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VarNet: Variational Neural Networks for the Solution of Partial Differential Equations

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In this paper, we propose a new model-based unsupervised learning method, called VarNet, for the solution of partial differential equations (PDEs) using deep neural networks (NNs). Particularly, we propose a novel loss function that relies on the variational (integral) form of PDEs as apposed to their differential form which is commonly used in the literature. Our loss function is discretization-free, highly parallelizable, and more effective in capturing the solution of PDEs since it employs lower order derivatives and trains over measure non-zero regions of space-time. Given this loss function, we also propose an approach to optimally select the space-time samples, used to train the NN, that is based on the feedback provided from the PDE residual. The models obtained using VarNet are smooth and do not require interpolation. They are also easily differentiable and can directly be used for control and optimization of PDEs. Finally, VarNet can straight-forwardly incorporate parametric PDE models making it a natural tool for model order reduction (MOR) of PDEs. We demonstrate the performance of our method through extensive numerical experiments for the advection-diffusion PDE as an important case-study.