1. This problem is similar to the worked example in lecture 15. We first compute that
\[ 1 - 2j = \sqrt{5} e^{j \sin^{-1}(-2)} = \sqrt{5} e^{-j 63.43^\circ} \]
which implies that this wave has \( E_x = E_0 \cos \omega t \) and \( E_y = \sqrt{5} E_0 \cos (\omega t - 63.43^\circ) \).
(a) Setting \( E_0 = 1 \), we make a table of the evolution of the two field components as \( \omega t \) increases from 0 to \( \pi \):

\[
\begin{array}{ccc}
\omega t = 0 & E_x & E_y \\
\omega t = \pi/4 & 0.71 & 2.12 \\
\omega t = \pi/2 & 0 & 2 \\
\omega t = 3\pi/4 & \text{No real values} & \text{No real values} \\
\omega t = \pi & -1 & -1 \\
\end{array}
\]

(b) As you can figure out from the sequence of plotted points, this makes a counter-clockwise rotation, but we are looking along the \(-z\) direction, which is opposite to the propagation direction. Thus, when looking along the propagation direction, this rotation is clockwise. That means that the light is right-hand elliptically polarized.
(c) The fraction of the power passed by an x-polarizer is
\[
\frac{I_x}{I_x + I_y} = \frac{E_x^2}{E_x^2 + E_y^2} = \frac{1}{6} = 16.7\%.
\]

2. (a) Using rotation matrices and the procedure described in lecture 17, we compute the Jones matrix for a quarter-wave plate rotated by an angle \( \theta \):
\[
A_{\text{QWP}}(\theta) = \begin{bmatrix}
\cos(\theta) & -\sin(\theta) \\
\sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & j
\end{bmatrix}
\begin{bmatrix}
\cos(\theta) & \sin(\theta) \\
-\sin(\theta) & \cos(\theta)
\end{bmatrix}
= \begin{bmatrix}
\cos^2(\theta) - j \sin(\theta) \cos(\theta) & 1 - j \sin(\theta) \\
- j \sin(\theta) \cos(\theta) & \cos^2(\theta) + j \sin^2(\theta)
\end{bmatrix}
\begin{bmatrix}
\cos(\theta) & \sin(\theta) \\
-\sin(\theta) & \cos(\theta)
\end{bmatrix}
= \begin{bmatrix}
\cos^2(\theta) + j \sin^2(\theta) & 1 - j \sin(\theta) \cos(\theta) \\
(1 - j) \sin(\theta) \cos(\theta) & \sin^2(\theta) + j \cos^2(\theta)
\end{bmatrix}
\]
Replace \( \theta \) by \( \epsilon \), use the small angle approximation, and throw away terms of order \( \epsilon^2 \), to find:
\[
A_{\text{QWP}}(\epsilon) \approx \begin{bmatrix}
1 & (1 - j) \epsilon \\
(1 - j) \epsilon & j
\end{bmatrix}
\]
(b) Applying this to an x-polarized input wave, we find
\[
\begin{bmatrix}
1 & (1 - j) \epsilon \\
(1 - j) \epsilon & j
\end{bmatrix}
\begin{bmatrix}
1 \\
0
\end{bmatrix}
= \begin{bmatrix}
1 \\
(1 - j) \epsilon
\end{bmatrix}
= \begin{bmatrix}
1 \\
\epsilon \sqrt{2} e^{-j \pi/4}
\end{bmatrix}.
\]
That is, the transmitted wave is no longer purely x-polarized, but is tilted, with a small y component proportional to \( \epsilon \). The ratio of the amplitude of the y component to that of the x component is \( |E_y/E_x| = \sqrt{2} \epsilon \).
3. This device is known as a Fresnel Rhomb. It is valuable for converting linear to circular polarization, as we shall prove here.

The incident beam has equal parts of parallel and perpendicular polarization. We know that the amplitudes of $r_{\parallel}$ and $r_{\perp}$ are both equal to one, because this is total internal reflection. Here, we are concerned with the phases of these quantities.

For the first bounce, we have $\theta_i = 50.23^\circ$. To compute the reflection coefficients, we must compute the (complex) values for $\sin(\theta_i)$ and $\cos(\theta_i)$, as described in lecture 14. From Snell’s law, we find $\sin(\theta_i) = n_i \sin(\theta_i)/n_t = 1.153$. This is bigger than one, as expected for the case under consideration where there is no transmitted ray. Thus, $\cos \theta_i = \sqrt{1 - \sin^2 \theta_i} = 0.574$. Now we can compute the reflection coefficients:

$$r_{\perp} = \frac{n_i \cos \theta_i - n_t \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_i} = e^{-j61.76^\circ} \quad \text{and} \quad r_{\parallel} = \frac{n_i \cos \theta_i - n_t \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_i} = e^{j73.24^\circ}$$

After one bounce, the parallel component leads the perpendicular component by a phase angle of: $\text{angle}(r_{\parallel}) - \text{angle}(r_{\perp}) = 135^\circ$. After the second bounce, this phase difference doubles, since the situation is identical. Thus, the emerging field has the parallel component leading by $270^\circ$, which is equivalent to lagging by $\pi/2$. The emerging wave is therefore circularly polarized.

4. Recognizing that $\theta_{\text{refl}} = \theta_i$, it is clear that $\tan \theta_i = 10/5 = 2$, so $\theta_i = 63.4^\circ$. Using Snell’s Law and using $n_t = 4/3$, we compute that the angle of the transmitted light is $\theta_t = 42.1^\circ$. We need this value in order to compute the reflection coefficients, like this:

$$r_{\perp} = \frac{n_i \cos \theta_i - n_t \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_i} = -0.377 \quad \text{and} \quad r_{\parallel} = \frac{n_i \cos \theta_i - n_t \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_i} = 0.108.$$  

The power ratio for horizontal/vertical is $|r_{\perp}/r_{\parallel}|^2 = 12.1$. For sunlight (which starts out with equal amounts of power in the s and p polarized components), the reflection off of the puddle changes the ratio dramatically. When the sunlight reaches your eyes, there is 12 times more power in the perpendicular component than in the parallel component. You can easily see this effect when you look at a puddle in the sunshine while wearing polarized sunglasses.

5(a) The Cauchy equation, $n = A + B \lambda^{-2}$, can be used to find both the ordinary and extraordinary indices, using the appropriate values for $A$ and $B$. We wish to find a wavelength such that the extraordinary index at $\lambda$ is equal to the ordinary index at $0.5 \times \lambda$. This involves solving for $\lambda$ in the equation:

$$n_e(\lambda) = n_o(\lambda/2).$$
\[ A_c + B_c \lambda^{-2} = A_o + B_o \left( \frac{\lambda}{2} \right)^2 \]

Simple algebraic rearrangement of terms leads us to: \[ \lambda^2 = \frac{4B_o - B_c}{A_c - A_o} \] or \( \lambda = 1225 \text{ nm} \).

Note the interesting implication here: ordinary-polarized light at 612.5 nm travels the same speed as extraordinary-polarized light at 1225 nm.

(b) This problem requires us to use Snell’s law. For both ordinary and extraordinary rays, we have \( n_i = 1 \) and \( \theta_i = 20^\circ \). At \( \lambda = 500 \text{ nm} \), we have two different values for the refractive index of the transmitted ray: \( n_o = 1.5616 \) and \( n_e = 1.5694 \). Therefore we have two different angles for the transmitted ray.

For the ordinary ray:
\[ 1.5616 \sin \theta_o = \sin 20^\circ \] which gives \( \theta_o = 12.65^\circ \)

For the extraordinary ray:
\[ 1.5694 \sin \theta_e = \sin 20^\circ \] which gives \( \theta_e = 12.59^\circ \)

Thus the angular spread between the two rays is 0.06\(^\circ\), which is quite a small angle. Therefore this is not a particularly good way to separate out the two polarization components.