We introduce a gravitational waveform inversion strategy that discovers mechanical models of binary black hole (BBH) systems. We show that only a single time series of (possibly noisy) waveform data is necessary to construct the orbital trajectories and corresponding equations of motion for a BBH system. Starting with a general class of universal differential equations, our strategy involves the construction of a space of plausible mechanical models and a physics-informed constrained optimization within that space to minimize the waveform error. We choose to parameterize the space of models with feed-forward neural networks. This leads to efficient, flexible, and highly accurate algorithms which can be easily implemented with modern software libraries. We apply our method to various BBH systems including extreme and comparable mass ratio systems in eccentric and non-eccentric orbits. We show the resulting differential equations are applicable to time durations larger than the training interval, and a variety of relativistic effects, such as perihelion precession, radiation reaction, and orbital plunge, are automatically accounted for. The methods outlined here provide a new, data-driven approach to studying the dynamics of binary black hole systems. Various applications in gravitational wave astronomy will be discussed as time permits.