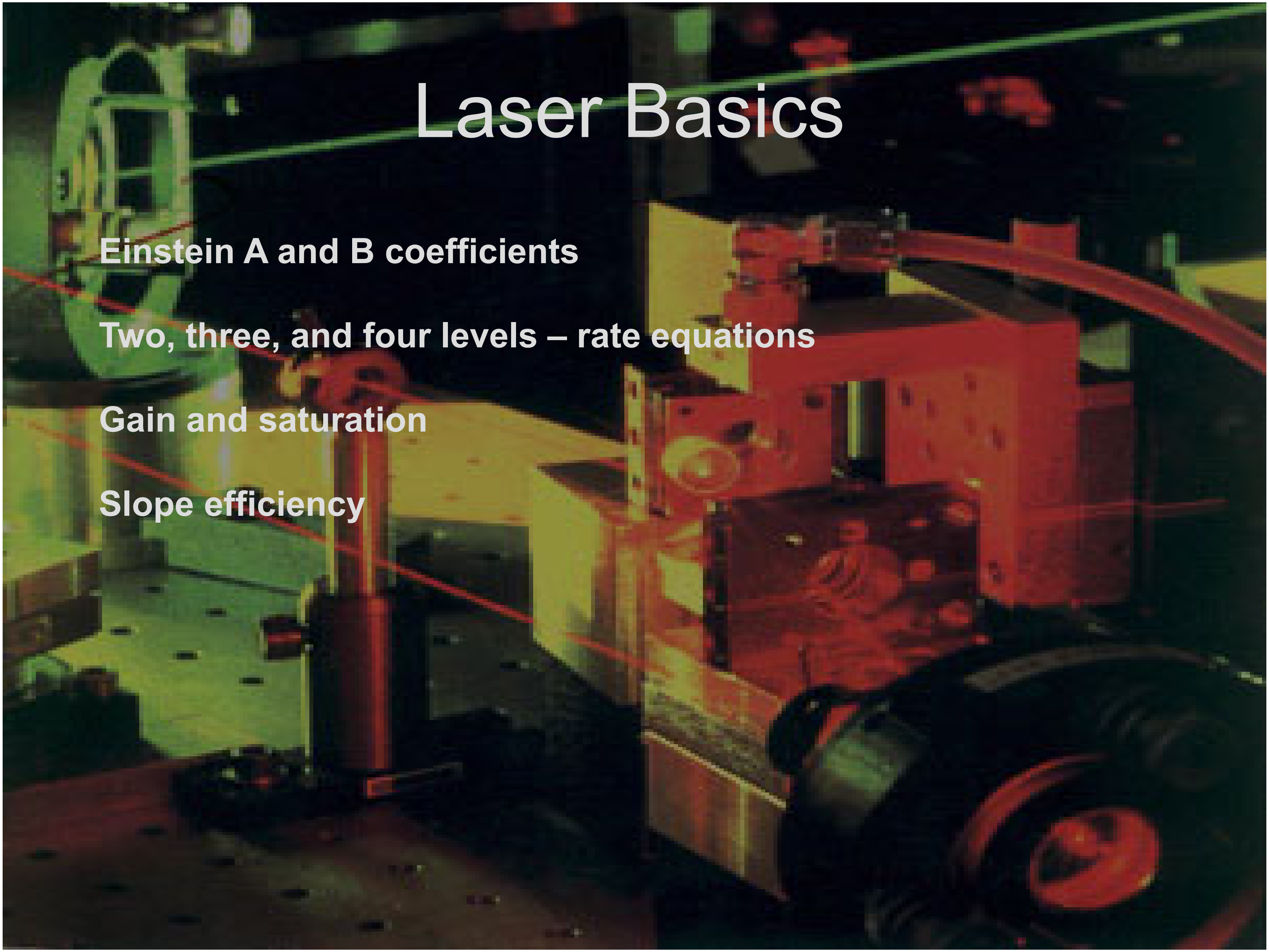
Laser Basics



Einstein A and B coefficients

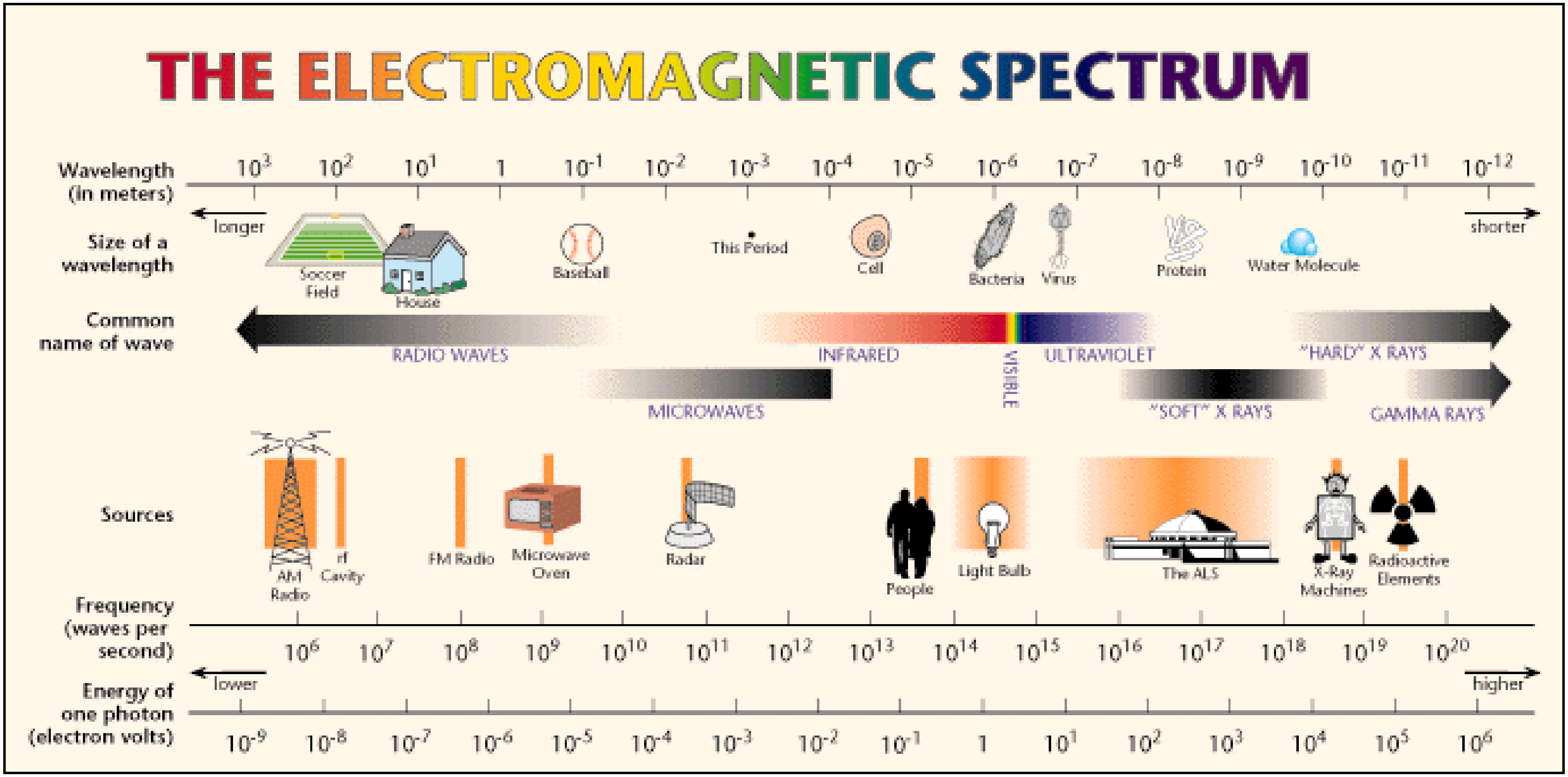
Two, three, and four levels – rate equations

Gain and saturation

Slope efficiency

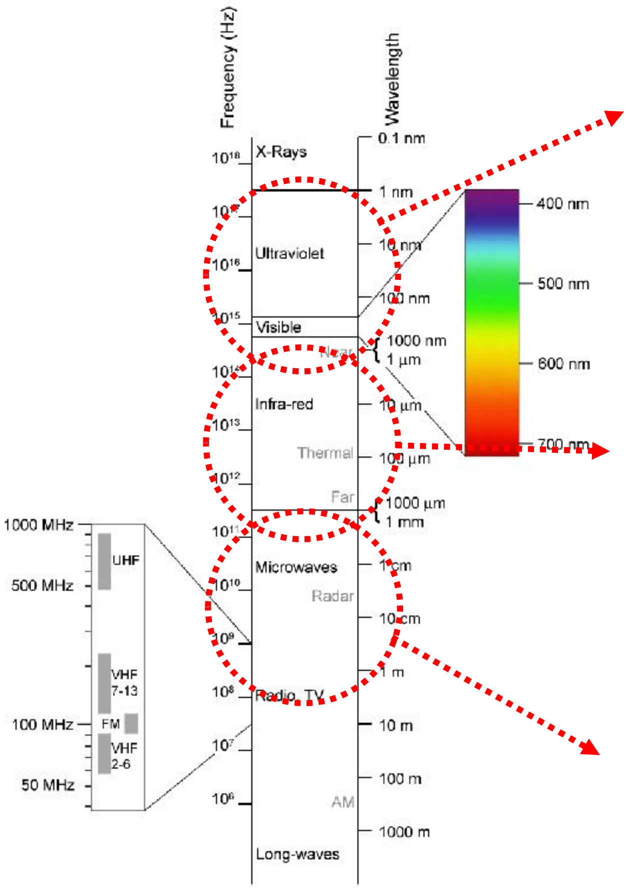
The electromagnetic spectrum

All of these are electromagnetic waves. The amazing diversity is a result of the fact that the interaction of radiation with matter depends on the frequency of the wave.



The boundaries between regions are a bit arbitrary…

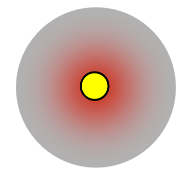
Most light in the universe is emitted by atomic and molecular vibrations.



Electrons vibrate in their motion around nuclei.

High frequency:

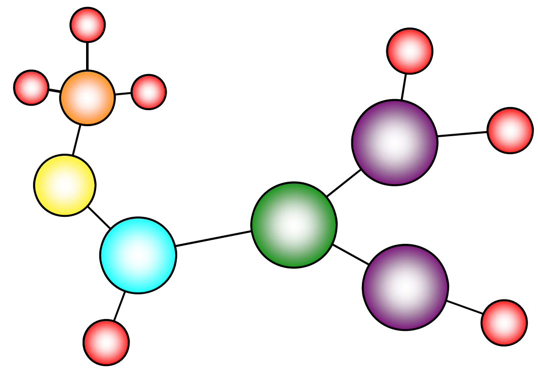
~10^{14} - 10^{17} Hz



Nuclei in molecules vibrate with respect to each other.

Intermediate frequency:

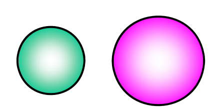
~10^{11} - 10^{13} Hz



Nuclei in molecules rotate.

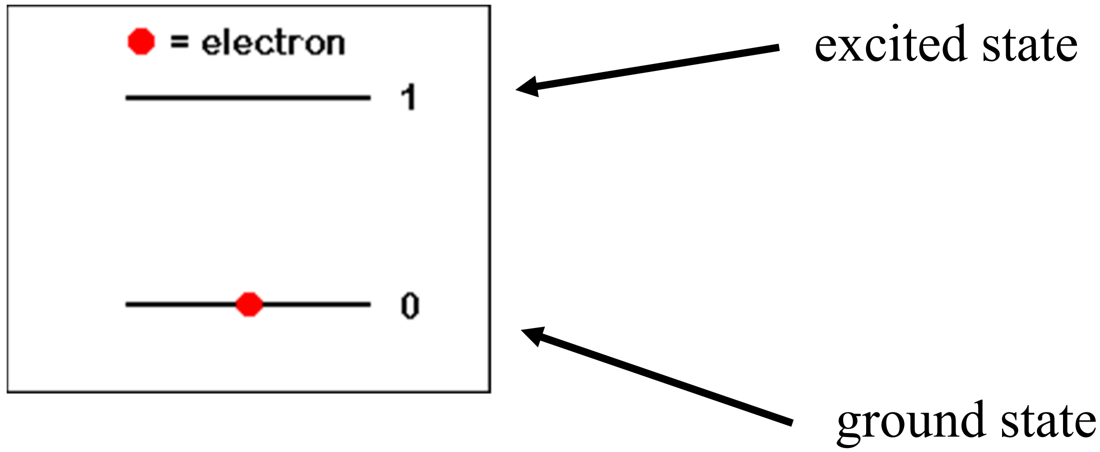
Low frequency:

~10^{9} - 10^{10} Hz



The two-level system

Quantum energy levels



Absorption: promotes an electron from the ground to the excited state
Emission: drops the electron back to the ground state

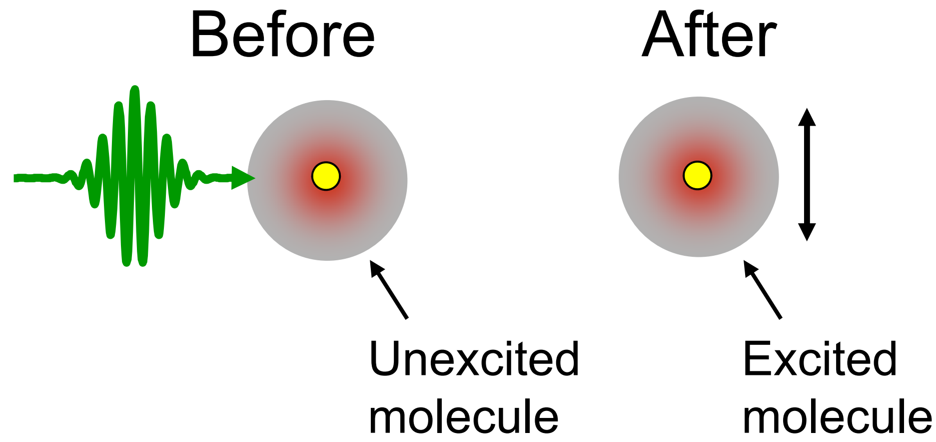
"spontaneous emission" - the decay of an excited state to the ground state with the corresponding emission of a photon

Conservation of energy: $E_{excited} - E_{ground} = E_{photon}$

Actually, there are three types of interactions

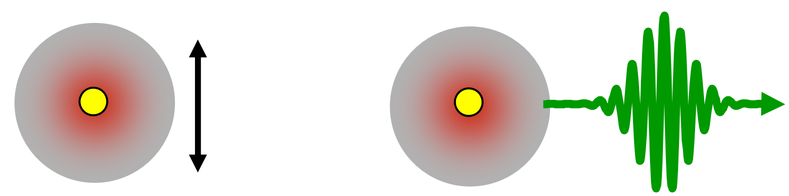
key idea: conservation of energy

Absorption



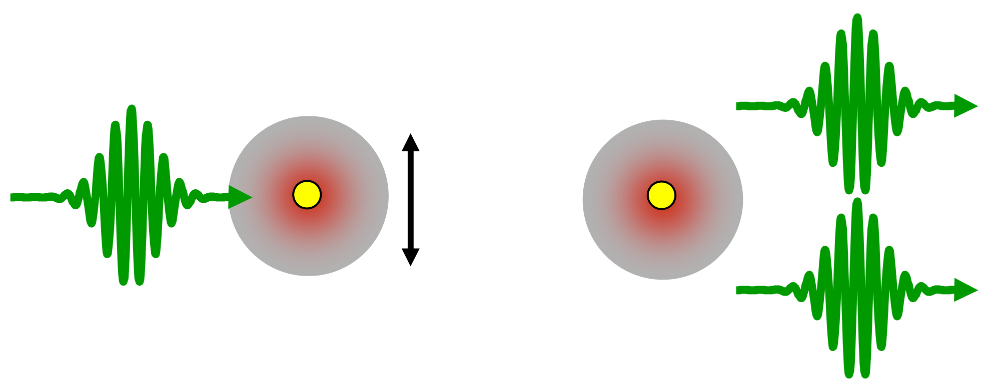
- Promotes molecule to a higher energy state
- Decreases the number of photons

Spontaneous Emission



- Molecule drops from a high energy state to a lower state
- Increases the number of photons

Stimulated Emission

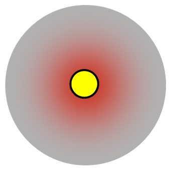


- Molecule drops from a high energy state to a lower state
- The presence of one photon stimulates the emission of a second one
- This process has no analog in classical physics - it can only be understood with quantum mechanics!

Einstein A and B Coefficients

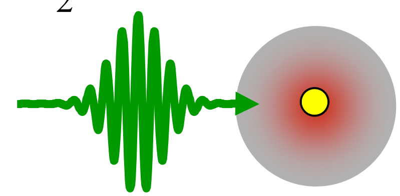
In 1916, Einstein considered the various **transition rates** between molecular states (say, 1 and 2) involving light of irradiance, I:

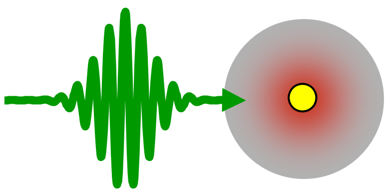
at frequency ν



← Spontaneous emission rate = $A\, N_2$

Absorption rate = $B_{12}\, N_1\, I$ →





← Stimulated emission rate = $B_{21}\, N_2\, I$

With this, we can write a **rate equation** for the population density of the two states. For the upper level:

Absorption

Stimulated emission

Spontaneous emission

$$\frac{dN_2}{dt} = \left(B_{12}N_1 - B_{21}N_2\right) I - AN_2$$

light intensity

Einstein B Coefficients

We can make a connection between B_{12} and B_{21} using a quantum mechanical argument based on time-dependent perturbation theory.

As a result of a perturbation h(t), a system in quantum state 1 makes a transition to quantum state 2 with probability given by:

$$P_{1\to 2} = \frac{1}{\hbar^2}\left| \int_{-\infty}^{t} e^{iE_{21}t'/\hbar} h(t')\,dt' \right|^2 \qquad Notation: \quad \omega_{21} = \frac{E_{21}}{\hbar}$$

<u>Key example</u>: suppose we subject a two-level system, initially in state 1, to a harmonic perturbation (like a light wave), of the form:

$$h(t) = \begin{cases} 0 & t < 0 \\ 2E_0 \sin \omega t & t > 0 \end{cases}$$

(and suppose that the frequency of the perturbation, ω, is close to ω_{21})

The transition probability from state 1 to state 2 is:

$$P_{1\to 2} = \frac{E_0^2}{\hbar^2}\left| \int_{0}^{t} e^{i\omega_{21}t'} \left(e^{i\omega t'} - e^{-i\omega t'} \right) dt' \right|^2$$

Einstein B Coefficients are equal

$$P_{1\to 2} = \frac{A_0^2}{\hbar^2}\left|\int_0^t e^{i\omega_{21}t'}\left(e^{i\omega t'} - e^{-i\omega t'}\right)dt'\right|^2$$

Evaluating this integral:

$$= \frac{4A_0^2}{\hbar^2}\left|\left(\frac{e^{i(\omega_{21}-\omega)t/2}\sin\left(\omega_{21}-\omega\right)t/2}{\left(\omega_{21}-\omega\right)}\right) - \left(\frac{e^{i(\omega_{21}+\omega)t/2}\sin\left(\omega_{21}+\omega\right)t/2}{\left(\omega_{21}+\omega\right)}\right)\right|^2$$

$$\approx \frac{4A_0^2}{\hbar^2}\cdot\frac{\sin^2\left[\left(\omega_{21}-\omega\right)t/2\right]}{\left(\omega_{21}-\omega\right)^2}$$

(neglecting the 2nd term is called the “rotating wave approximation”)

$$= \frac{A_0^2 t^2}{\hbar^2}\cdot\frac{\sin^2\left[\left(\omega_{21}-\omega\right)t/2\right]}{\left(\omega_{21}-\omega\right)^2\left(t^2/4\right)}$$

Note that $P_{1\to 2} = P_{2\to 1}$

On a per-atom, per-photon basis: Absorption and stimulated emission are equally likely!

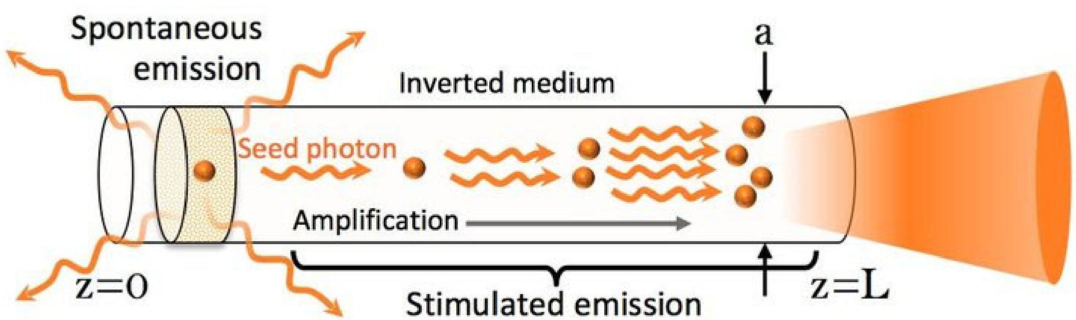
The Einstein coefficients B_{12} and B_{21} are equal.

Back to our rate equation analysis

$$\frac{dN_2}{dt} = BI(N_1 - N_2) - AN_2$$

Stimulated emission: photons go only into the direction of the incident light

spontaneous emission: photons go in all directions



So, we can ignore the spontaneous emission contribution to the photon number, which thus varies according to:

$$\frac{dI}{dt} = -BI(N_1 - N_2)$$

Population inversion

$$\frac{dI}{dt} = -BI(N_1 - N_2) \Rightarrow I = I_0 e^{-B(N_1-N_2)t} = I_0 e^{-B(N_1-N_2)nz/c}$$

$$= I_0 e^{-\alpha z} \text{ or } I_0 e^{gz}$$

g = gain coefficient

α = absorption coefficient

→ Number of photons grows exponentially with propagation distance z, if $\boxed{N_2 > N_1}$

This condition is known as "population inversion".

Population inversion is a necessary condition for lasing to occur between levels 1 and 2.

A common definition: the gain (or absorption) cross-section:

If $N_2 > N_1$: $\quad g \equiv \frac{1}{2}[N_2 - N_1]\sigma$

If $N_2 < N_1$: $\quad \alpha \equiv \frac{1}{2}[N_1 - N_2]\sigma$

$\sigma \approx \frac{\alpha}{N_1}$ units of σ are m^2 (i.e., area)

σ is something like the absorption per molecule. It is the effective size (area) of a molecule as seen by an incoming photon.

Population inversion isn't easy to achieve

In thermal equilibrium, we can use Boltzmann statistics:

$$N_2 = e^{-\Delta E/kT} \cdot N_1 < N_1$$

$\longrightarrow$ Population inversion is impossible in equilibrium.

In any steady-state situation, the derivative is zero:

$$\frac{dN_2}{dt} = BI(N_1 - N_2) - AN_2 = 0$$

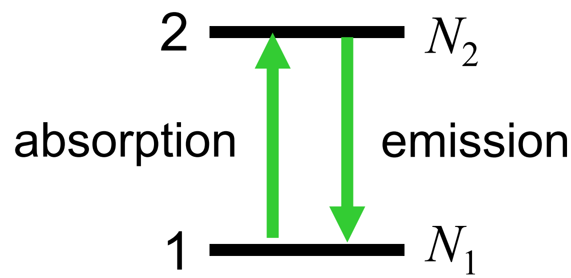
$$N_2 = \frac{BI}{BI + A} N_1 < N_1$$

$\longrightarrow$ Population inversion is impossible in steady-state.

Rate equations for atomic population, continued

$$\frac{dN_2}{dt} = BI(N_1 - N_2) - AN_2$$

$$\frac{dN_1}{dt} = BI(N_2 - N_1) + AN_2$$



If the total number of molecules is N:

$$N \equiv N_1 + N_2 \qquad \text{total population (per cm}^3\text{)}$$

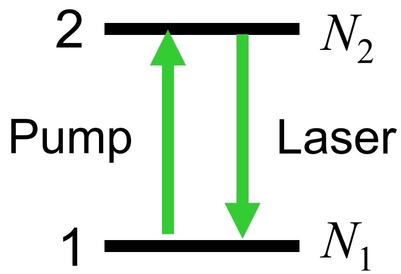
$$\Delta N \equiv N_1 - N_2 \qquad \text{population difference (per cm}^3\text{)}$$

$$\Rightarrow \quad \frac{d\Delta N}{dt} = -2BI\Delta N + 2AN_2$$

$$2N_2 = (N_1 + N_2) - (N_1 - N_2)$$
$$= N - \Delta N$$

$$\Rightarrow \quad \frac{d\Delta N}{dt} = -2BI\Delta N + AN - A\Delta N$$

How does the population difference depend on pump intensity?



$$\frac{d\Delta N}{dt} = -2BI\Delta N + AN - A\Delta N$$

In steady-state: $0 = -2BI\Delta N + AN - A\Delta N$

Solve for ΔN: $(A + 2BI)\Delta N = AN$

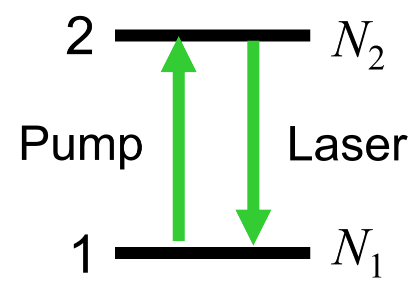
$$\Rightarrow \quad \Delta N = AN/(A + 2BI) = \frac{N}{1 + \frac{2B}{A}I}$$

$$\Rightarrow \boxed{\Delta N = \frac{N}{1 + 2I/I_{sat}}}$$

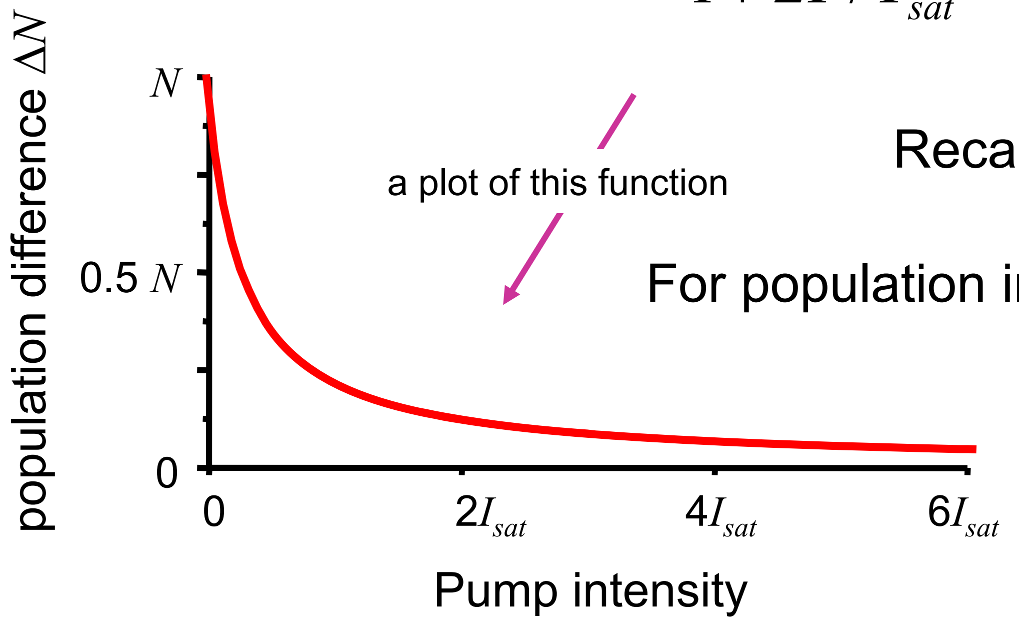
where: $I_{sat} = A/B$

I_{sat} is called the **saturation intensity**.

Why inversion is impossible in a two-level system



Population difference $\Delta N = \dfrac{N}{1 + 2I / I_{sat}}$



Recall that $\Delta N \equiv N_1 - N_2$

For population inversion, we require $\Delta N < 0$

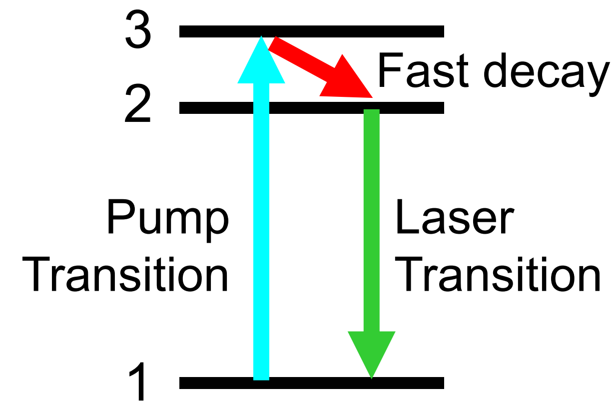
ΔN is always positive, no matter how hard we pump on the system!

It's impossible to achieve a steady-state inversion in a two-level system!

Why? Because absorption and stimulated emission are equally likely.

Even for an infinite pump intensity, the best we can do is $N_1 = N_2$ (i.e., $\Delta N = 0$)

Rate equations for a three-level system



So, if we can't make a laser using two levels, what if we try it with three?

Assume we pump to a state 3 that rapidly decays to level 2.

$$\frac{dN_2}{dt} = BIN_1 - AN_2$$

$$\frac{dN_1}{dt} = -BIN_1 + AN_2$$

The total number of molecules is N:

$$N \equiv N_1 + N_2$$

$$\Delta N \equiv N_1 - N_2$$

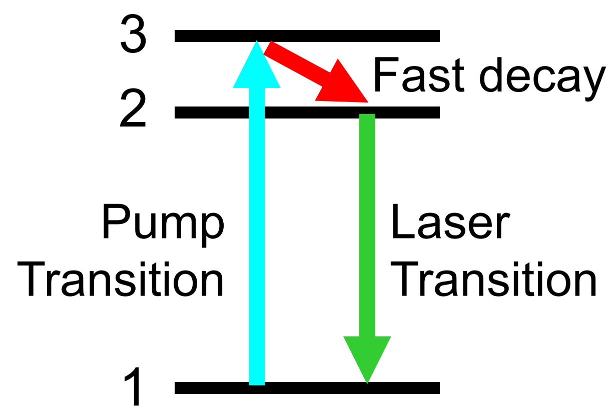
Level 3 decays fast and so $N_3 = 0$.

$$\frac{d\Delta N}{dt} = -2BIN_1 + 2AN_2$$

$$2N_2 = N - \Delta N$$

$$2N_1 = N + \Delta N$$

Why inversion is possible in a three-level system



$$\frac{d\Delta N}{dt} = -BIN - BI\Delta N + AN - A\Delta N$$

In steady-state: $0 = -BIN - BI\Delta N + AN - A\Delta N$

Solve for ΔN: $$\Delta N = N\frac{A - BI}{A + BI} = N\frac{1-(B/A)I}{1+(B/A)I}$$

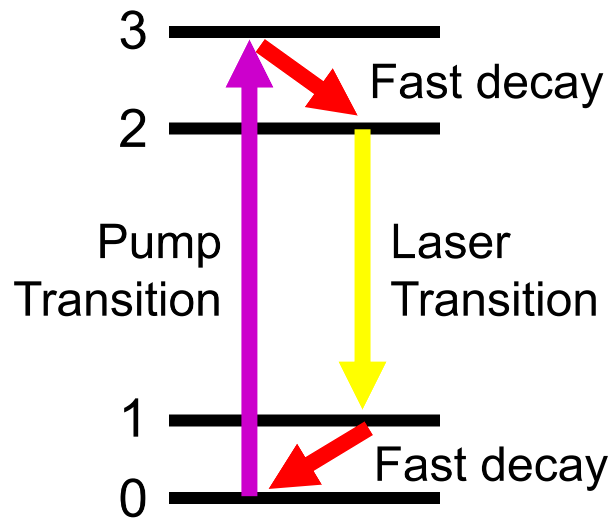
$$\Rightarrow \boxed{\Delta N = N\frac{1 - I/I_{sat}}{1 + I/I_{sat}}}$$

where, as before: $I_{sat} = A/B$

I_{sat} is the **saturation intensity**.

Now if $I > I_{sat}$, ΔN is negative!

Rate equations for a four-level system



Now assume the lower laser level 1 also rapidly decays to a ground level 0.

As before: $$\frac{dN_2}{dt} = BIN_0 - AN_2$$

The total number of molecules is N:

$$N \equiv N_0 + N_2$$

$$N_0 = N - N_2$$

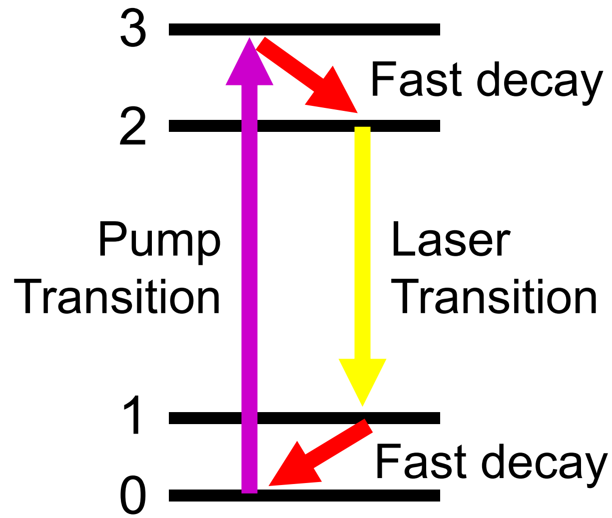
$$\frac{dN_2}{dt} = BI(N - N_2) - AN_2$$

Because $N_1 \approx 0$, $\quad \Delta N \approx -N_2$

$$-\frac{d\Delta N}{dt} = BIN + BI\Delta N + A\Delta N$$

At steady state: $$0 = BIN + BI\Delta N + A\Delta N$$

Why inversion is easy in a four-level system



$$0 = BIN + BI\Delta N + A\Delta N$$

Solve for ΔN:

$$\Rightarrow \quad \Delta N = -BIN/(A+BI) = -N\frac{(B/A)I}{1+(B/A)I}$$

$$\Rightarrow \quad \boxed{\Delta N = -N\frac{I/I_{sat}}{1+I/I_{sat}}}$$

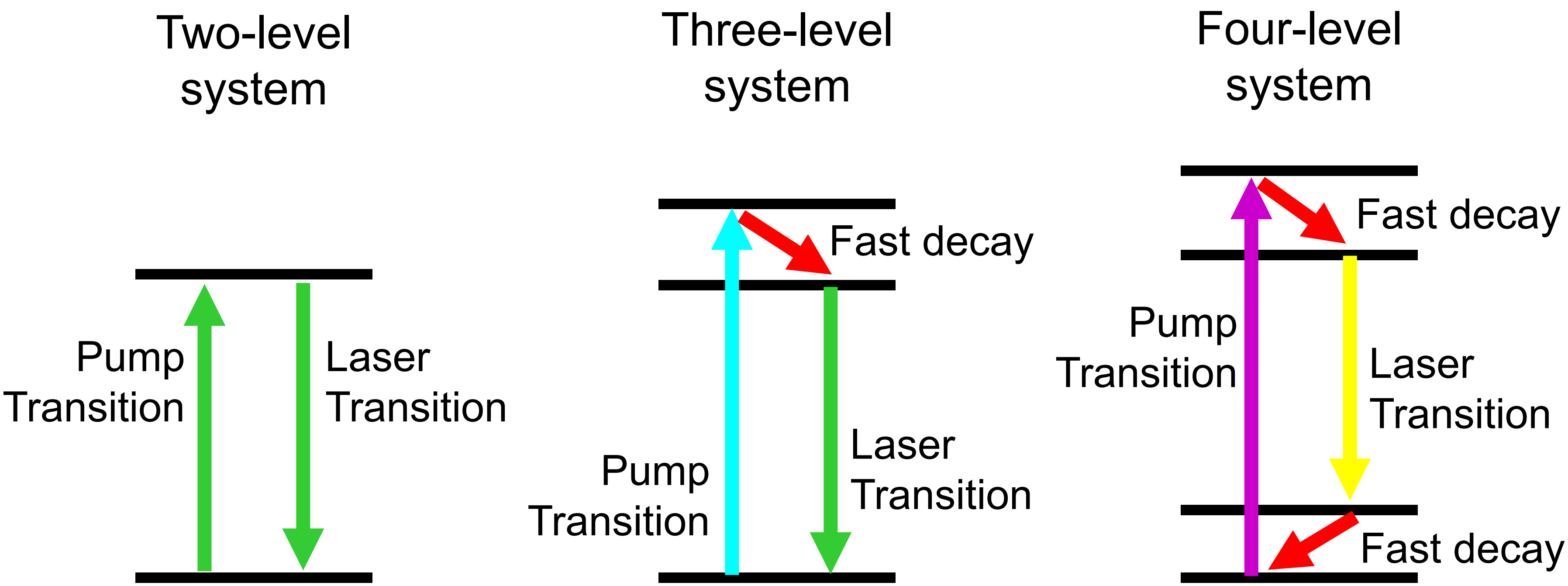
where: $I_{sat} = A/B$

I_{sat} is the **saturation intensity**.

Now, ΔN is negative—for any non-zero value of I!

Two-, three-, and four-level systems

Four-level systems are best.

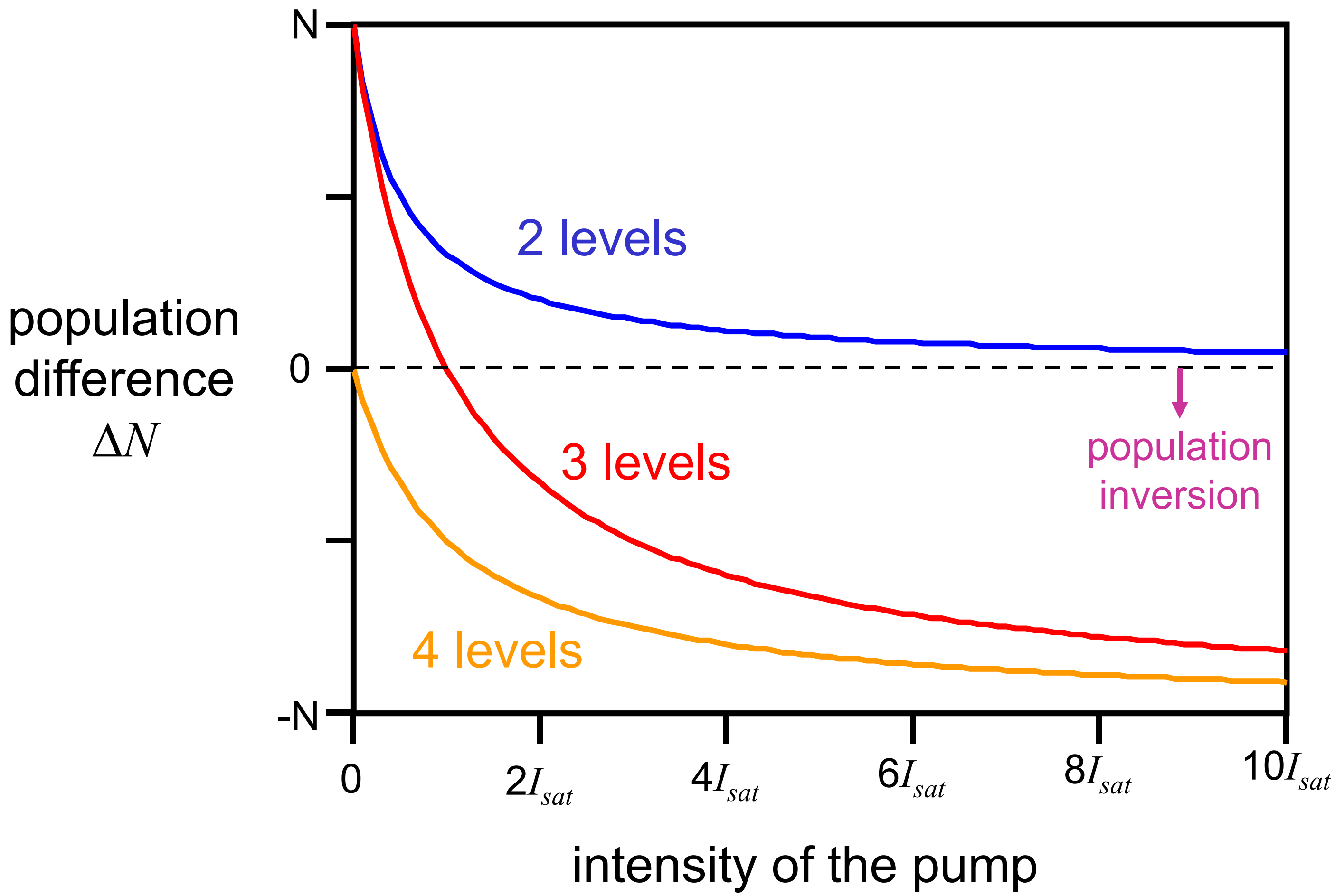


At best, you get equal populations. No lasing.

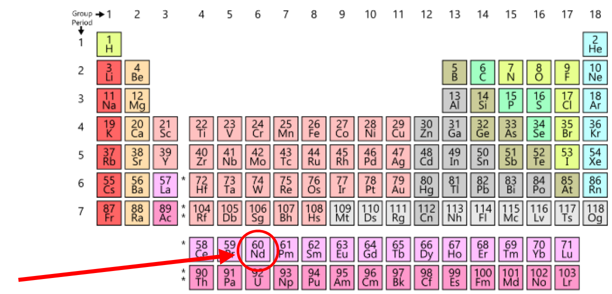
If you hit it hard, you get lasing.

Lasing is easy!

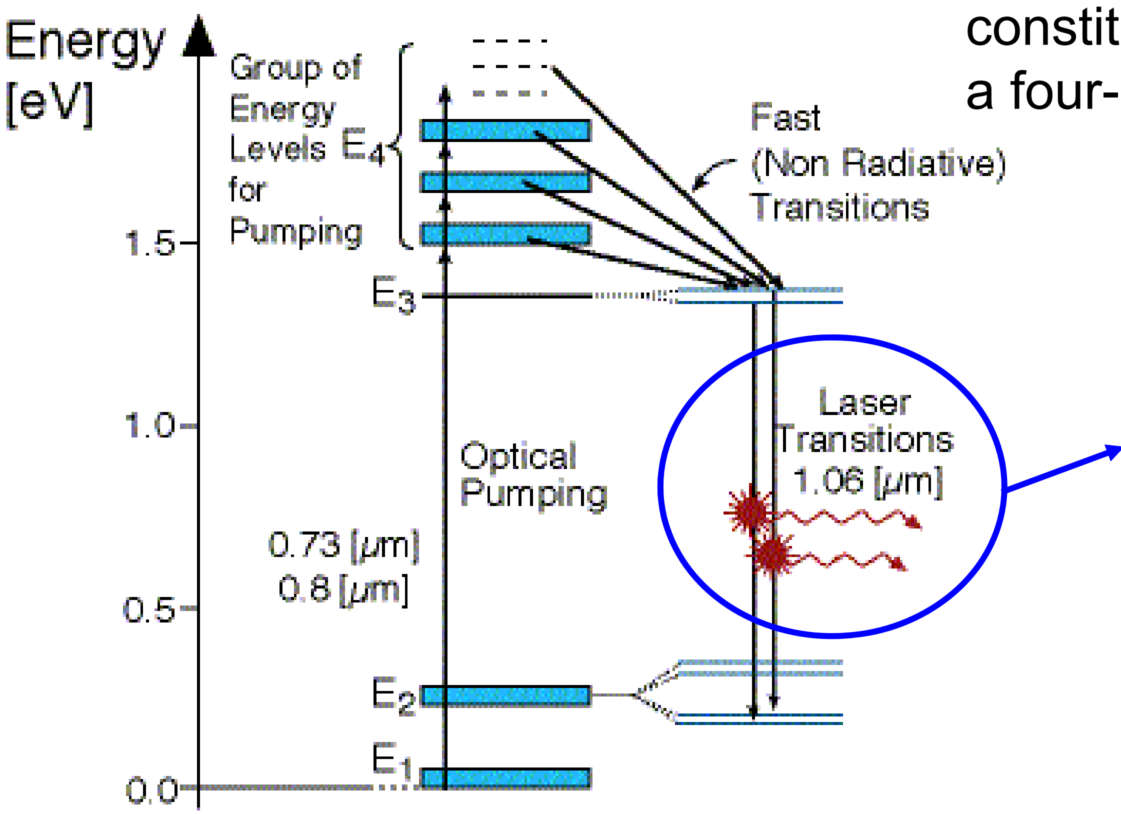
Population inverstion in two-, three-, and four-level systems



Many lasers are almost ideal 4-level systems



For example, green laser pointers are Nd^{+3}:YAG lasers. The energy levels of Nd^{+3} constitute a classic example of a four-level system.



Infrared laser emission is converted to green light by a nonlinear optical process: frequency doubling! (to be discussed later this semester)

What is the saturation intensity?

$$I_{sat} = A / B$$

A is the excited-state relaxation rate: $1/\tau$

B is the absorption cross-section, σ, divided by the energy per photon, $\hbar\omega$: $\sigma / \hbar\omega$

Both σ and τ depend on the molecule, and also the frequency of the light.

$$\boxed{I_{sat} = \frac{\hbar\omega}{\sigma\tau}}$$

$\hbar\omega$ ~10^{-19} J for visible/near IR light

τ ~10^{-12} to 10^{-8} s for molecules

σ ~10^{-20} to 10^{-16} cm^2 for molecules (on resonance)

10^5 to 10^{13} W/cm^2

The saturation intensity plays a key role in laser theory. It is the intensity which corresponds to one photon incident on each molecule, within its cross-section σ, per recovery time τ.

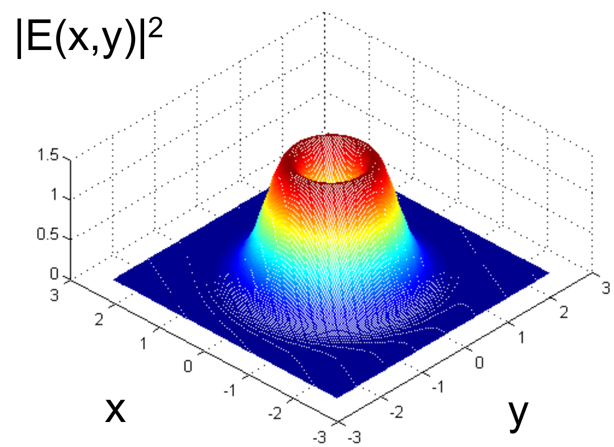
For Ti:sapphire, I_{sat} ~ 300 kW/cm^2

There are 3 conditions for steady-state laser operation.

$\boxed{\text{Amplitude condition}}$

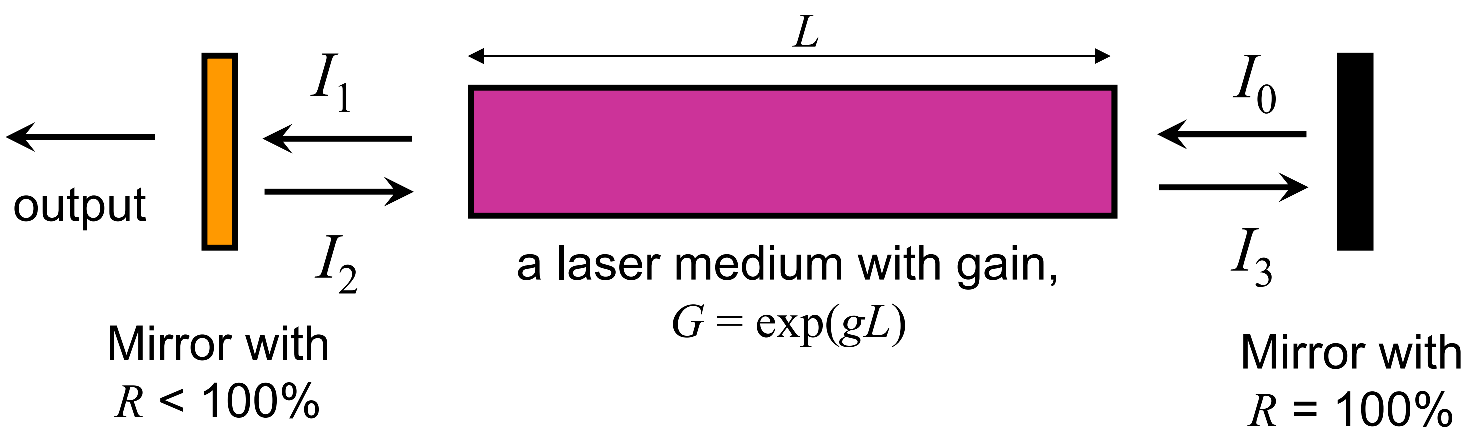
Phase condition

Transverse spatial mode condition



Amplitude condition: gain must exceed loss

Having a population inversion ($\Delta N < 0$) isn't enough. Additional **losses** in intensity occur, due to absorption, scattering, and reflections. Also, there is an output beam…



The laser will lase if the beam maintains its intensity after one round trip, that is, if:

Gain = Loss

This means: $I_3 = I_0$. Here, it means a condition on I_3:

$$I_3 = I_0 \exp(gL) \; R \exp(gL) = I_0$$

In this expression, we are ignoring sources of loss other than the mirror reflectivity R. This is an approximation, because there are always other sources of loss.

ΔN < 0 is necessary, but not sufficient.

Solving for g, we find:

$$\Rightarrow \quad g = \frac{1}{2L}\ln(1/R)$$

This is the minimum value of the gain which is required in order to turn on a laser. This can be thought of as a threshold gain value, $g_{threshold}$.

Now, recall that the gain is proportional to the population inversion:

$$\text{If } N_2 > N_1:\ g \equiv \frac{1}{2}\left[N_2 - N_1\right]\sigma = -\frac{1}{2}\Delta N \cdot \sigma$$

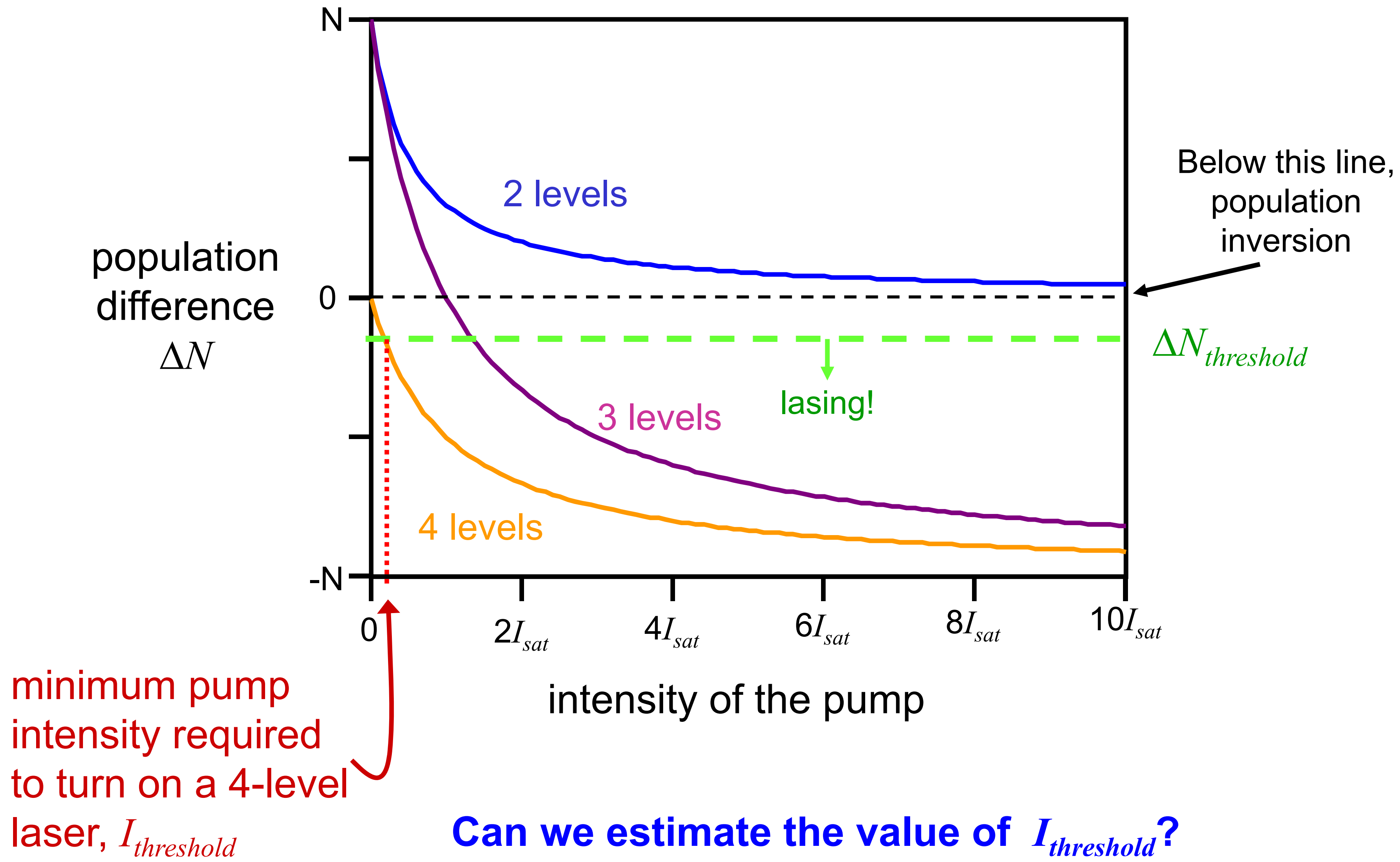
remember: ΔN < 0 for a population inversion, so g > 0.

Thus, there is a threshold value of the population inversion which is required:

$$\Delta N_{threshold} = -\left(\frac{1}{2L\sigma}\right)\ln\left(\frac{1}{R}\right) < 0$$

ΔN < 0 is not sufficient to make the laser operate. We need ΔN < $\Delta N_{threshold}$.

A threshold value of pump intensity

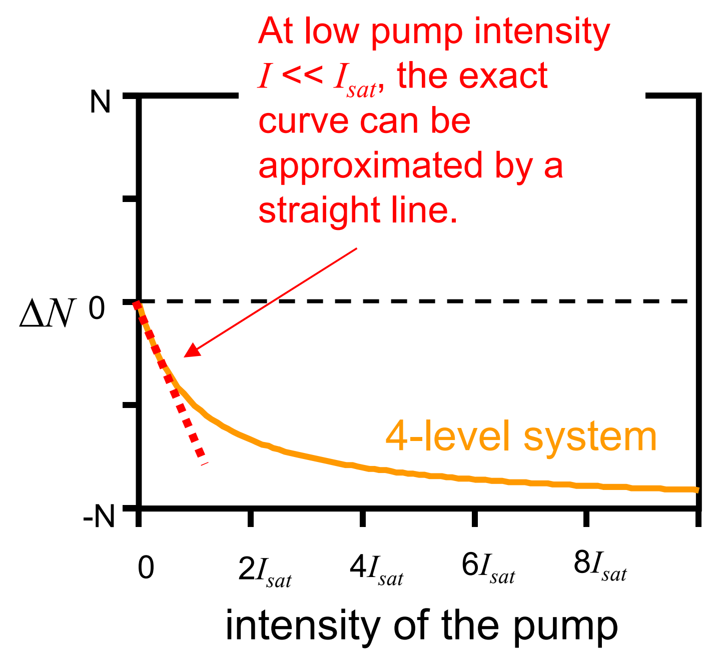


Can we estimate the value of $I_{threshold}$?

Achieving Laser Threshold in a four-level system

For a four-level system:

$$\Delta N = -N \frac{I_{pump} / I_{sat}}{1 + I_{pump} / I_{sat}}$$



If the pump intensity is low (well below the saturation intensity), then this can be approximated by:

$$\Delta N \approx -N \frac{I_{pump}}{I_{sat}}$$

i.e., population inversion is negative and proportional to the pump intensity.

→ Even for a four-level system, lasers have a threshold pump intensity, in order to achieve sufficient gain to overcome the loss and begin to lase.

$$I_{th} = \frac{I_{sat}}{2NL\sigma} \ln(1/R)$$

This is valid if the output coupler (with R < 1) is the only source of loss.

Lasing behavior above threshold

In most lasers, the product gL is a small number - the gain per pass through the gain medium is small. Then the gain per pass, e^{gL}, is approximately equal to: $1 + gL$. The power circulating inside the laser is therefore proportional to the gain g.

Thus: $I_{out} = (1-R) \cdot I_{circulating} \propto g$

i.e., the power emerging from the laser varies linearly with g.

So, for pump intensities above threshold but well below the saturation regime, we have:

$$\text{output power} \propto g \propto -\Delta N \propto \text{pump power}$$

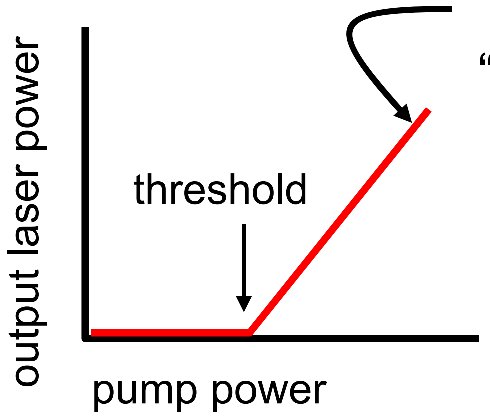
As we turn up the pump (from zero), there will be essentially no laser photons until we reach threshold. At that point, the laser turns on.

Then, the output power increases linearly with the pump power, until the pump approaches and exceeds I_{sat}.

Slope efficiency

$$I_{out} \propto \frac{I_{pump} / I_{sat}}{1 + I_{pump} / I_{sat}} \approx I_{pump} / I_{sat}$$

Above threshold (but not too far above), the output laser power is proportional to the input pump power.



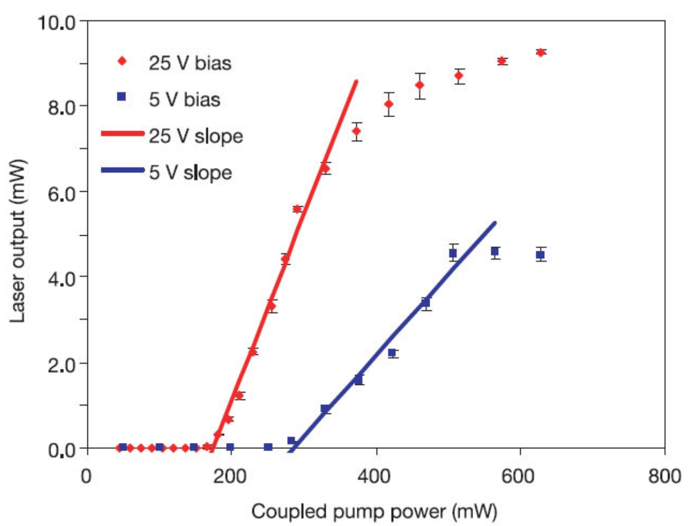
The slope of this line is called the "slope efficiency" of the laser.

For example, a slope efficiency of 50% means that for every two additional pump photons we add, one additional laser photon is generated.

The concept of slope efficiency only applies for $I_{th} < I_{pump} << I_{sat}$

Essentially all lasers exhibit this behavior. Here's an example: the silicon laser (2005)

In these data, you can see all three regimes: below threshold, above threshold, and saturation



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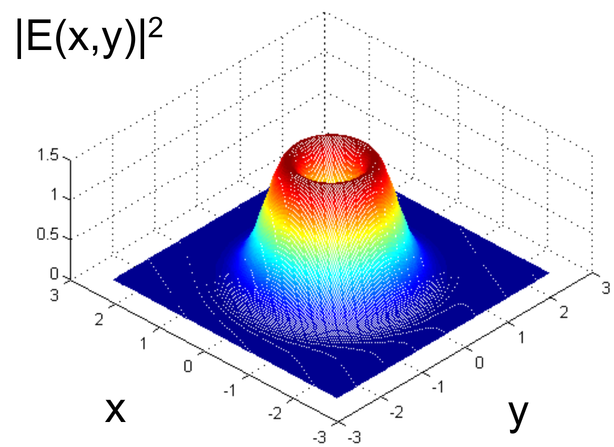


Another steady-state condition: the phase

In addition to requiring that gain exceed loss, (i.e., that the laser field amplitude must be constant on each round trip) we also require that the phase of the laser field must reproduce itself on each round trip.

$$\frac{angle(E_{after})}{angle(E_{before})} = \exp\left(-j\omega L_{RT}/c\right) = 1 \quad \Rightarrow \quad \omega_m = \frac{2\pi c}{L_{RT}} \cdot m$$

(m = an integer)

Since $\omega = \dfrac{2\pi c}{\lambda}$, this is the same as saying that an integer number of wavelengths must fit in the cavity.

Only certain specific frequencies can satisfy this condition and lase.

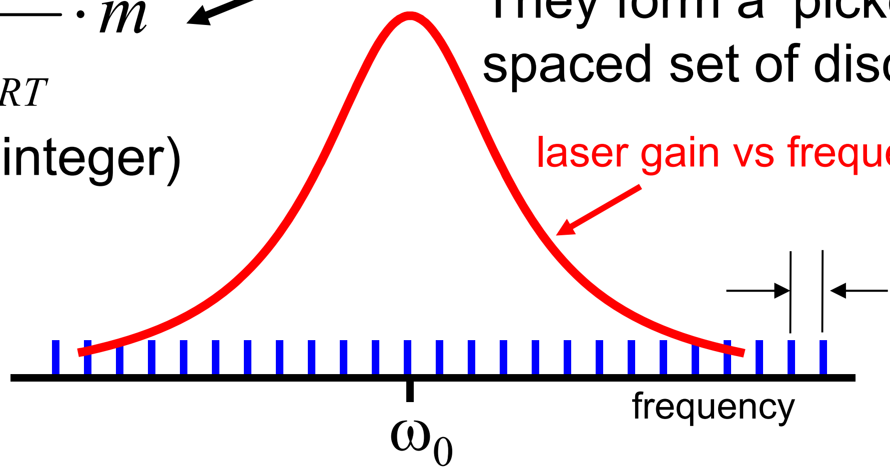
Longitudinal modes

$$\omega_m = \frac{2\pi c}{L_{RT}} \cdot m$$

(m = an integer)

These are called the axial or longitudinal modes of the laser cavity.

They form a 'picket fence', an equally spaced set of discrete frequencies.



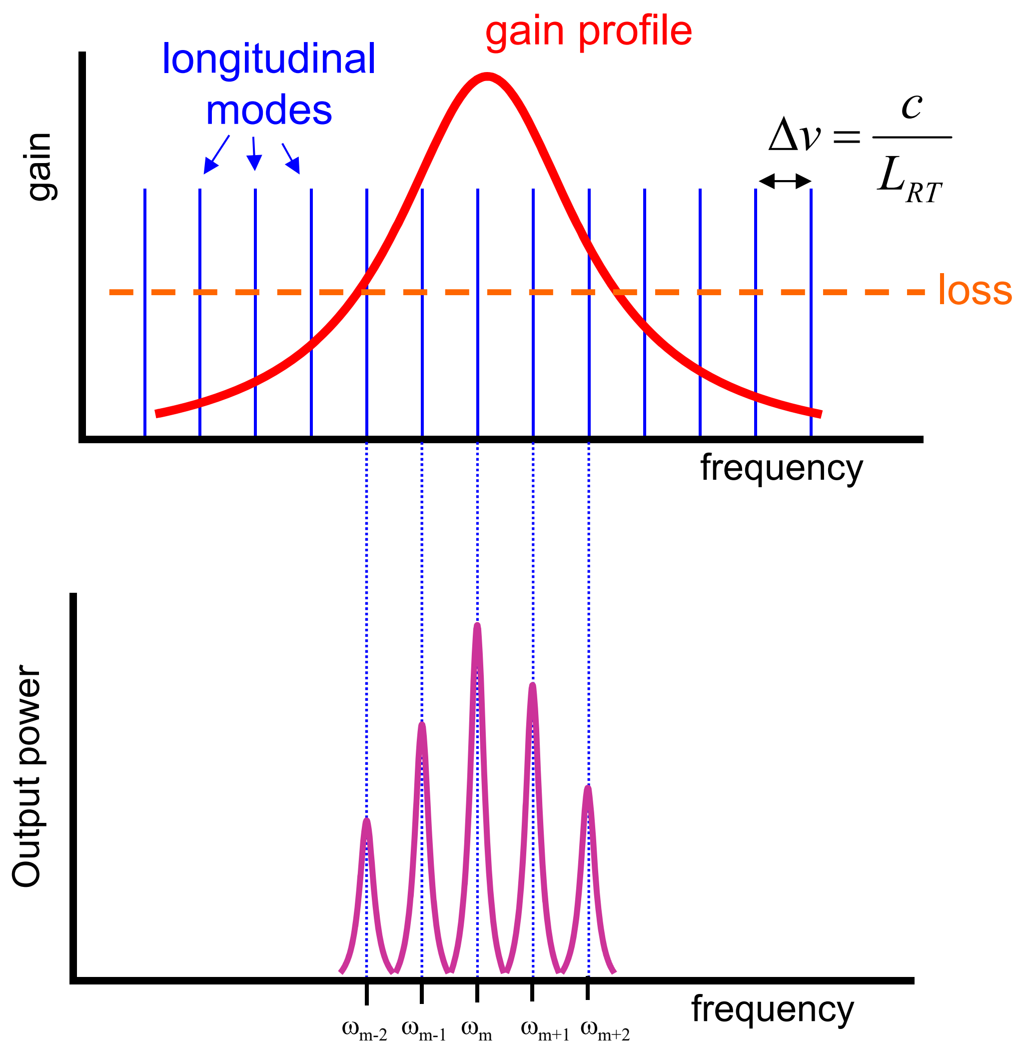
space between adjacent modes: $\Delta\nu = \frac{c}{L_{RT}}$

This is the inverse of the cavity round-trip time

single-mode laser - a laser which lases at only one frequency. Only one longitudinal mode lases.

multi-mode laser - when more than one longitudinal mode lases. Most lasers are multi-mode lasers.

Longitudinal modes



The number of modes oscillating inside a laser depends on the mode spacing $\Delta\nu$, and how it compares to the spectral width of the gain curve – in other words, the range of frequencies where the gain is large.

All modes that experience gain > loss will lase.

This number can range from one or just a few all the way up to millions.