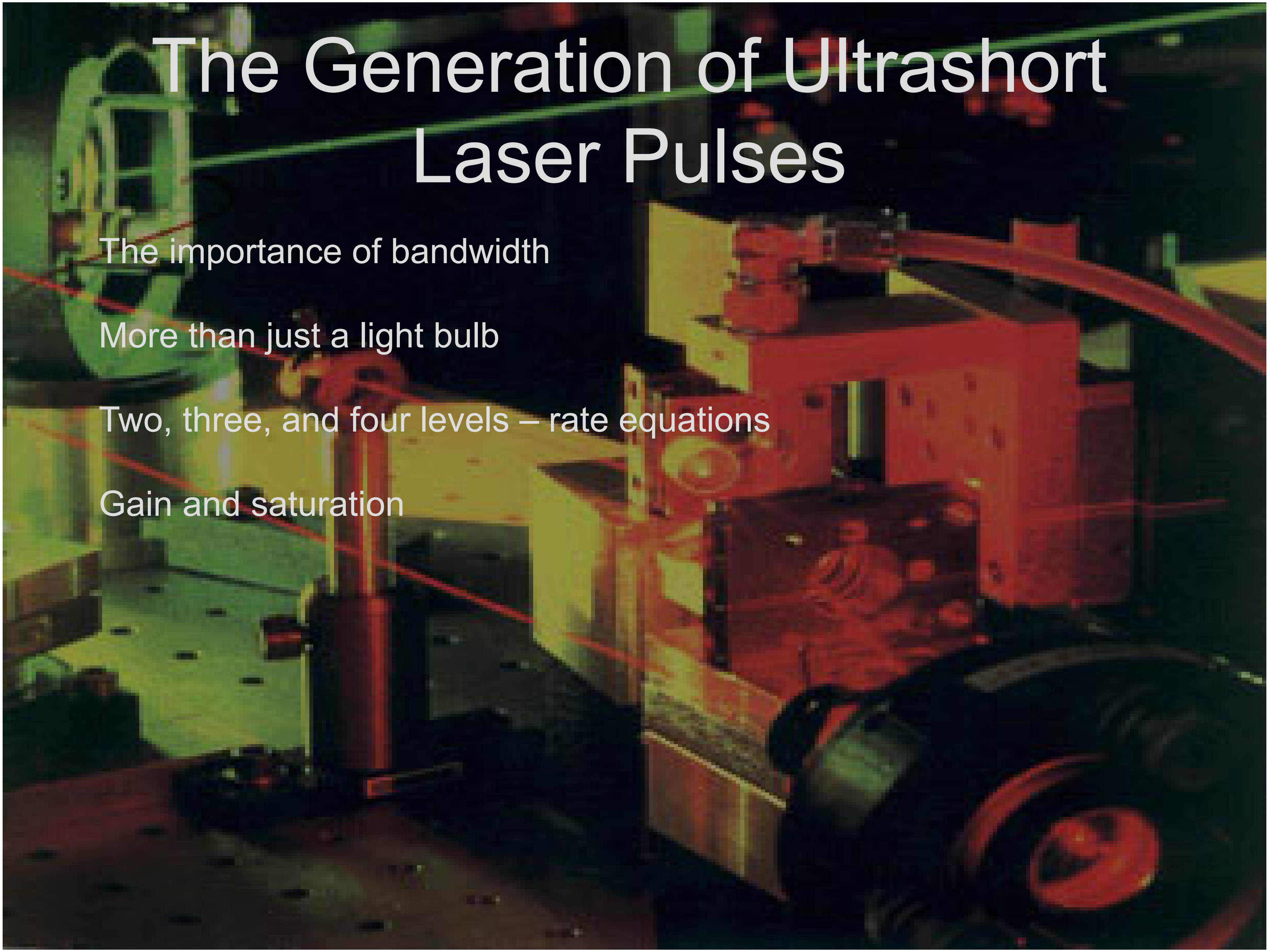


The Generation of Ultrashort Laser Pulses

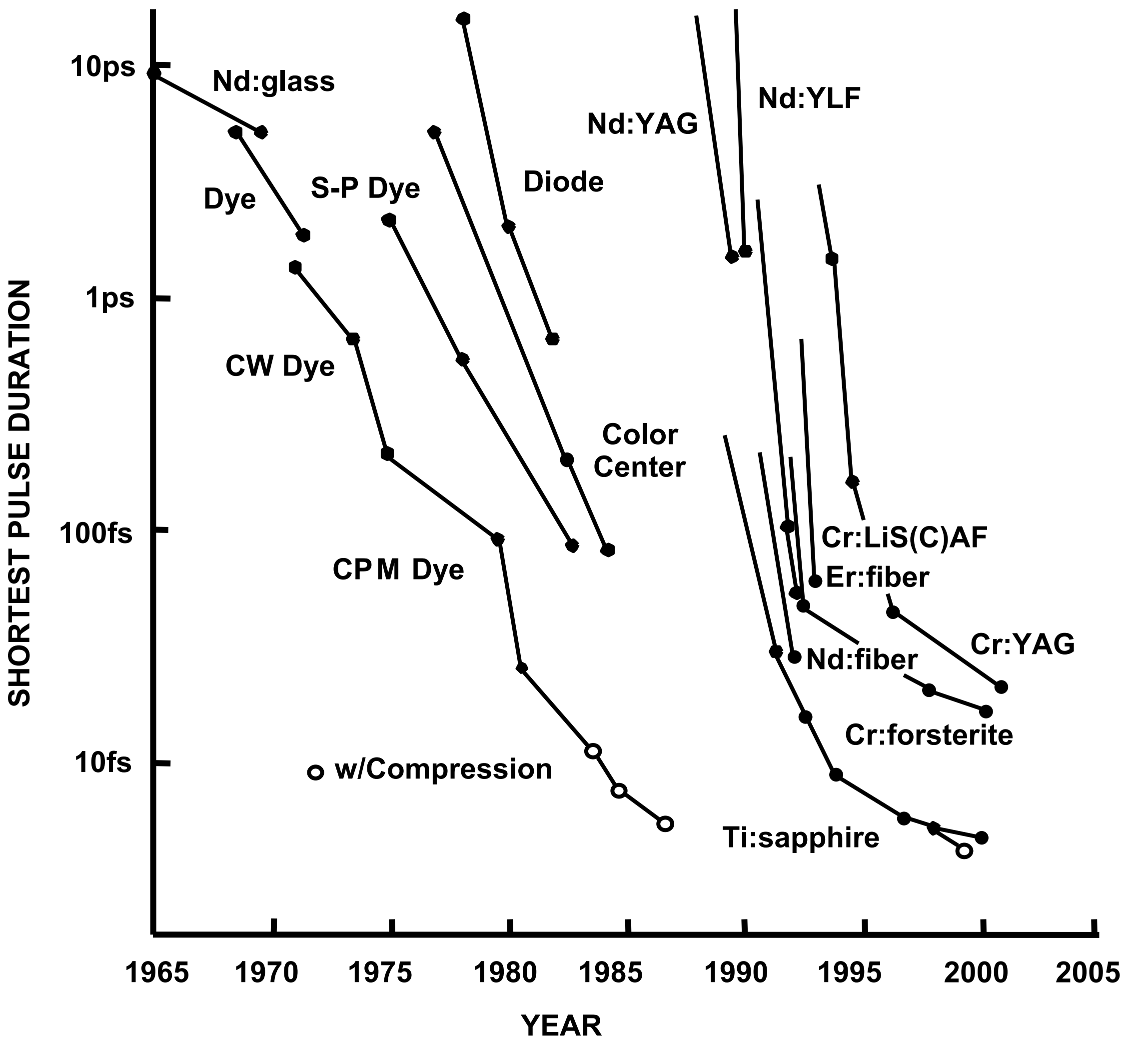
The importance of bandwidth

More than just a light bulb

Two, three, and four levels – rate equations

Gain and saturation

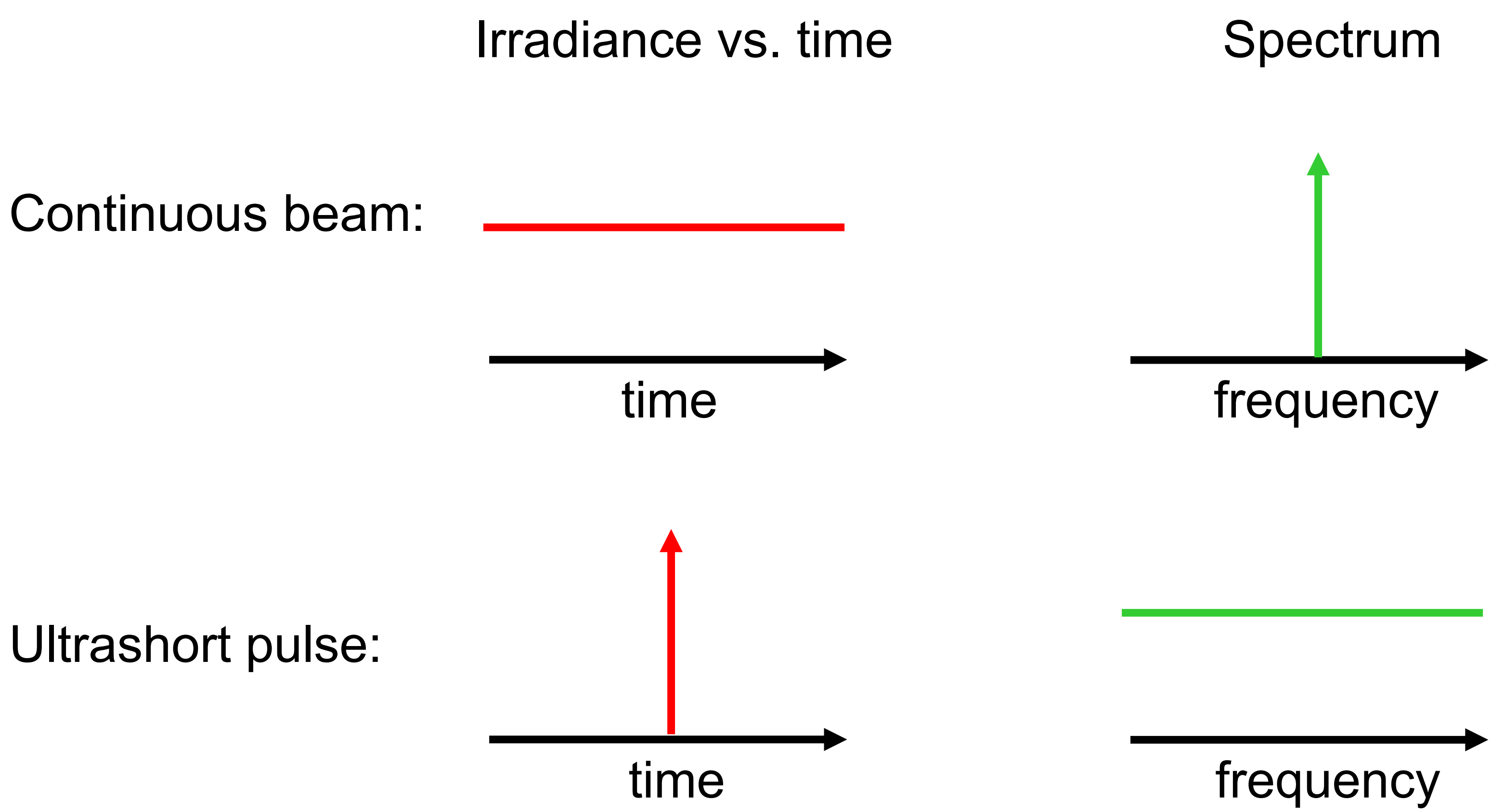
First point: the progress has been amazing!



The shortest pulse vs. year (for different media)

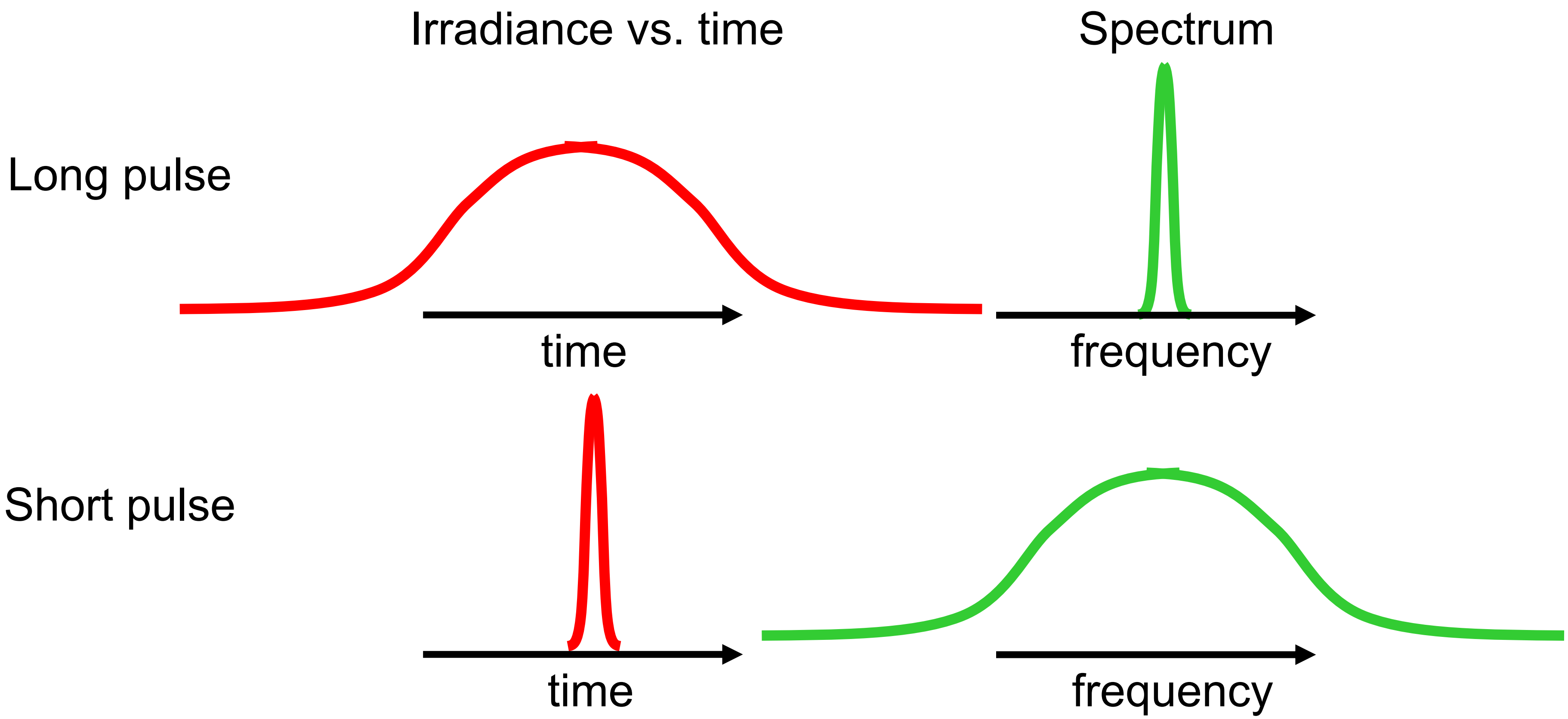
Continuous vs. ultrashort pulses of light

A constant and a delta-function are a Fourier-Transform pair.



Long vs. short pulses of light

The uncertainty principle says that the product of the temporal and spectral pulse widths is greater than or equal to ~1.



Light bulbs, lasers, and ultrashort pulses

But a light bulb is also broadband.

What exactly is required to make an ultrashort pulse?

Answer: A Mode-locked Laser

Okay, what's a laser, what are modes, and what does it mean to lock them?

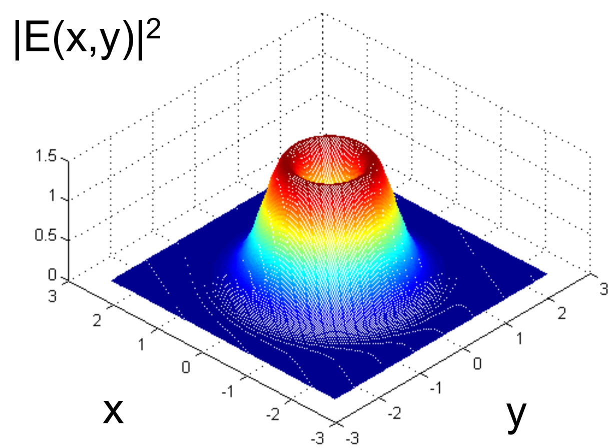
→ We'll get to this part in a later lecture.

There are 3 conditions for steady-state laser operation.

Amplitude condition
- threshold
- slope efficiency

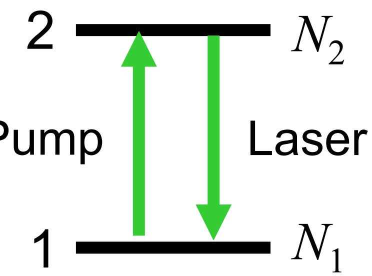
Phase condition
- axial modes
- homogeneous vs. inhomogeneous gain media

Transverse modes
- Hermite Gaussians
- the "donut" mode



Rate equations for a two-level system

Rate equations for the population densities of the two states:



Absorption — Stimulated emission — Spontaneous emission

$$\frac{dN_2}{dt} = BI(N_1 - N_2) - AN_2$$

Pump intensity

$$\frac{dN_1}{dt} = BI(N_2 - N_1) + AN_2$$

If the total number of molecules is N:

$$N \equiv N_1 + N_2$$

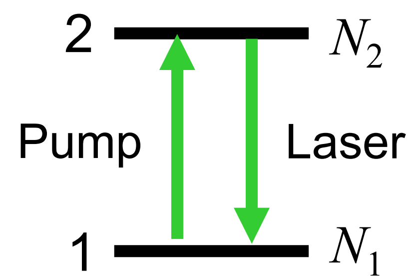
$$\Delta N \equiv N_1 - N_2$$

$$\Rightarrow \quad \frac{d\Delta N}{dt} = -2BI\Delta N + 2AN_2$$

$$2N_2 = (N_1 + N_2) - (N_1 - N_2) = N - \Delta N$$

$$\Rightarrow \quad \frac{d\Delta N}{dt} = -2BI\Delta N + AN - A\Delta N$$

How does the population difference depend on pump intensity?



$$\frac{d\Delta N}{dt} = -2BI\Delta N + AN - A\Delta N$$

In steady-state: $0 = -2BI\Delta N + AN - A\Delta N$

Solve for ΔN: $(A + 2BI)\Delta N = AN$

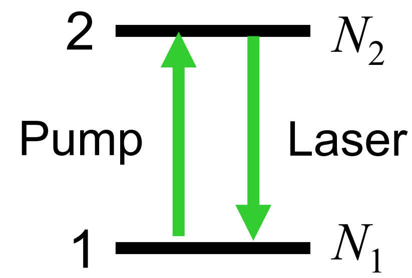
$$\Rightarrow \quad \Delta N = AN/(A + 2BI) = \frac{N}{1 + \frac{2B}{A} I}$$

$$\Rightarrow \boxed{\Delta N = \frac{N}{1 + 2I / I_{sat}}}$$

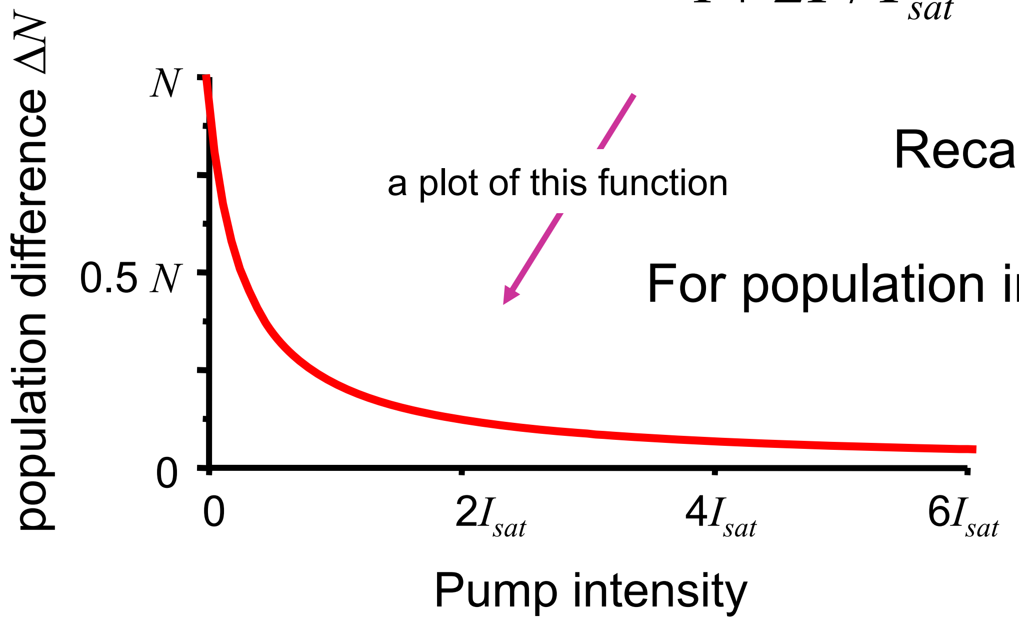
where: $I_{sat} = A / B$

I_{sat} is called the **saturation intensity**.

Why inversion is impossible in a two-level system



Population difference $\Delta N = \dfrac{N}{1 + 2I / I_{sat}}$



Recall that $\Delta N \equiv N_1 - N_2$

For population inversion, we require $\Delta N < 0$

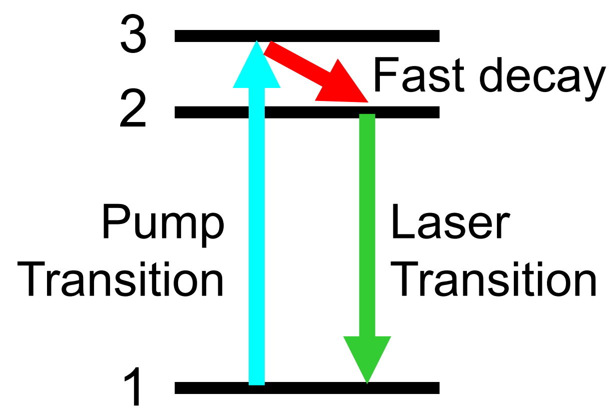
ΔN is always positive, no matter how hard we pump on the system!

It's impossible to achieve a steady-state inversion in a two-level system!

Why? Because absorption and stimulated emission are equally likely.

Even for an infinite pump intensity, the best we can do is $N_1 = N_2$ (i.e., $\Delta N = 0$)

Rate equations for a three-level system



So, if we can't make a laser using two levels, what if we try it with three?

Assume we pump to a state 3 that rapidly decays to level 2.

$$\frac{dN_2}{dt} = BIN_1 - AN_2$$

$$\frac{dN_1}{dt} = -BIN_1 + AN_2$$

The total number of molecules is N:

$$N \equiv N_1 + N_2$$

$$\Delta N \equiv N_1 - N_2$$

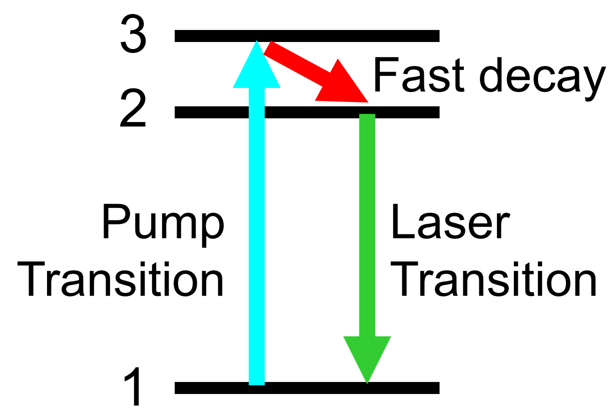
Level 3 decays fast and so $N_3 = 0$.

$$\frac{d\Delta N}{dt} = -2BIN_1 + 2AN_2$$

$$2N_2 = N - \Delta N$$

$$2N_1 = N + \Delta N$$

Why inversion is possible in a three-level system



$$\frac{d\Delta N}{dt} = -BIN - BI\Delta N + AN - A\Delta N$$

In steady-state: $0 = -BIN - BI\Delta N + AN - A\Delta N$

Solve for ΔN: $$\Delta N = N\frac{A - BI}{A + BI} = N\frac{1-(B/A)I}{1+(B/A)I}$$

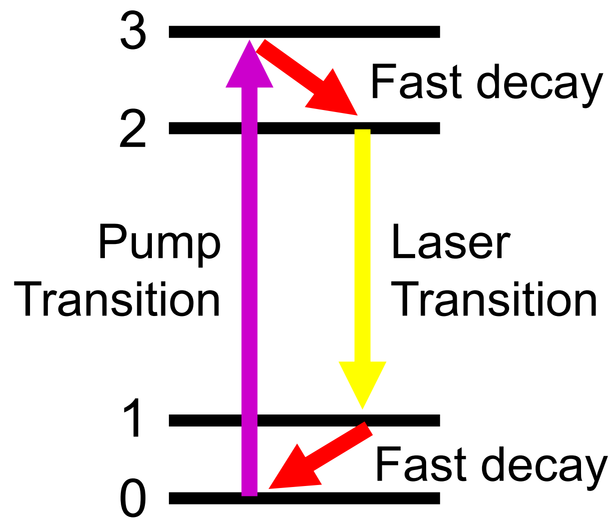
$$\Rightarrow \boxed{\Delta N = N\frac{1 - I/I_{sat}}{1 + I/I_{sat}}}$$

where, as before: $I_{sat} = A/B$

I_{sat} is the **saturation intensity**.

Now if $I > I_{sat}$, ΔN is negative!

Rate equations for a four-level system



Now assume the lower laser level 1 also rapidly decays to a ground level 0.

As before: $$\frac{dN_2}{dt} = BIN_0 - AN_2$$

The total number of molecules is N:

$$N \equiv N_0 + N_2$$

$$N_0 = N - N_2$$

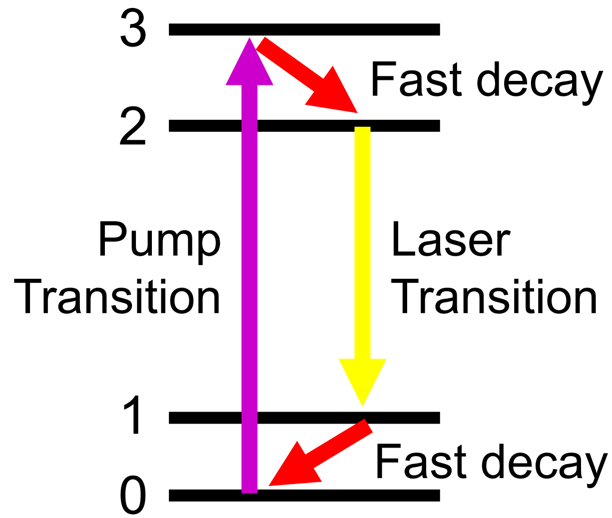
$$\frac{dN_2}{dt} = BI(N - N_2) - AN_2$$

Because $N_1 \approx 0$, $\Delta N \approx -N_2$

$$-\frac{d\Delta N}{dt} = BIN + BI\Delta N + A\Delta N$$

At steady state: $$0 = BIN + BI\Delta N + A\Delta N$$

Why inversion is easy in a four-level system



$$0 = BIN + BI\Delta N + A\Delta N$$

Solve for ΔN:

$$\Rightarrow \quad \Delta N = -BIN/(A + BI) = -N\frac{(B/A)I}{1+(B/A)I}$$

$$\Rightarrow \quad \boxed{\Delta N = -N\frac{I/I_{sat}}{1+I/I_{sat}}}$$

where: $I_{sat} = A/B$

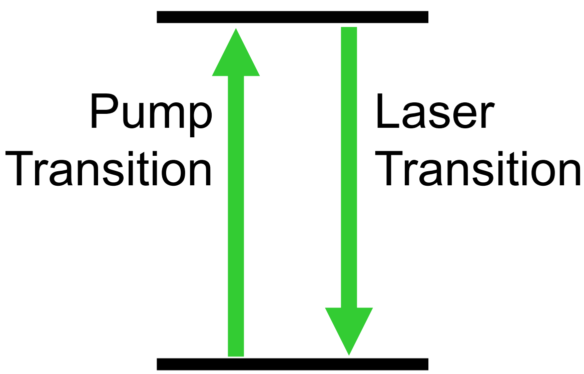
I_{sat} is the **saturation intensity**.

Now, ΔN is negative—for any non-zero value of I!

Two-, three-, and four-level systems

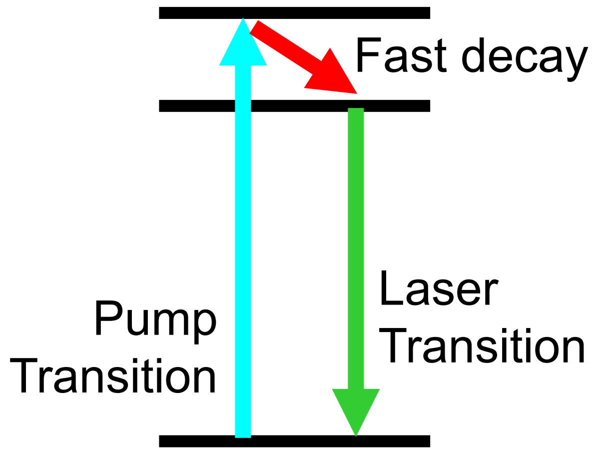
Four-level systems are best.

Two-level system



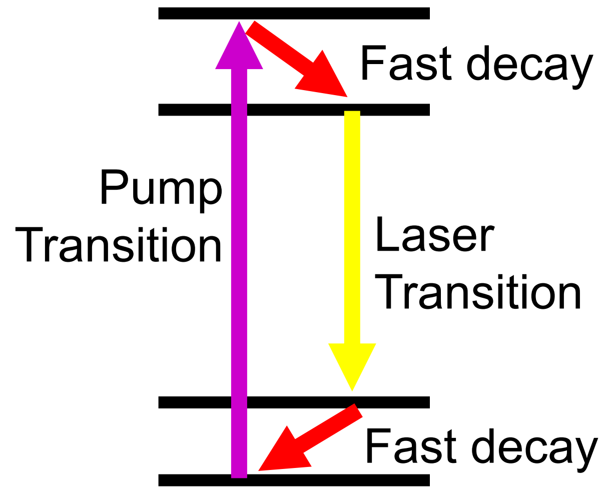
At best, you get equal populations. No lasing.

Three-level system



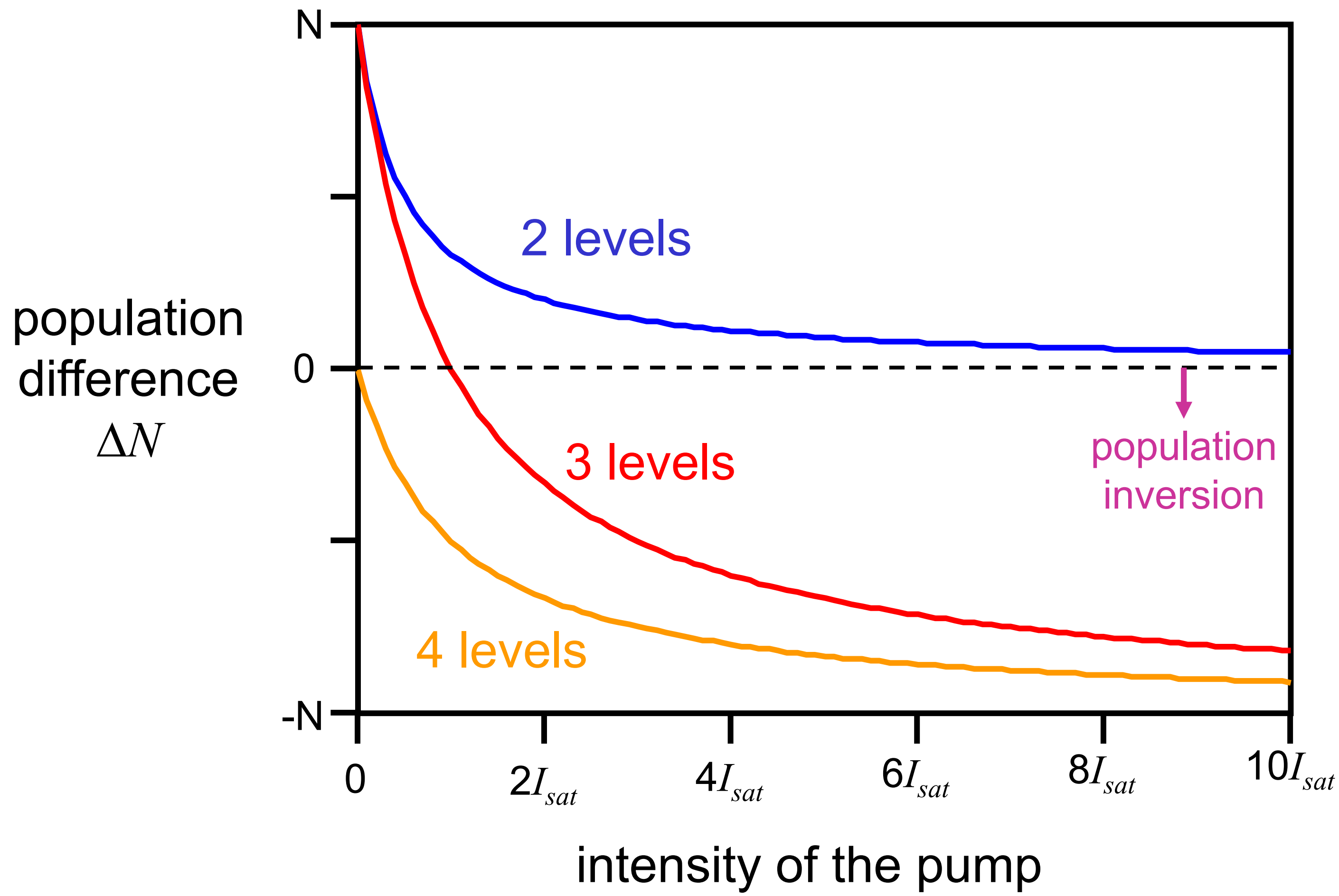
If you hit it hard, you get lasing.

Four-level system



Lasing is easy!

Population inverstion in two-, three-, and four-level systems



What is the saturation intensity?

$$I_{sat} = A / B$$

A is the excited-state relaxation rate: $1/\tau$

B is the absorption cross-section, σ, divided by the energy per photon, $\hbar\omega$: $\sigma / \hbar\omega$

Both σ and τ depend on the molecule, and also the frequency of the light.

$$I_{sat} = \frac{\hbar\omega}{\sigma\tau}$$

$\hbar\omega$ ~10^{-19} J for visible/near IR light

τ ~10^{-12} to 10^{-8} s for molecules

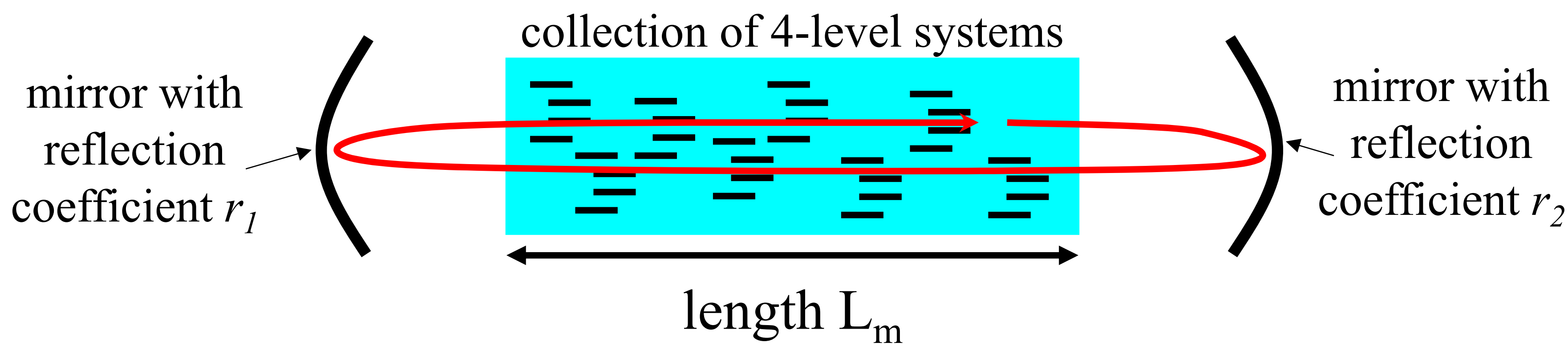
σ ~10^{-20} to 10^{-16} cm^2 for molecules (on resonance)

10^5 to 10^{13} W/cm^2

The saturation intensity plays a key role in laser theory. It is the intensity which corresponds to one photon incident on each molecule, within its cross-section σ, per recovery time τ.

For Ti:sapphire, I_{sat} ~ 300 kW/cm^2

Threshold condition: what about losses?



Steady-state condition #1:

Amplitude is invariant after each round trip

$$\left|\frac{E_{after}}{E_{before}}\right| = |r_1 \cdot r_2| \exp(2gL_m) = 1 \;\Rightarrow\; g = \frac{1}{2L_m}\ln\left[\frac{1}{|r_1 \cdot r_2|}\right]$$

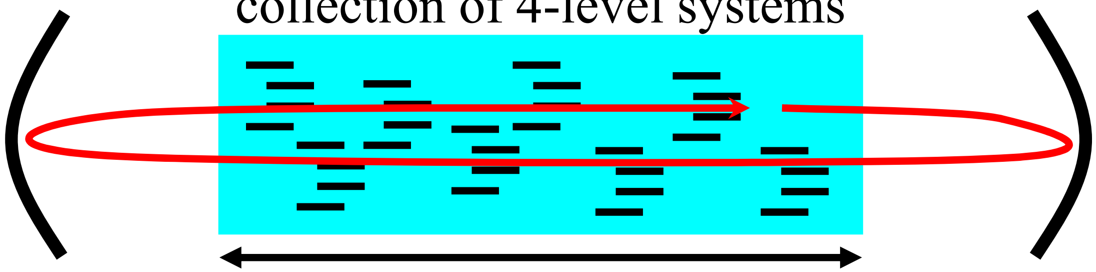
gain

Gain g must be large enough to compensate for reflection losses R_1 and R_2 at the mirrors.

Note: $R_j = |r_j|^2$

Note: this ignores other sources of loss in the laser.

Threshold population inversion



$$g = \frac{1}{4L_m}\ln\left[\frac{1}{R_1R_2}\right]$$

But we have seen that the gain at resonance is:

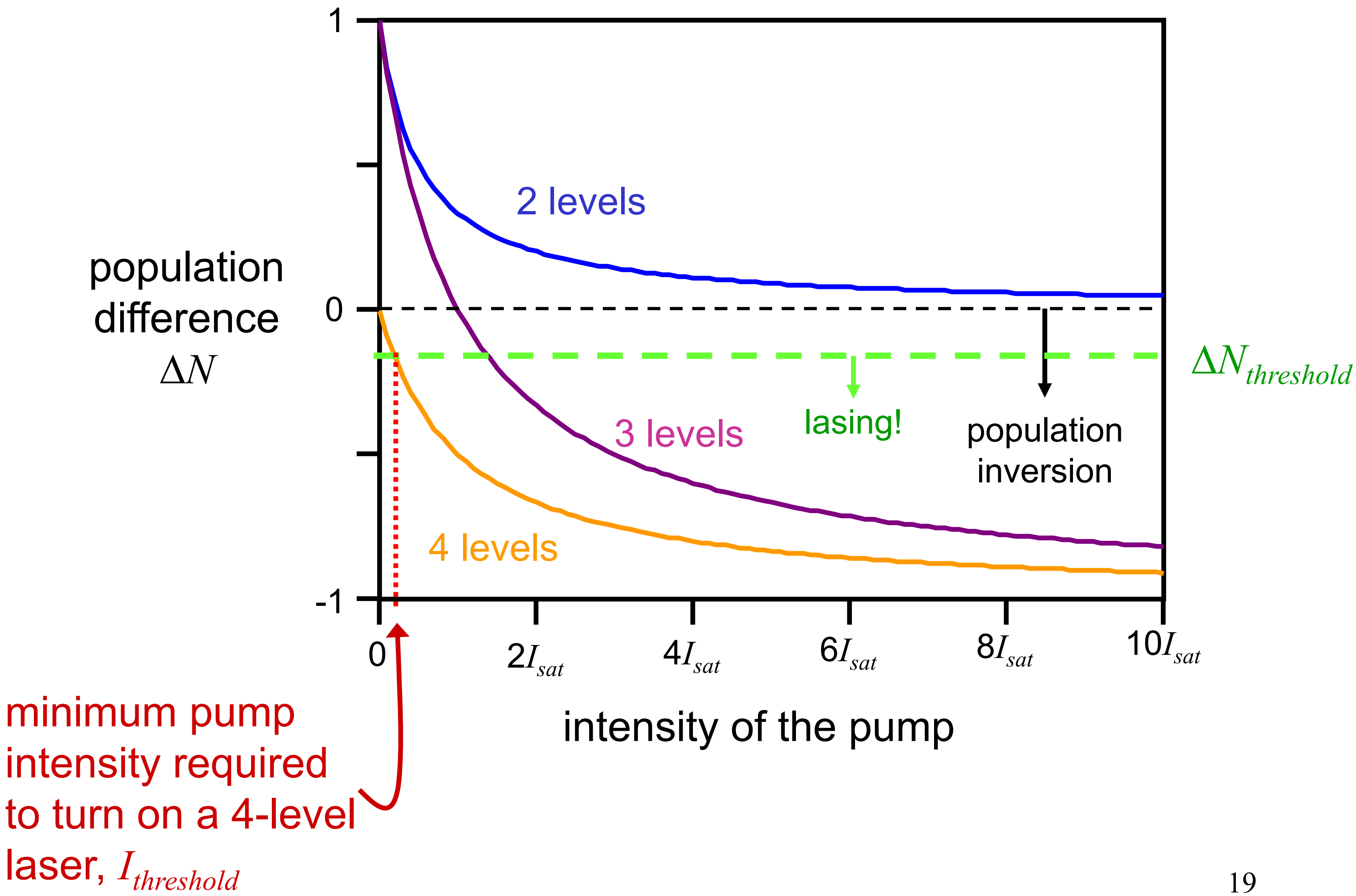
$$g(\omega_0) = \frac{1}{2}\Delta N \cdot \sigma_0$$

Threshold inversion density: $$\Delta N_{thresh} = \frac{1}{2L_m\sigma_0}\ln\left(\frac{1}{R_1R_2}\right)$$

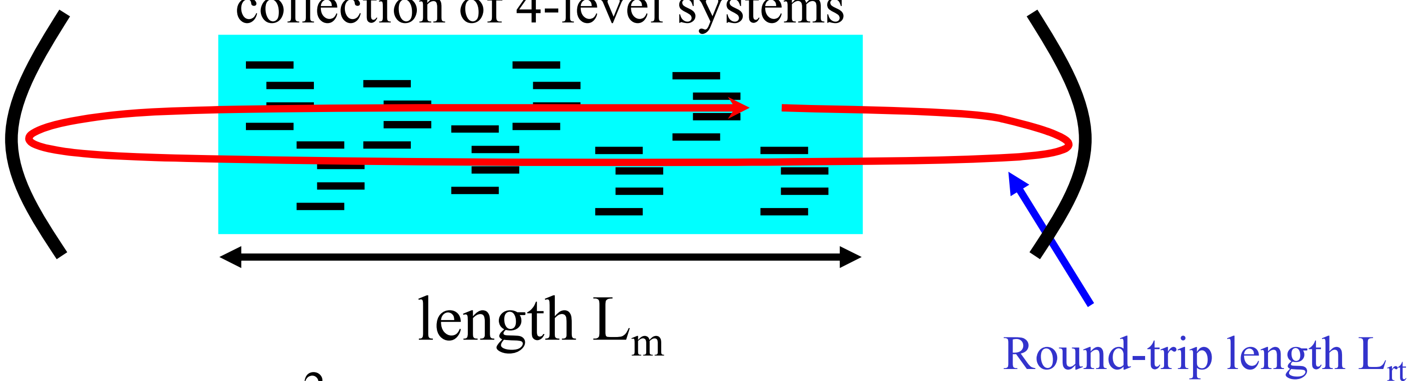
Example: A 5 mm Ti:sapphire crystal, in a cavity with one high reflector and one 5% output coupler ⟶ $\Delta N_{th} \approx \text{-}2\cdot 10^{17} cm^{-3}$

This corresponds to ~0.5% of the Ti^{3+} ions in the crystal.

A threshold value of pump intensity



Gain vs. loss in a laser cavity



More generally: $$\left|\frac{E_{after}}{E_{before}}\right|^2 = R_1 R_2 e^{\delta_m - \delta_0} = e^{\delta_m - \delta_c}$$

δ_m: cavity gain per round trip

δ_c: net cavity loss per round trip, including mirrors

Cavity Q-factor: $Q = 2\pi L_{rt} / (\lambda \delta_c)$ — the fractional power loss per optical cycle

Cavity lifetime: $\tau_c = T_{rt} / \delta_c$

Laser output power

$$\left|\frac{E_{after}}{E_{before}}\right|^2 = e^{\delta_m - \delta_c}$$

Threshold inversion condition becomes $\delta_c = \delta_m$

or $$\Delta N_{thresh} = \frac{\delta_c}{2L_m\sigma_0}$$

Output power is determined by gain saturation:

$$\delta_m = 4gL_m = \frac{4g_0L_m}{1+\dfrac{I_{circ}}{I_{sat}}} = \delta_c = \delta_0 + \delta_{oc}$$

gain

loss (separated into the output coupler plus all other losses)

$$I_{out} = \delta_{oc}I_{circ} = I_{sat}\delta_{oc}\left(\frac{4g_0L_m}{\delta_0+\delta_{oc}} - 1\right) \propto I_{sat}$$

(valid only above threshold, $I_{out} \geq 0$)

One consequence: the dependence on the output coupler

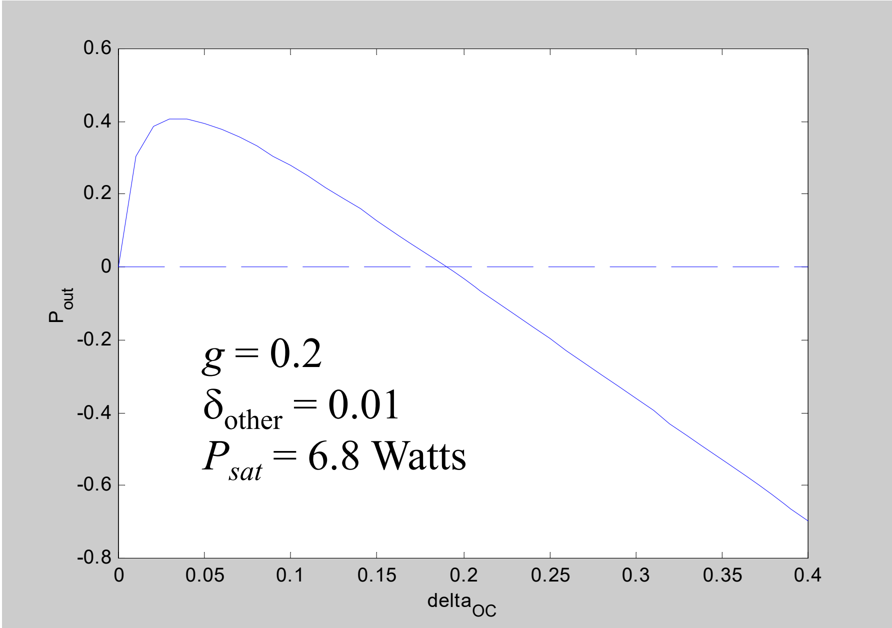
We've just shown this: $$P_{out} = P_{sat}\delta_{oc}\left(\frac{2G}{\delta_0 + \delta_{oc}} - 1\right)$$

What output coupler should we use to maximize the output power?

$$\frac{\partial P_{out}}{\partial \delta_{OC}} = P_{sat}\left(\frac{2G}{\delta_{OC} + \delta_{other}} - 1\right) - \delta_{OC} P_{sat} \frac{2G}{\left(\delta_{OC} + \delta_{other}\right)^2} = 0$$

$$\delta_{OC} = \sqrt{2G\delta_{other}} - \delta_{other}$$

There is an optimum value for the output coupler, which depends on the gain and loss factors.



Slope efficiency

Recall that the *unsaturated* gain is proportional to the pumping rate R_p:

$$g_0 \propto \Delta N_0 = \left(\frac{\gamma_{21} - \gamma_{10}}{\gamma_{21}\gamma_{10}} \right) \cdot R_p$$

Thus, output power can also be written:

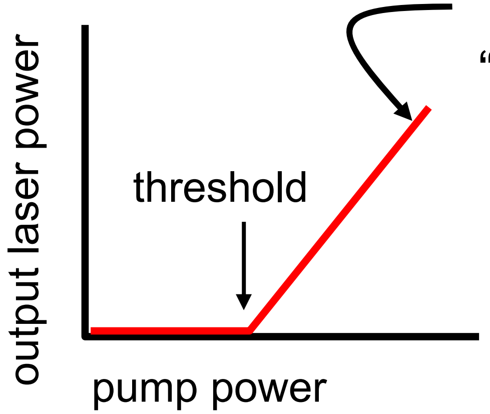
$$I_{out} = I_{sat}\delta_{oc}\left(\frac{R_p}{R_{p,th}} - 1 \right)$$

Output power varies linearly with the pump rate (above threshold but below saturation)

$$\text{“Slope efficiency”} = \frac{\text{output power}}{\text{pump power}}$$

Slope efficiency

Above threshold (but not too far above), the output laser power is a linear function of the input pump power.



The slope of this line is called the "slope efficiency" of the laser.

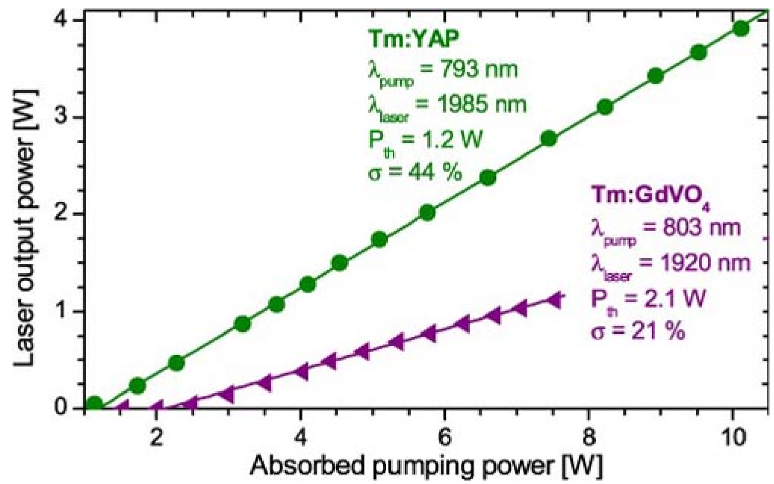
For example, a slope efficiency of 50% means that for every two additional pump photons we add, one additional laser photon is generated.

The concept of slope efficiency only applies for $I_{th} < I_{pump} < I_{sat}$.

Essentially all lasers exhibit this behavior.

Slope efficiency – some randomly chosen examples

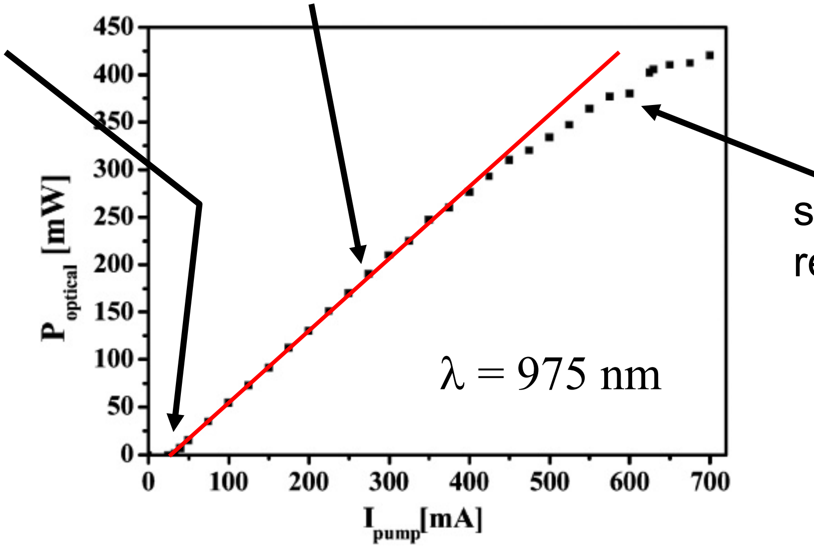
diode-pumped thulium lasers
P. Černý and H. Jelínková, SPIE 2006



slope efficiency: 0.74 W/A

threshold: 33 mA

distributed feedback diode laser
A. Jechow, et al., Opt. Lett (2007)



saturation regime