Dispersion and Ultrashort Pulses

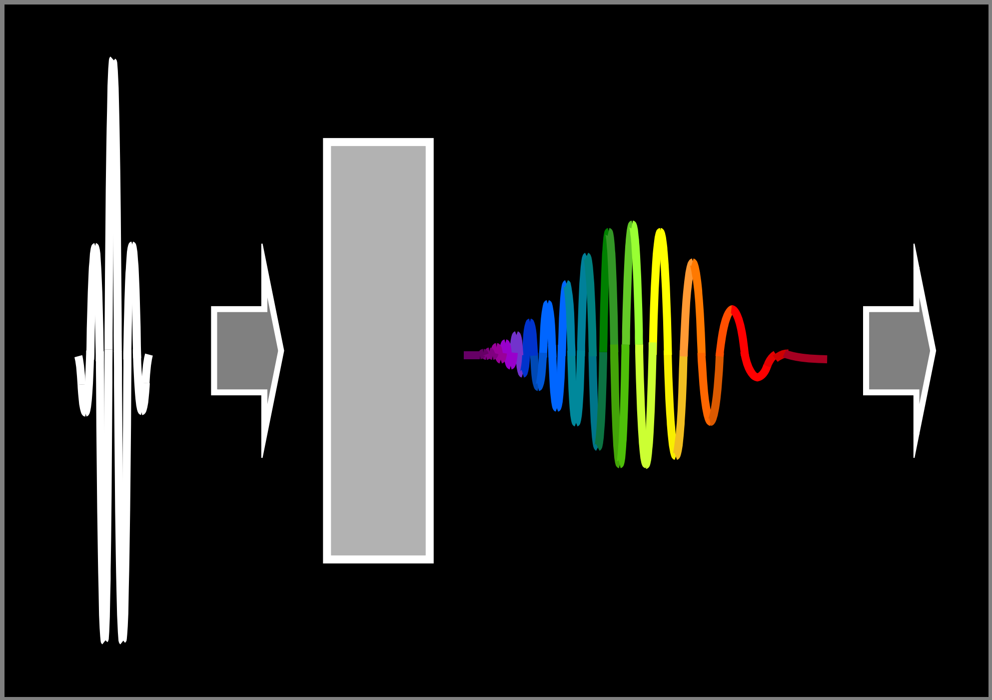
Angular dispersion and group-velocity dispersion

Phase and group velocities

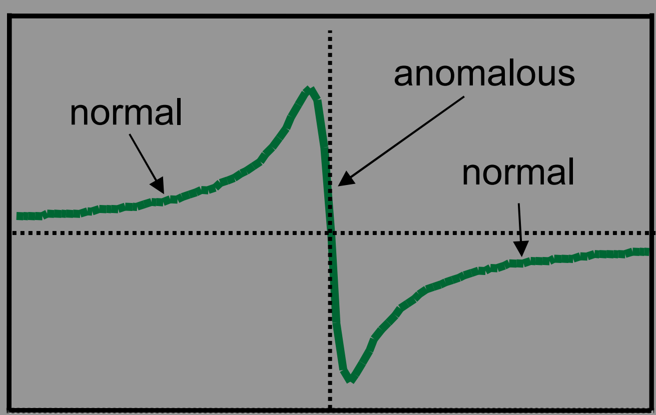
Group-delay dispersion

Pulse propagation

The need for negative GDD



Reminder: what is dispersion

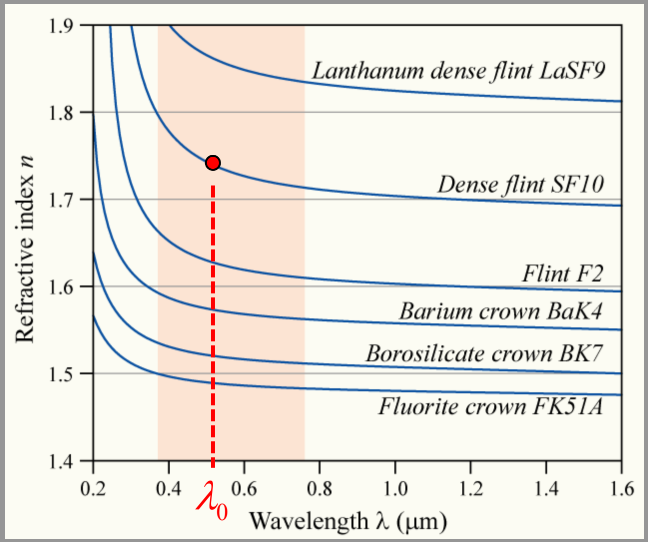


The word "dispersion" refers to the frequency-dependence of the refractive index of a medium.

"normal dispersion" : $\dfrac{dn}{d\omega} > 0$

The refractive index of most materials varies with ω in some nonlinear way.

It has frequency (or wavelength) derivatives that are all generally non-zero.



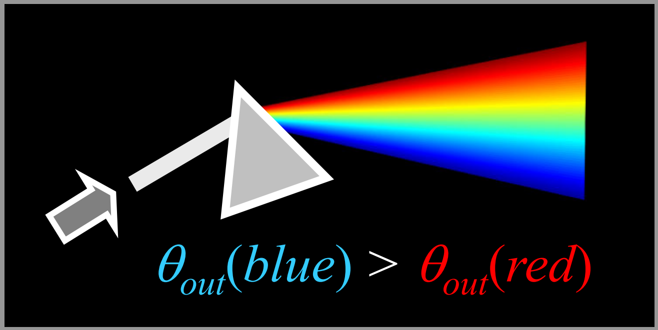
Taylor expansion:

$$n(\lambda) \approx n(\lambda_0) + (\lambda - \lambda_0)\left.\frac{dn}{d\lambda}\right|_{\lambda_0} + \frac{1}{2}(\lambda - \lambda_0)^2 \left.\frac{d^2 n}{d\lambda^2}\right|_{\lambda_0} + \ldots$$

Dispersion in Optics

The dependence of the refractive index on wavelength has two effects on a pulse, one in space and the other in time.

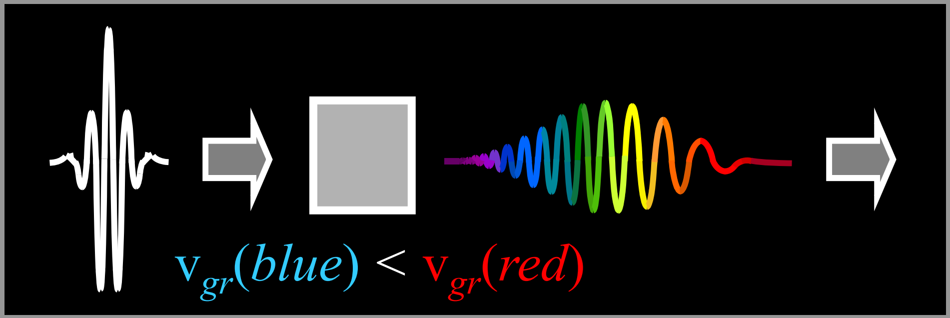
Dispersion can disperse a pulse in space (angle):



Angular dispersion

$dn/d\lambda$

Dispersion also can disperse a pulse in time:



Group delay dispersion (or Chirp)

$d^2n/d\lambda^2$

Both of these effects play major roles in ultrafast optics.

When two functions of different frequency interfere, the result is beats.

$$\mathscr{E}_{tot}(t) = \text{Re}\{E_0 \exp(i\omega_1 t) + E_0 \exp(i\omega_2 t)\}$$

Suppose that ω_1 and ω_2 are large, and not too different from each other (e.g., two different visible frequencies)

Let: $\omega_{ave} = \dfrac{\omega_1 + \omega_2}{2}$ and $\Delta\omega = \dfrac{\omega_1 - \omega_2}{2}$

$$\omega_1 = \omega_{ave} + \Delta\omega$$

$$\omega_2 = \omega_{ave} - \Delta\omega$$

$$\Rightarrow \mathscr{E}_{tot}(t) = \text{Re}\{E_0 \exp i(\omega_{ave} t + \Delta\omega t) + E_0 \exp i(\omega_{ave} t - \Delta\omega t)\}$$

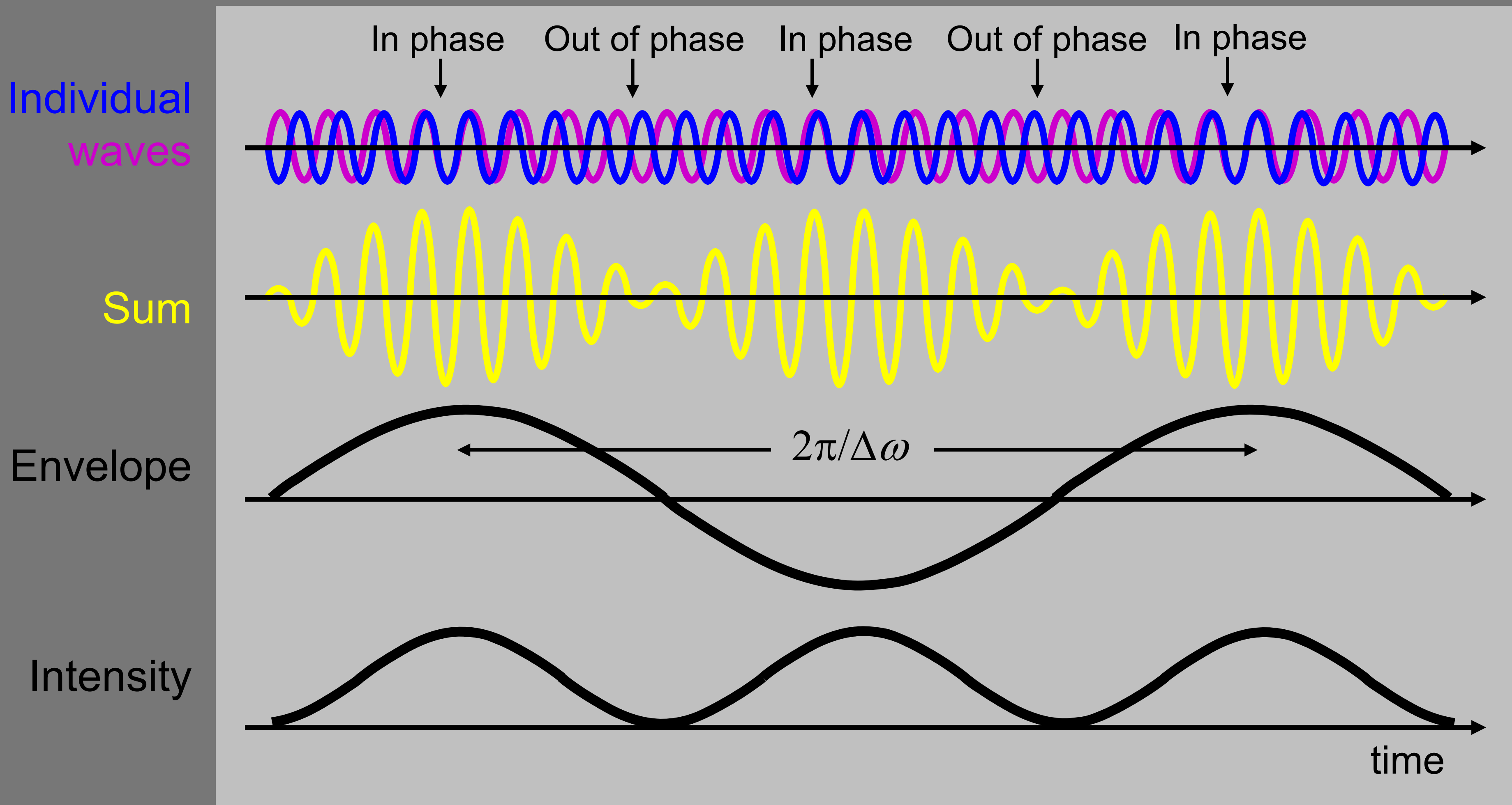
$$= \text{Re}\{E_0 \exp(i\omega_{ave} t)[\exp(i\Delta\omega t) + \exp(-i\Delta\omega t)]\}$$

$$= \text{Re}\{2E_0 \exp(i\omega_{ave} t)\cos(\Delta\omega t)\}$$

$$\Rightarrow \mathscr{E}_{tot}(t) = 2E_0 \cos(\omega_{ave} t)\cos(\Delta\omega t)$$

Adding oscillations of two different frequencies yields the product of a rapidly varying cosine (ω_{ave}) and a slowly varying cosine ($\Delta\omega$).

When two functions of different frequency interfere, the result is beats.



When two waves of different frequency interfere, they also produce beats.

$$\mathscr{E}_{tot}(z,t) = \mathrm{Re}\{E_0 \exp i(k_1 z - \omega_1 t) + E_0 \exp i(k_2 z - \omega_2 t)\}$$

Same assumption about ω_1 and ω_2, and similarly for k_1 and k_2

Let $k_{ave} = \dfrac{k_1 + k_2}{2}$ and $\Delta k = \dfrac{k_1 - k_2}{2}$

Similiarly, $\omega_{ave} = \dfrac{\omega_1 + \omega_2}{2}$ and $\Delta\omega = \dfrac{\omega_1 - \omega_2}{2}$

So:

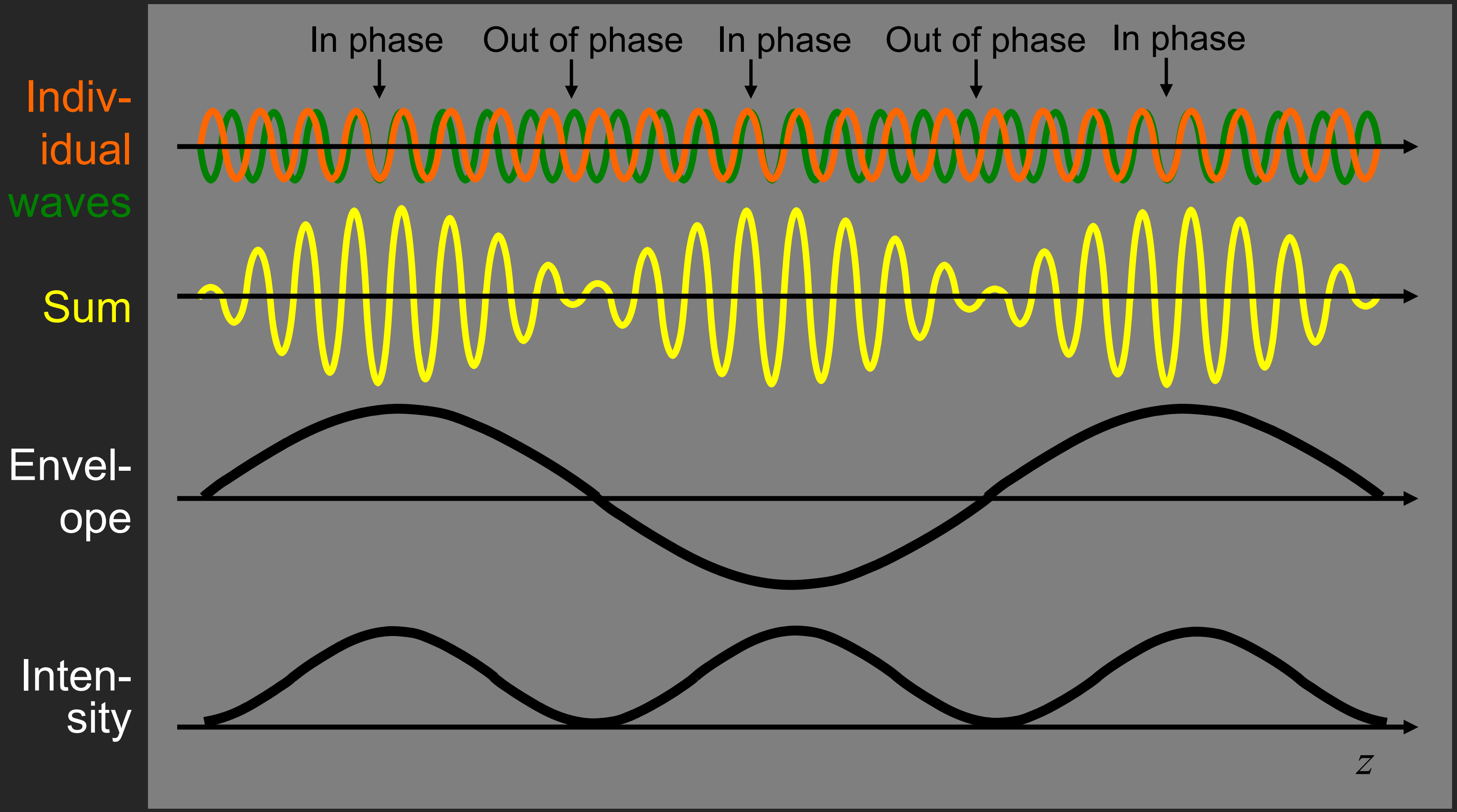
$$\mathscr{E}_{tot}(z,t) = \mathrm{Re}\{E_0 \exp i(k_{ave} z + \Delta k z - \omega_{ave} t - \Delta\omega t) + E_0 \exp i(k_{ave} z - \Delta k z - \omega_{ave} t + \Delta\omega t)\}$$

$$= \mathrm{Re}\{E_0 \exp i(k_{ave} z - \omega_{ave} t)\left[\exp i(\Delta k z - \Delta\omega t) + \exp[-i(\Delta k z - \Delta\omega t)]\right]\}$$

$$= \mathrm{Re}\{2E_0 \exp i(k_{ave} z - \omega_{ave} t)\cos(\Delta k z - \Delta\omega t)\}$$

$$\boxed{= 2E_0 \cos(k_{ave} z - \omega_{ave} t)\cos(\Delta k z - \Delta\omega t)}$$

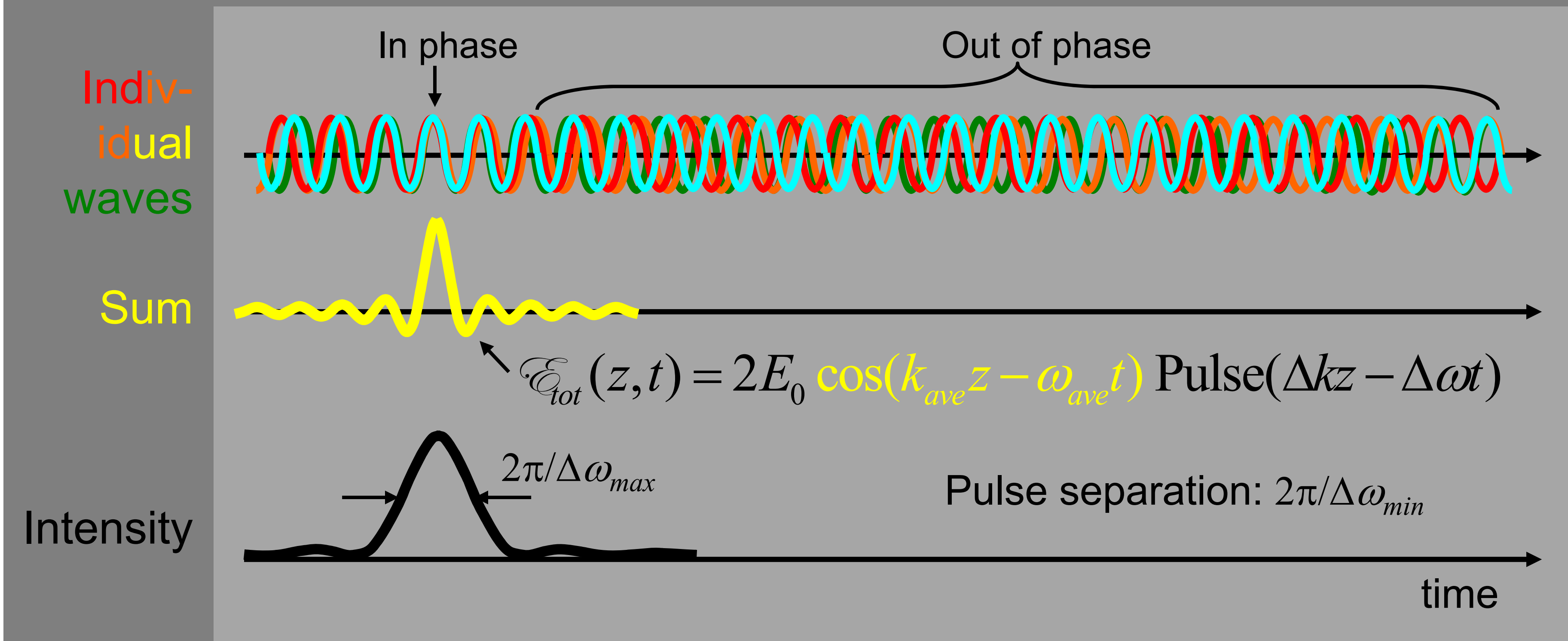
Traveling-Wave Beats



Seeing Beats

It's usually impossible to see optical beats because they occur on a time scale that's too fast to detect. This is why we say that light waves of different colors don't interfere, and we only see the average intensity.

However, a sum of many frequencies will yield a train of well-separated pulses:



Group Velocity and phase velocity

Light-wave beats (continued):

$$\mathscr{E}_{tot}(z,t) = 2E_0 \cos(k_{ave}z - \omega_{ave}t)\ \text{Pulse}(\Delta kz - \Delta\omega t)$$

carrier wave

This is a rapidly oscillating wave: $[\cos(k_{ave}z - \omega_{ave}t)]$

amplitude

with a slowly varying amplitude: $[2E_0\,\text{Pulse}(\Delta kz - \Delta\omega t)]$

The **phase velocity** comes from the rapidly varying part: $\text{v} = \omega_{ave} / k_{ave}$

What about the other velocity—**the velocity of the pulse amplitude**?

Define the **group velocity**: $\text{v}_\text{g} \equiv \Delta\omega / \Delta k$

Taking the continuous limit,
we define the **group velocity** as:

$$\boxed{\mathbf{v}_g \equiv d\omega / dk}$$

Group velocity is not equal to phase velocity if the medium is dispersive (i.e., if n varies).

Evaluate the group velocity for the case of just two frequencies:

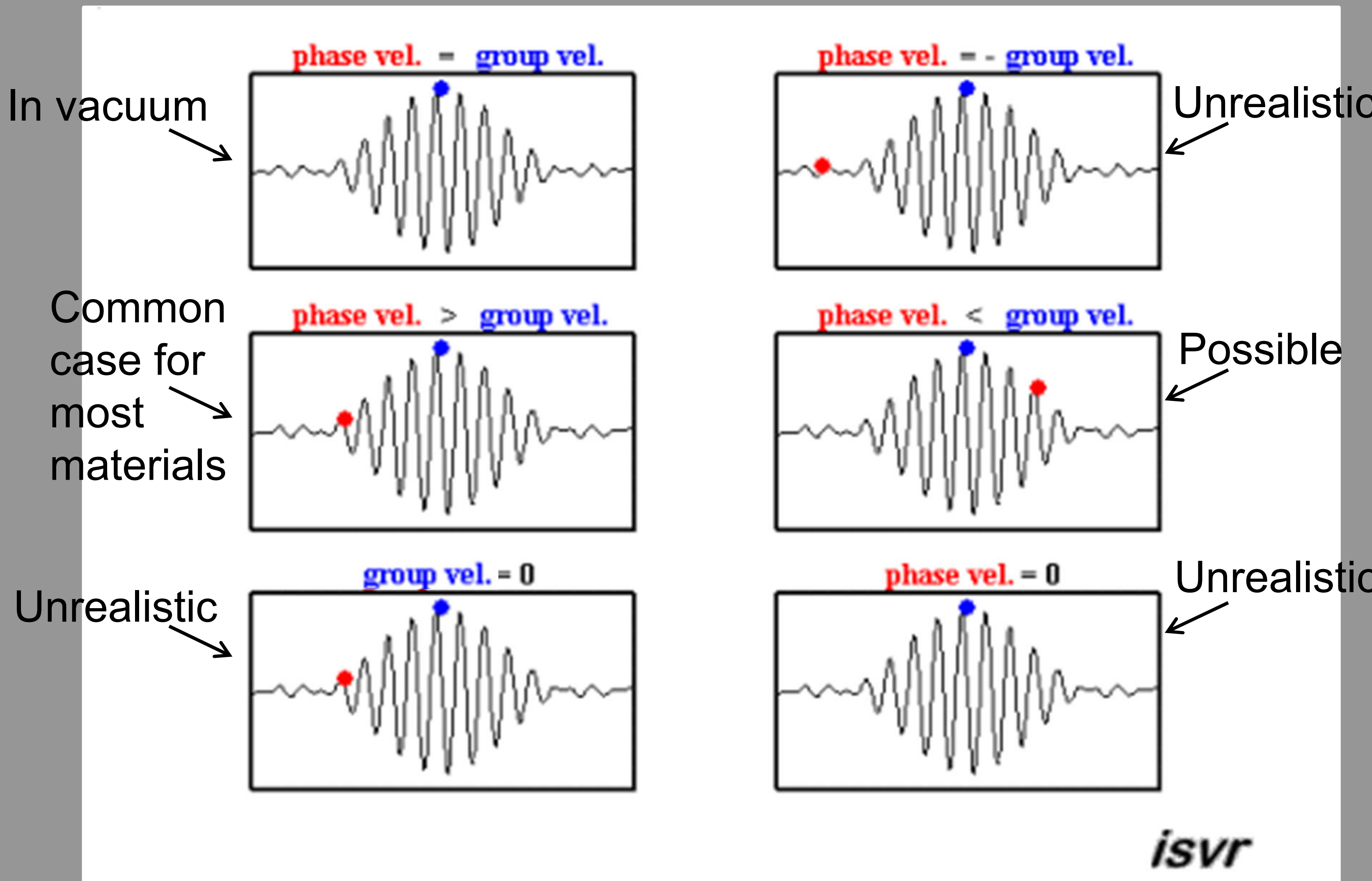
$$\mathrm{v}_g \equiv \frac{\Delta\omega}{\Delta k} = \frac{c_0 k_1 - c_0 k_2}{n_1 k_1 - n_2 k_2}$$

where k_1 and k_2 are the k-vector magnitudes in vacuum, and where n_1 and n_2 are the refractive indices of the medium in which the wave is propagating, at frequencies ω_1 and ω_2: $n(\omega_1) = n_1$ and $n(\omega_2) = n_2$.

If $n_1 = n_2 = n$, $\quad \mathrm{v}_g = \dfrac{c_0}{n}\dfrac{k_1 - k_2}{k_1 - k_2} = \dfrac{c_0}{n} = \mathrm{v}_\phi =$ phase velocity

If $n_1 \neq n_2$, $\quad \mathrm{v}_g \neq \mathrm{v}_\phi = c_0 / n$

Phase and Group Velocities



Calculating the group velocity

$$\mathrm{v}_g \equiv d\omega / dk$$

Now, ω is the same in or out of the medium, but $k = k_0\, n$, where k_0 is the k-vector in vacuum, and n depends on the medium.
So it's easier to think of ω as the independent variable:

$$\boxed{\mathrm{v}_g \equiv \left[dk / d\omega \right]^{-1}}$$

Using $k = \omega\, n(\omega) / c_0$, calculate: $dk / d\omega = (n + \omega\, dn/d\omega) / c_0$

$$\mathrm{v}_g = c_0 / (n + \omega\, dn/d\omega) = (c_0 / n) / (1 + \omega / n\; dn/d\omega)$$

Finally:

$$\boxed{\mathrm{v}_g = \mathrm{v}_\phi \Big/ \left(1 + \frac{\omega}{n} \frac{dn}{d\omega} \right)}$$

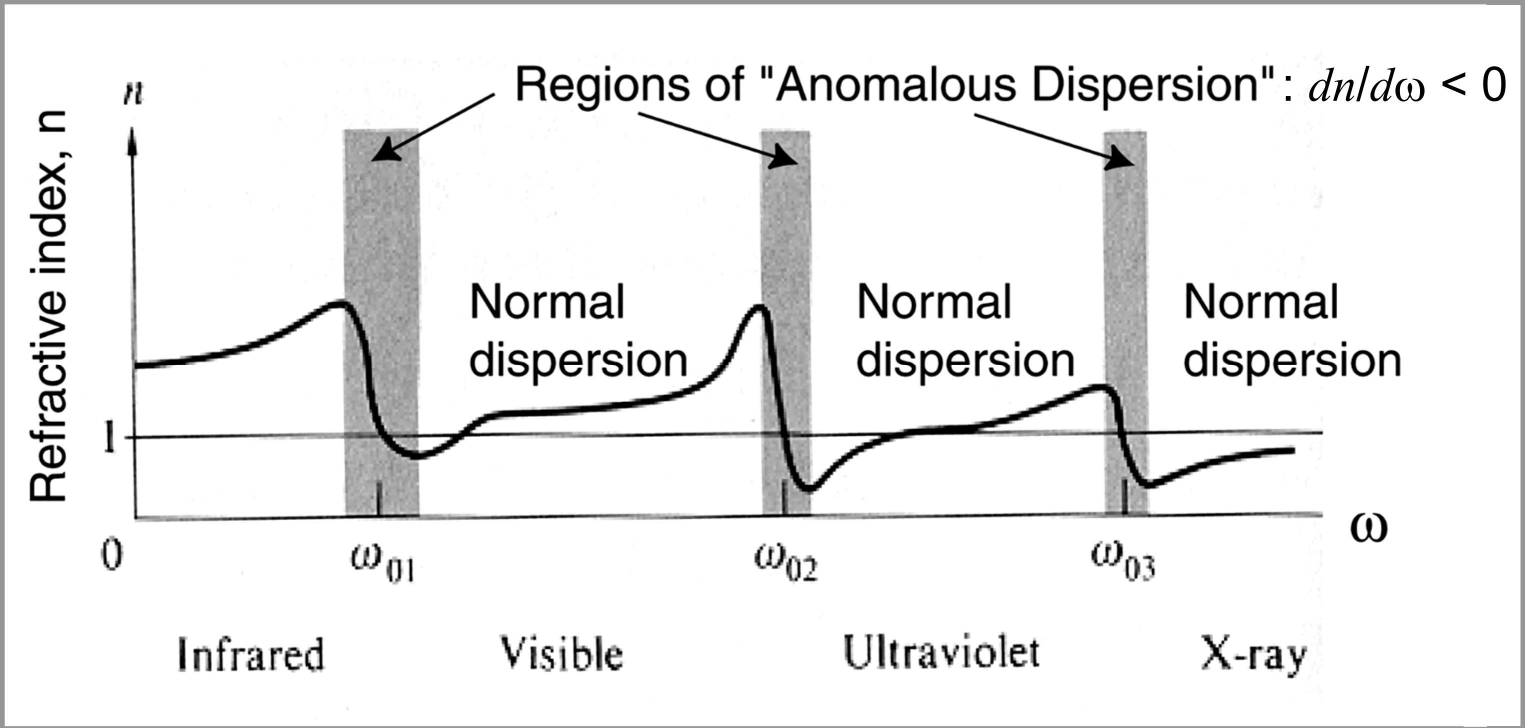
So the group velocity equals the phase velocity only when $dn/d\omega = 0$, such as in vacuum. But for most materials, n usually varies with ω.

Usually group velocity < phase velocity.

$$v_g = \frac{c_0/n}{1+\dfrac{\omega}{n}\dfrac{dn}{d\omega}}$$

Except in regions of anomalous dispersion (near a resonance, where absorption is often large), $dn/d\omega$ is positive.

So $v_g < v_\phi$ for most frequencies!



Calculating group velocity vs. wavelength

We more often think of the refractive index in terms of wavelength, so let's write the group velocity in terms of the vacuum wavelength λ_0.

Use the chain rule: $$\frac{dn}{d\omega} = \frac{dn}{d\lambda_0}\frac{d\lambda_0}{d\omega}$$

Now, $\lambda_0 = 2\pi c_0 / \omega$, so: $$\frac{d\lambda_0}{d\omega} = \frac{-2\pi c_0}{\omega^2} = \frac{-2\pi c_0}{(2\pi c_0/\lambda_0)^2} = \frac{-\lambda_0^2}{2\pi c_0}$$

Recalling that: $$v_g = \left(\frac{c_0}{n}\right) \Big/ \left[1 + \frac{\omega}{n}\frac{dn}{d\omega}\right]$$

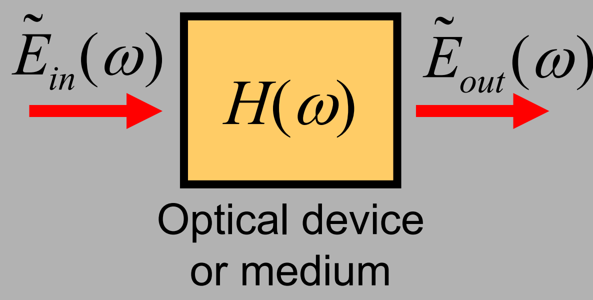
we have: $$v_g = \left(\frac{c_0}{n}\right) \Big/ \left[1 + \frac{\cancel{2\pi c_0}}{n\lambda_0}\left\{\frac{dn}{d\lambda_0}\left(\frac{-\lambda_0^2}{\cancel{2\pi c_0}}\right)\right\}\right]$$

or:

$$\boxed{v_g = \left(\frac{c_0}{n}\right) \Big/ \left(1 - \frac{\lambda_0}{n}\frac{dn}{d\lambda_0}\right)}$$

Spectral Phase and Optical Devices

Recall that the effect of a linear passive optical device (i.e., windows, filters, etc.) on a pulse is to **multiply** the frequency-domain field by a transfer function:



$$\tilde{E}_{out}(\omega) \;=\; H(\omega)\,\tilde{E}_{in}(\omega)$$

where $H(\omega)$ is the transfer function of the device/medium:

for a material with absorption coefficient $\alpha(\omega)$

$$H(\omega) \;=\; \exp[-\alpha(\omega)L/2]\exp[-i\varphi_H(\omega)]$$

Since we also write $\tilde{E}(\omega) = \sqrt{S(\omega)}\,\exp[-i\varphi(\omega)]$, the spectral phase of the output light will be:

$$\boxed{\varphi_{out}(\omega) \;=\; \varphi_H(\omega) + \varphi_{in}(\omega)}$$

We simply add spectral phases.

Note that we CANNOT add the temporal phases!

$$\phi_{out}(t) \;\neq\; \phi_H(t) + \phi_{in}(t)$$

The Group-Velocity Dispersion (GVD)

The phase due to a medium is: $\varphi_H(\omega) = n(\omega)\, k_0\, L = k(\omega)\, L$ where $k_0 = \frac{\omega}{c_0}$

To account for dispersion, expand the phase in a Taylor series:

$$k(\omega)L = k(\omega_0)L + k'(\omega_0)\left[\omega-\omega_0\right]L + \tfrac{1}{2}k''(\omega_0)\left[\omega-\omega_0\right]^2 L + \ldots$$

$$k(\omega_0) = \frac{\omega_0}{\mathrm{v}_\phi(\omega_0)} \qquad k'(\omega_0) = \frac{1}{\mathrm{v}_g(\omega_0)} \qquad k''(\omega) = \frac{d}{d\omega}\left[\frac{1}{\mathrm{v}_g}\right]$$

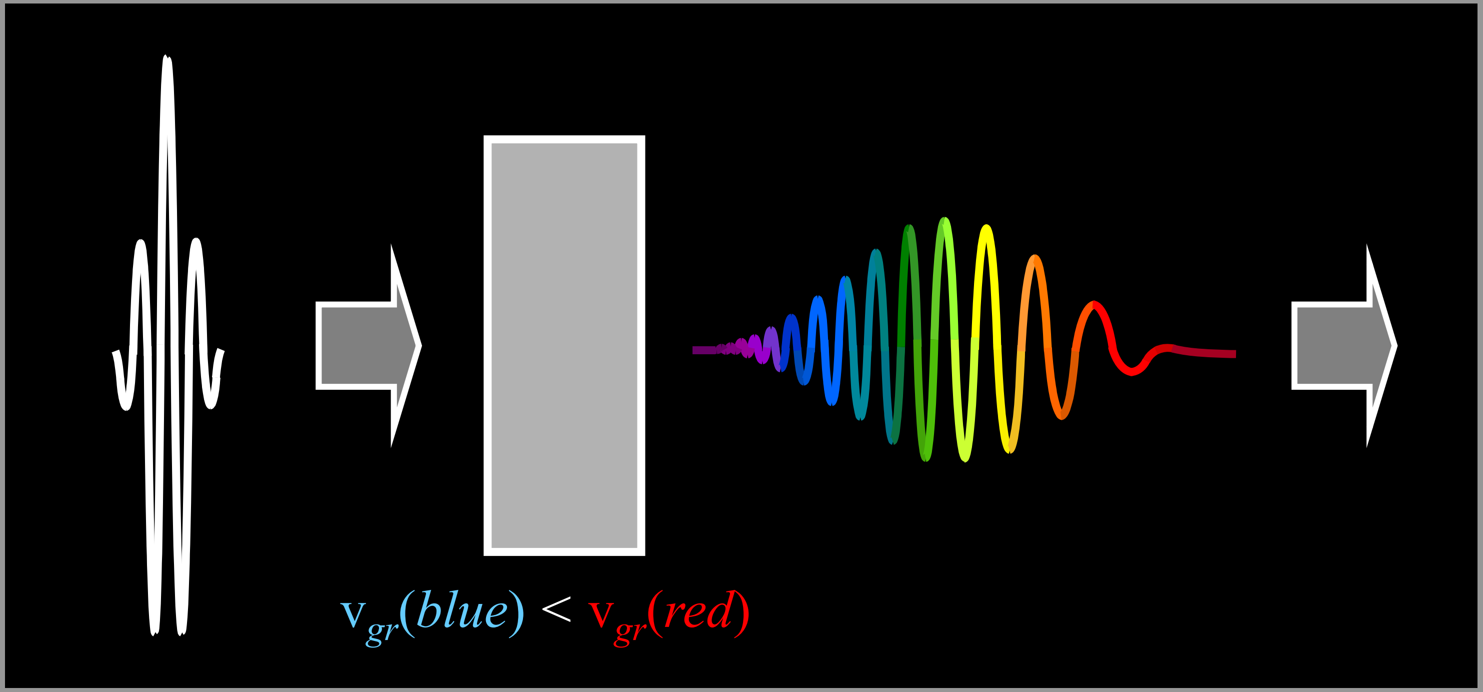
The first few terms are all related to important quantities.

The third one in particular: the variation in group velocity with frequency

$$\boxed{k''(\omega) = \frac{d}{d\omega}\left[\frac{1}{\mathrm{v}_g}\right]}$$ is the **group velocity dispersion**.

The effect of group velocity dispersion

GVD means that the group velocity will be different for different wavelengths in the pulse.



Because ultrashort pulses have such large bandwidths, GVD is a bigger issue here than it is for cw light.

Calculation of the GVD (in terms of wavelength)

Recall that: $$\frac{d\lambda_0}{d\omega} = \frac{-\lambda_0^2}{2\pi c_0} \qquad \frac{d}{d\omega} = \frac{d\lambda_0}{d\omega}\frac{d}{d\lambda_0} = \frac{-\lambda_0^2}{2\pi c_0}\frac{d}{d\lambda_0}$$

and $$\mathrm{v}_g = c_0 \Big/ \left(n - \lambda_0 \frac{dn}{d\lambda_0} \right)$$

Okay, the GVD is:

$$\frac{d}{d\omega}\left[\frac{1}{\mathrm{v}_g}\right] = \frac{-\lambda_0^2}{2\pi c_0}\frac{d}{d\lambda_0}\left[\frac{1}{c_0}\left(n - \lambda_0\frac{dn}{d\lambda_0}\right)\right] = \frac{-\lambda_0^2}{2\pi c_0^2}\frac{d}{d\lambda_0}\left[n - \lambda_0\frac{dn}{d\lambda_0}\right]$$

$$= \frac{-\lambda_0^2}{2\pi c_0^2}\left[\frac{dn}{d\lambda_0} - \lambda_0\frac{d^2 n}{d\lambda_0^2} - \frac{dn}{d\lambda_0}\right]$$

Simplifying: $$\boxed{GVD \equiv k''(\omega_0) = \frac{\lambda_0^3}{2\pi c_0^2}\frac{d^2 n}{d\lambda_0^2}}$$

Units:
ps^2/m or
(s/m)/Hz or
s/Hz/m

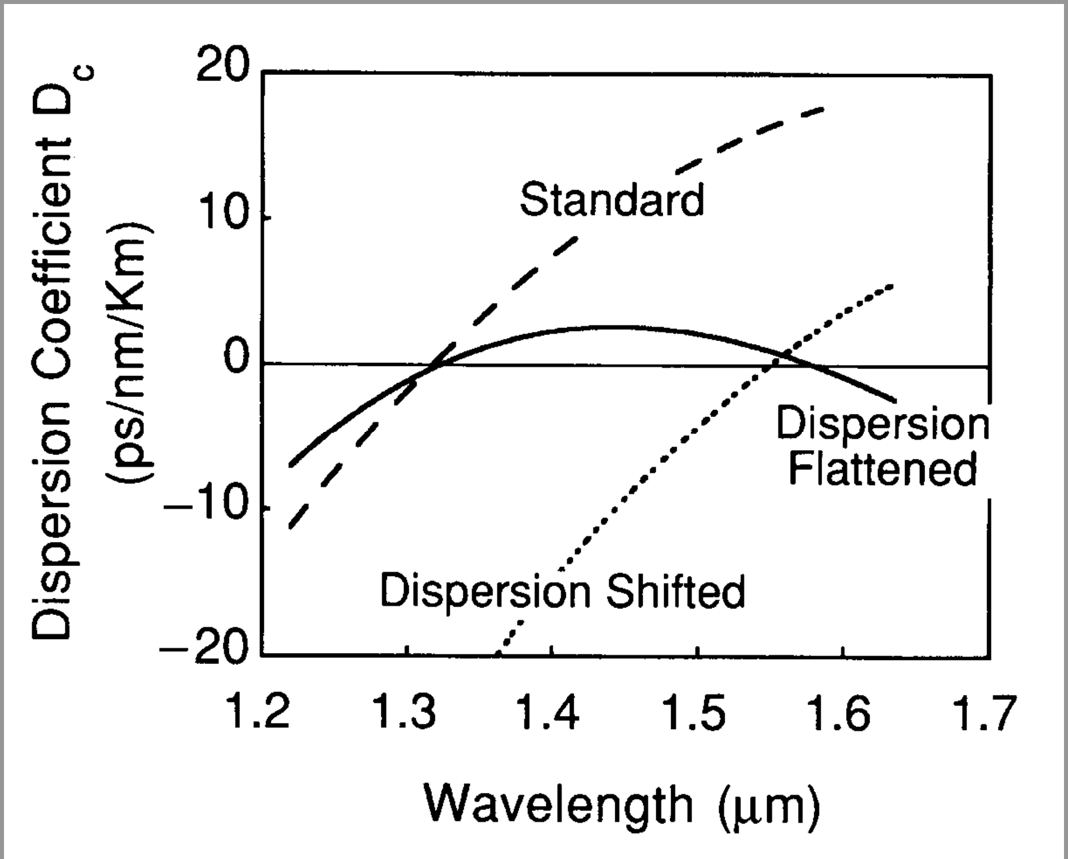
GVD in optical fibers

"Dispersion parameter"

psec / km - nm

$$\boxed{D} = \frac{1}{L}\frac{d}{d\lambda}\left(\frac{L}{V_g}\right) = -\frac{2\pi c}{\lambda^2}\boxed{k''}$$

psec^2 / cm



Note that fiber folks define their "dispersion parameter" as proportional to the negative of the definition of GVD that we've been using.

Sophisticated fiber structures, i.e., index profiles, have been designed and optimized to produce a waveguide dispersion that modifies the bulk material dispersion

GVD yields group delay dispersion (GDD).

The delay is just the medium length L divided by the velocity.

The phase delay:

$$k(\omega_0) = \frac{\omega_0}{\mathrm{v}_\phi(\omega_0)} \qquad \text{so:} \qquad t_\phi(\omega_0) = \frac{L}{\mathrm{v}_\phi(\omega_0)} = \frac{k(\omega_0)L}{\omega_0}$$

The group delay:

$$k'(\omega_0) = \frac{1}{\mathrm{v}_g(\omega_0)} \qquad \text{so:} \qquad t_g(\omega_0) = \frac{L}{\mathrm{v}_g(\omega_0)} = k'(\omega_0)L$$

The group delay dispersion (GDD):

GDD = GVD×L

$$k''(\omega) = \frac{d}{d\omega}\left[\frac{1}{\mathrm{v}_g}\right] \qquad \text{so:} \qquad \boxed{GDD = \frac{d}{d\omega}\left[\frac{1}{\mathrm{v}_g}\right]L = k''(\omega)L}$$

Units: fs^2 or fs/Hz

Dispersion parameters for various materials

material	λ [nm]	$n(\lambda)$	$\frac{dn}{d\lambda} \cdot 10^{-2} \left[\frac{1}{\mu m}\right]$	$\frac{d^2n}{d\lambda^2} \cdot 10^{-1} \left[\frac{1}{\mu m^2}\right]$	$\frac{dn^3}{d\lambda^3} \left[\frac{1}{\mu m^3}\right]$	$T_g \left[\frac{fs}{mm}\right]$	$GDD \left[\frac{fs^2}{mm}\right]$	$TOD \left[\frac{fs^3}{mm}\right]$
BK7	400	1,5308	-13,17	10,66	-12,21	5282	120,79	40,57
	500	1,5214	-6,58	3,92	-3,46	5185	86,87	32,34
	600	1,5163	-3,91	1,77	-1,29	5136	67,52	29,70
	800	1,5108	-1,97	0,48	-0,29	5092	43,96	31,90
	1000	1,5075	-1,40	0,15	-0,09	5075	26,93	42,88
	1200	1,5049	-1,23	0,03	-0,04	5069	10,43	66,12
SF10	400	1,7783	-52,02	59,44	-101,56	6626	673,68	548,50
	500	1,7432	-20,89	15,55	-16,81	6163	344,19	219,81
	600	1,7267	-11,00	6,12	-4,98	5980	233,91	140,82
	800	1,7112	-4,55	1,58	-0,91	5830	143,38	97,26
	1000	1,7038	-2,62	0,56	-0,27	5771	99,42	92,79
	1200	1,6992	-1,88	0,22	-0,10	5743	68,59	107,51
Sapphire	400	1,7866	-17,20	13,55	-15,05	6189	153,62	47,03
	500	1,7743	-8,72	5,10	-4,42	6064	112,98	39,98
	600	1,7676	-5,23	2,32	-1,68	6001	88,65	37,97
	800	1,7602	-2,68	0,64	-0,38	5943	58,00	42,19
	1000	1,7557	-1,92	0,20	-0,12	5921	35,33	57,22
	1200	1,7522	-1,70	0,04	-0,05	5913	13,40	87,30
Quartz	300	1,4878	-30,04	34,31	-54,66	5263	164,06	46,49
	400	1,4701	-11,70	9,20	-10,17	5060	104,31	31,49
	500	1,4623	-5,93	3,48	-3,00	4977	77,04	26,88
	600	1,4580	-3,55	1,59	-1,14	4934	60,66	25,59
	800	1,4533	-1,80	0,44	-0,26	4896	40,00	28,43
	1000	1,4504	-1,27	0,14	-0,08	4880	24,71	38,73
	1200	1,4481	-1,12	0,03	-0,03	4875	9,76	60,05

Manipulating the phase of light

Recall that we expand the spectral phase of the pulse in a Taylor Series:

$$\varphi(\omega) \quad = \quad \varphi_0 + \varphi_1\,[\omega-\omega_0] + \varphi_2\,[\omega-\omega_0]^2/2! + \ldots$$

and we do the same for the spectral phase of the optical medium, H:

$$\varphi_H(\omega) \quad = \quad \underbrace{\varphi_{H0}}_{\text{phase}} + \underbrace{\varphi_{H1}}_{\text{group delay}}\,[\omega-\omega_0] + \underbrace{\varphi_{H2}}_{\text{group delay dispersion (GDD)}}\,[\omega-\omega_0]^2/2! + \ldots$$

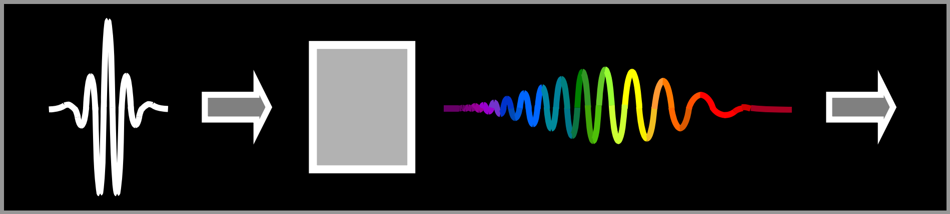
So, to manipulate light, we must add or subtract **spectral-phase terms**.

For example, to eliminate the linear chirp (second-order spectral phase), we must design an optical device whose second-order spectral phase cancels that of the pulse:

$$\varphi_2 + \varphi_{H2} = 0 \qquad \text{i.e.,} \qquad \left.\frac{d^2\varphi}{d\omega^2}\right|_{\omega_0} + \left.\frac{d^2\varphi_H}{d\omega^2}\right|_{\omega_0} = 0$$

Propagation of the pulse manipulates it.

Dispersive pulse broadening is unavoidable.



If φ_2 is the pulse 2nd-order spectral phase on entering a medium, and $k''L$ is the 2nd-order spectral phase of the medium, then the resulting pulse 2nd-order phase will be the sum: $\varphi_2 + k''L$.

A linearly chirped input pulse has 2nd-order phase: $\varphi_{2,in} = \dfrac{\beta/2}{\alpha^2+\beta^2}$

(This result pulls out the ½ in the Taylor Series.)

Emerging from a medium, its 2nd-order phase will be:

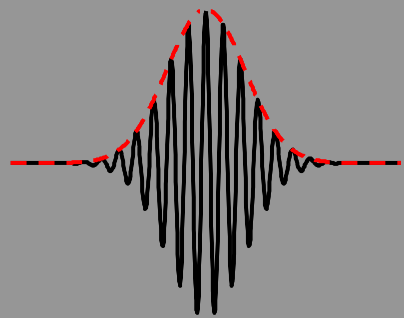
$$\varphi_{2,out} = \frac{\beta/2}{\alpha^2+\beta^2} + GDD = \frac{\beta/2}{\alpha^2+\beta^2} + \frac{\lambda_0^3}{2\pi c_0^2}\frac{d^2 n}{d\lambda_0^2}L \longleftarrow$$

This result, with the spectrum, can be inverse Fourier-transformed to yield the pulse.

Let's do that!

Since GDD is generally positive (for transparent materials in the visible and near-IR), a positively chirped pulse will broaden further; a negatively chirped pulse will shorten.

Posing the problem



$\alpha(\omega)$, $n(\omega)$

?

Suppose we have a short pulse, with (initially) zero chirp.

It traverses through a block of something transparent (with known n, and assume $\alpha=0$).

What does it look like when it emerges?

To analyze:

start with $E_{in}(t)$ (known)

convert to $E_{in}(\omega)$

(ignore absorption)

propagate forward in the frequency domain: $E_{out}(\omega)$

back to time domain: $E_{out}(t)$

Propagation of a Gaussian pulse

Start with temporal amplitude of the form:

$$\mathscr{E}(t) \propto E_0 \exp\left[-\alpha t^2\right]\exp\left[i\omega_0 t\right] + c.c$$

i.e., an unchirped Gaussian pulse with center frequency ω_0, and duration

$$\tau_{FWHM} = \sqrt{2\ln 2/\alpha}$$

Fourier-Transforming yields:

As usual, neglecting the negative-frequency term due to the c.c.

$$\tilde{E}(\omega) \propto E_0 \exp\left[-\frac{1}{4\alpha}(\omega - \omega_0)^2\right]$$

Let's write this as:

$$\tilde{E}(\omega) \propto E_0 \exp\left[-\frac{1}{4}(\omega - \omega_0)^2 t_G{}^2\right] \quad \text{where} \quad t_G = \frac{1}{\sqrt{\alpha}} = \frac{\tau_{FWHM}}{\sqrt{2\ln 2}}$$

Propagation of a Gaussian pulse

So we have the spectral amplitude: $$E\left(\omega, z=0\right) = E_0 \cdot e^{-\frac{(\omega-\omega_0)^2 t_G{}^2}{4}}$$

After propagation a distance z:

$$E(\omega, z) = E_{z=0} \cdot e^{-ik(\omega)z} = E_0 \cdot \exp\left[-\frac{(\omega-\omega_0)^2 t_G{}^2}{4} - ik(\omega)z\right]$$

Taylor expansion of $k(\omega)$ at $\omega = \omega_0$: (keeping only terms up to order ω^2)

$$E(\omega, z) = E_0 \cdot \exp\left[-\frac{(\omega-\omega_0)^2 t_G{}^2}{4} - ik(\omega_0)z - ik'z \cdot (\omega-\omega_0) - \frac{ik''z}{2} \cdot (\omega-\omega_0)^2\right]$$

This is still a quadratic in ω

Time-domain field at z is found via inverse Fourier transform:

$$E(t, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(\omega, z) e^{i\omega t} d\omega$$

Propagation of a Gaussian pulse

$$E(t,z) \propto E_0 \cdot \frac{1}{t_G(z)} \exp\left[i\omega_0 \left(t - z / V_\phi(\omega_0) \right) - \left(\frac{t - z / V_g(\omega_0)}{t_G(z)} \right)^2 \right]$$

where: $t_G(z) = \sqrt{t_G{}^2 + 2ik''z}$ pulse width increases with propagation

$$V_\varphi(\omega_p) = \frac{\omega_p}{k(\omega_p)}$$ phase velocity

$$V_g(\omega_0) = \frac{1}{k'(\omega_p)} = \left.\frac{d\omega}{dk}\right|_{\omega_p}$$ group velocity - speed of pulse envelope

And as we have seen: $V_\varphi(\omega) = \dfrac{c}{n(\omega)}$ $\quad V_g(\omega) = \dfrac{c}{n(\omega) + \omega \dfrac{dn}{d\omega}}$ generally positive

$$k''(\omega) = \frac{d^2k}{d\omega^2} = \frac{d}{d\omega}\left(\frac{1}{V_g(\omega)} \right)$$ Group velocity dispersion

Propagation of a Gaussian pulse

$$E(t,z) \propto E_0 \cdot \frac{1}{t_G(z)} \exp\left[i\omega_0 \left(t - z / \mathrm{V}_\phi(\omega_0) \right) - \left(\frac{t - z / \mathrm{V}_g(\omega_0)}{t_G(z)} \right)^2 \right]$$

We can define a retarded time coordinate t_R which moves along the z axis at the speed of the pulse envelope:

$$t_R = t - \frac{z}{\mathrm{V}_g(\omega_0)}$$

We can also define a scaled GVD parameter, $\xi = \frac{2k''}{t_G{}^2}$, which has units of (length)$^{-1}$.

Then, the propagation-distance-dependent pulse duration becomes:

$$t_G(z) = t_G \sqrt{1 + i\xi z}$$

Note that this (complex) quantity appears in two places: the width of the Gaussian envelope (complex width = chirp!) AND the prefactor in front of the expression (peak intensity goes down as width increases)

Propagation of a Gaussian pulse

Plugging these into the expression:

$$E(t,z) = \frac{E_0 / \sqrt{\pi}}{t_G \sqrt{1+i\xi z}} \boxed{\exp\left[-\frac{1}{1+i\xi z}\left(\frac{t_R}{t_G}\right)^2\right]} \exp\left\{-i\omega_0\left[t_R + z\left(\frac{1}{V_g} - \frac{1}{V_\phi}\right)\right]\right\}$$

This term looks like a Gaussian with a complex width parameter! Sound familiar?

$$\exp\left[-\frac{1}{1+i\xi z}\left(\frac{t_R}{t_G}\right)^2\right] = \exp\left[-\frac{t_R{}^2}{t_G{}^2\left(1+\boxed{\xi^2 z^2}\right)}\left(\boxed{1-i\xi z}\right)\right]$$

Because of this, the pulse width increases with increasing z, regardless of the sign of ξ

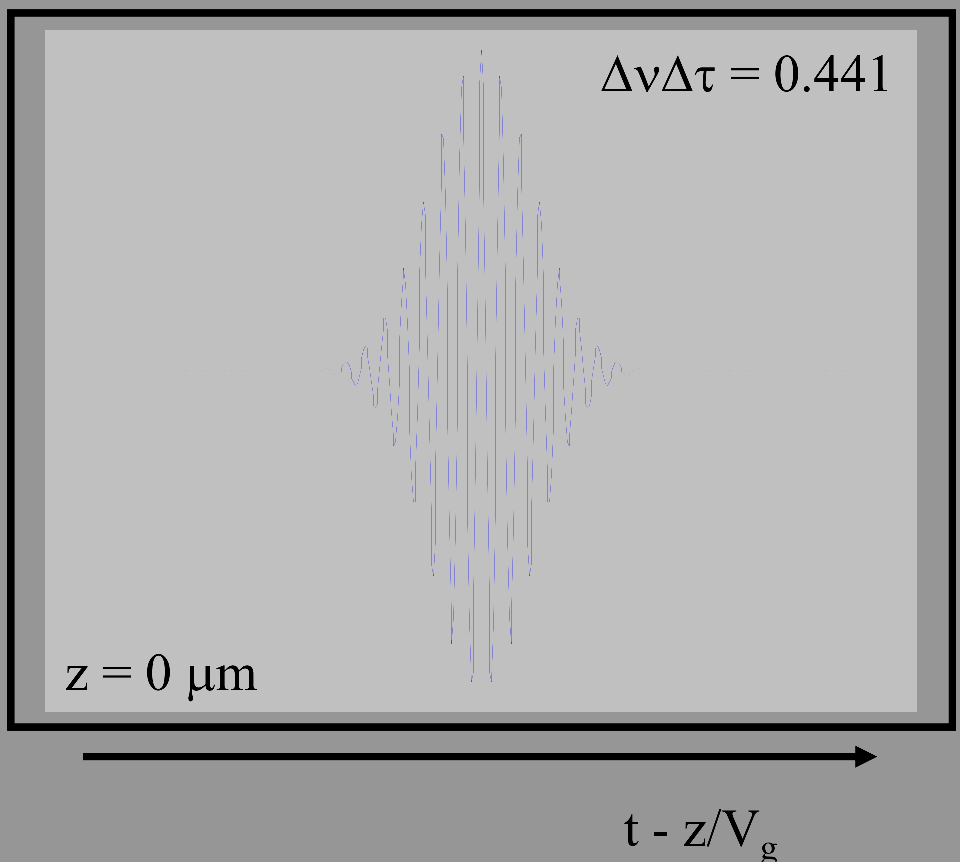
This looks just like our expression for a linearly chirped pulse from last lecture, with $(\alpha - i\beta)$ replaced by $(1 - i\xi z)$

Let's define a dimensionless linear chirp parameter $\gamma = \xi z$

Group velocity dispersion (GVD)

Gaussian part of the exponential:

$$\exp\left[-\frac{t_R^{\,2}}{t_G^{\,2}\left(1+\gamma^2\right)}\left(1-i\gamma\right)\right] \qquad \gamma = \frac{2k''}{t_G^{\,2}}z$$



SF6 glass

- n = 1.78
- n' = 2×10^{-5} psec
- n'' = 8×10^{-9} psec^2

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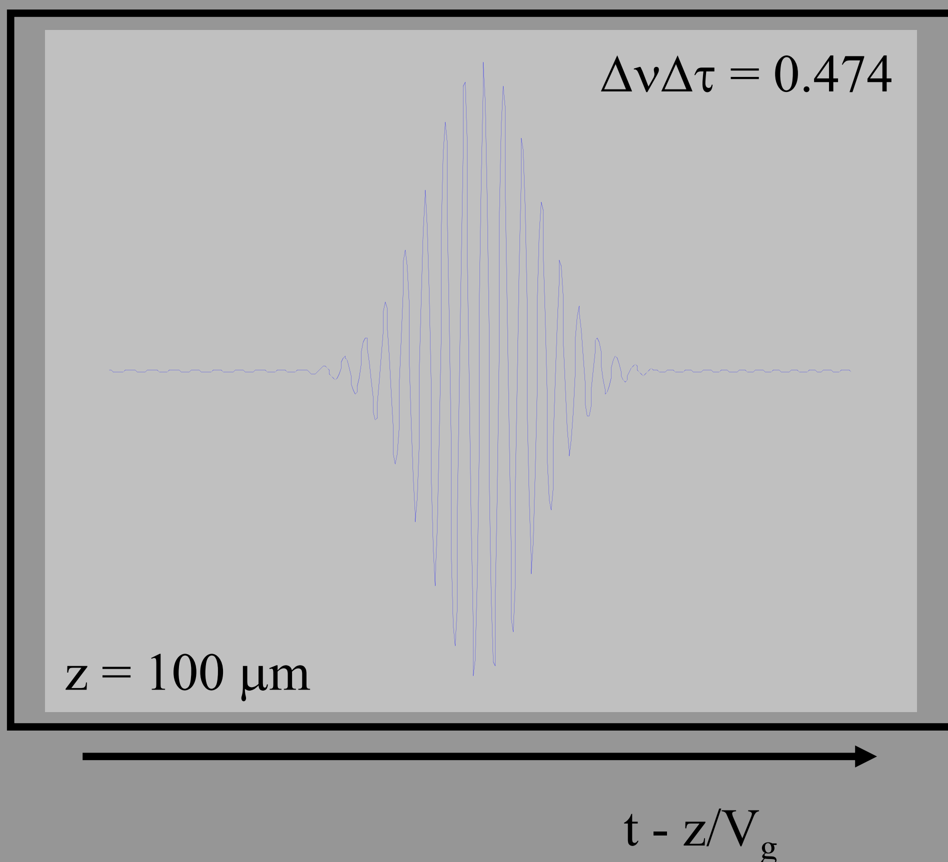

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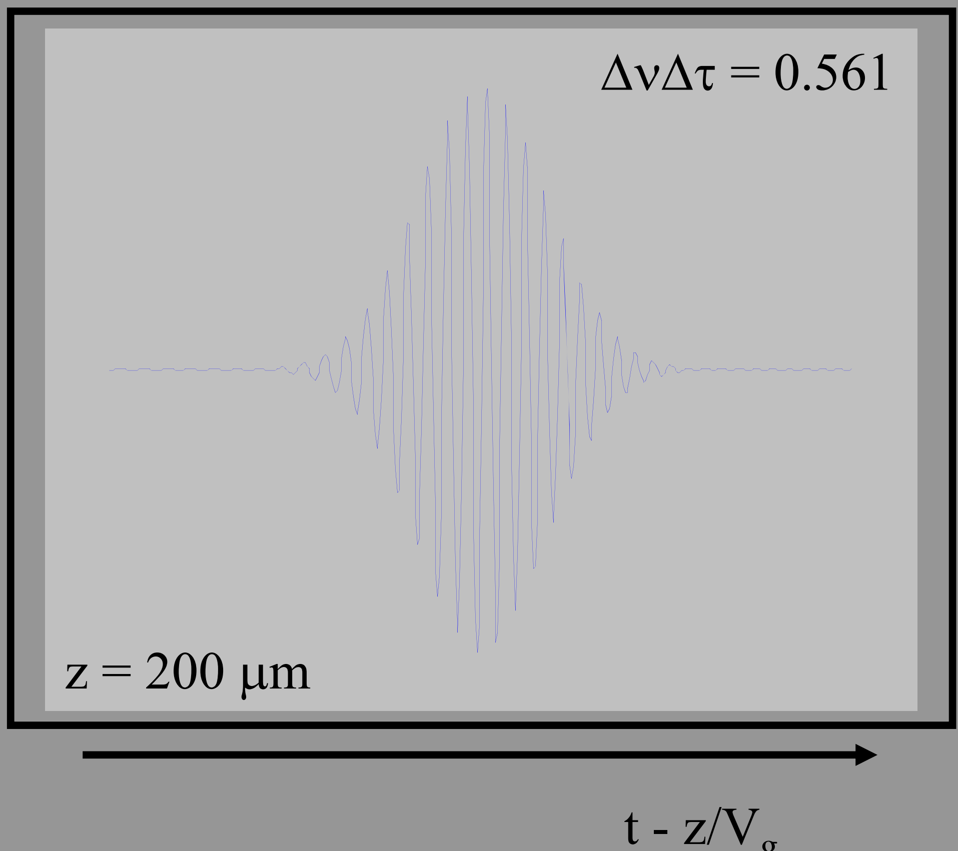

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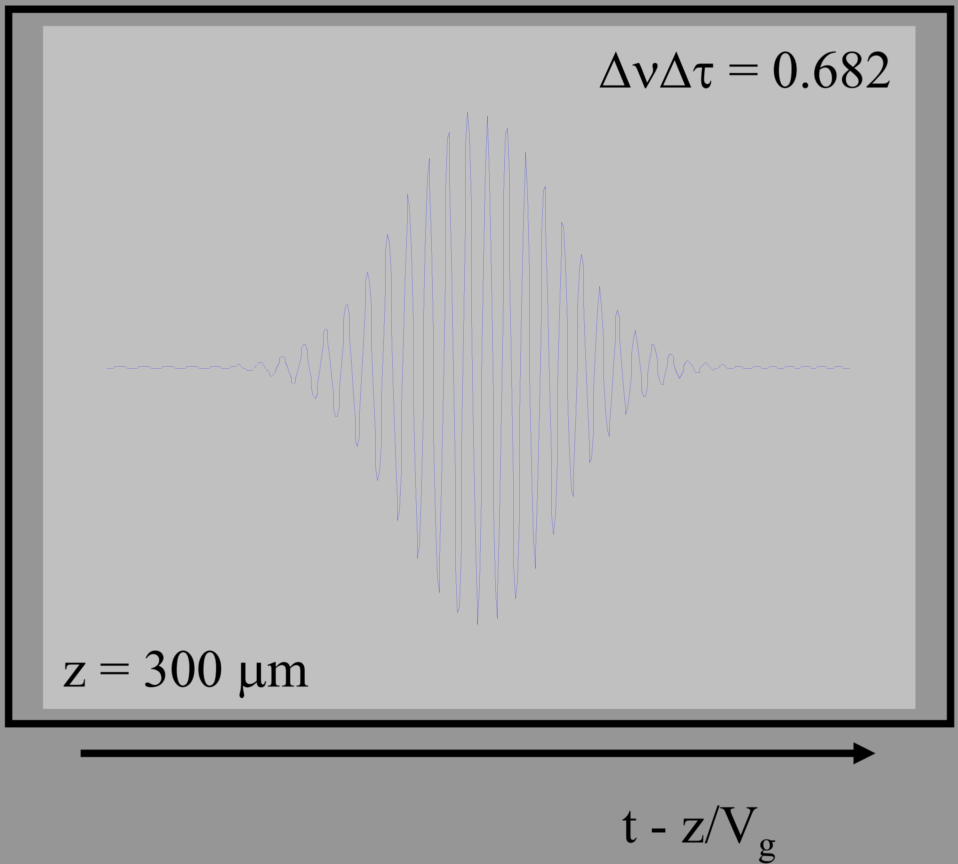

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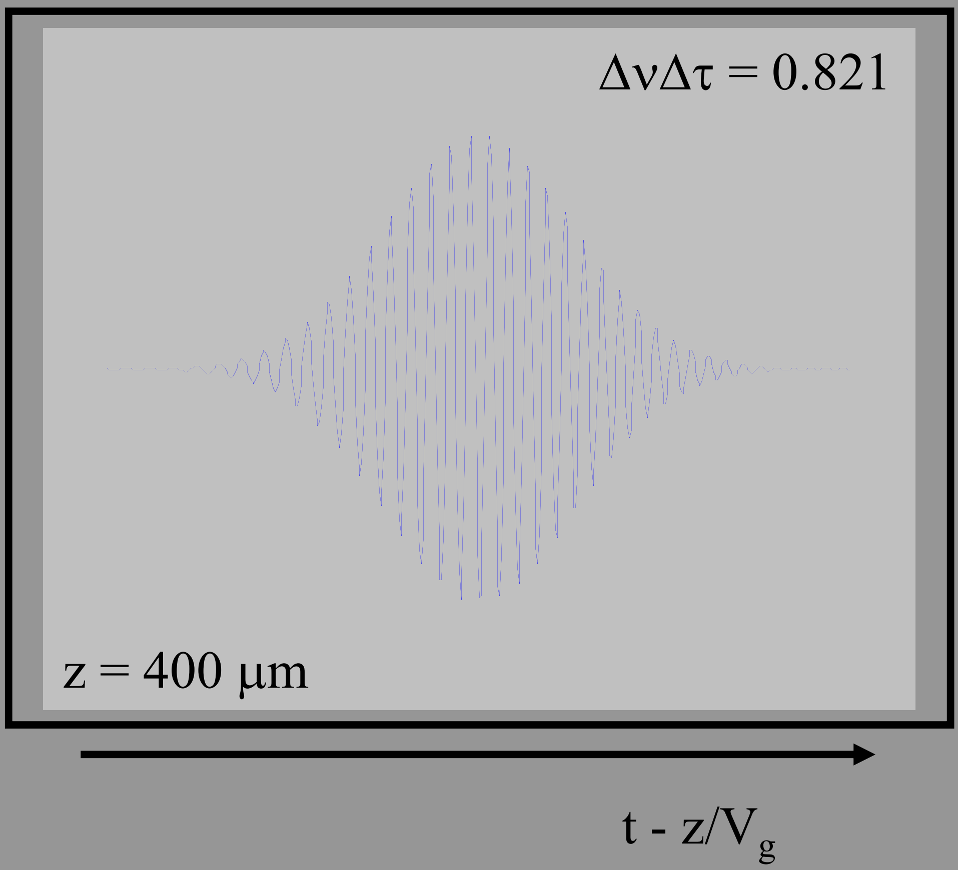

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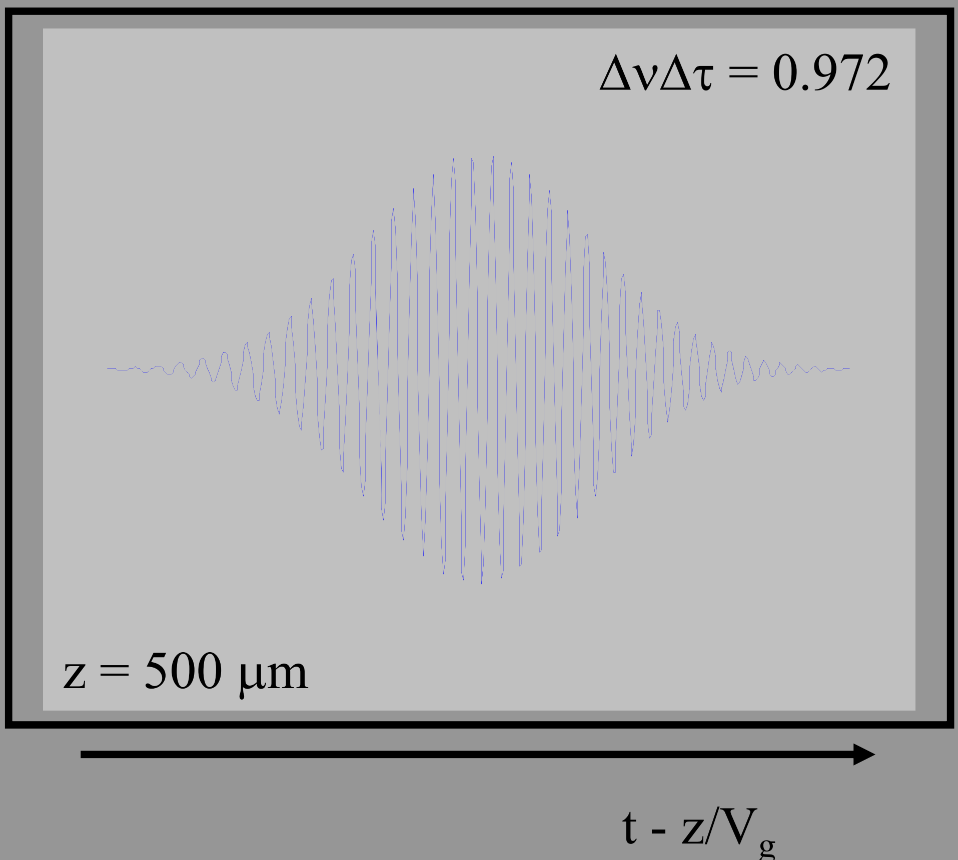


SF6 glass

$n = 1.78$

$n' = 2\times10^{-5}$ psec

$n'' = 8\times10^{-9}$ psec^2

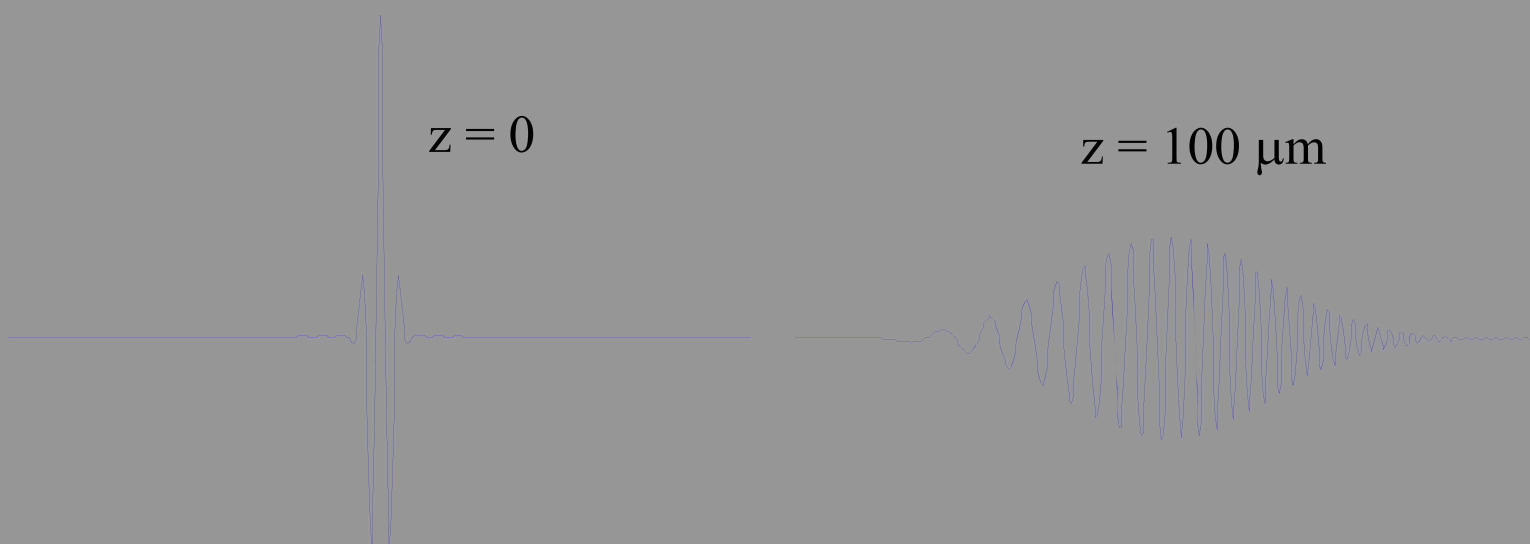
Group velocity dispersion (GVD)

$$\xi = \frac{2k''}{t_G^{\,2}}$$

units of (length)$^{-1}$

pulse width doubles after propagation through a length $\sqrt{3}/\xi$

- group velocity dispersion k'' distorts pulses
- typical materials have $k'' > 0$, which induces an up chirp
- initially shorter pulses distort much more readily (larger bandwidth)



Dispersion vs. absorption

Is it reasonable to neglect absorption?

If we include absorption, then k has an imaginary part: $\mathrm{Im}(k) = \alpha/2$

Fractional change in pulse energy:

$$\Delta U_{rel} = \frac{U(z)}{U(0)} - 1 \approx \alpha \cdot z$$

Fractional change in pulse duration:

$$\Delta t_{rel} = \frac{t(z)}{t(0)} - 1 \approx 2\left(\frac{z \cdot \mathrm{Re}(k'')}{t_G{}^2}\right)^2$$

Example:

Typical fiber optics have: α ~ 0.11 / km (= 1 dB/km)

and: k'' ~ 21 psec^2 / km at λ = 1 μm

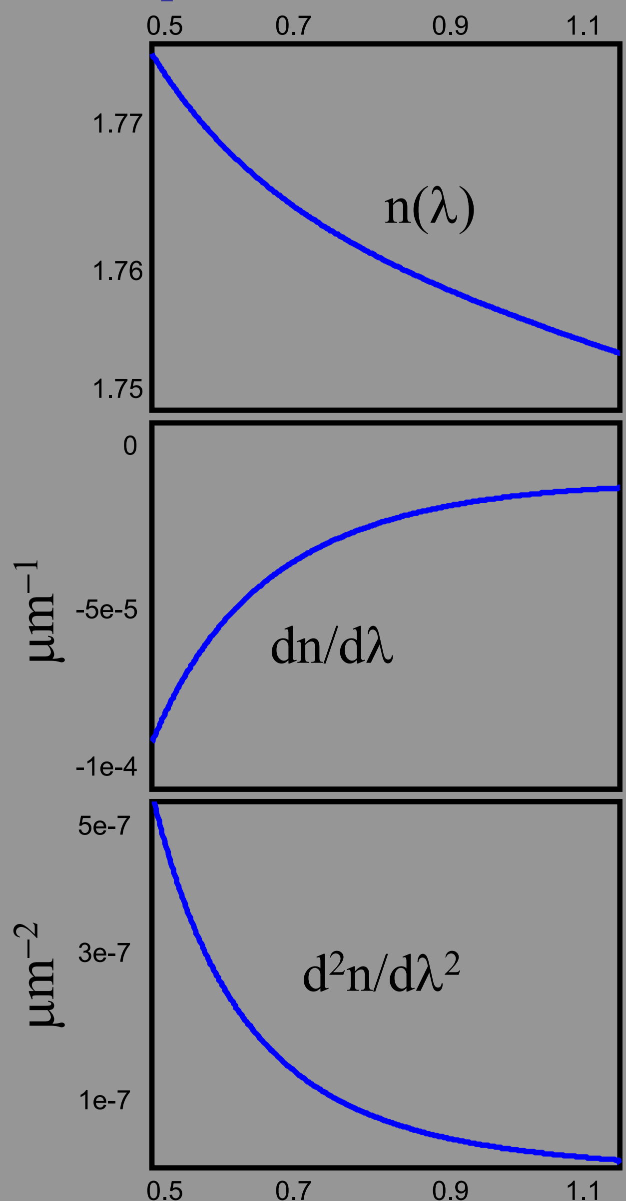
Thus, in 10 m of fiber, a 100 fsec pulse experiences:

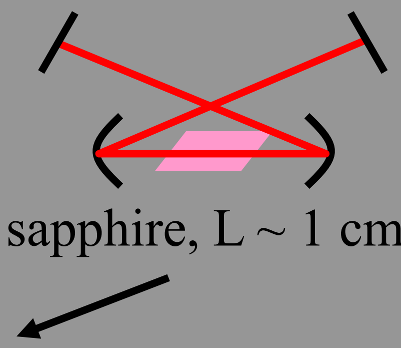
0.2% absorption loss

& pulse width broadening by a factor of ~900!

In the non-resonant regime: Dispersion is Everything

Dispersion in a laser cavity





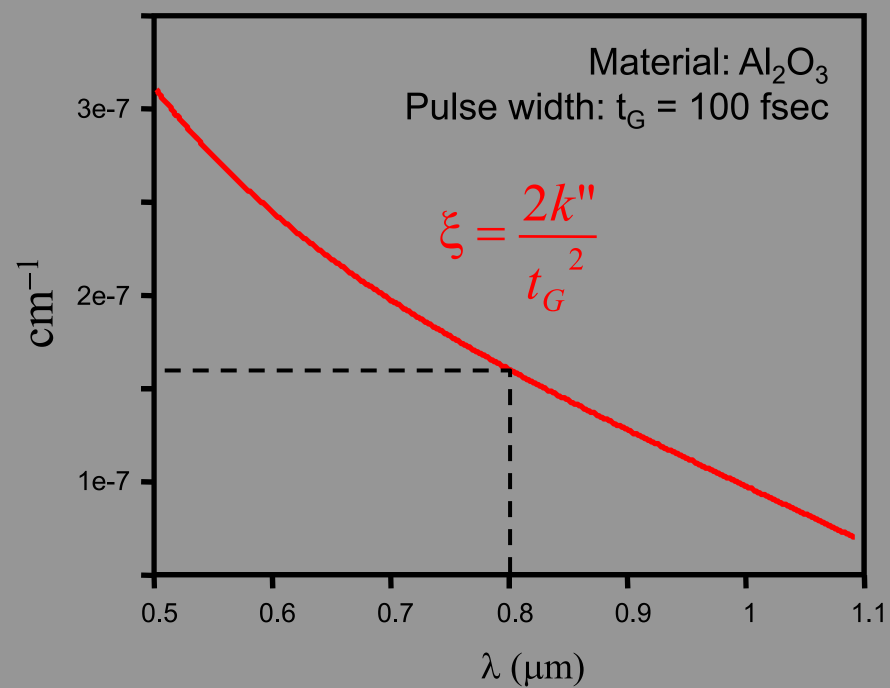
At λ = 800 nm:

chirp parameter $\gamma = 2\xi L$

$= 3.2\times10^{-7}$

(per round trip)

It is small, but **not zero!**



So how can we generate negative GDD?

This is a big issue because pulses spread further and further as they propagate through materials.

We need a way of generating negative GDD to compensate.

Negative GDD
Device