The Generation of Ultrashort Laser Pulses

Spatial modes

The phase condition, revisited

Trains of pulses – the Shah function

Laser modes and mode locking

Homogeneous vs. inhomogeneous gain media

Classical electron oscillator line shapes

In a material where a significant fraction of the atoms are in an excited state, we must include the population difference factor ΔN in place of N:

absorption becomes *gain* when the sign of ΔN changes!

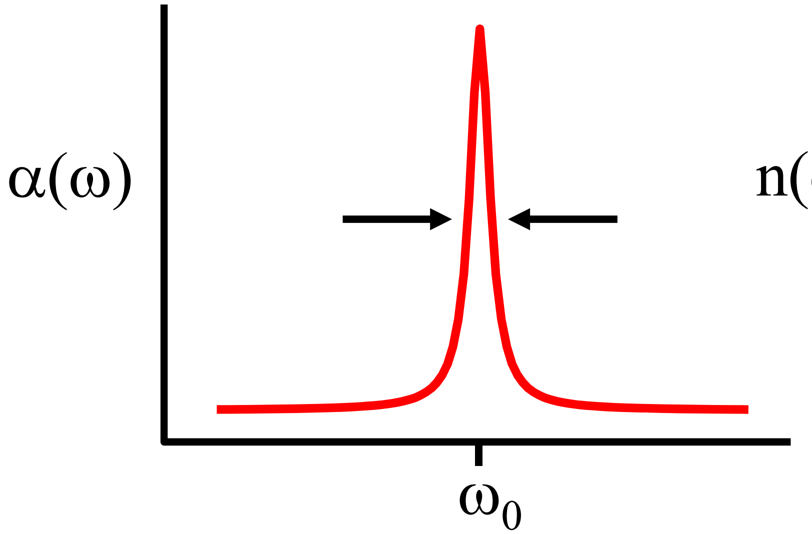
Lorentzian line shape:
(approximate)

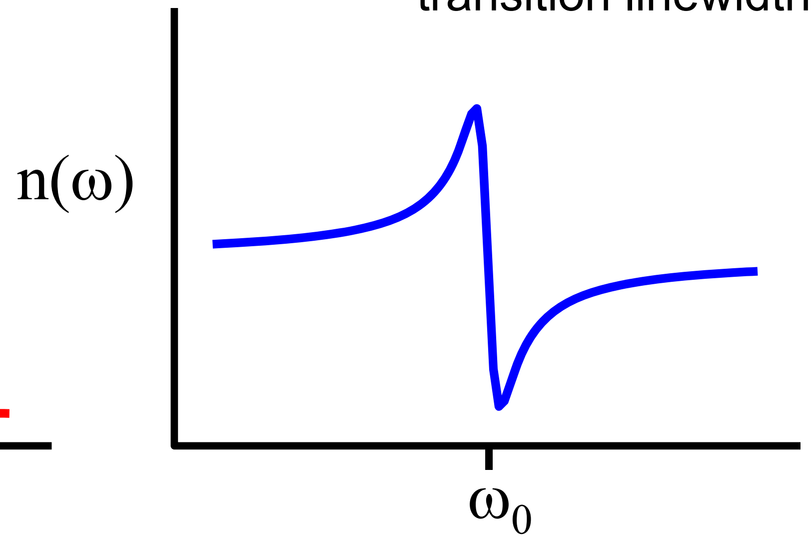
$$\alpha(\omega) \propto \frac{\Delta N}{1+\zeta^2}$$

$$n(\omega) \propto \Delta N \frac{\zeta}{1+\zeta^2}$$

$$\zeta = \frac{2(\omega-\omega_0)}{\boxed{\Gamma}}$$

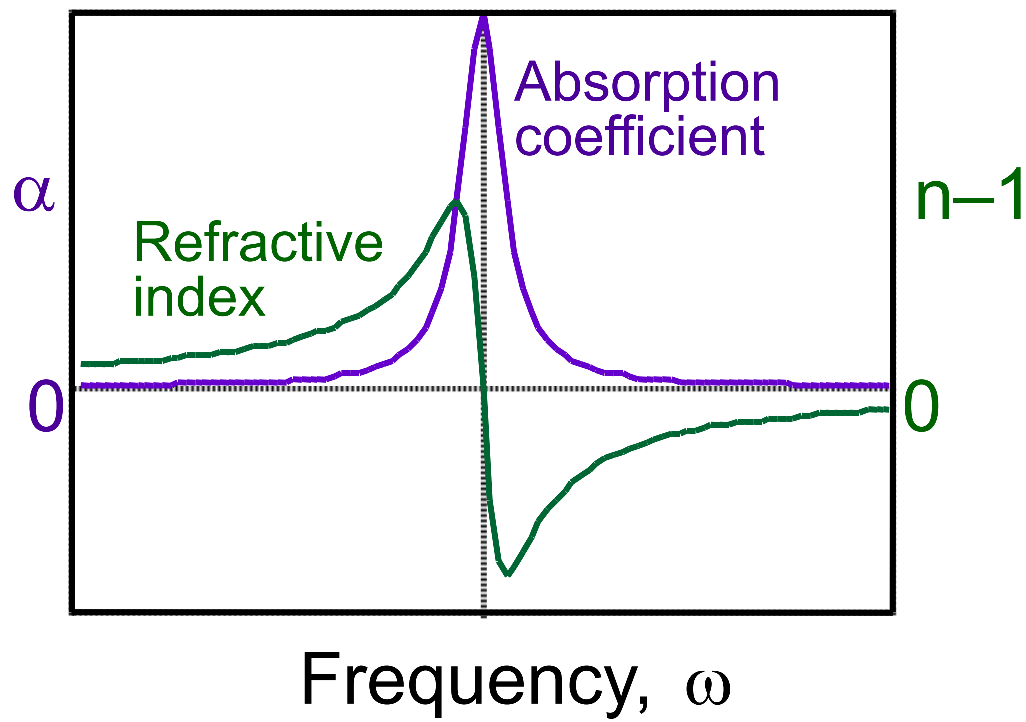
transition linewidth





α and n both depend on frequency

These functions are, together, a Complex Lorentzian (with some constants in front).



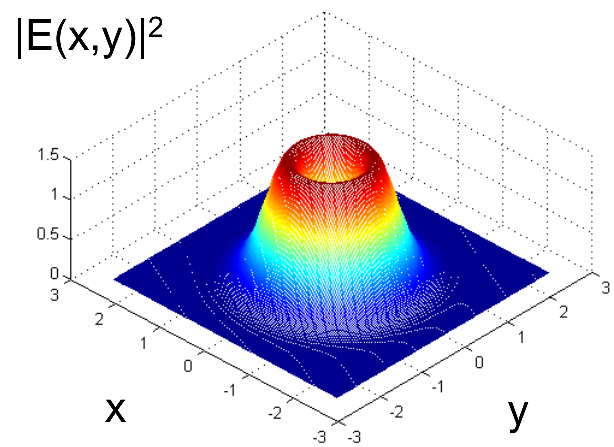
$$\alpha \propto \frac{\Gamma}{\left(\omega_0 - \omega\right)^2 + \Gamma^2} \qquad n - 1 \propto \frac{\left(\omega_0 - \omega\right)}{\left(\omega_0 - \omega\right)^2 + \Gamma^2}$$

There are 3 conditions for steady-state laser operation.

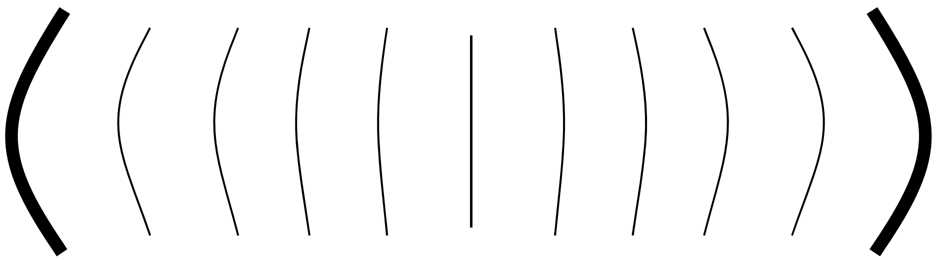
Amplitude condition

Phase condition

$\boxed{\text{Transverse spatial mode condition}}$



Condition on the transverse profile



Steady-state condition #3:

Transverse profile reproduces on each round trip

"transverse modes": those which reproduce themselves on each round trip, except for overall amplitude and phase factors

Determining these modes can be cast as an eigenvalue problem:

$$\beta_{nm} \cdot E_{nm}(x,y) = \iint \underbrace{K(x,y;x_0,y_0)}_{\text{propagation kernel}} \cdot E_{nm}(x_0,y_0)\,dx_0 dy_0$$

Transverse modes

Solutions are the product of two functions, one for each transverse dimension:

$$E_{nm}(x,y) = u_n(x) \cdot u_m(y)$$

where the u_n's are Hermite Gaussians:

$$\left|u_n(x)\right| = H_n\left(\frac{\sqrt{2}x}{w}\right) \cdot \exp\left(-\frac{x^2}{w^2}\right)$$

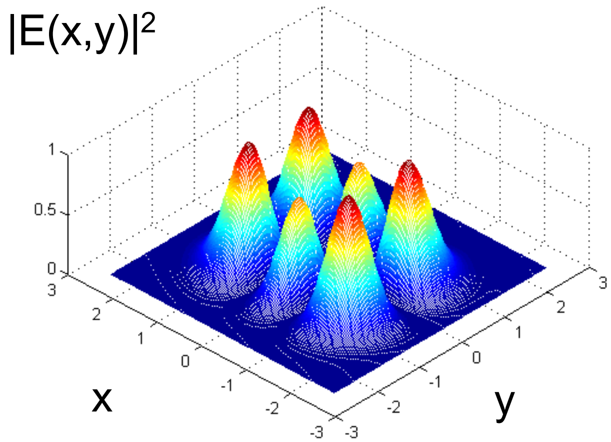
w = beam waist parameter

Notation:

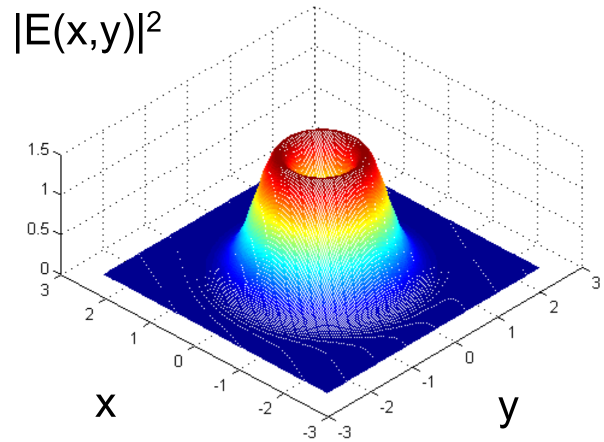
"TEM" = transverse electric and magnetic

"nm" = number of nodes along two principal axes

Transverse modes - examples

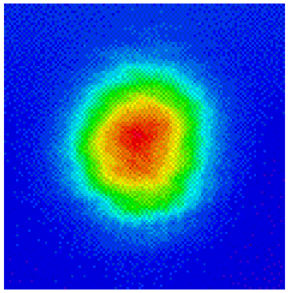


12 mode

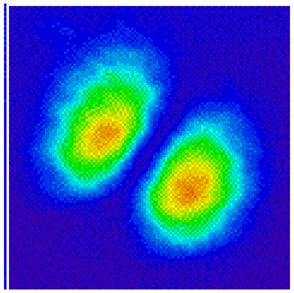


A superposition of the 10 and 01 modes: the “donut mode”

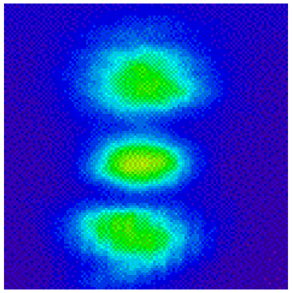
TEM00



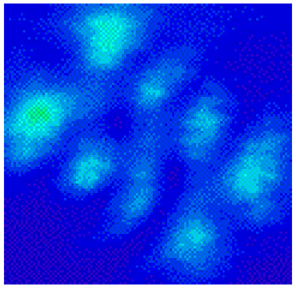
TEM01



TEM02



TEM13



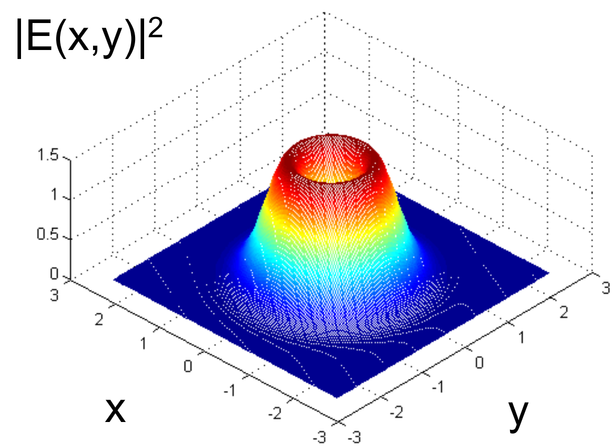
http://www.physics.adelaide.edu.au/optics/

There are 3 conditions for steady-state laser operation.

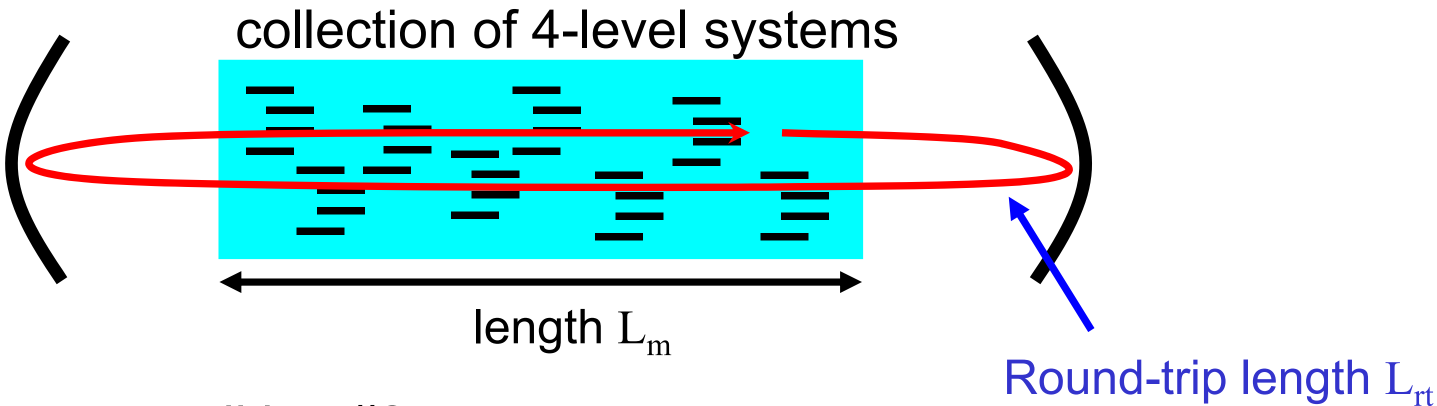
Amplitude condition

$\boxed{\text{Phase condition}}$ Let's revisit this one…

Transverse spatial mode condition



Phase condition



Steady-state condition #2:

Phase is invariant after each round trip

$$\frac{angle(E_{after})}{angle(E_{before})} = \exp\left(-i\omega L_{rt}/c\right) = 1 \quad \Rightarrow \quad \omega_q = \frac{2\pi c}{L_{rt}} \cdot q \quad \text{(q = an integer)}$$

Technically, this should be:
$\left(L_{rt} - 2L_m\right) n_{air} + 2L_m n_{sapphire}$

And for a general laser layout, this is often written: $L_{rt} n_{effective}$

An integer number of wavelengths must fit in the laser cavity.

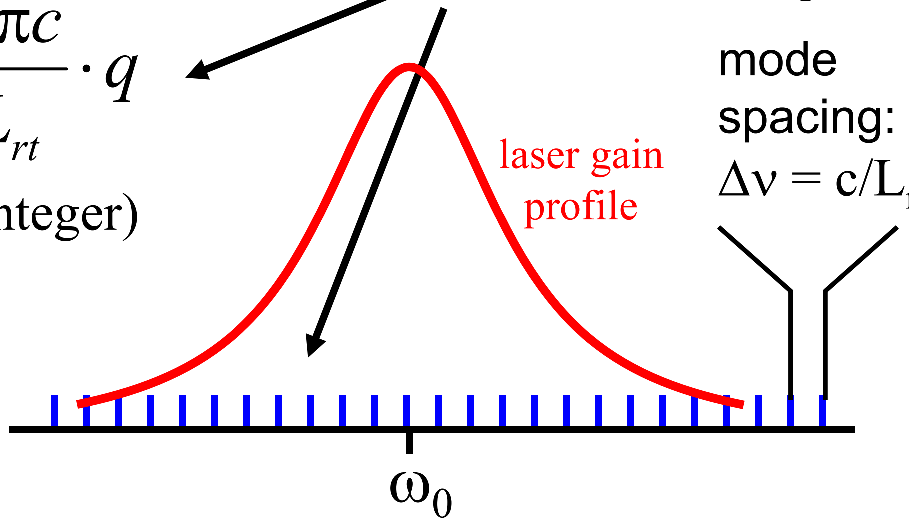
Longitudinal modes

"axial" or "longitudinal" cavity modes

$$\omega_q = \frac{2\pi c}{L_{rt}} \cdot q$$

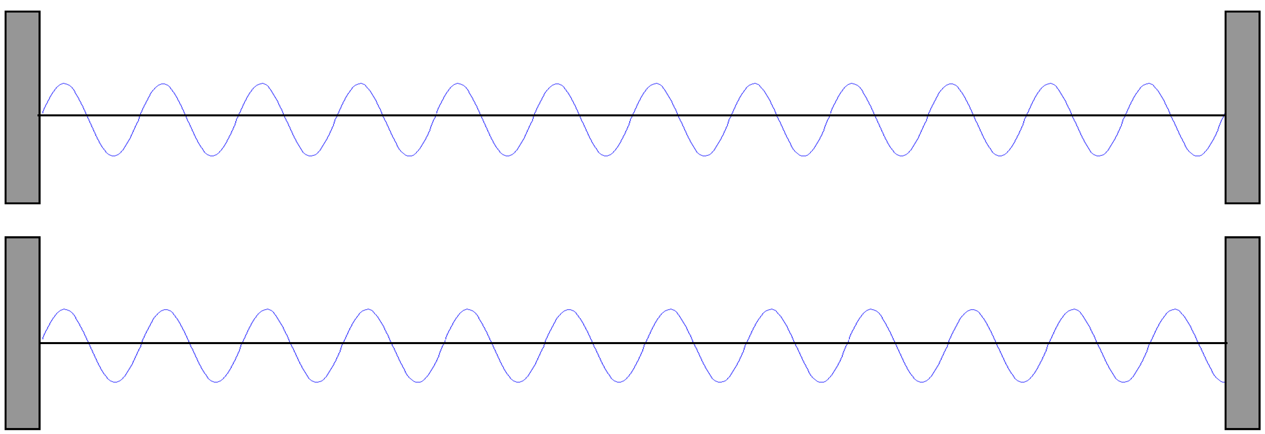
(q = an integer)

mode spacing:
$\Delta\nu = c/L_{rt}$



The q^{th} mode is just:

$$E(x,y)\,e^{ik_q z - i\omega_q t}$$



fits

does not fit

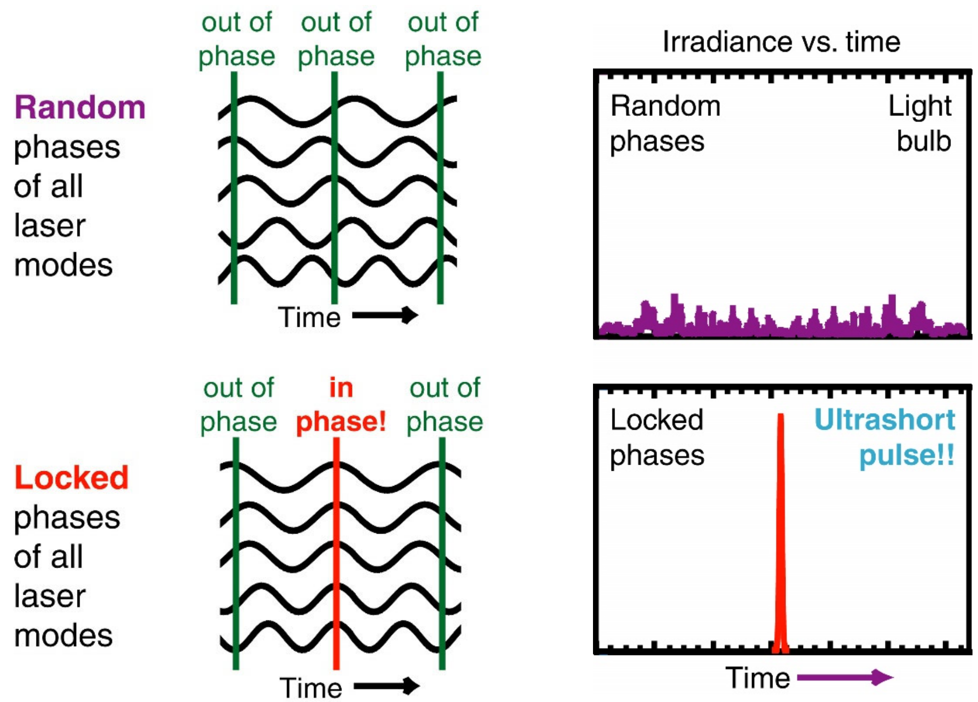
But how does this translate to the case of short pulses?

Mode locking

For a laser with multiple modes lasing simultaneously, the output is the superposition of all of these modes.

If we can "lock" all of these phases together, we get a short pulse!

Techniques for doing this are called "mode locking". They can be used to generate **absurdly short** pulses (the more modes that are locked, the shorter the pulse).

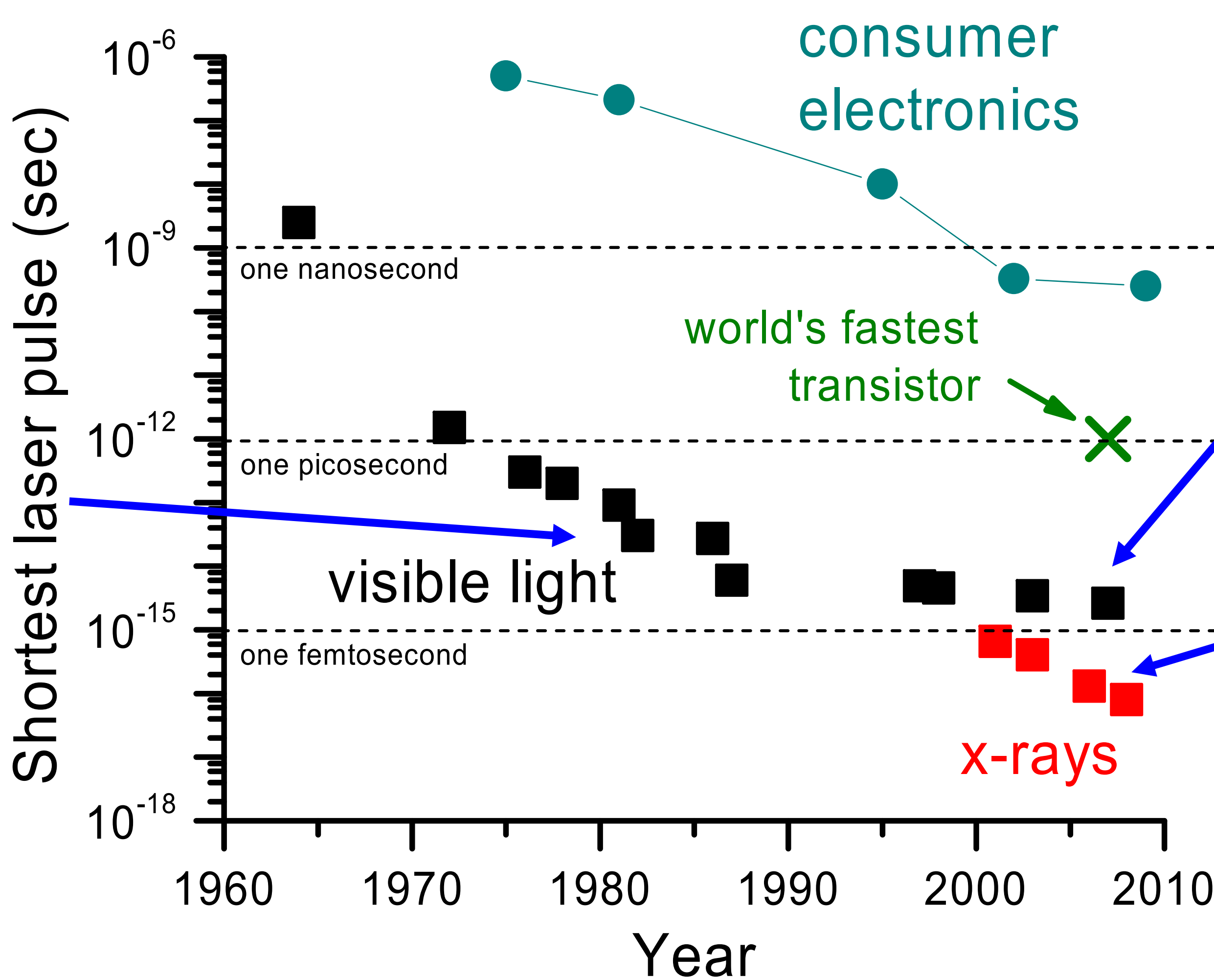


How short is "absurdly short"?
10 femtoseconds is to one second as one second is to 3.1 million years.

Short optical pulses are the fastest events ever created (or measured)

Charles Shank

Erich Ippen

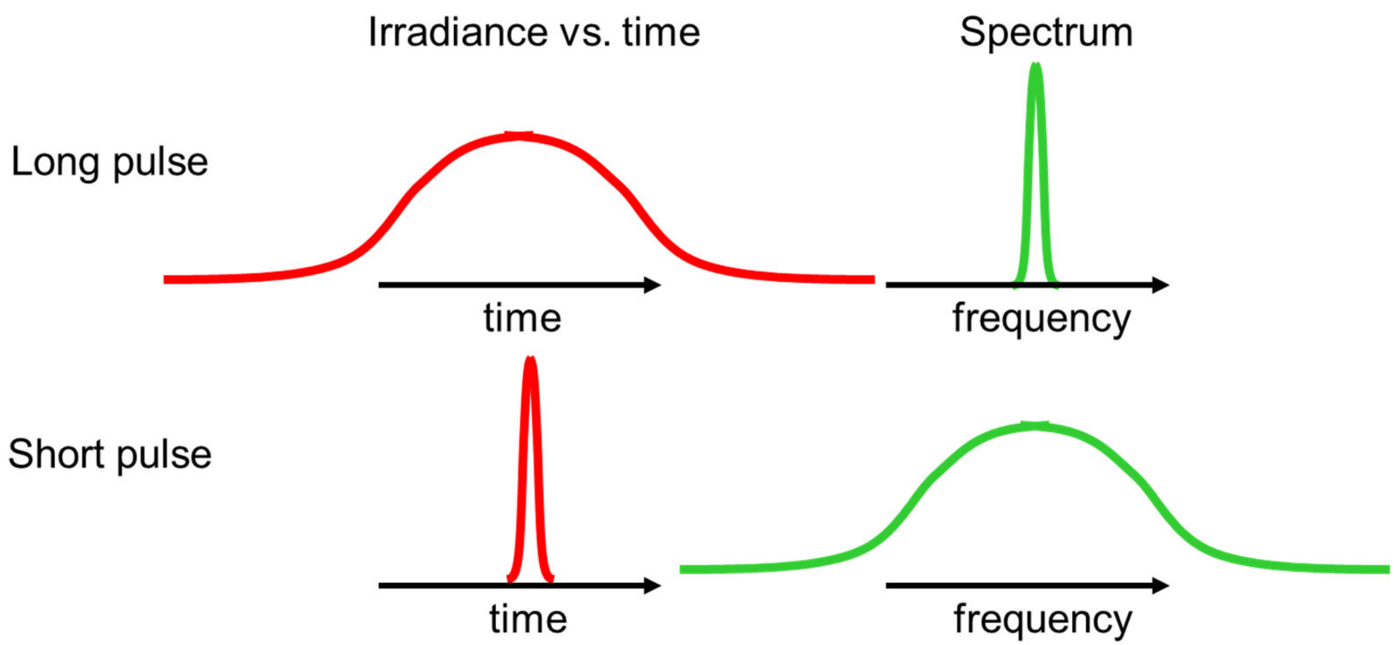


Mikio Yamashita

Ferenc Krausz

The spectrum of a **single** pulse

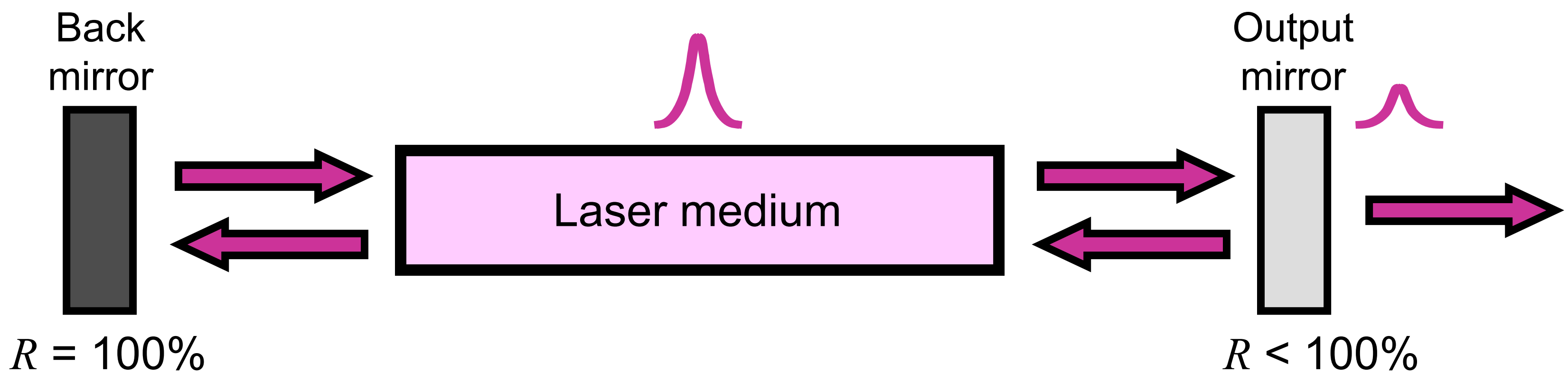
The uncertainty principle says that the product of the temporal and spectral pulse widths is greater than ~1. So a short pulse has a broad bandwidth.



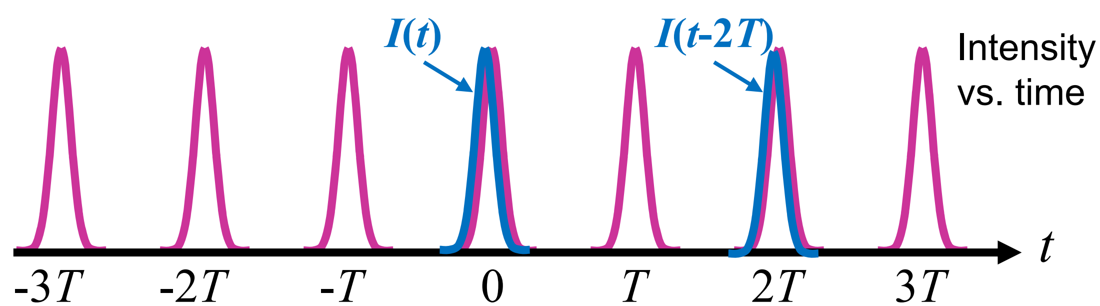
But femtosecond lasers do not emit just one single pulse…

Femtosecond lasers emit **trains** of (nominally) identical pulses.

Every time the laser pulse hits the output mirror, some of it emerges.



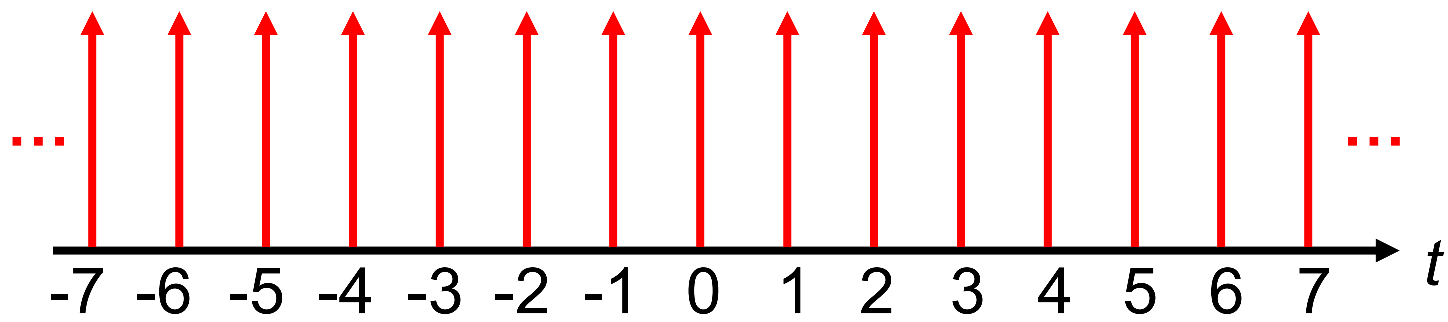
The output of a typical ultrafast laser is a train of identical very short pulses:



where $I(t)$ represents a single pulse intensity vs. time and T is the time between pulses.

To describe this, we use the Shah Function

The Shah function, $\text{III}(t)$, is an infinitely long train of equally spaced delta-functions.



$$\boxed{\text{III}(t) \equiv \sum_{m=-\infty}^{\infty} \delta(t-m)}$$

The symbol III is pronounced *shah* after the Cyrillic character Ш, which is said to have been modeled on the Hebrew letter ש (shin) which, in turn, may derive from the Egyptian , a hieroglyph depicting papyrus plants along the Nile.

The Fourier Transform of the Shah Function

$$\mathscr{F}\{\mathrm{III}(t)\} =$$

$$= \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta(t-m)\exp(-i\omega t)\,dt$$

$$= \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(t-m)\exp(-i\omega t)\,dt$$

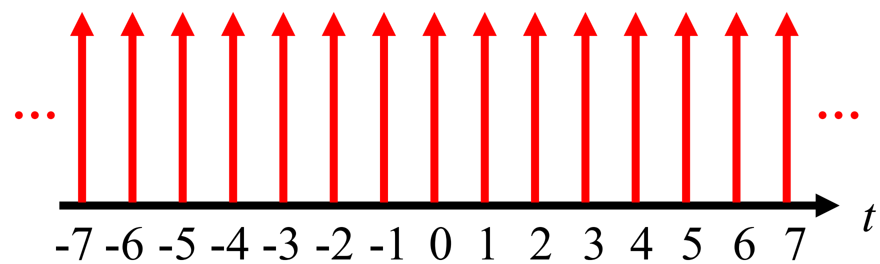
$$= \sum_{m=-\infty}^{\infty} \exp(-i\omega m)$$

If $\omega = 2n\pi$, where n is an integer, the sum diverges; otherwise, cancellation occurs and the sum vanishes.

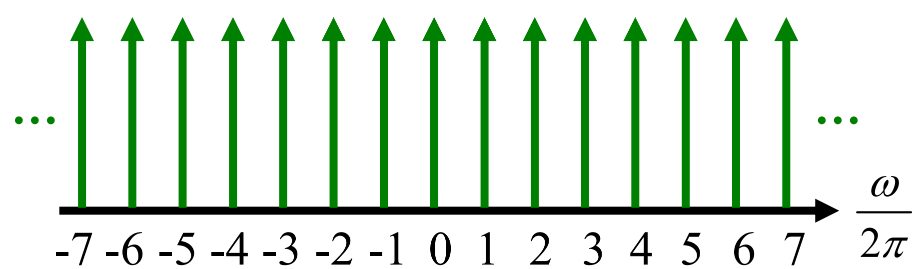
So:

$$\boxed{\mathscr{F}\{\mathrm{III}(t)\} \propto \mathrm{III}(\omega / 2\pi)}$$

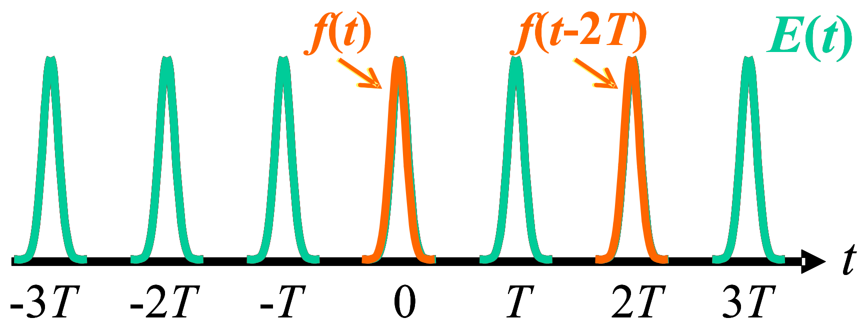
III(*t*)



F{III(*t*)}



The Shah Function and a Pulse Train



An infinite train of identical pulses can be written:

$$E(t) = \sum_{m=-\infty}^{\infty} f(t - mT)$$

where $f(t)$ is the shape of each pulse and T is the time between pulses.

But $E(t)$ can also can be written:

$$E(t) = \text{III}(t/T) \underbrace{*}_{\text{convolution}} f(t)$$

Proof:

$$\text{III}(t/T) * f(t) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(t'/T - m)\, f(t - t')\, dt'$$

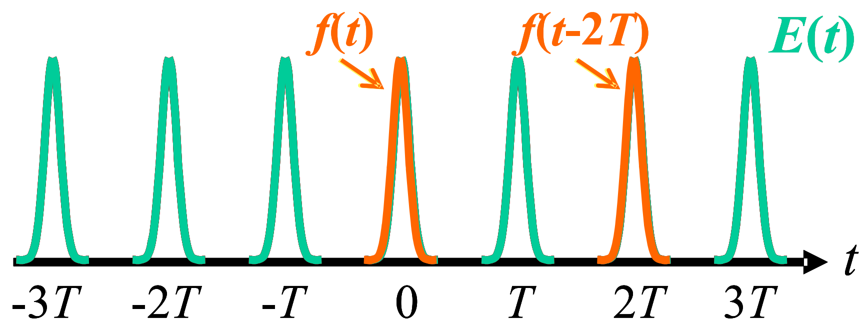
To do the integral, set:
$t'/T = m$ or $t' = mT$

$$= \sum_{m=-\infty}^{\infty} f(t - mT)$$

The Fourier Transform of an Infinite Train of Pulses

An infinite train of identical pulses can be written:

$$E(t) = \text{III}(t/T) * f(t)$$

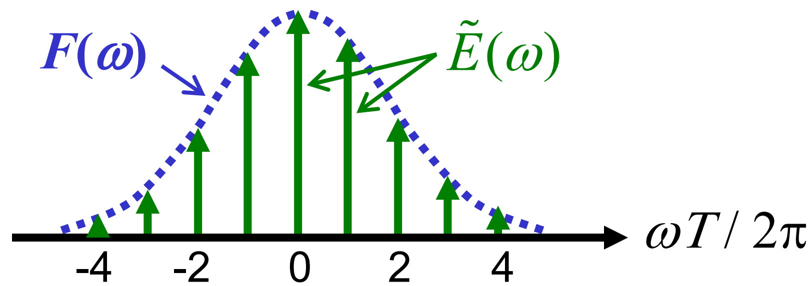


The Convolution Theorem says that the Fourier Transform of a convolution is the **product** of the Fourier Transforms. So the spectrum of the infinite train of pulses is given by:

$$S(\omega) = \left|\tilde{E}(\omega)\right|^2$$

where:

$$\tilde{E}(\omega) \propto \text{III}(\omega T / 2\pi)\, F(\omega)$$



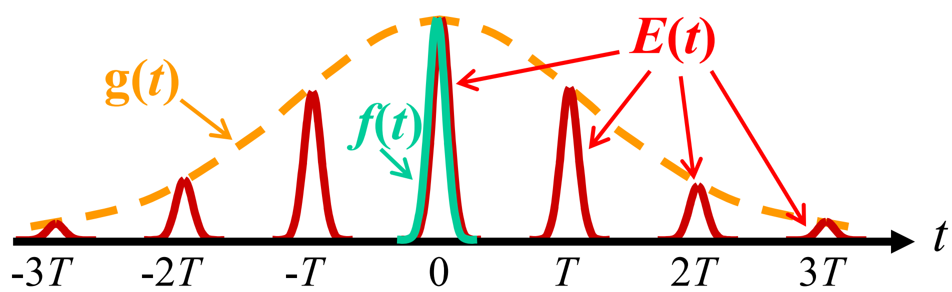
The spacing between frequencies (**modes**) is then $\delta\omega = 2\pi/T$ or $\boldsymbol{\delta\nu = 1/T}$.

The Fourier Transform of a Finite Pulse Train

A ***finite*** train of identical pulses can be written:

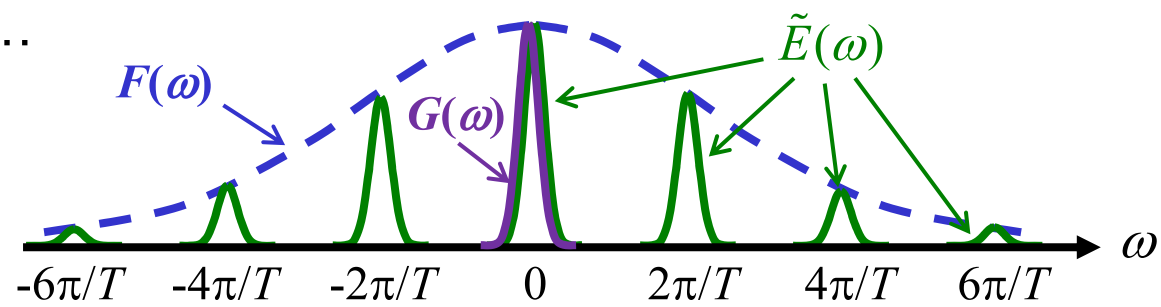
$$E(t) = [\text{III}(t/T)\, g(t)] * f(t)$$

where $g(t)$ is a finite-width envelope over the pulse train.

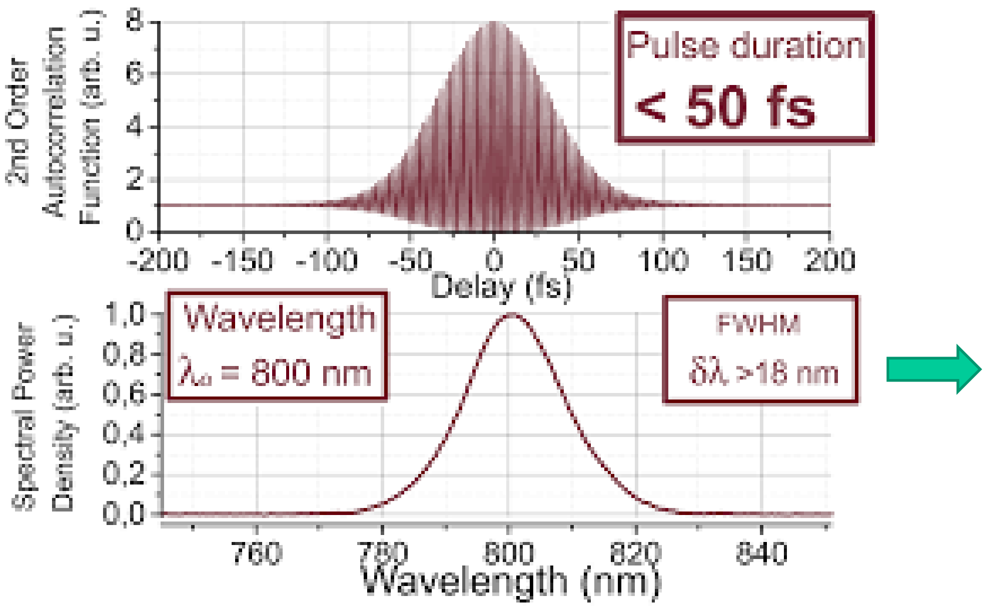


Use the fact that the Fourier transform of a product is a convolution…

$$\tilde{E}(\omega) \propto [\text{III}(\omega T/2\pi) * G(\omega)]\, F(\omega)$$



The spectrum of a short pulse



So where are the modes?

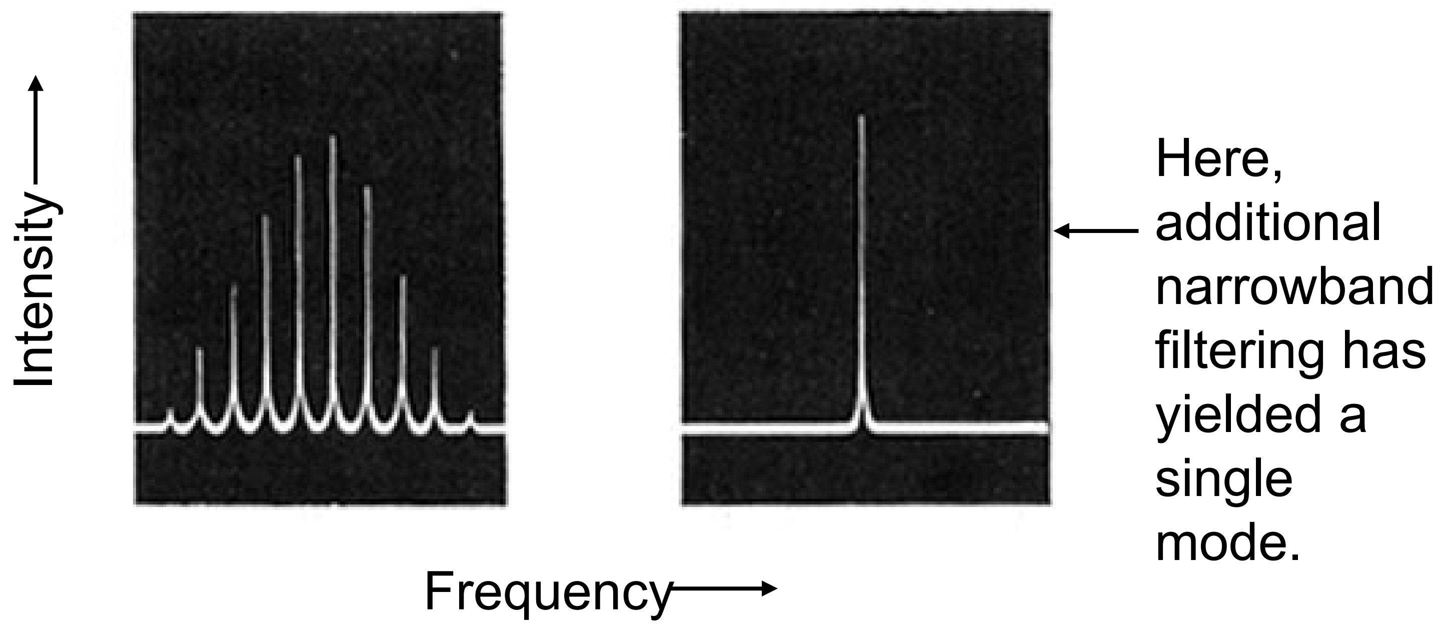
They are there. They are just too close together to resolve with most spectrometers.

Actual Laser Modes

A laser's frequencies are called **longitudinal modes**.

They're separated by $1/T = c/2L$, where L is the length of the laser.

Which modes lase depends on the gain and loss profiles.



Light bulbs, lasers, and ultrashort pulses

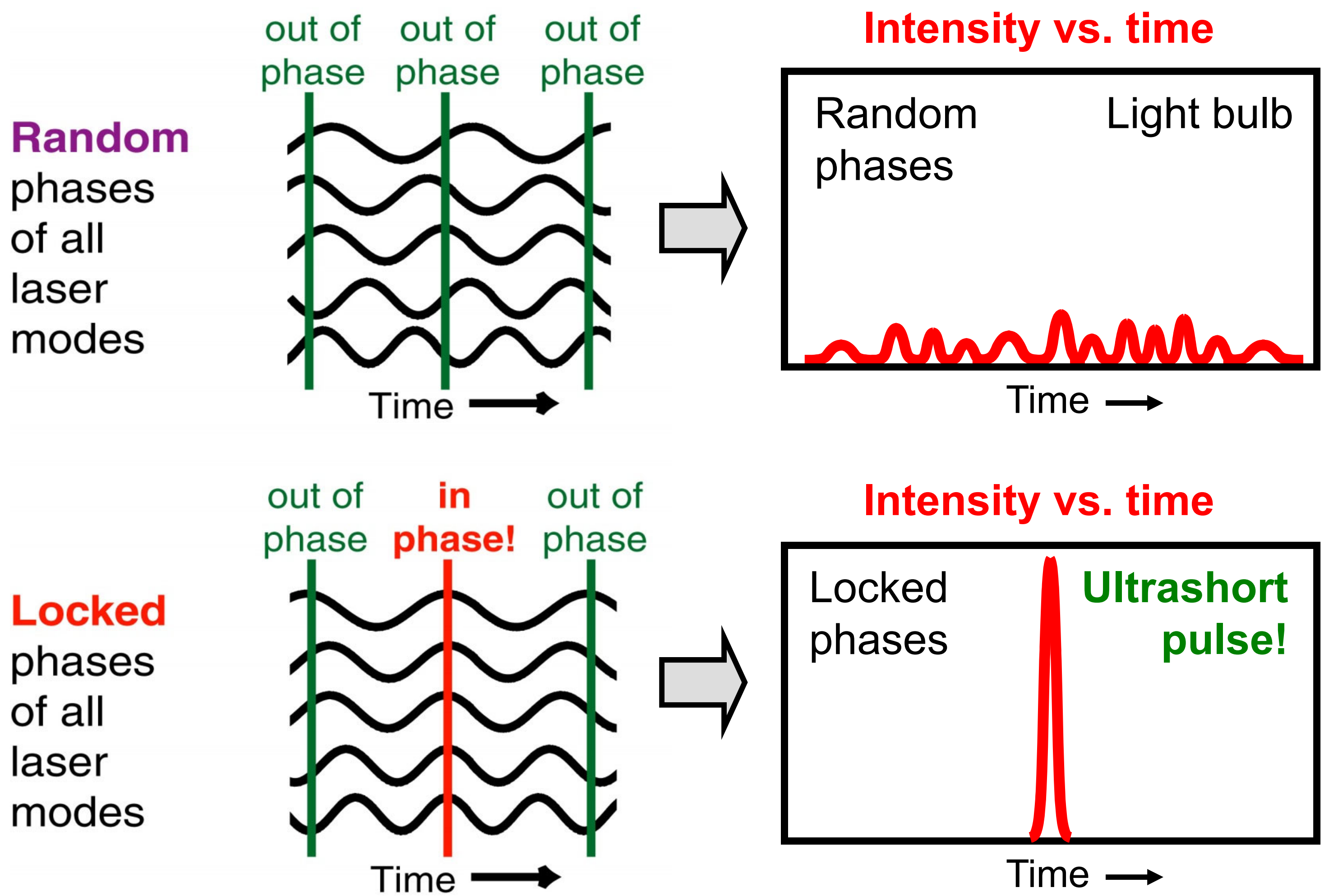
But a light bulb is also broadband.

What exactly is required to make an ultrashort pulse?

Answer:
BOTH a laser with a very broadband output (many modes), **and** a way to make all the modes have the same phase as each other.

Generating short pulses = Mode-locking

Locking vs. not locking the phases of the laser modes (frequencies)



Mode-locked vs. non-mode-locked light

Mode-locked pulse train:

$$\tilde{E}(\omega) = F(\omega)\,\mathrm{III}(\omega T / 2\pi)$$

$$= F(\omega) \sum_{m=-\infty}^{\infty} \delta(\omega - 2\pi m / T)$$

A train of short pulses

Non-mode-locked pulse train:

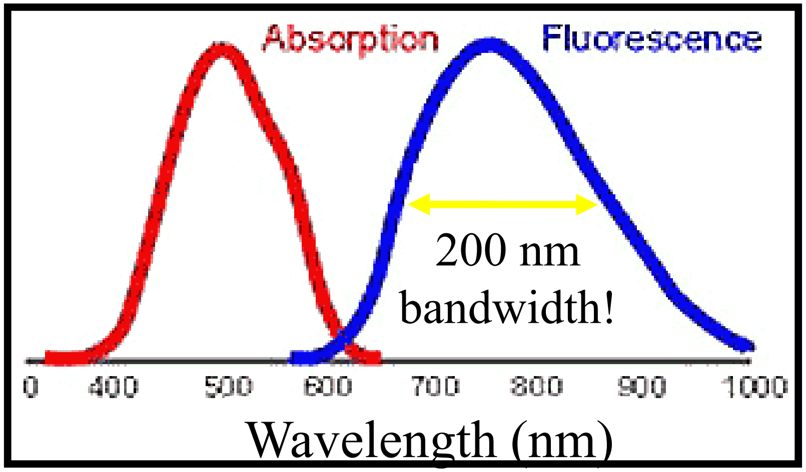
Random phase for each mode

$$\tilde{E}(\omega) = \sum_{m=-\infty}^{\infty} F(\omega) \exp(i\varphi_m)\, \delta(\omega - 2\pi m / T)$$

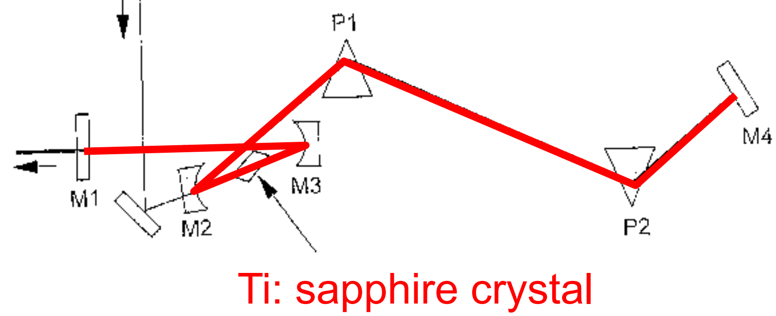
$$= F(\omega) \sum_{m=-\infty}^{\infty} \exp(i\varphi_m)\, \delta(\omega - 2\pi m / T)$$

A mess…

Ti:sapphire: how many modes lock?



Typical cavity layout:
length of path from
M1 to M4 = 1.5 m



Therefore mode spacing is $\Delta\nu = c/L_{rt} \approx$ 100 MHz

Q: How many different modes can oscillate simultaneously in a 1.5 meter Ti:sapphire laser?

A: Gain bandwidth $\Delta\lambda = 200$ nm $\Rightarrow$ $\Delta\nu = (c/\lambda^2)\,\Delta\lambda \sim 10^{14}$ Hz

$\Delta\nu_{bandwidth}/\Delta\nu_{mode} = 10^6$ modes

That seems like a lot. Can this really happen?

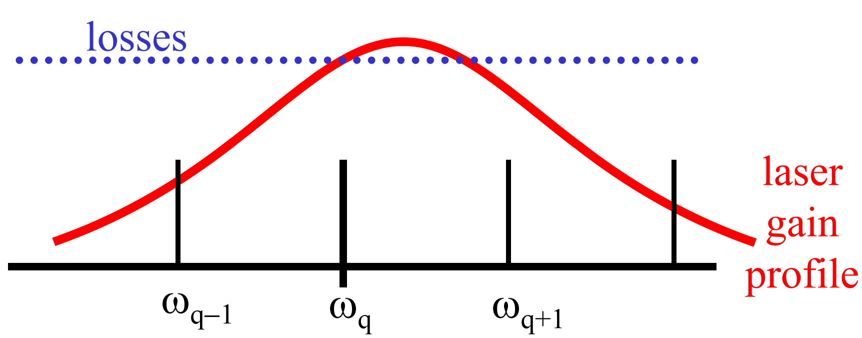
Homogeneous gain medium

We have seen that the gain is given by:

$$g(\omega) = \boxed{\frac{1}{2} \cdot \frac{\Delta N_0}{1 + I / I_{sat}}} \cdot \frac{\sigma_0}{1+\zeta^2} \quad \text{where} \quad \zeta = \frac{2(\omega - \omega_0)}{\Gamma}$$

independent of ω

Suppose the gain is increased to a point where it equals the loss at a particular frequency, ω_q which is one of the cavity mode frequencies.



Q: Suppose the gain is increased further. Can it be increased so that the mode at ω_{q+1} oscillates in steady state?

A: In an ideal laser, NO!
At ω_q, Gain = Loss!

Inhomogeneous gain medium

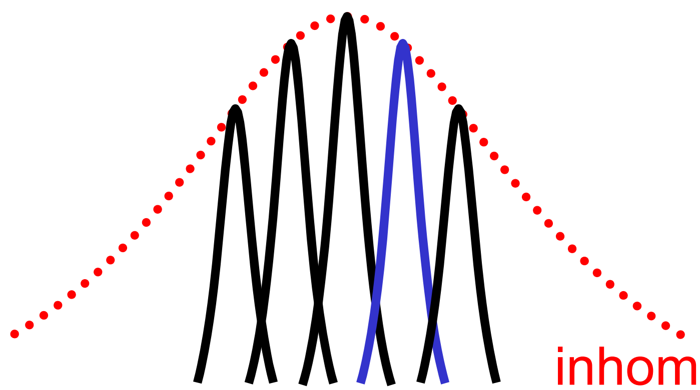
Suppose that the collection of 4-level systems do *not* all share the same ω_0

Consider a collection of sites, with fractional number between ω_0 and $\omega_0 + d\omega_0$:

$$dN(\omega_0) = Ng(\omega_0)d\omega_0$$

The modified susceptibility is:

$$\chi(\omega) = \int d\omega_0 \chi_h(\omega;\omega_0) g(\omega_0)$$



The "packets" are mutually independent
- they can saturate independently!

Inhomogeneous broadening

Choosing a particular form for $g(\omega_0)$: a Gaussian distribution:

Lorentzian homogeneous line shape

Gaussian inhomogeneous distribution, of width $\Delta\omega$

$$\chi(\omega) = \int d\omega_0 \boxed{\chi_h(\omega;\omega_0)} \cdot \boxed{e^{-4\ln(2)\left(\frac{\omega_0-\bar{\omega}_0}{\Delta\omega}\right)^2}}$$

No closed-form solution to this integral

For strong inhomogeneous broadening $(\Delta\omega >> \gamma)$:

$\chi''(\omega)$ = Gaussian, with width = $\Delta\omega$ (NOT γ)
$\chi'(\omega)$: no simple form, but it resembles $\chi_h'(\omega)$

Examples:

Nd:YAG - weak inhomogeneity
Nd:glass - strong inhomogeneity
Ti:sapphire - absurdly strong inhomogeneity

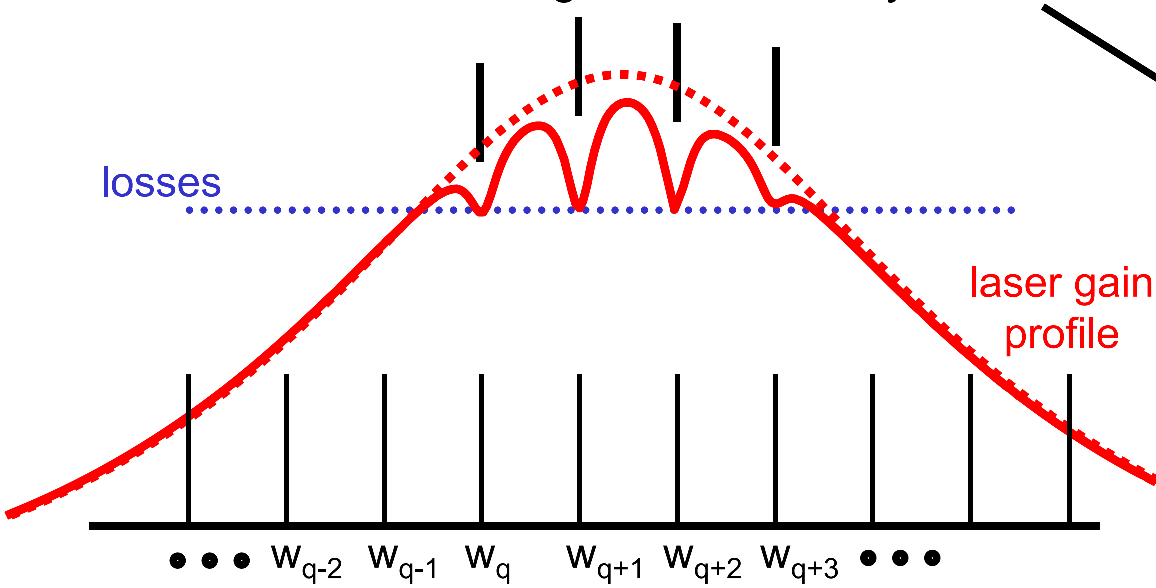
Spectral hole burning

Q: Suppose the gain is increased above threshold in an inhomogeneously broadened laser. Can it be increased so that the mode at ω_{q+1} oscillates in steady state?

New answer: Yes! Each homogeneous packet saturates independently

multiple cavity modes oscillating simultaneously

There are ‘holes’ burned in the gain spectrum.



a priori, these modes are completely independent, and need not have any particular phase relationship to one another