The Generation of Ultrashort Laser Pulses II

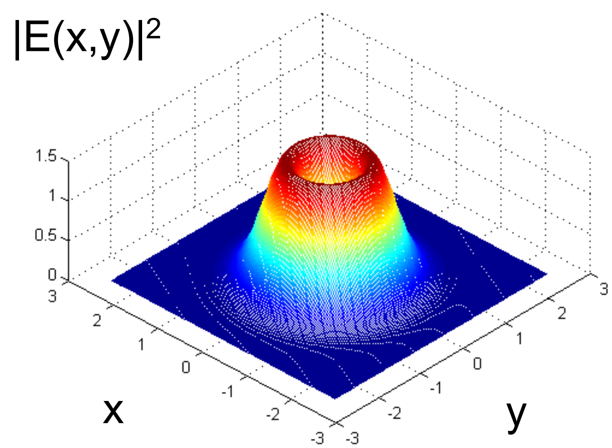
- The phase condition
- Trains of pulses – the Shah function
- Laser modes and mode locking
- Homogeneous vs. inhomogeneous gain media
- Spatial modes

There are 3 conditions for steady-state laser operation.

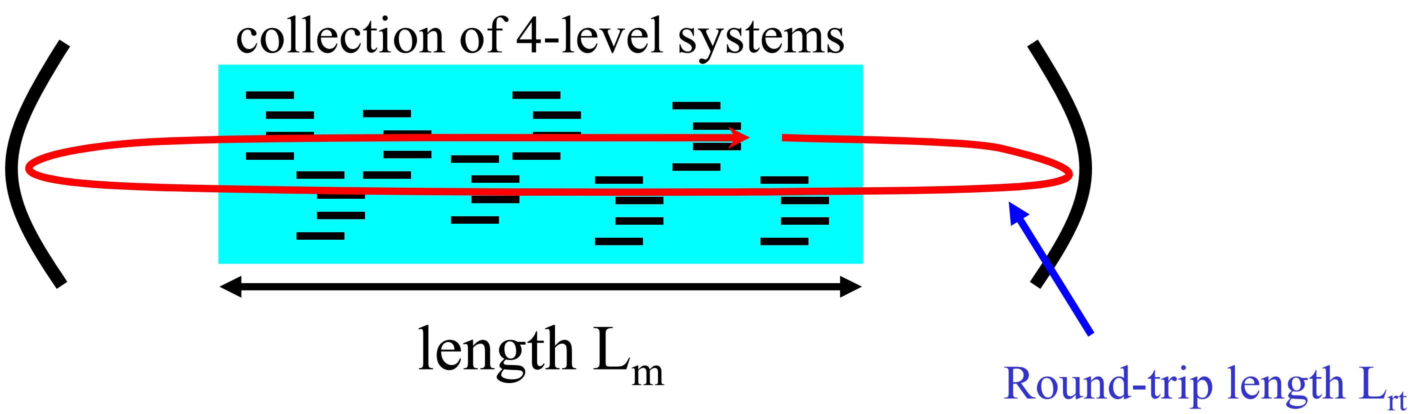
Amplitude condition
- threshold
- slope efficiency

Phase condition
- axial modes
- homogeneous vs. inhomogeneous gain media

Transverse modes
- Hermite Gaussians
- the “donut” mode



Phase condition



Steady-state condition #2:

Phase is invariant after each round trip

$$\frac{angle(E_{after})}{angle(E_{before})} = \exp\left(-i\omega L_{rt} / c\right) = 1 \quad \Rightarrow \quad \omega_q = \frac{2\pi c}{L_{rt}} \cdot q \quad \text{(q = an integer)}$$

Technically, this should be:

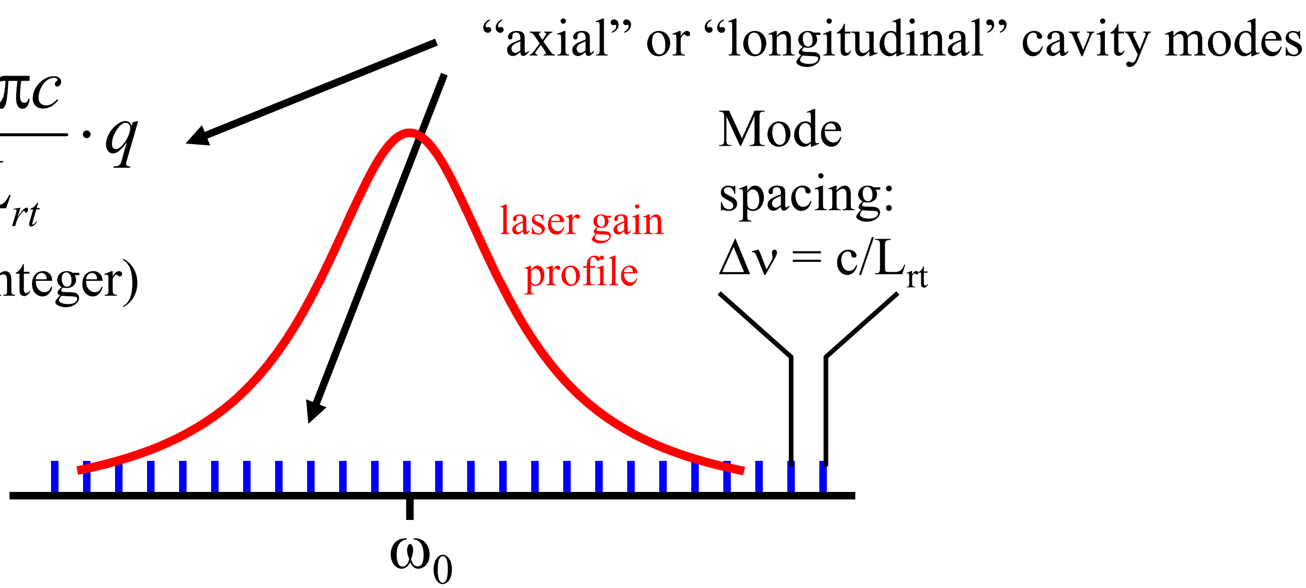
$$\left(L_{rt} - L_m\right) n_{air} + L_m n_{sapphire}$$

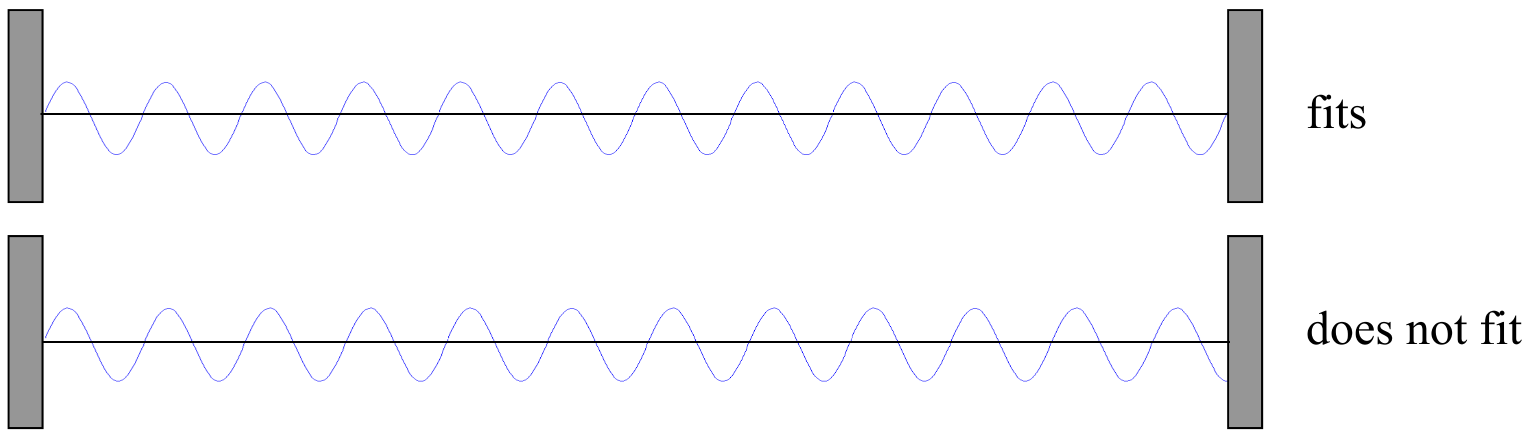
An integer number of wavelengths must fit in the cavity.

Longitudinal modes

$$\omega_q = \frac{2\pi c}{L_{rt}} \cdot q$$

(q = an integer)

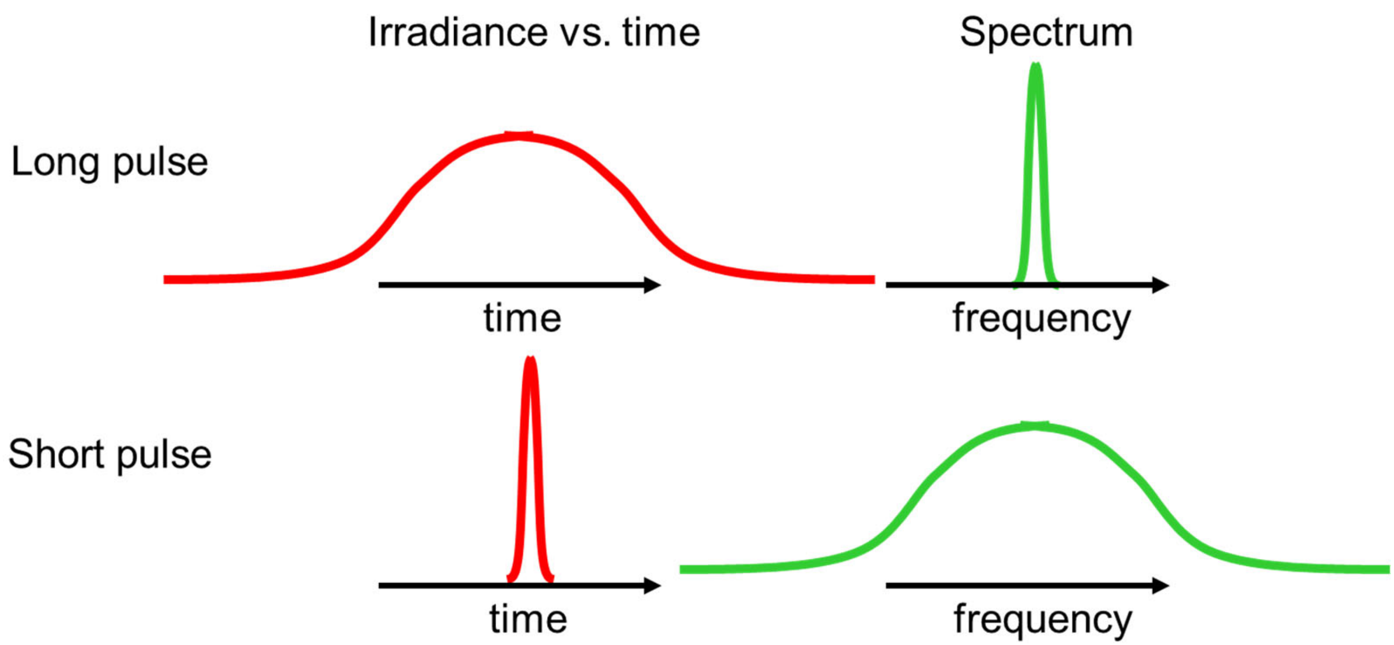




But how does this translate to the case of short pulses?

The spectrum of a **single** pulse

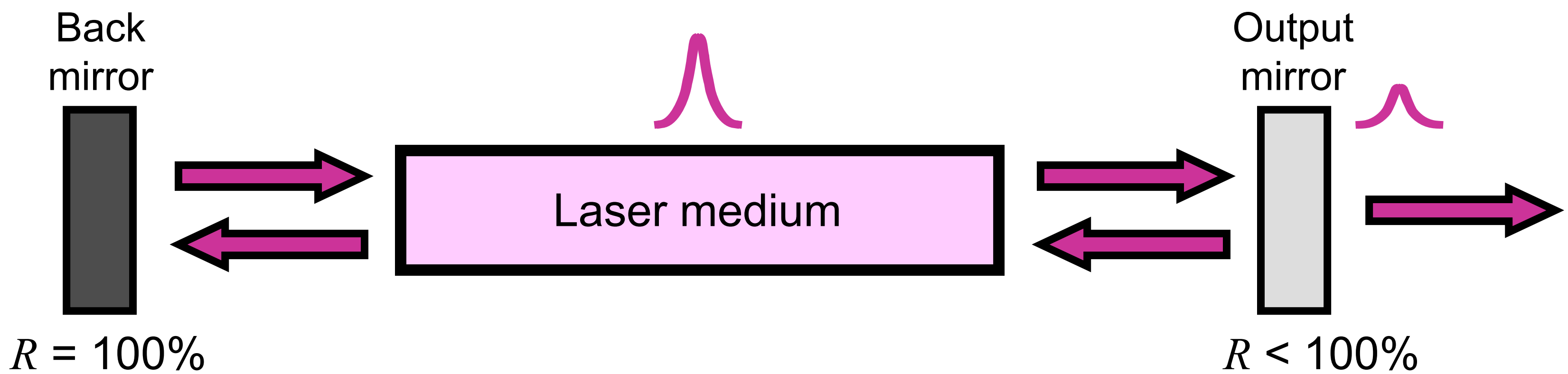
The uncertainty principle says that the product of the temporal and spectral pulse widths is greater than ~1. So a short pulse has a broad bandwidth.



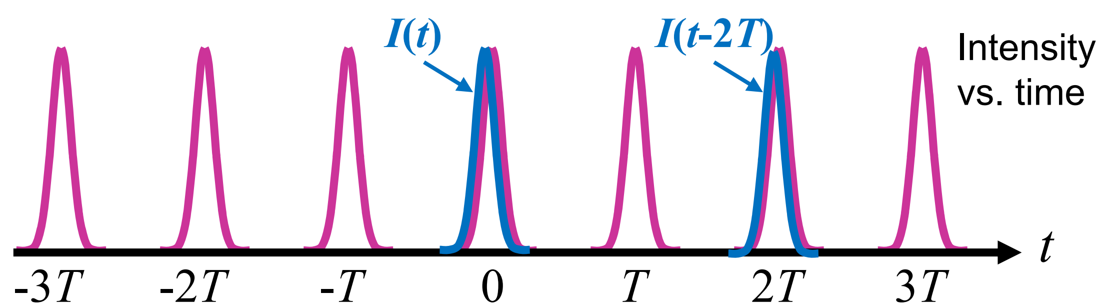
But femtosecond lasers do not emit just one single pulse…

Femtosecond lasers emit **trains** of (nominally) identical pulses.

Every time the laser pulse hits the output mirror, some of it emerges.



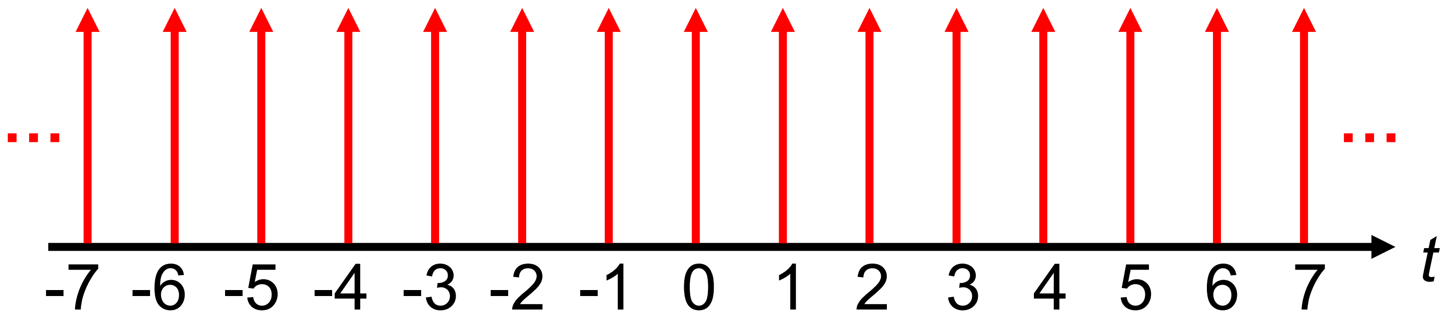
The output of a typical ultrafast laser is a train of identical very short pulses:



where $I(t)$ represents a single pulse intensity vs. time and T is the time between pulses.

The Shah Function

The Shah function, $\text{III}(t)$, is an infinitely long train of equally spaced delta-functions.



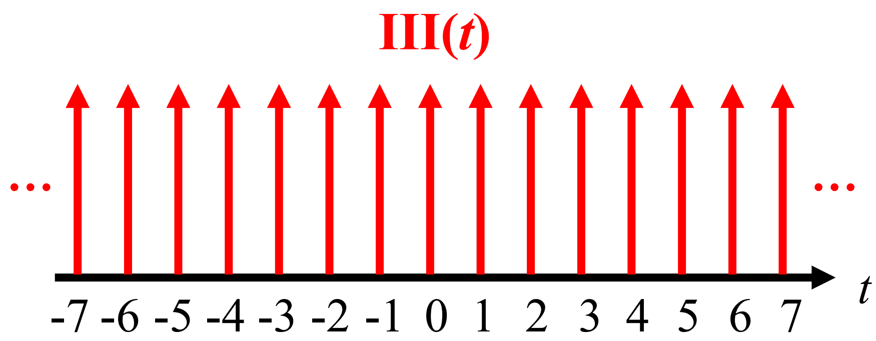
$$\text{III}(t) \equiv \sum_{m=-\infty}^{\infty} \delta(t-m)$$

The symbol III is pronounced *shah* after the Cyrillic character Ш, which is said to have been modeled on the Hebrew letter ש (shin) which, in turn, may derive from the Egyptian , a hieroglyph depicting papyrus plants along the Nile.

The Fourier Transform of the Shah Function

$$\mathscr{F}\{\text{III}(t)\} =$$

$$= \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta(t-m)\exp(-i\omega t)dt$$



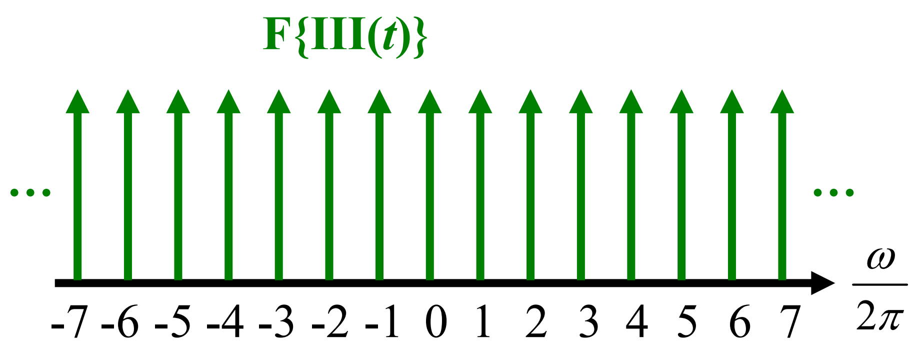
$$= \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(t-m)\exp(-i\omega t)dt$$

$$= \sum_{m=-\infty}^{\infty} \exp(-i\omega m)$$

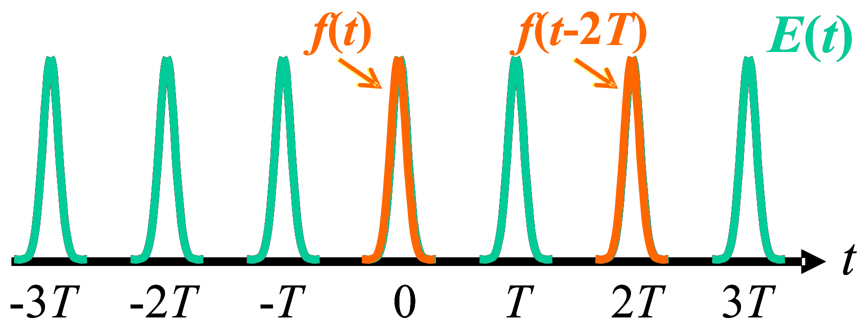
If $\omega = 2n\pi$, where n is an integer, the sum diverges; otherwise, cancellation occurs and the sum vanishes.

So:

$$\mathscr{F}\{\text{III}(t)\} \propto \text{III}(\omega / 2\pi)$$



The Shah Function and a Pulse Train



An infinite train of identical pulses can be written:

$$E(t) = \sum_{m=-\infty}^{\infty} f(t - mT)$$

where $f(t)$ is the shape of each pulse and T is the time between pulses.

But $E(t)$ can also can be written:

$$E(t) = \mathrm{III}(t/T) * f(t)$$

(convolution)

Proof:

$$\mathrm{III}(t/T) * f(t) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(t'/T - m)\, f(t - t')\, dt'$$

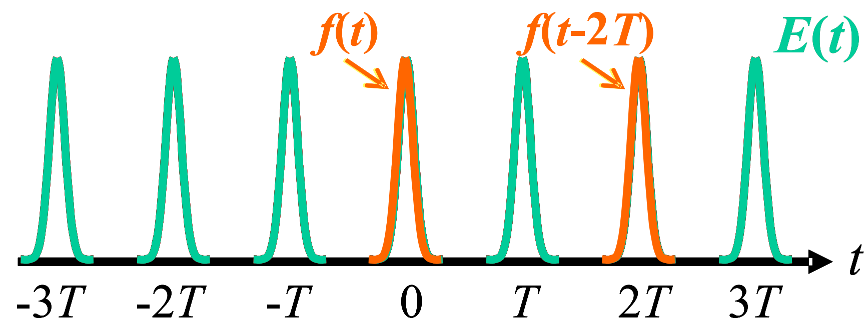
To do the integral, set:
$t'/T = m$ or $t' = mT$

$$= \sum_{m=-\infty}^{\infty} f(t - mT)$$

The Fourier Transform of an Infinite Train of Pulses

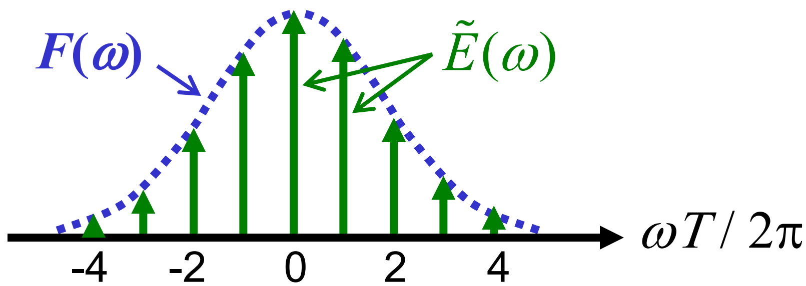
An infinite train of identical pulses can be written:

$$E(t) \;=\; \text{III}(t/T) * f(t)$$



The Convolution Theorem says that the Fourier Transform of a convolution is the **product** of the Fourier Transforms. So:

$$\tilde{E}(\omega) \propto \text{III}(\omega T / 2\pi)\, F(\omega)$$



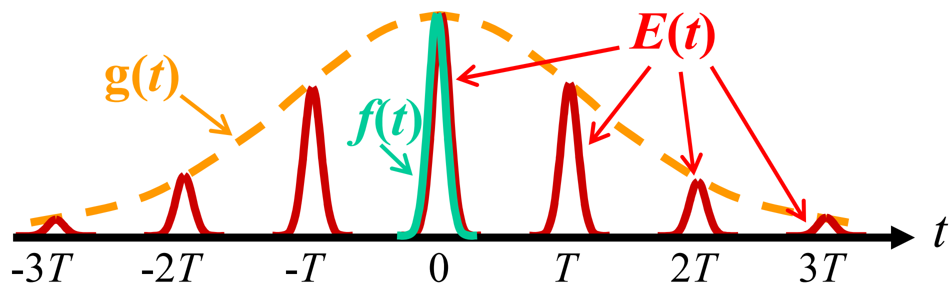
The spacing between frequencies (**modes**) is then $\delta\omega = 2\pi/T$ or $\delta\nu = 1/T$.

The Fourier Transform of a Finite Pulse Train

A ***finite*** train of identical pulses can be written:

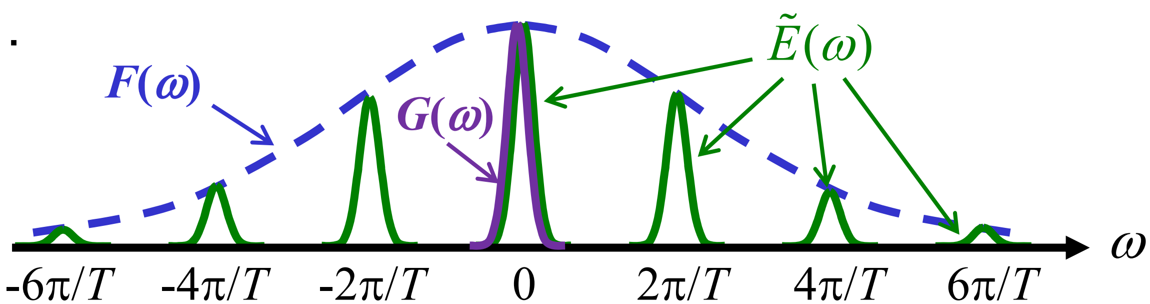
$$E(t) = [\text{III}(t/T)\, g(t)] * f(t)$$

where $g(t)$ is a finite-width envelope over the pulse train.



Use the fact that the Fourier transform of a product is a convolution…

$$\tilde{E}(\omega) \propto [\text{III}(\omega T/2\pi) * G(\omega)]\, F(\omega)$$

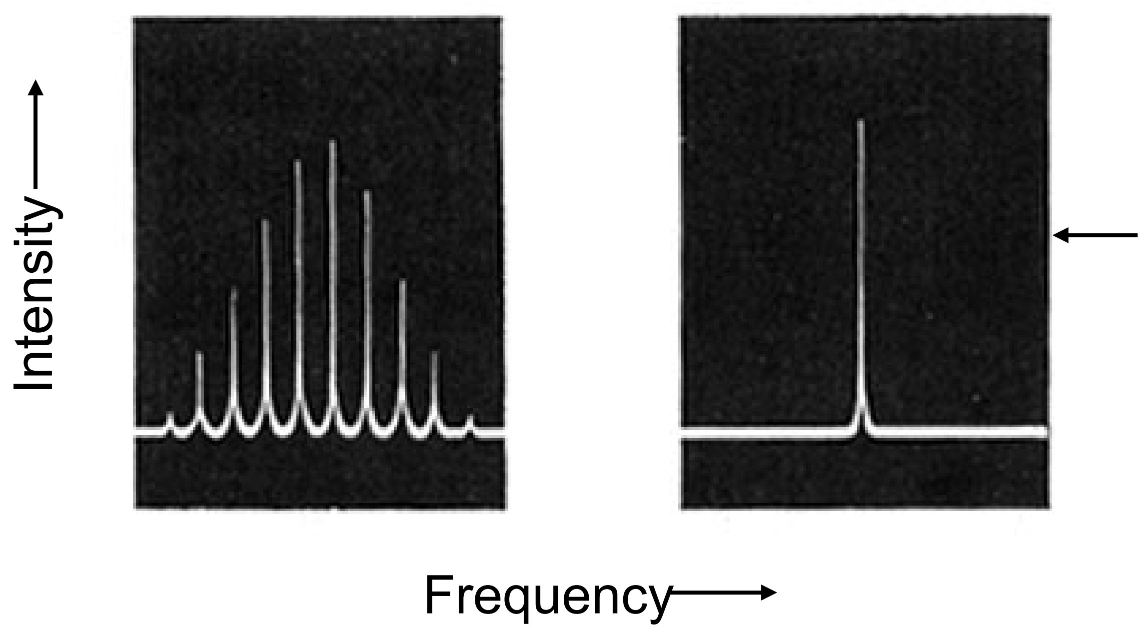


Actual Laser Modes

A laser's frequencies are often called **longitudinal modes**.

They're separated by $1/T = c/2L$, where L is the length of the laser.

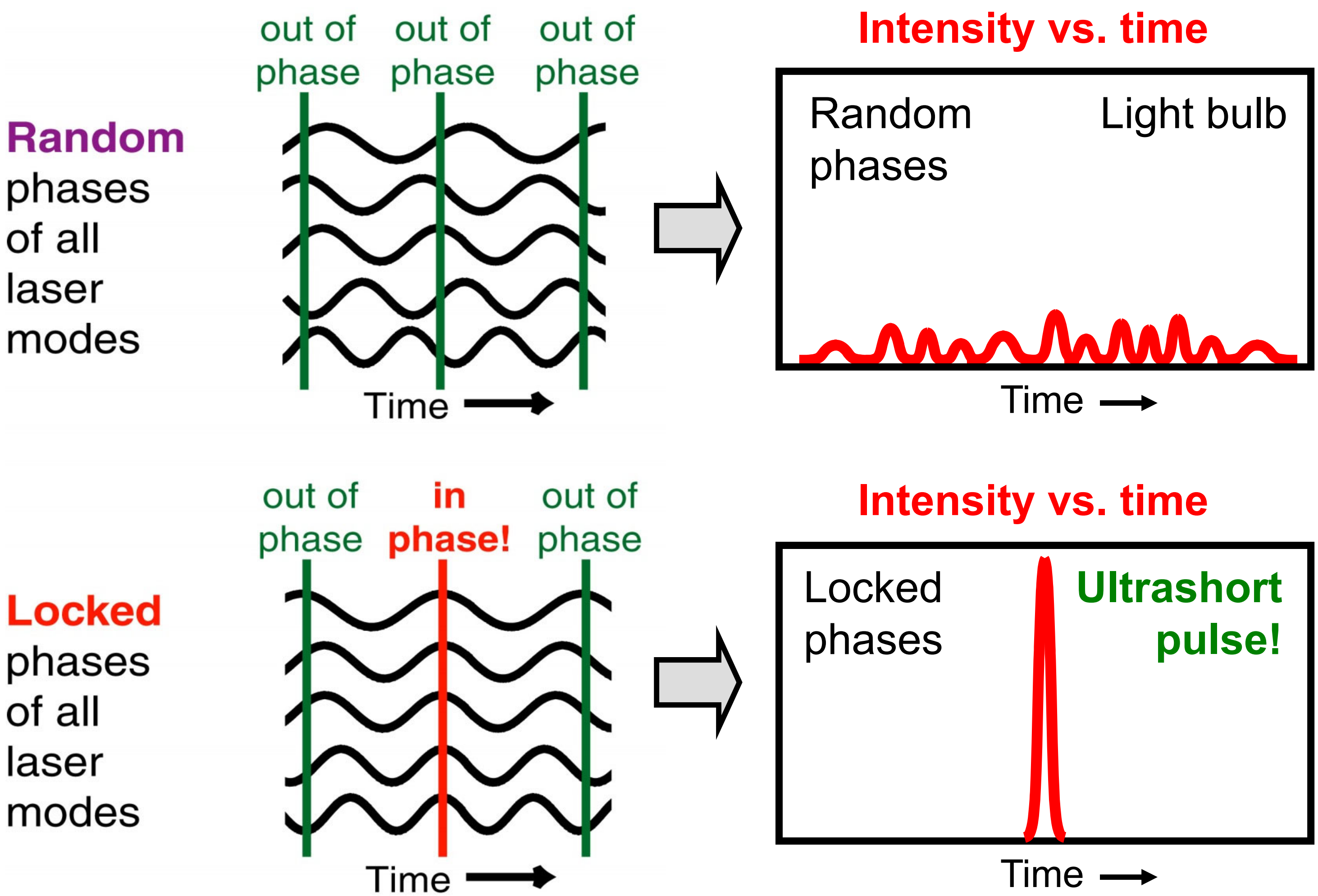
Which modes lase depends on the gain and loss profiles.



Here, additional narrowband filtering has yielded a single mode.

Generating short pulses = Mode-locking

Locking vs. not locking the phases of the laser modes (frequencies)



Mode-locked vs. non-mode-locked light

Mode-locked pulse train:

$$\tilde{E}(\omega) = F(\omega)\,\text{III}(\omega T / 2\pi)$$

$$= F(\omega) \sum_{m=-\infty}^{\infty} \delta(\omega - 2\pi m / T)$$

A train of short pulses

Non-mode-locked pulse train:

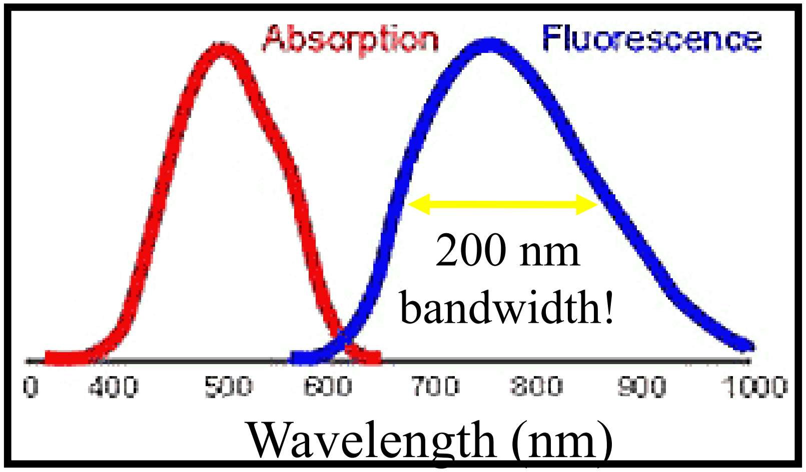
Random phase for each mode

$$\tilde{E}(\omega) = \sum_{m=-\infty}^{\infty} F(\omega) \exp(i\varphi_m)\, \delta(\omega - 2\pi m / T)$$

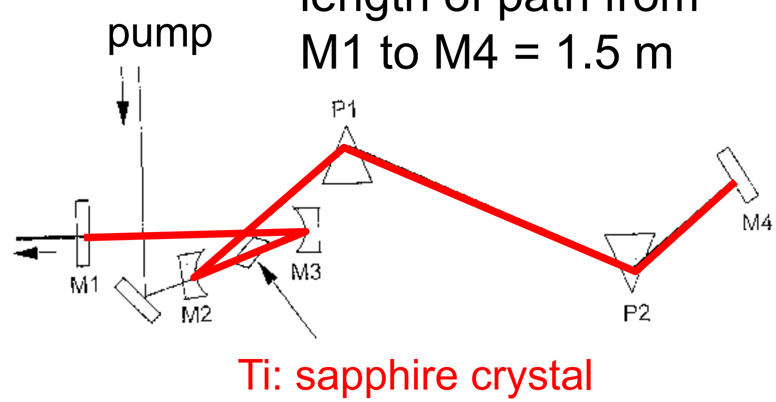
$$= F(\omega) \sum_{m=-\infty}^{\infty} \exp(i\varphi_m)\, \delta(\omega - 2\pi m / T)$$

A mess…

Ti:sapphire: how many modes lock?



Typical cavity layout: length of path from M1 to M4 = 1.5 m



Therefore $\Delta\nu = c/L_{rt} \approx 100$ MHz

Q: How many different modes can oscillate simultaneously in a 1.5 meter Ti:sapphire laser?

A: Gain bandwidth $\Delta\lambda = 200$ nm $\Rightarrow$ $\Delta\nu = (c/\lambda^2)\,\Delta\lambda \sim 10^{14}$ Hz

$\Delta\nu_{bandwidth}/\Delta\nu_{mode} = 10^6$ modes

That seems like a lot. Can this really happen?

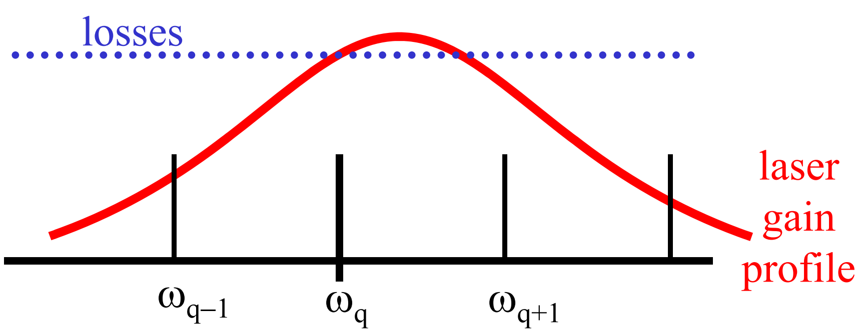
Homogeneous gain media

We have seen that the gain is given by:

$$g(\omega) = \frac{1}{2} \cdot \frac{\Delta N_0}{1 + I / I_{sat}} \cdot \frac{\sigma_0}{1 + \zeta^2} \quad \text{where} \quad \zeta = \frac{2(\omega - \omega_0)}{\gamma}$$

independent of ω

Suppose the gain is increased to a point where it equals the loss at a particular frequency, ω_q which is one of the cavity mode frequencies.



Q: Suppose the gain is increased further. Can it be increased so that the mode at ω_{q+1} oscillates in steady state?

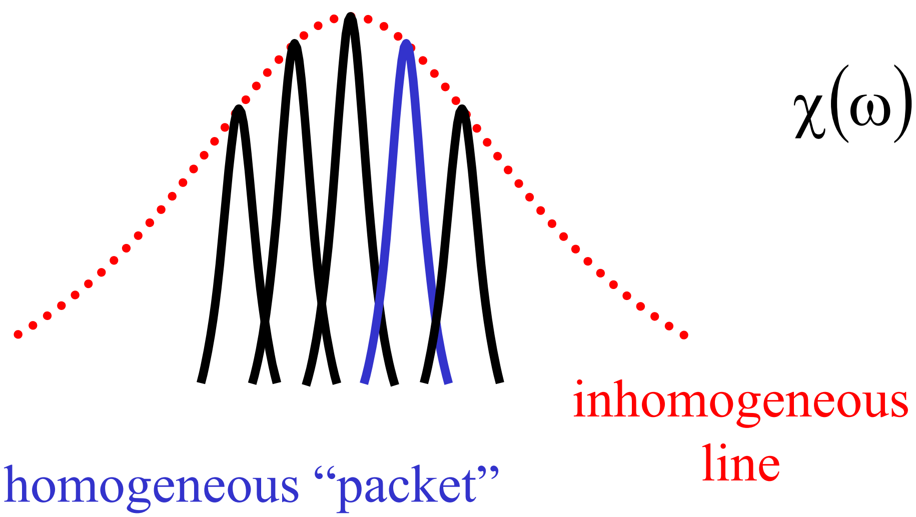
A: In an ideal laser, NO!
At ω_q, Gain = Loss!

Inhomogeneous gain media

Suppose that the collection of 4-level systems do *not* all share the same ω_0

Consider a collection of sites, with fractional number between ω_0 and $\omega_0 + d\omega_0$:

$$dN(\omega_0) = Ng(\omega_0)d\omega_0$$



The modified susceptibility is:

$$\chi(\omega) = \int d\omega_0 \chi_h(\omega; \omega_0) g(\omega_0)$$

"packets" are mutually independent -
they can saturate independently!

Inhomogeneous broadening

Lorentian homogeneous line shape

Gaussian inhomogeneous distribution of width $\Delta\omega$

$$\chi(\omega) = \int d\omega_0 \chi_h(\omega;\omega_0) \cdot e^{-4\ln(2)\left(\frac{\omega_0 - \bar{\omega}_0}{\Delta\omega}\right)^2}$$

No closed-form solution

For strong inhomogeneous broadening ($\Delta\omega >> \gamma$):

$\chi''(\omega)$ = Gaussian, with width = $\Delta\omega$ (NOT γ)
$\chi'(\omega)$: no simple form, but it resembles $\chi_h'(\omega)$

Examples:

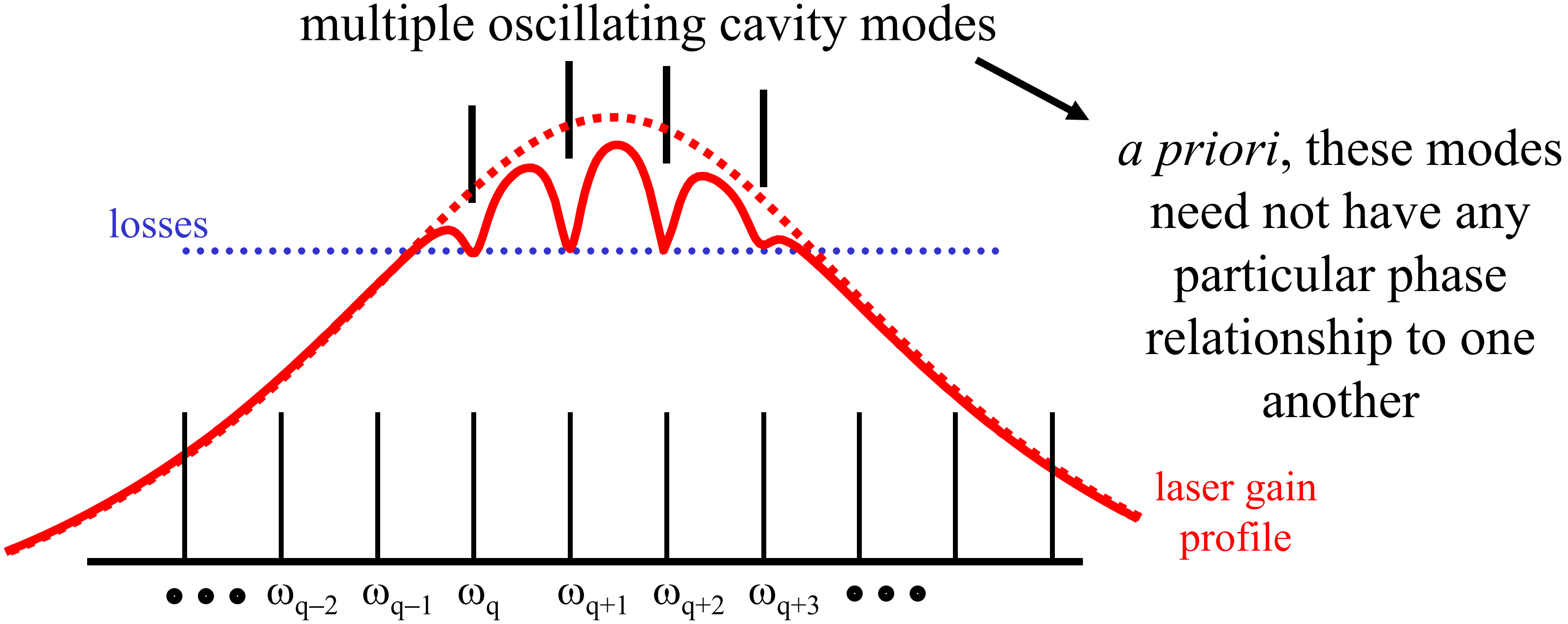
Nd:YAG - weak inhomogeneity
Nd:glass - strong inhomogeneity
Ti:sapphire - absurdly strong inhomogeneity

Hole burning

Q: Suppose the gain is increased above threshold in an inhomogeneously broadened laser. Can it be increased so that the mode at ω_{q+1} oscillates cw?

A: Yes! Each homogeneous packet saturates independently

"spectral hole burning"

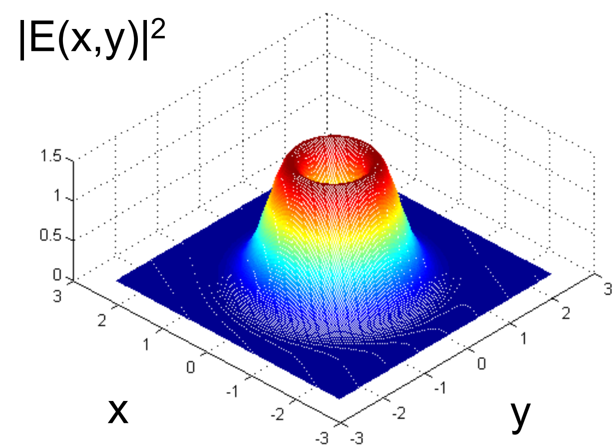


There are 3 conditions for steady-state laser operation.

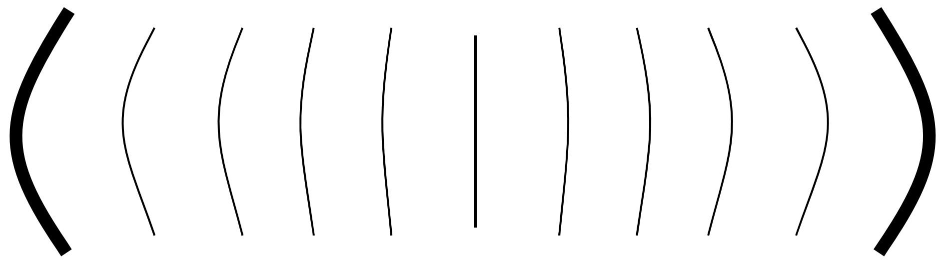
Amplitude condition
- threshold
- slope efficiency

Phase condition
- axial modes
- homogeneous vs. inhomogeneous gain media

Transverse modes
- Hermite Gaussians
- the "donut" mode



Condition on the transverse profile



Steady-state condition #3:

Transverse profile reproduces on each round trip

"transverse modes": those which reproduce themselves on each round trip, except for overall amplitude and phase factors

How would we determine these modes?

An eigenvalue problem:

$$\beta_{nm} \cdot E_{nm}(x,y) = \iint K(x,y;x_0,y_0) \cdot E_{nm}(x_0,y_0)\,dx_0 dy_0$$

propagation kernel

Transverse modes

Solutions are the product of two functions, one for each transverse dimension:

$$E_{nm}(x,y) = u_n(x) \cdot u_m(y)$$

where the u_n's are Hermite Gaussians:

$$\left|u_n(x)\right| = H_n\left(\frac{\sqrt{2}x}{w}\right) \cdot \exp\left(-\frac{x^2}{w^2}\right)$$

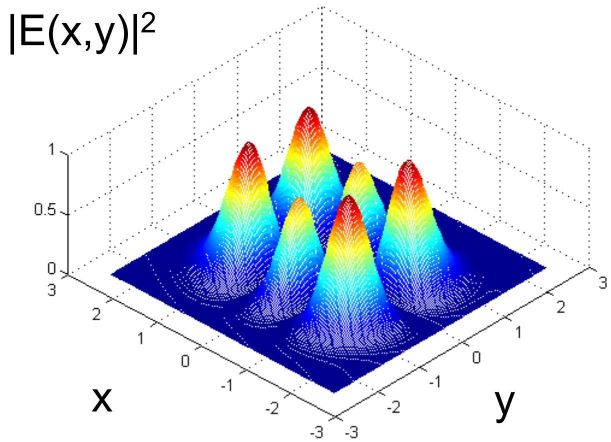
w = beam waist parameter

Notation:

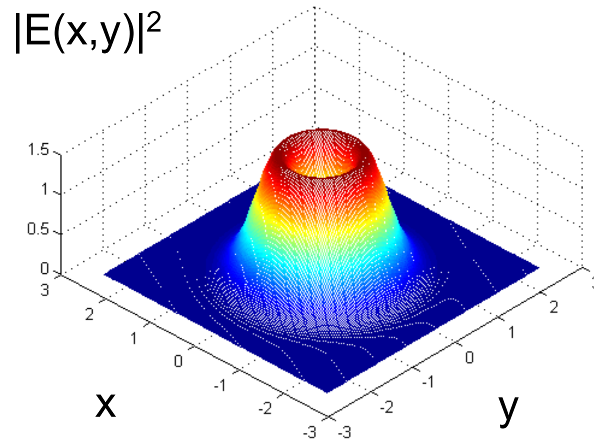
"TEM" = transverse electric and magnetic

"nm" = number of nodes along two principal axes

Transverse modes - examples

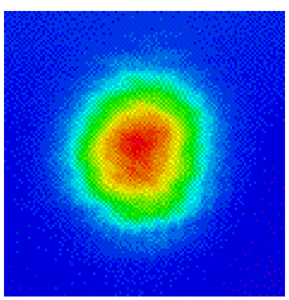


12 mode

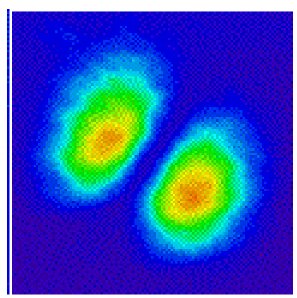


A superposition of the 10 and 01 modes: the “donut mode”

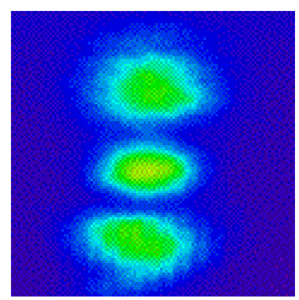
TEM00



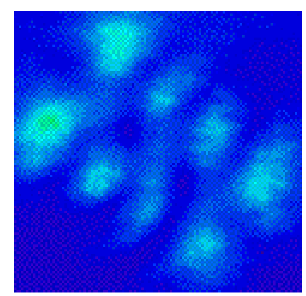
TEM01



TEM02



TEM13



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