Fractional statistics of Laughlin quasiparticles in quantum antidots

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In two dimensions, fractionally charged particles must possess fractional exchange statistics. In experiments on quantum antidots in the quantum Hall regime the charge of the tunneling particles can be determined directly as a measure of the gate voltage needed to attract one particle. In the same experiments, when the magnetic field is varied, it is observed that the fundamental Aharonov-Bohm period is h/e even for fractionally charged Laughlin quasiparticles. In this paper we analyze these experiments, explicitly taking into account the fractional statistical Berry's phase contribution.

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The fundamental "elementary" particles exist in three spatial dimensions, and thus all have either bosonic or fermionic integer exchange statistics. However, in two spatial dimensions the laws of physics allow existence of particles with fractional statistics, dubbed *anyons*.[1,2] This is so because in two dimensions (2D) a closed loop executed by a particle around another particle is topologically distinct from a loop which encloses no particles, unlike the three-dimensional case. An exchange of two particles is equivalent to one particle executing a half loop around the other, so that a closed loop is equivalent to exchange squared. The particles are said to have statistics Θ if upon exchange the two-particle wave function acquires a phase factor of $\exp(i\pi\Theta)$, and, upon a closed loop, a factor of $\exp(i2\pi\Theta)$. The integer values $\Theta_B=2j$ and $\Theta_F=2j+1$, where $j=0,\pm1,\pm2,\cdots$, describe the familiar boson and fermion exchange statistics: $\exp(i2\pi j)=(-1)^{2j}=+1$ and $\exp[i\pi(2j+1)]=(-1)^{2j+1}=-1$, respectively. Upon execution of a closed loop both bosons and fermions produce a phase factor of +1, which is unobservable, so usually the statistical contribution can be safely neglected when describing an interference experiment, such as the Aharonov-Bohm effect.[3]

Any particles having fractional statistics must be elementary collective excitations of a nontrivial system of many integer statistics particles confined to move in 2D. Thinking in terms of a few of such weakly-interacting, fractional effective particles instead of in terms of very complex collective motions of all the underlying strongly-interacting, integer statistics particles greatly simplifies the description of relevant physics. In particular, the elementary charged excitations (Laughlin quasiparticles[4]) of a fractional quantum Hall (FQH) electron fluid[5,4] have a fractional electric charge[6] and therefore, as pointed out by Halperin,[7] are expected to obey fractional statistics.

Arovas, Schrieffer, and Wilczek[8] have used the adiabatic theorem to calculate the Berry's phase[9] γ of a charge $e/3$ Laughlin quasiparticle at position $\mathfrak{R}$ encircling a closed path C containing another $e/3$ quasiparticle at $\mathfrak{R}'$ in the filling $f=1/3$ FQH condensate,

$$\gamma = i\oint_C d\,\mathfrak{R}\left\langle \Psi(\mathfrak{R},\mathfrak{R}';z_j)\left|\frac{\partial}{\partial\,\mathfrak{R}}\Psi(\mathfrak{R},\mathfrak{R}';z_j)\right.\right\rangle, \qquad (1)$$

where $\Psi(\mathfrak{R},\mathfrak{R}';z_j)$ is the many-electron Laughlin wave function[4] with the electron complex coordinates z_j. If the path is executed counterclockwise, the difference between an "empty" loop and a loop containing another quasiparticle, identified as the statistical contribution, is

$$\Delta\gamma = 2\pi\Theta_{1/3} = 4\pi/3, \qquad (2)$$

where $\Theta_{1/3}$ is the statistics of a quasihole excited in the filling $f=1/3$ FQH condensate.

Indeed, it is possible to assign definite fractional statistics (mod 1) to quasiparticles of certain simple FQH fluids based only on the same plausible assumptions that allow to assign their charge.[10] Without loss of generality, we can use the transparent composite fermion mapping[11] of the integer QH states at $f=p$ to the fractional QH fluid at $f=p/(2jp+1)$. For example, for the one electron layer FQH fluids corresponding to the main composite fermion sequence $f=p/(2jp+1)$, with p and j positive integers, the charge $q=e/(2jp+1)$ quasiparticle statistics is expected to be

$$\Theta_{p/(2jp+1)} = 2j/(2jp+1)(\mathrm{mod}\ 1). \qquad (3)$$

Equation (3) is easy to derive following Ref. 2: two identical anyons of charge q and $2j(2\pi\hbar/e)$ of flux attached have relative statistics $\Theta=2jq/e$.

A quantum antidot electrometer[6,12] has been used in the direct observation of the charge $e/3$ and $e/5$ quasiparticles, subsequently confirmed in shot noise measurements.[13] A quantum antidot is a potential hill lithographically defined in a 2D electron layer in the quantum Hall regime. The wave functions of a charge q particle encircling the antidot are quantized by the Aharonov-Bohm condition (explicitly including the statistical contribution):

$$\gamma_m = \frac{q}{\hbar}\Phi + 2\pi\Theta N = 2\pi m, \qquad (4)$$

where m is an integer, Φ is the enclosed magnetic flux, and N is the number of particles being encircled.[6] Addition of one

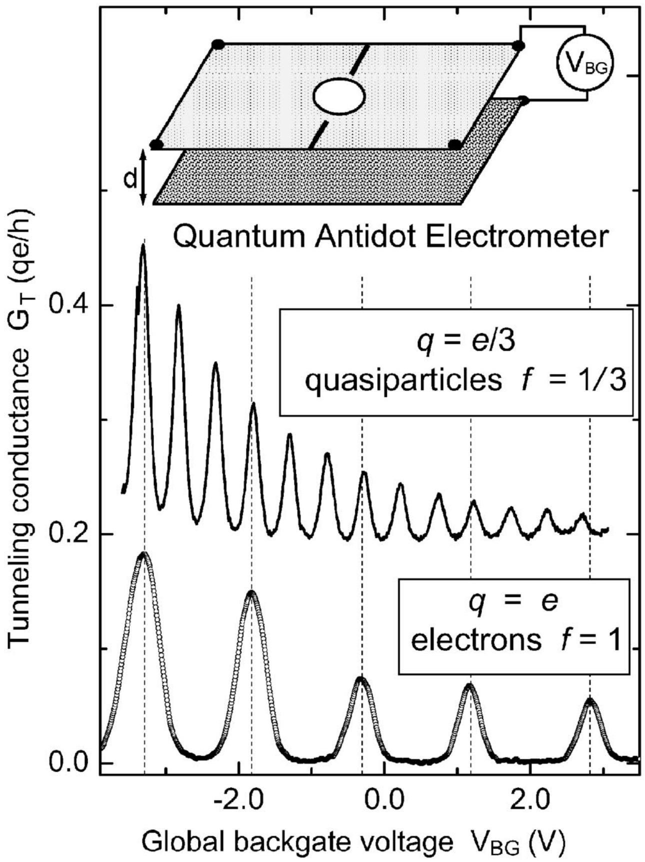

FIG. 1. Quantum antidot electrometer. In the experiment, a conductance peak occurs when the occupation of the antidot increments by one particle: an electron on the integer $f=1$ QH plateau, and a Laughlin quasiparticle on the fractional $f=1/3$ plateau. The measured charge of the particle q is directly proportional to the change in backgate voltage between the conductance peaks. It takes the same gate voltage to attract three $q=e/3$ quasiparticles as one $q=e$ electron. The $f=1/3$ conductance data is offset vertically by $0.2e^2/3h$ for clarity. The dashed vertical lines are guides for the eye.

vortex to the $f=p/(2jp+1)$ FQH condensate creates p quasiholes, one in each of the p "composite fermion Landau levels." When the chemical potential μ moves between two successive quasiparticle orbitals ψ_m and ψ_{m+1}, so that $\Delta\Phi=h/e$ and $\Delta N=p$, the change in the phase of the wave function is

$$\Delta\gamma \equiv \gamma_{m+1} - \gamma_m = \frac{q}{\hbar}\Delta\Phi + 2\pi\Theta\Delta N = 2\pi\left(\frac{1}{2jp+1} + \frac{2jp}{2jp+1}\right) = 2\pi. \tag{5}$$

Note that total of p nondegenerate QP states correspond to each orbital ψ_m.

In experiments, the occupation of the antidot-bound quasiparticle states ψ_m is monitored by the interedge two-step resonant tunneling. The tunneling proceeds via the antidot-bound QP states, each step being essentially a phase-coherent tunneling between two many body configurations: one in which a QP is localized on the extended edge, and one in which it is localized on the antidot.[14] A peak of the tunneling conductance G_T is thus observed when an antidot-bound QP state crosses μ. The two different ways to shift ψ_m relative to μ are: (i) to vary the applied magnetic field B, in a kind of the Aharonov-Bohm (AB) effect, and (ii) to vary the global backgate voltage V_{BG}, which produces an electric field normal to the 2D electron layer.

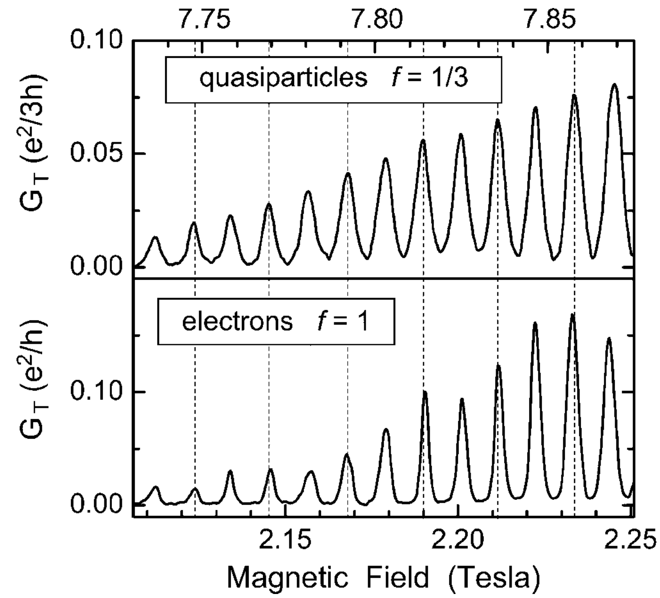

FIG. 2. Aharonov-Bohm periodicity. In the experiment, a conductance peak occurs when an antidot-bound single-particle state crosses the chemical potential. As B is varied, so is the flux Φ through the antidot area (the area encircled by the tunneling particle). Thus the period ΔB is a direct measure of the flux period $\Delta\Phi=h/e$ on both $f=1$ and $f=1/3$ QH plateaus. Single-valuedness of the wave function requires the Berry's phase difference between successive states be an integer multiple of 2π. This condition is satisfied by fermionic electrons, but requires a fractional statistical phase contribution by the fractionally-charged Laughlin quasiparticles, as discussed in the text. The dashed vertical lines are guides for the eye.

Figure 1 shows the experimental G_T versus V_{BG} data obtained on the integer $f=1$ and the fractional $f=1/3$ quantum Hall plateaus at 12 mK. The quantum antidot samples and the experimental procedure were described in Refs. 12,15. As discussed previously,[6,12] the backgate voltage induces a small change in the charge density of the 2D electron layer. The induced charge is quantized in units of elementary charged excitations of the surrounding quantum Hall condensate within the area of the antidot. Thus, as seen in Fig. 1, it takes 1/3 as much variation in V_{BG} to attract a charge $e/3$ Laughlin quasiparticle to the antidot, as it takes to attract an electron. This experiment provides direct proof that on thefractional $f=1/3$ quantum Hall plateau the charge of the antidot-bound elementary excitations is $q=e/3$.

Having established that the charge of the $f=1/3$ antidot-bound particles is $e/3$, we now turn to the determination of their statistics. Figure 2 shows the experimental G_T versus B data obtained on the integer $f=1$ and the fractional $f=1/3$ quantum Hall plateaus at 12 mK. The separation between the conductance peaks ΔB gives the flux period through the area encircled by the tunneling particle. The most significant observation here is that the Aharonov-Bohm periods $\Delta\Phi$ are equal for both the integer $f=1$ and the fractional $f=1/3$ plateaus: $\Delta\Phi_{1/3}=\Delta\Phi_1=h/e$.

It is easy to see that the Aharonov-Bohm flux period of a particle of charge q in vacuum is $\Delta\Phi=2\pi\hbar/q$. Thus, if a charge $e/3$ particle existed in vacuum, its flux period would be $3h/e$. However, the $e/3$ Laughlin quasiparticle encircling the antidot does not exist in vacuum, and thus the statistical contribution from the enclosed particles must be taken into account. It is easier to depict the antidot as a disk of completely filled quasihole states on top of the $f=1/3$ FQH condensate.[16] Addition of flux $\Delta\Phi_{1/3}$ to the area of the antidot then adds one more vortex in the many-electron condensate wave function, that is, excites one more quasihole. This corresponds to the transition $m\rightarrow m+1$ of the tunneling quasihole encircling $\Delta N=1$ more quasihole, so that

$$\Delta\gamma \equiv \gamma_{m+1}-\gamma_m = \frac{q}{\hbar}\Delta\Phi_{1/3} + 2\pi\Theta_{1/3}\Delta N = 2\pi(q/e + 1\Theta_{1/3}) = 2\pi. \tag{6}$$

The equivalent description of the same process in terms of the tunneling $-e/3$ quasielectrons must still include contribution from the quasihole disk, but gives the same (mod 2π) result,

$$\Delta\gamma \equiv \gamma_{m+1}-\gamma_m = \frac{q}{\hbar}\Delta\Phi_{1/3} + 2\pi\Theta_{1/3}^{-1/3}\Delta N = 2\pi(-1/3 + 1\Theta_{1/3}^{-1/3}) = -2\pi, \tag{7}$$

where we use the quasielectron-quasihole relative statistics[10] $\Theta_{1/3}^{-1/3}=-\Theta_{1/3}$.

Thus the experimentally observed Aharonov-Bohm $\Delta\Phi_{1/3}=h/e$ for the charge $e/3$ quasiparticles requires a fractional quasihole statistics of $\Theta_{1/3}=2/3$ in order to ensure single-valuedness of the quasiparticle wave function.[2] It has been argued[17] that this experiment, however, is not entirely satisfactory as a *direct* demonstration of the fractional statistics of Laughlin quasiparticles, because in a quantum antidot the tunneling quasiparticle encircles physical vacuum devoid of 2D electrons. The most important ingredient, the experimental fact that in quantum antidots the period $\Delta\Phi=h/e$, and not $\Delta\Phi=h/q$, even for fractionally charged particles, is then ensured by the gauge invariance argument of the Byers-Yang theorem.[18] Since no known experiment violates gauge invariance, the assignment of the degree of the directness is perhaps a matter of taste. We note that an experiment has been reported on a quasiparticle interferometer which contains no region devoid of electrons within the interference path, so that the Byers-Yang theorem is not applicable.[19]

In summary, we have presented experimental single-particle tunneling data obtained in quantum antidots on the integer $f=1$ and the fractional $f=1/3$ QH plateaus. The results directly demonstrate, in the same experiment, the fractional charge of Laughlin quasiparticles and their magnetic flux period of one h/e, the same as for electrons. An explicit Berry's phase analysis of these data implies a fractional statistical angle $2\pi\Theta_{1/3}=4\pi/3$ of the $f=1/3$ FQH quasiparticles.

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