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Variable-Order Approach to Nonlocal Elasticity: Theoretical Formulation and Order Identification via Deep Learning Techniques

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This study presents the application of variable-order (VO) fractional calculus to the modeling of nonlo- cal solids. The reformulation of nonlocal fractional-order continuum mechanic framework, by means of VO kinematics, enables a unique approach to the modeling of solids exhibiting position-dependent nonlocal behavior. The frame-invariance of the strain tensor is leveraged to identify constraints on the definition of the VO. The VO nonlocal continuum formulation is then applied to model the response of Euler-Bernoulli type beams whose governing equations are derived in strong form by means of variational principles. The VO formulation is shown to be self-adjoint and positive-definite, which ensure that the governing equations are well-posed and free from boundary effects. These characteristics stand in contrast to classical integral approaches to nonlocal elasticity, where it is not always possible to obtain positive-definite and self-adjoint systems. A key step in promoting the use of VO approaches is to identify methodologies to determine the VO describing a given physical system. This study presents a deep learning based framework capable of solving the inverse problem consisting in the identification of the VO describing the behavior of the nonlocal beam on the basis of its response. It is established that the internal architecture of bidirectional recurrent neural networks makes them suitable for nonlocal boundary value problems, similar to the one treated in this study. Results show that the network accurately solves the inverse problem even for non- local beams with VO patterns inconsistent with the network training data set. Although presented in the context of a 1D Euler-Bernoulli beam, both the VO nonlocal formulation and the deep learning techniques are very general and could be extended to the solution of any general higher-dimensional VO boundary value problem.