Meshless modeling and simulation (MMS) methods are ubiquitous across the sciences and engineering. For instance, the smooth particle hydrodynamics (SPH) method can provide effective modeling in fluid dynamics, additive manufacturing, and astrophysics. Vortex methods, another class of meshless methods, are widely used in the aerospace community to model the near-wakes of rotorcraft and fixed-wing aircraft, and have also been used in biomimetic studies of ocean swimmers. MMS methods operate vastly different than their mesh-based counterparts in the sense that they capture the physics of a problem by releasing collocation points (discrete structures or particles that carry physical properties) into a Lagrangian numerical domain, where each collocation point reacts and adapts to surrounding collocation points and complex domain geometries. As a result, MMS has become an attractive approach for modeling complex multiphysics, multiphase, and free-surface flow problems. Unfortunately, MMS methods often suffer from high costs in large-scale settings due to poor operational count complexity (OCC) that is associated with pairwise interactions of collocation points. Specifically, the OCC of pairwise interactions generally scale quadratically or super-linearly with the number of collocation points in the domain, and the OCC of MMS tends to be much higher than counterpart mesh-based techniques. This talk will showcase recent advances made by the CMSL group at US-NRL and collaborators to drastically reduce the overhead cost and accelerate MMS computations. Specifically, a new class of model-order reduction will be presented: projection-tree reduced-order modeling (PTROM). The presented approach combines methods employed in traditional data hierarchical decomposition/clustering and data-driven projection-based reduced-order modeling to reduce the cost and accelerate MMS computations. The PTROM will be presented on vortex dynamics problems, in which results have shown an unprecedented decrease in MMS OCC that can reach sub-linear scaling and less than 0.1% error in quantities of interest, both of which are features not available in traditional meshless acceleration algorithms.