

**CRUNCH Seminars at Brown, Division of Applied Mathematics**

**Friday – March 15, 2019**

**Paper Review: Optimal approximation of continuous functions by very deep ReLU networks by  
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We consider approximations of general continuous functions on finite-dimensional cubes by general deep ReLU neural networks and study the approximation rates with respect to the modulus of continuity of the function and the total number of weights  $W$  in the network. We establish the complete phase diagram of feasible approximation rates and show that it includes two distinct phases. One phase corresponds to slower approximations that can be achieved with constant-depth networks and continuous weight assignments. The other phase provides faster approximations at the cost of depths necessarily growing as a power law [in the number of weights) and with necessarily discontinuous weight assignments. In particular, we prove that constant-width fully-connected networks of depth  $L$  [proportional to]  $W$  provide the fastest possible approximation rate [which] cannot be achieved with less deep networks.