The Generation of Ultrashort Laser Pulses

The importance of bandwidth

More than just a light bulb

Two, three, and four levels – rate equations

Gain and saturation
But first: the progress has been amazing!

The shortest pulse vs. year (for different media)
Continuous vs. ultrashort pulses of light

A constant and a delta-function are a Fourier-Transform pair.
The uncertainty principle says that the product of the temporal and spectral pulse widths is greater than \( \sim 1 \).
Light bulbs, lasers, and ultrashort pulses

But a light bulb is also broadband.

What exactly is required to make an ultrashort pulse?

Answer: A Mode-locked Laser

Okay, what’s a laser, what are modes, and what does it mean to lock them?
There are 3 conditions for steady-state laser operation.

Amplitude condition
    threshold
    slope efficiency

Phase condition
    axial modes
    homogeneous vs. inhomogeneous gain media

Transverse modes
    Hermite Gaussians
    the “donut” mode

|E(x,y)|²
Rate equations for a two-level system

**Rate equations** for the population densities of the two states:

\[
\frac{dN_2}{dt} = BI(N_1 - N_2) - AN_2
\]

\[
\frac{dN_1}{dt} = BI(N_2 - N_1) + AN_2
\]

\[\Rightarrow \quad \frac{d\Delta N}{dt} = -2BI\Delta N + 2AN_2\]
\[\Rightarrow \quad \frac{d\Delta N}{dt} = -2BI\Delta N + AN - A\Delta N\]

If the total number of molecules is \(N\):

\[N \equiv N_1 + N_2\]

\[\Delta N \equiv N_1 - N_2\]

\[2N_2 = (N_1 + N_2) - (N_1 - N_2) = N - \Delta N\]
How does the population difference depend on pump intensity?

\[
\frac{d\Delta N}{dt} = -2BI\Delta N + AN - A\Delta N
\]

In steady-state: \[0 = -2BI\Delta N + AN - A\Delta N\]

Solve for \(\Delta N\):

\[(A + 2BI)\Delta N = AN\]

\[\Rightarrow \Delta N = AN / (A + 2BI) = \frac{N}{2B / I} 1 + \frac{2B}{A}
\]

\[\Rightarrow \Delta N = \frac{N}{1 + 2I / I_{sat}}\]

where: \(I_{sat} = A / B\)

\(I_{sat}\) is called the saturation intensity.
Why inversion is impossible in a two-level system

Population difference $\Delta N = \frac{N}{1 + 2I / I_{sat}}$

Recall that $\Delta N \equiv N_1 - N_2$

For population inversion, we require $\Delta N < 0$

$\Delta N$ is always positive, no matter how hard we pump on the system!

It’s impossible to achieve a steady-state inversion in a two-level system!

Why? Because absorption and stimulated emission are equally likely.

Even for an infinite pump intensity, the best we can do is $N_1 = N_2$ (i.e., $\Delta N = 0$)
Rate equations for a three-level system

So, if we can’t make a laser using two levels, what if we try it with three?

Assume we pump to a state 3 that rapidly decays to level 2.

\[
\frac{dN_2}{dt} = BIN_1 - AN_2
\]

\[
\frac{dN_1}{dt} = -BIN_1 + AN_2
\]

\[
\frac{d\Delta N}{dt} = -2BIN_1 + 2AN_2 \quad \Rightarrow \quad 2N_2 = N - \Delta N
\]

\[
2N_1 = N + \Delta N
\]

The total number of molecules is \( N \):

\[
N \equiv N_1 + N_2
\]

\[
\Delta N \equiv N_1 - N_2
\]

Level 3 decays fast and so \( N_3 = 0 \).
Why inversion is possible in a three-level system

\[
\frac{d\Delta N}{dt} = -BIN - BI\Delta N + AN - A\Delta N
\]

In steady-state: \(0 = -BIN - BI\Delta N + AN - A\Delta N\)

Solve for \(\Delta N\):

\[
\Delta N = N \frac{A - BI}{A + BI} = N \frac{1 - (B/A)I}{1 + (B/A)I}
\]

\[
\Rightarrow \Delta N = N \frac{1 - I/I_{sat}}{1 + I/I_{sat}}
\]

where, as before: \(I_{sat} = A/B\)

\(I_{sat}\) is the saturation intensity.

Now if \(I > I_{sat}\), \(\Delta N\) is negative!
Rate equations for a four-level system

Now assume the lower laser level 1 also rapidly decays to a ground level 0.

As before: \[ \frac{dN_2}{dt} = B I N_0 - A N_2 \]

\[ \frac{dN_2}{dt} = B I (N - N_2) - A N_2 \]

Because \( N_1 \approx 0 \), \( \Delta N \approx -N_2 \)

\[ -\frac{d\Delta N}{dt} = B I N + B I \Delta N + A \Delta N \]

At steady state: \( 0 = B I N + B I \Delta N + A \Delta N \)
Why inversion is easy in a four-level system

\[ 0 = B\Delta N + BI\Delta N + A\Delta N \]

Solve for \( \Delta N \):

\[ \Rightarrow \Delta N = -B\Delta N / (A + BI) = -N \frac{(B/A)I}{1 + (B/A)I} \]

\[ \Rightarrow \Delta N = -N \frac{I / I_{\text{sat}}}{1 + I / I_{\text{sat}}} \]

where: \( I_{\text{sat}} = A / B \)

\( I_{\text{sat}} \) is the saturation intensity.

Now, \( \Delta N \) is negative—for any non-zero value of \( I \)!
Two-, three-, and four-level systems

Four-level systems are best.

- **Two-level system**: At best, you get equal populations. No lasing.
- **Three-level system**: If you hit it hard, you get lasing.
- **Four-level system**: Lasing is easy!
Population inversion in two-, three-, and four-level systems
What is the saturation intensity?

\[ I_{\text{sat}} = \frac{A}{B} \]

* \( A \) is the excited-state relaxation rate: \( 1/\tau \)

* \( B \) is the absorption cross-section, \( \sigma \), divided by the energy per photon, \( \hbar \omega \): \( \sigma / \hbar \omega \)

Both \( \sigma \) and \( \tau \) depend on the molecule, and also the frequency of the light.

\[ \begin{align*}
I_{\text{sat}} &= \frac{\hbar \omega}{\sigma \tau} \\
\hbar \omega &\sim 10^{-19} \text{ J for visible/near IR light} \\
\tau &\sim 10^{-12} \text{ to } 10^{-8} \text{ s for molecules} \\
\sigma &\sim 10^{-20} \text{ to } 10^{-16} \text{ cm}^2 \text{ for molecules (on resonance)} \\
10^5 \text{ to } 10^{13} \text{ W/cm}^2
\end{align*} \]

The saturation intensity plays a key role in laser theory. It is the intensity which corresponds to one photon incident on each molecule, within its cross-section \( \sigma \), per recovery time \( \tau \).

For Ti:sapphire, \( I_{\text{sat}} \sim 300 \text{ kW/cm}^2 \)
Threshold condition: what about losses?

Steady-state condition #1:

Amplitude is invariant after each round trip

\[ \frac{E_{after}}{E_{before}} = |r_1 \cdot r_2| \exp(2gL_m) = 1 \Rightarrow g = \frac{1}{2L_m} \ln \left[ \frac{1}{|r_1 \cdot r_2|} \right] \]

Gain \( g \) must be large enough to compensate for reflection losses \( R_1 \) and \( R_2 \) at the mirrors.

Note: this ignores other sources of loss in the laser.

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Threshold population inversion

A 5 mm Ti:sapphire crystal, in a cavity with one high reflector and one 5% output coupler.

Example:

$$g(\omega_0) = \frac{1}{2} \Delta N \cdot \sigma_0$$

Threshold inversion density:

$$\Delta N_{thresh} = \frac{1}{2L_m \sigma_0} \ln \left( \frac{1}{R_1 R_2} \right)$$

Example: A 5 mm Ti:sapphire crystal, in a cavity with one high reflector and one 5% output coupler.

$$\Delta N_{th} \approx -2 \cdot 10^{17} \text{cm}^{-3}$$

This corresponds to \(~0.5\%) of the Ti\(^{3+}\) ions in the crystal.
A threshold value of pump intensity

Population difference $\Delta N$

Intensity of the pump

Population inversion

Lasing!

$\Delta N_{\text{threshold}}$

Minimum pump intensity required to turn on a 4-level laser, $I_{\text{threshold}}$
Gain vs. loss in a laser cavity

More generally:

\[ \left| \frac{E_{\text{after}}}{E_{\text{before}}} \right|^2 = R_1 R_2 e^{\delta_m - \delta_0} = e^{\delta_m - \delta_c} \]

Cavity Q-factor: \[ Q = \frac{2\pi L_{\text{rt}}}{(\lambda \delta_c)} \]

Cavity lifetime: \[ \tau_c = \frac{T_{\text{rt}}}{\delta_c} \]

Round-trip length: \( L_{\text{rt}} \)

Cavity lifetime: \( \tau_c \)

Collection of 4-level systems

Length: \( L_m \)
Laser output power

*Threshold inversion* condition becomes $\delta_c = \delta_m$

or $\Delta N_{\text{thresh}} = \frac{\delta_c}{2L_m \sigma_0}$

Output power is determined by gain saturation:

$$\delta_m = 4gL_m = \frac{4g_0L_m}{1 + \frac{I_{\text{circ}}}{I_{\text{sat}}}} = \delta_c = \delta_0 + \delta_{oc}$$

Gain

$$I_{\text{out}} = \delta_{oc} I_{\text{circ}} = I_{\text{sat}} \delta_{oc} \left( \frac{4g_0L_m}{\delta_0 + \delta_{oc}} - 1 \right) \propto I_{\text{sat}}$$

(valid only above threshold, $I_{\text{out}} \geq 0$)
Recall that the *unsaturated* gain is proportional to the pumping rate $R_p$:

$$g_0 \propto \Delta N_0 = \left(\frac{\gamma_{21} - \gamma_{10}}{\gamma_{21} \gamma_{10}}\right) \cdot R_p$$

Thus, output power can also be written:

$$I_{out} = I_{sat} \delta_{oc} \left(\frac{R_p}{R_{p,th}} - 1\right)$$

Output power is linear in the pump rate (above threshold but below saturation)

"Slope efficiency" = \underline{output power} / \underline{pump power}
Slope efficiency

Above threshold (but not too far above), the output laser power is a linear function of the input pump power.

The concept of slope efficiency only applies for $I_{th} < I_{pump} < I_{sat}$.

For example, a slope efficiency of 50% means that for every two additional pump photons we add, one additional laser photon is generated.

Essentially all lasers exhibit this behavior.
Slope efficiency - examples

diode-pumped thulium lasers
P. Černý and H. Jelínková, SPIE 2006

threshold: 33 mA

distributed feedback diode laser

λ = 975 nm

slope efficiency: 0.74 W/A

saturation regime